entropy
Supplementary Material

Interpretation of Entropy Algorithms in the Context of Biomedical Signal Analysis and Their Application to EEG Analysis in Epilepsy

Lampros Chrysovalantis Amarantidis [1,2] and Daniel Abásolo [1,*]

[1] Centre for Biomedical Engineering, Department of Mechanical Engineering Sciences, Faculty of Engineering and Physical Sciences, University of Surrey, Guildford, GU2 7XH, UK
[2] Ericsson, Thames Tower, Station Road, Reading, RG1 1LX, UK
* Correspondence: d.abasolo@surrey.ac.uk; Tel.: +44-(0)1483-682971

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Table S1. Electroencephalogram and Epilepsy studies (*I*): *p* – values from Wilcoxon signed rank tests for group A. Tests that rejected the hypothesis at the $p < 0.01$ level are highlighted in bold.

n	3	4	5	6				
PEn	**7.72e-14**	**4.11e-14**	**2.11e-14**	**1.90e-14**	-	-	-	-
MPEn	**7.72e-14**	**4.11e-14**	**2.11e-14**	-	-	-	-	-
(*m*,*r*)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
SEn	0.0118	0.0120	0.0108	**0.0098**	**2.74e-10**	**5.92e-10**	**3.27e-09**	**4.85e-08**
(*m*,*r*)	(1, 0.2)	(1, 0.4)	(1, 0.6)	(1, 0.8)	(2, 0.2)	(2, 0.4)	(2, 0.6)	(2, 0.8)
QSEn	**0.0089**	**0.0077**	**0.0073**	**0.0057**	0.0230	0.2094	0.0131	**0.0037**
(*m*,*r*)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
FEn, *n*=1	**0.0037**	**0.0035**	**0.0033**	**0.0036**	0.0900	0.4151	0.9233	0.6253
FEn, *n*=2	**0.0037**	**0.0037**	**0.0036**	**0.0036**	0.4555	0.1496	0.0643	0.0313
FEn, *n*=3	**0.0037**	**0.0038**	**0.0041**	**0.0040**	0.0313	0.0142	**0.0075**	**0.0048**

Table S2. Electroencephalogram and Epilepsy studies (*I*): *p* – values from the Kruskal-Wallis tests for group B. Tests that rejected the hypothesis at the $p < 0.01$ level are highlighted in bold.

n	3	4	5	6				
PEn	**1.46e-14**	**1.30e-14**	**7.07e-15**	**4.22e-15**	-	-	-	-
MPEn	**1.46e-14**	**1.30e-14**	**7.07e-15**	-	-	-	-	-
(*m*,*r*)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
SEn	0.2363	0.6699	0.2932	0.0221	**1.73e-12**	**1.75e-11**	**3.54e-10**	**2.89e-08**
(*m*,*r*)	(1, 0.2)	(1, 0.4)	(1, 0.6)	(1, 0.8)	(2, 0.2)	(2, 0.4)	(2, 0.6)	(2, 0.8)
QSEn	**3.64e-07**	**6.16e-12**	**4.70e-15**	**4.29e-18**	0.0298	0.0787	**6.81e-08**	**1.44e-14**
(*m*,*r*)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
FEn, *n*=1	**2.92e-12**	**3.13e-15**	**1.56e-17**	**3.58e-19**	0.1148	**0.0033**	**3.13e-05**	**2.88e-07**

FEn, n=2	**3.45e-27**	**3.99e-29**	**3.25e-30**	**6.52e-31**	**2.67e-08**	**9.97e-13**	**3.68e-16**	**6.82e-19**
FEn, n=3	**4.39e-34**	**5.26e-35**	**2.06e-35**	**1.11e-35**	**5.94e-19**	**5.43e-24**	**1.69e-26**	**3.19e-28**

Table S3. Electroencephalogram and Epilepsy studies (*I*): Mann-Whitney U tests at $p < 0.01$ level for three pairs of data sets in group B: data sets (3,4), data sets (3,5), data sets (4,5). Results of tests for the three pairs are given as x_1, x_2, x_3 where x_i is equal to 1 if the hypothesis was rejected for that test and equal to 0 otherwise. Combinations that rejected the relevant Kruskal-Wallis test are underlined. Combinations that rejected all three Mann-Whitney U tests are highlighted in bold.

n	3	4	5	6				
PEn	**1,1,1**	**1,1,1**	**1,1,1**	**1,1,1**	-	-	-	-
MPEn	**1,1,1**	**1,1,1**	**1,1,1**	**-**	-	-	-	-
(m,r)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
SEn	0,0,0	0,0,0	0,0,0	0,0,0	0,1,1	0,1,1	0,1,1	0,1,1
(m,r)	(1, 0.2)	(1, 0.4)	(1, 0.6)	(1, 0.8)	(2, 0.2)	(2, 0.4)	(2, 0.6)	(2, 0.8)
QSEn	0,1,1	0,1,1	0,1,1	0,1,1	0,0,0	0,0,0	0,1,1	0,1,1
(m,r)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
FEn, n=1	0,1,1	0,1,1	0,1,1	0,1,1	0,0,0	0,0,0	0,1,1	0,1,1
FEn, n=2	0,1,1	0,1,1	0,1,1	0,1,1	0,1,1	0,1,1	0,1,1	0,1,1
FEn, n=3	0,1,1	0,1,1	0,1,1	0,1,1	0,1,1	0,1,1	0,1,1	0,1,1

Table S4. Electroencephalogram and Epilepsy studies (*II*): p – values from then Mann-Whitney U tests. Tests that rejected the hypothesis at the $p < 0.01$ level are highlighted in bold.

n	3	4	5	6				
PEn	0.0641	0.0929	0.1476	0.2124	-	-	-	-
MPEn	0.0641	0.0929	0.1476	-	-	-	-	-
(m,r)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
SEn	**5.75e-35**	**6.51e-35**	**1.27e-34**	**2.40e-34**	**5.90e-33**	**2.81e-34**	**9.31e-35**	**6.59e-35**
(m,r)	(1, 0.2)	(1, 0.4)	(1, 0.6)	(1, 0.8)	(2, 0.2)	(2, 0.4)	(2, 0.6)	(2, 0.8)
QSEn	**1.93e-33**	**4.29e-32**	**1.06e-30**	**2.94e-29**	**2.89e-34**	**1.00e-32**	**3.77e-31**	**1.37e-29**
(m,r)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
FEn, n=1	**8.16e-37**	**2.20e-36**	**5.70e-36**	**9.36e-36**	**9.08e-36**	**4.08e-36**	**4.25e-36**	**4.26e-36**
FEn, n=2	**6.81e-30**	**1.93e-28**	**1.61e-27**	**7.95e-27**	**2.14e-35**	**1.85e-34**	**1.26e-33**	**6.22e-33**
FEn, n=3	**7.31e-17**	**2.76e-15**	**2.56e-14**	**1.17e-13**	**1.66e-29**	**2.68e-27**	**9.58e-26**	**1.79e-24**

Table S5. Electroencephalogram and Epilepsy studies (*I*): Classification accuracy results in group A using one of the entropy algorithms realisations as a feature and k-NN with *k*=3. Maximum accuracy achieved is highlighted in bold.

n	3	4	5	6				
PEn	70%	68.5%	73%	64%	-	-	-	-
MPEn	68%	69.5%	**73.5%**	-	-	-	-	-
(*m*,*r*)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
SEn	56%	60%	64%	59%	57.5%	62%	61.5%	62.5%
(*m*,*r*)	(1, 0.2)	(1, 0.4)	(1, 0.6)	(1, 0.8)	(2, 0.2)	(2, 0.4)	(2, 0.6)	(2, 0.8)
QSEn	61.5%	58%	64%	69.5%	46%	57%	61.5%	61%
(*m*,*r*)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
FEn, *n*=1	64.5%	59.5%	57%	60%	57.5%	52.5%	59%	55%
FEn, *n*=2	63.5%	63%	62.5%	56.5%	55.5%	58%	58%	66%
FEn, *n*=3	56%	53.5%	52.5%	53%	57.5%	63%	60%	57%

Table S6. Electroencephalogram and Epilepsy studies (*I*): Classification accuracy results in group B using one of the entropy algorithms realisations as a feature and k-NN with *k*=3. Maximum accuracy achieved is highlighted in bold.

n	3	4	5	6				
PEn	51%	50%	50%	49%	-	-	-	-
MPEn	48%	50%	48%	-	-	-	-	-
(*m*,*r*)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
SEn	40%	41%	36%	37%	41%	41%	40%	50%
(*m*,*r*)	(1, 0.2)	(1, 0.4)	(1, 0.6)	(1, 0.8)	(2, 0.2)	(2, 0.4)	(2, 0.6)	(2, 0.8)
QSEn	43%	44%	50%	47%	36%	42%	45%	48%
(*m*,*r*)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
FEn, *n*=1	46%	48%	51%	51%	46%	44%	44%	48%
FEn, *n*=2	48%	54%	52%	52%	50%	50%	50%	53%
FEn, *n*=3	58%	57%	62%	**63%**	46%	51%	55%	50%

Table S7. Electroencephalogram and Epilepsy studies (*I*): Best classification accuracy results in group A and group B for varying number of features used. Best in group accuracies are highlighted in bold.

1 feature	Features	Group A	Group B
Best in group A	**PEn** (*n*=5)	**73.5%**	48%
Best in group B	**FEn** (*n*=3, *m*=1, *r*=0.25)	53%	**63%**
2 features			
Best in group A	**PEn** (*n*=5), **FEn** (*n*=1, *m*=1, *r*=0.2)	**92.5%**	71%
Best in group B	**SEn** (*m*=2, *r*=0.25), **FEn** (*n*=2, *m*=1, *r*=0.1)	92.5%	**83%**
3 features			
Best in group A	**PEn** (*n*=3), **MPEn** (n=3), **FEn** (*n*=1, *m*=1, *r*=0.2)	**93.5%**	69%
Best in group B	**PEn** (*n*=4), **SEn** (*m*=2, *r*=0.25), **FEn** (*n*=3, *m*=2, *r*=0.15)	89%	**87%**
4 features			
Best in group A	**MPEn** (*n*=5), **SEn** (*m*=2, *r*=0.15), **QSEn** (*m*=2, *r*=0.25), **FEn** (*n*=3, *m*=1, *r*=0.1)	**94%**	81%
Best in group B	**PEn** (*n*=3), **SEn** (*m*=2, *r*=0.2), **QSEn** (*m*=2, *r*=0.25), **FEn** (*n*=3, *m*=2, *r*=0.15)	89%	**87%**

Table S8. Electroencephalogram and Epilepsy studies (*II*): Classification accuracy results using one of the entropy algorithms realisations as a feature and k-NN with k=3. Maximum accuracy achieved is highlighted in bold.

n	3	4	5	6				
PEn	50%	58.5%	52%	56%	-	-	-	-
MPEn	50.5%	59.5%	53.5%	-	-	-	-	-
(m,r)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
SEn	59.5%	58%	56%	61%	52%	61.5%	57.5%	57%
(m,r)	(1, 0.2)	(1, 0.4)	(1, 0.6)	(1, 0.8)	(2, 0.2)	(2, 0.4)	(2, 0.6)	(2, 0.8)
QSEn	57%	60%	58%	54%	58%	55.5%	57.5%	56.5%
(m,r)	(1, 0.1)	(1, 0.15)	(1, 0.2)	(1, 0.25)	(2, 0.1)	(2, 0.15)	(2, 0.2)	(2, 0.25)
FEn, n=1	**63%**	62.5%	58%	59%	58.5%	58.5%	57%	57%
FEn, n=2	52.5%	55.5%	56.5%	57%	58.5%	59.5%	58%	55%
FEn, n=3	51%	51.5%	50%	55%	55%	58%	58.5%	55%

Table S9. Electroencephalogram and epilepsy studies (*II*): Best classification accuracy results for varying number of entropies used as a feature vector.

Number of features	Features	Best accuracy
1	**FEn** (n=1, m=2, r=0.15)	**63%**
2	**MPEn** (n=5), **SEn** (m=1, r=0.15)	**73%**
3	**MPEn** (n=3), **QSEn** (m=2, r=0.15), **FEn** (n=2, m=1, r=0.2)	**75%**
5	**PEn** (n=5), **MPEn** (n=4), **SEn** (m=2, r=0.15), **QSEn** (m=2, r=0.25), **FEn** (n=1, m=1, r=0.15)	**75.5%**