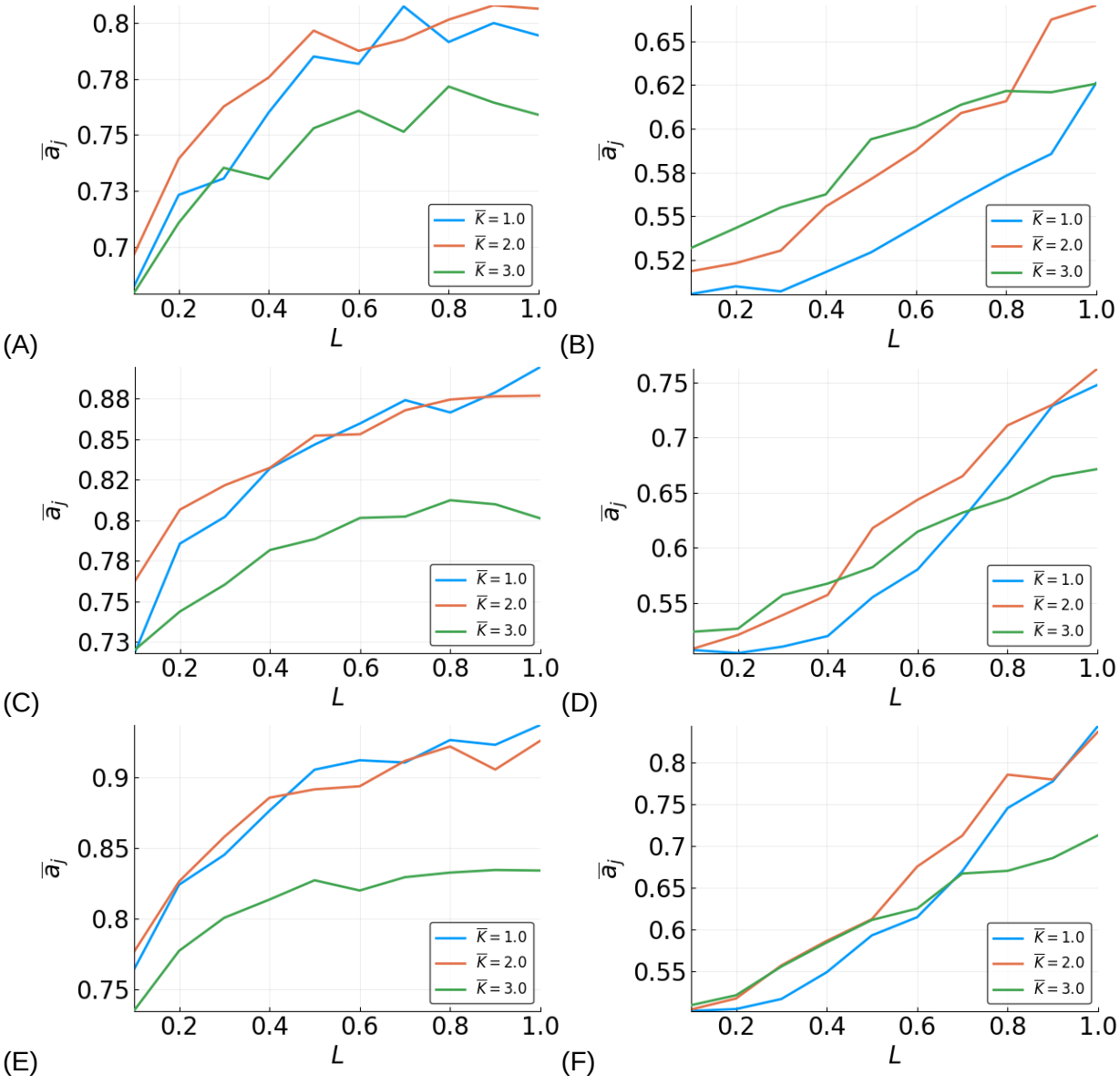
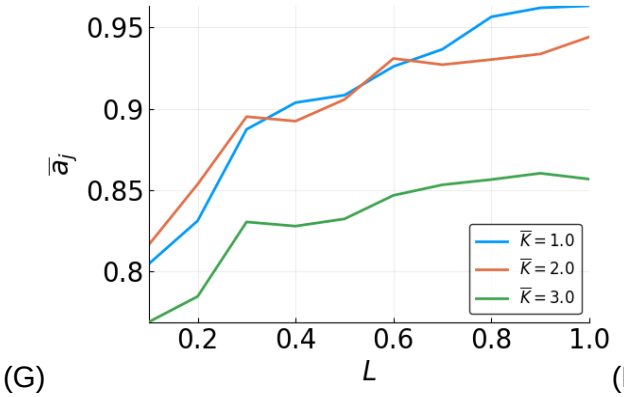
Supplementary Materials

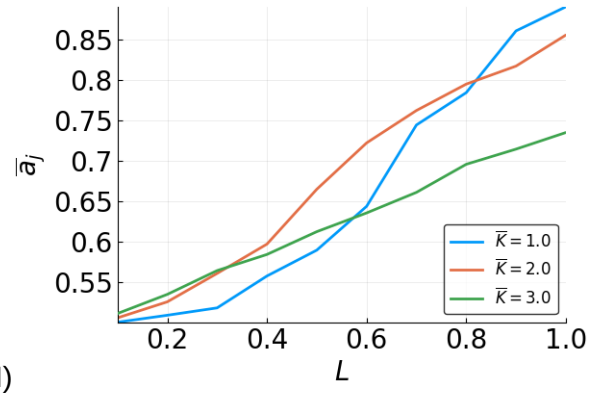
Flexibility of Boolean Network Reservoir Computers in Approximating Arbitrary Recursive and Non-recursive Binary Filters

Figure S1.




0.95
0.9
0.85
0.8
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

(G)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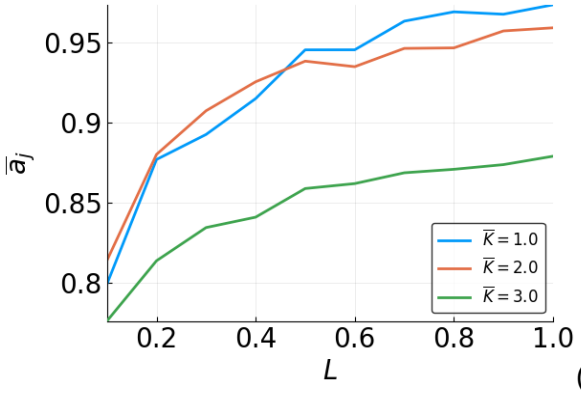
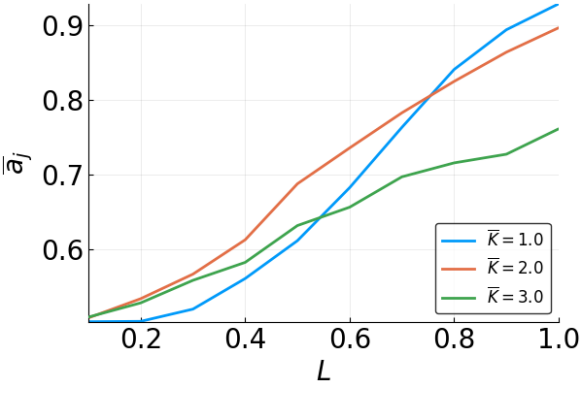
0.8
0.75
0.7
0.65
0.6
0.55
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

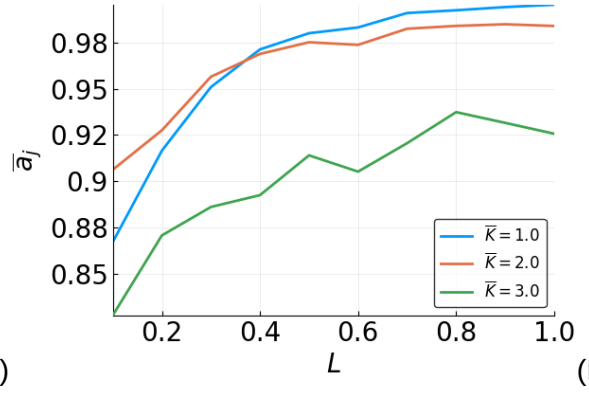
(H)


0.95
0.9
0.85
0.8
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

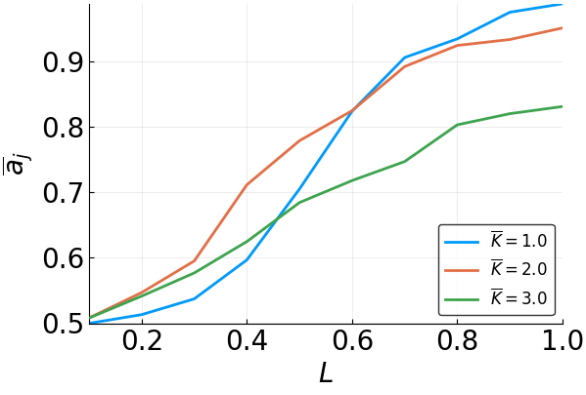
(I)


0.9
0.8
0.7
0.6
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

(J)


0.98
0.95
0.92
0.9
0.88
0.85
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

(K)


0.9
0.8
0.7
0.6
0.5
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

(L)

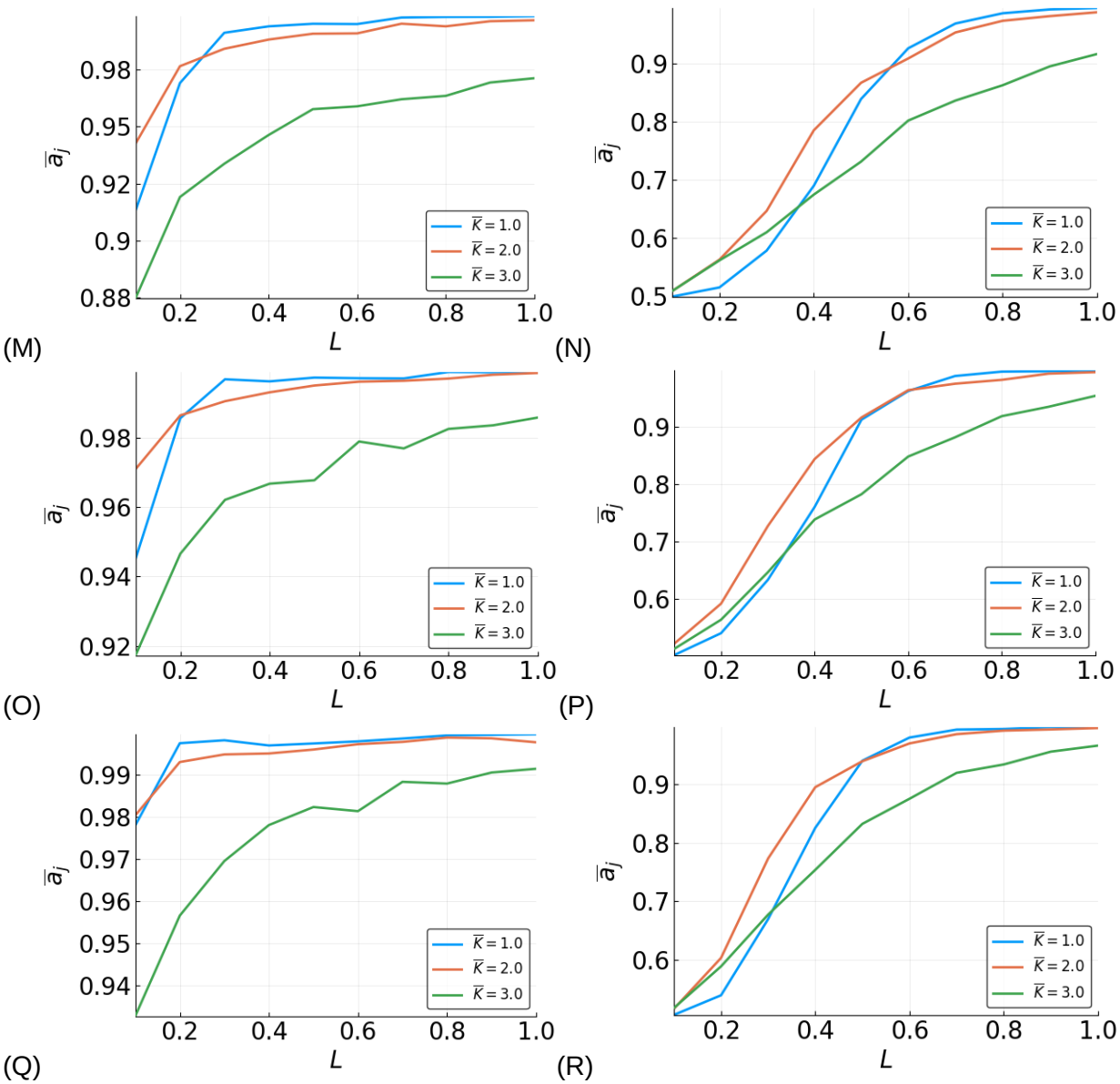
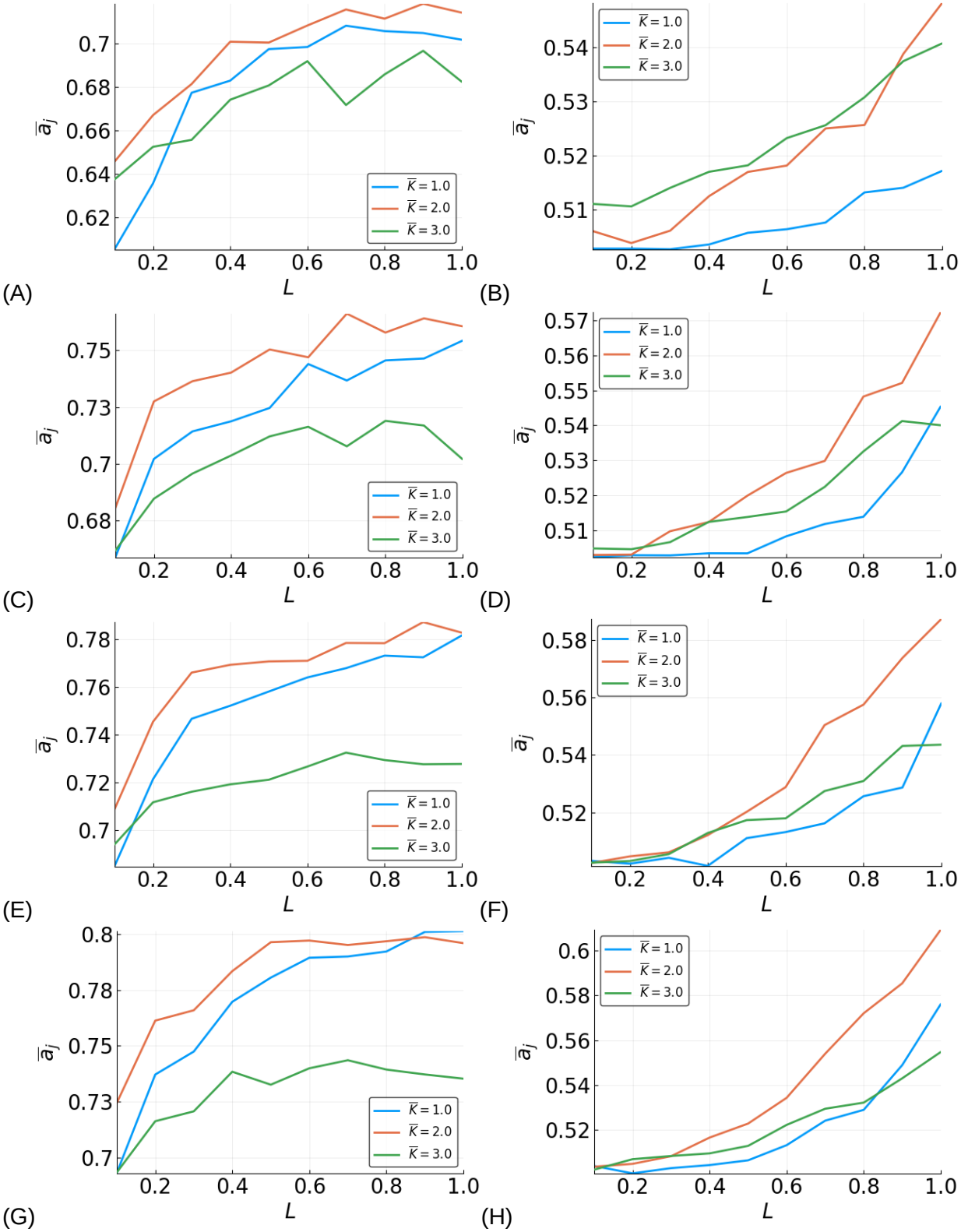
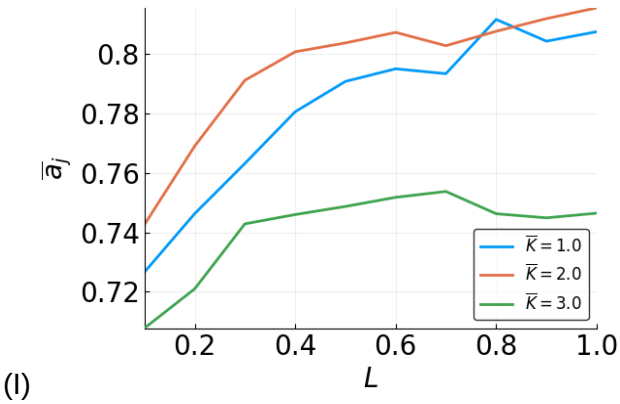


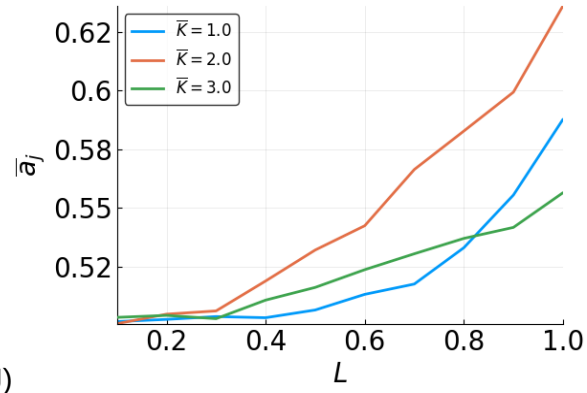
Figure S1. Mean accuracy, $\bar{a}_j$ vs L for the 3-bit median(Left Column) and parity(Right Column) functions for different $\bar{K}$-valued reservoirs with N= 10(A,B), 20(C,D), 30(E,F), 40(G,H), 50(I,J), 100(K,L), 200(M,N), 300(O,P), 400(Q,R).

Figure S2.

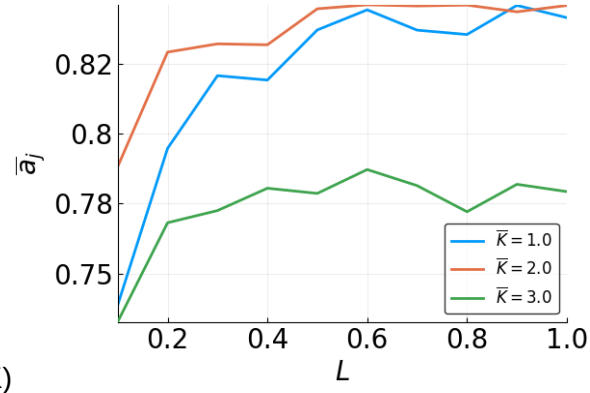

(A)
(B)
(C)
(D)
(E)
(F)
(G)
(H)
$\overline{a}_j$
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$


$\overline{a}_j$
0.8
0.78
0.76
0.74
0.72
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

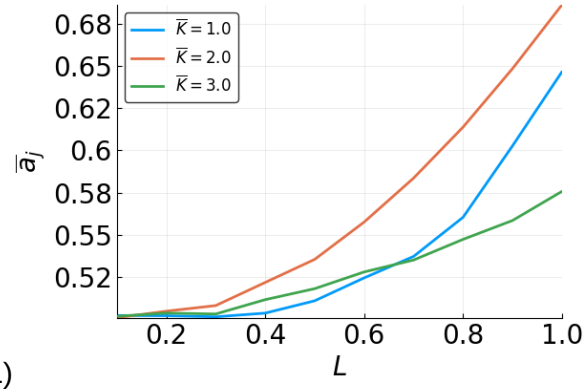
(I)


$\overline{a}_j$
0.62
0.6
0.58
0.55
0.52
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

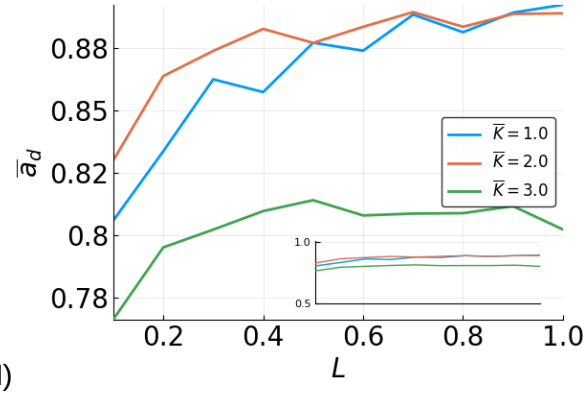
(J)


$\overline{a}_j$
0.82
0.8
0.78
0.75
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

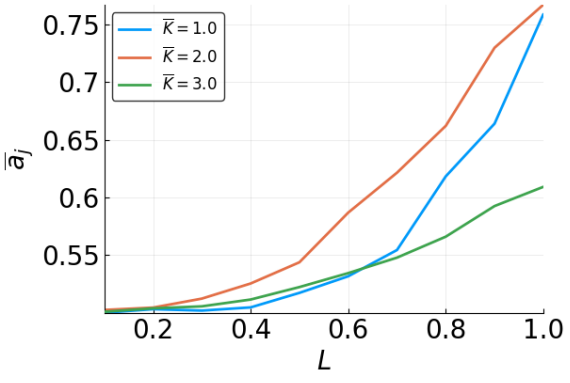
(K)


$\overline{a}_j$
0.68
0.65
0.62
0.6
0.58
0.55
0.52
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

(L)


$\overline{a}_d$
0.88
0.85
0.82
0.8
0.78
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$
1.0
0.5

(M)


$\overline{a}_j$
0.75
0.7
0.65
0.6
0.55
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

(N)

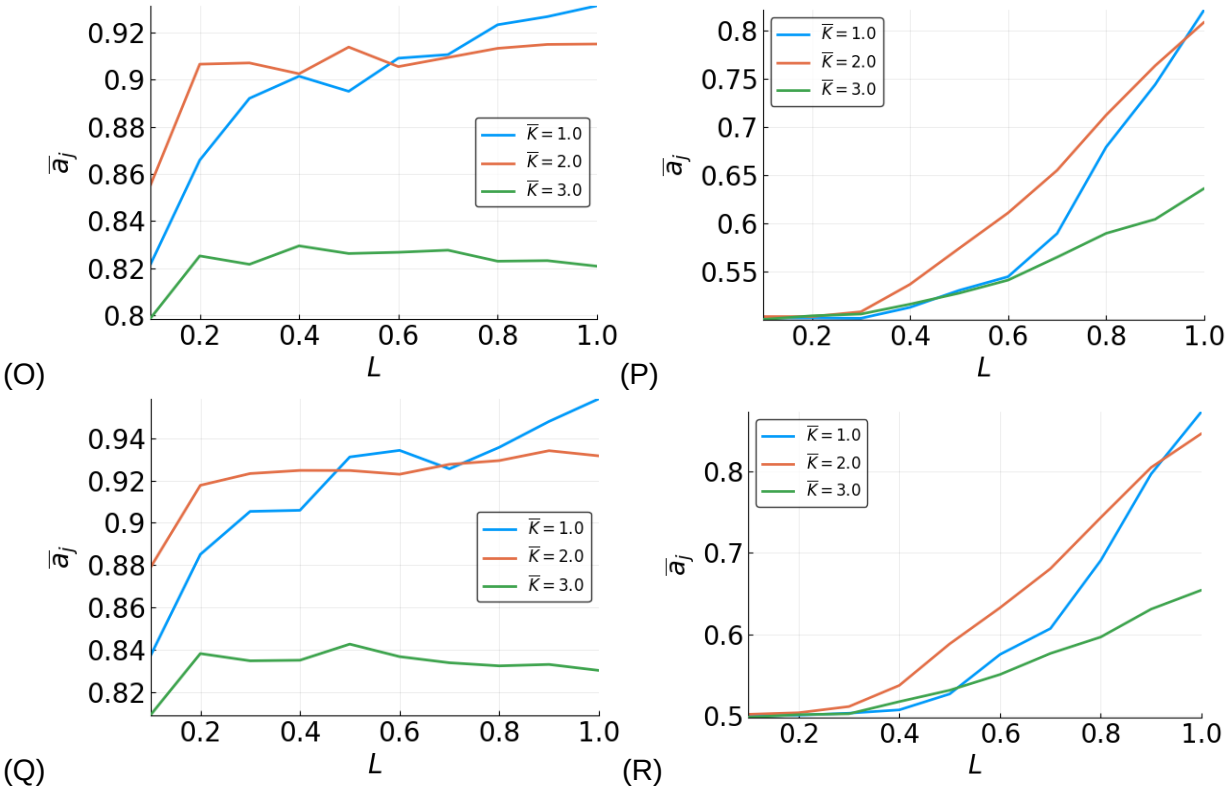
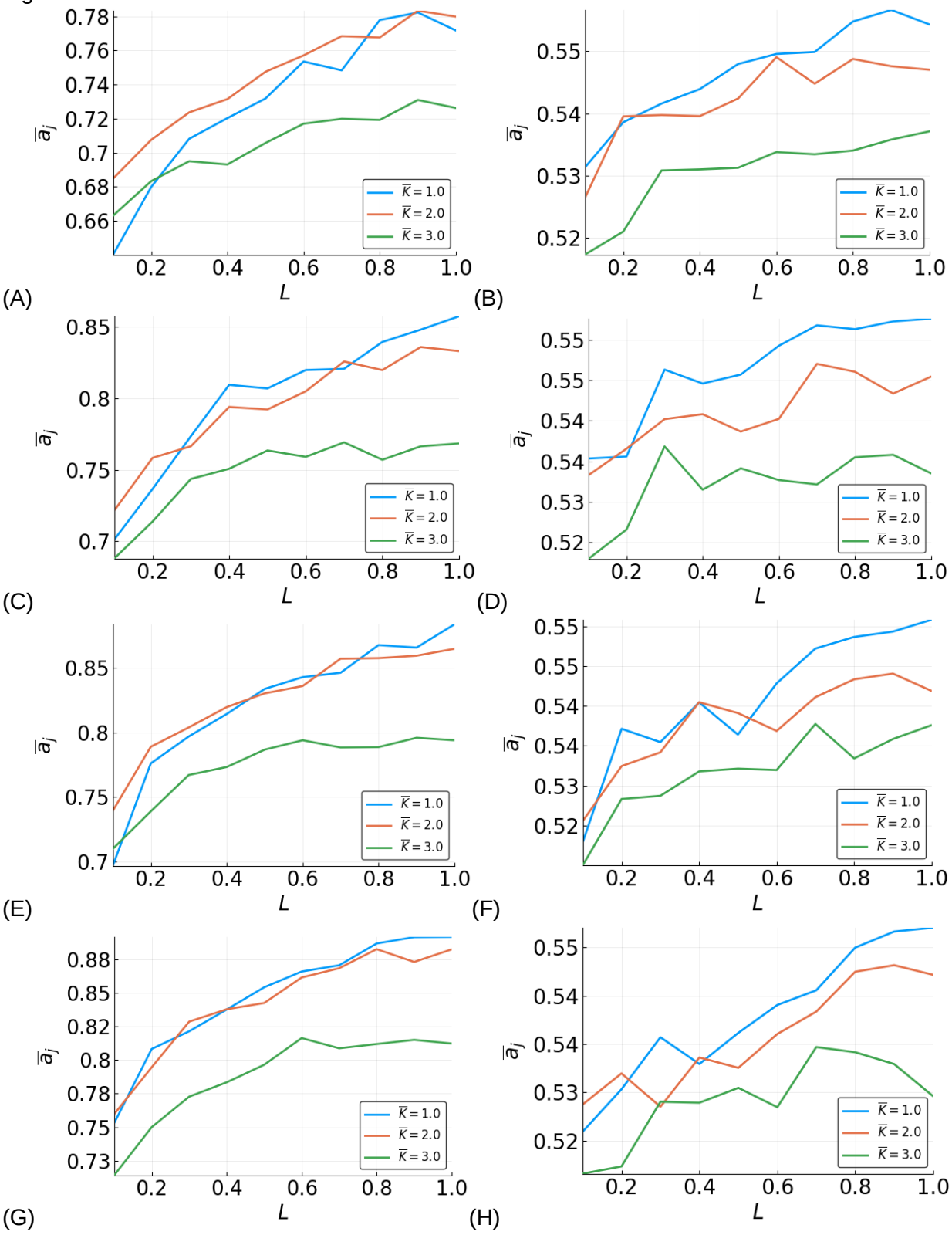
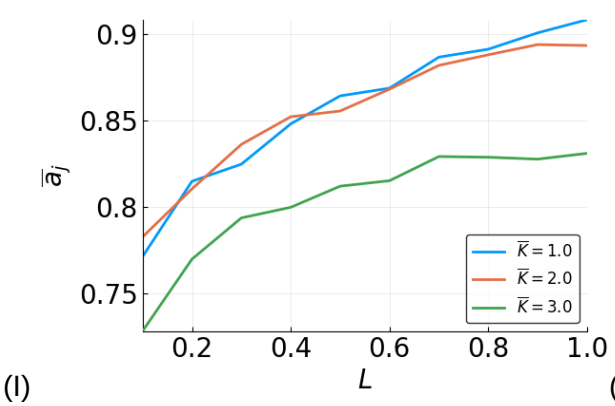


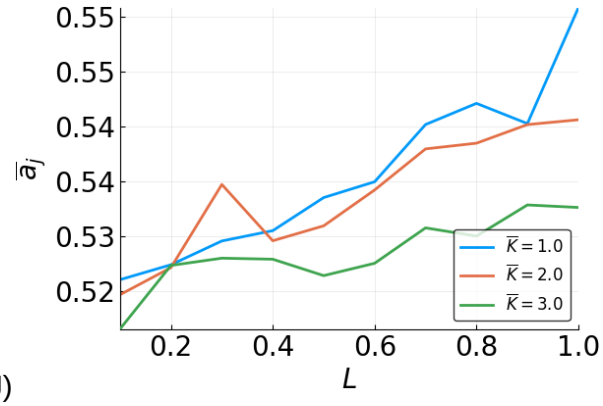
Figure S2. Mean accuracy, $\bar{a}_j$, vs L for the 5-bit median(Left Column) and parity(Right Column) functions for different $\bar{K}$-valued reservoirs with N= 10(A,B), 20(C,D), 30(E,F), 40(G,H), 50(I,J), 100(K,L), 200(M,N), 300(O,P), 400(Q,R).

Figure S3

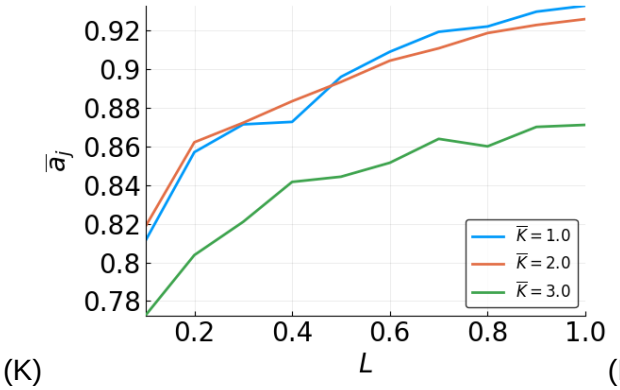

(A)
(B)
(C)
(D)
(E)
(F)
(G)
(H)
$\overline{a}_j$
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$


0.9
0.85
0.8
0.75
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

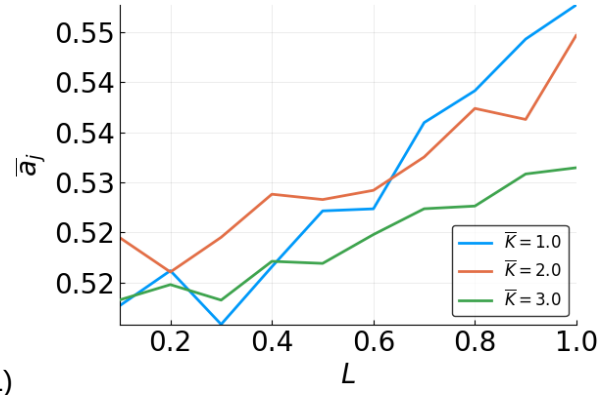
(I)


0.55
0.55
0.54
0.54
0.53
0.52
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

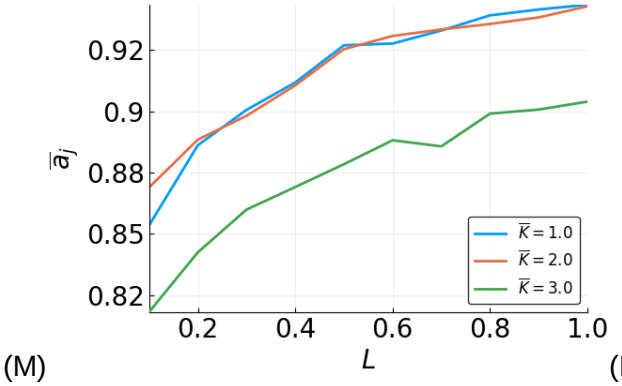
(J)


0.92
0.9
0.88
0.86
0.84
0.82
0.8
0.78
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

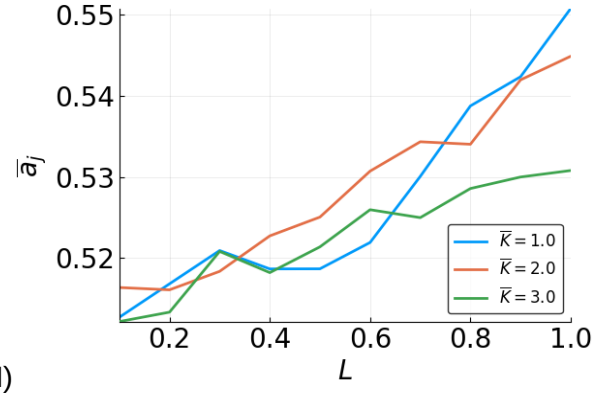
(K)


0.55
0.54
0.54
0.53
0.52
0.52
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

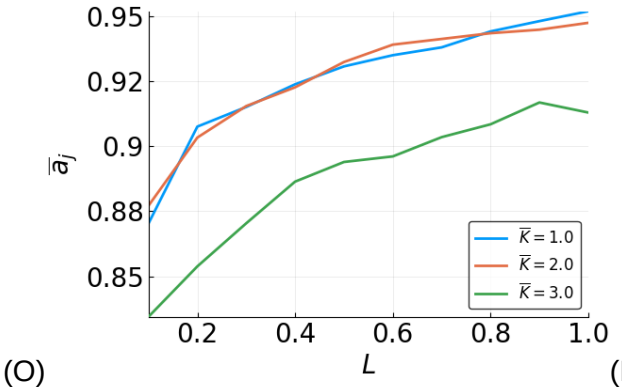
(L)


0.92
0.9
0.88
0.85
0.82
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

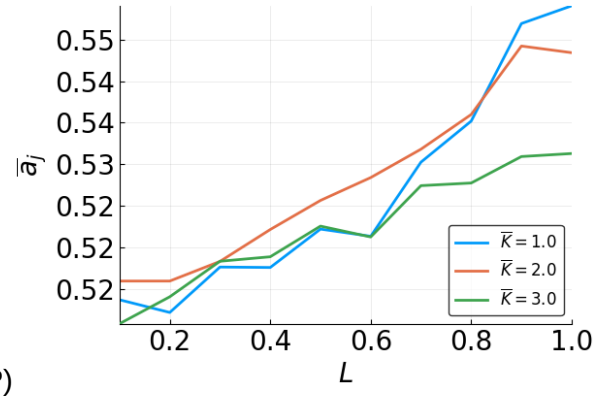
(M)


0.55
0.54
0.53
0.52
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

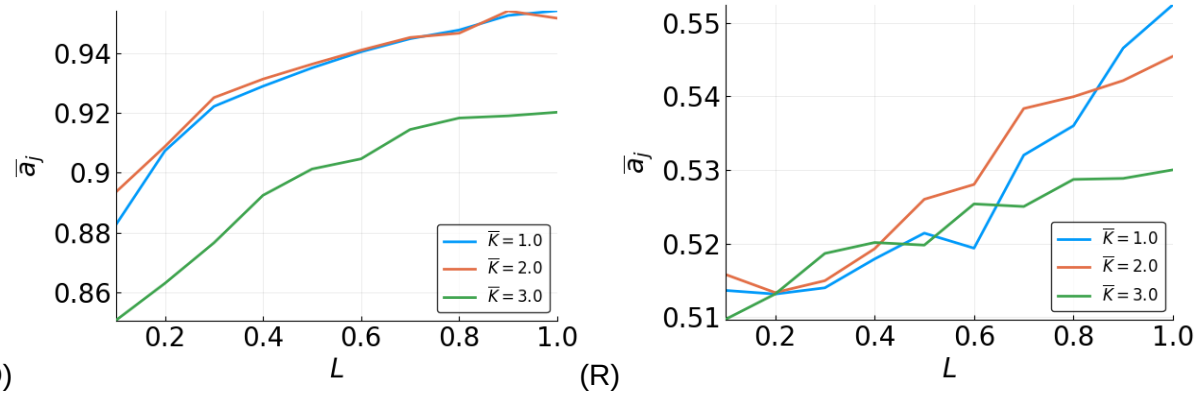
(N)


0.95
0.92
0.9
0.88
0.85
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

(O)


0.55
0.54
0.54
0.53
0.52
0.52
$\overline{a}_j$
0.2
0.4
0.6
0.8
1.0
L
$\overline{K} = 1.0$
$\overline{K} = 2.0$
$\overline{K} = 3.0$

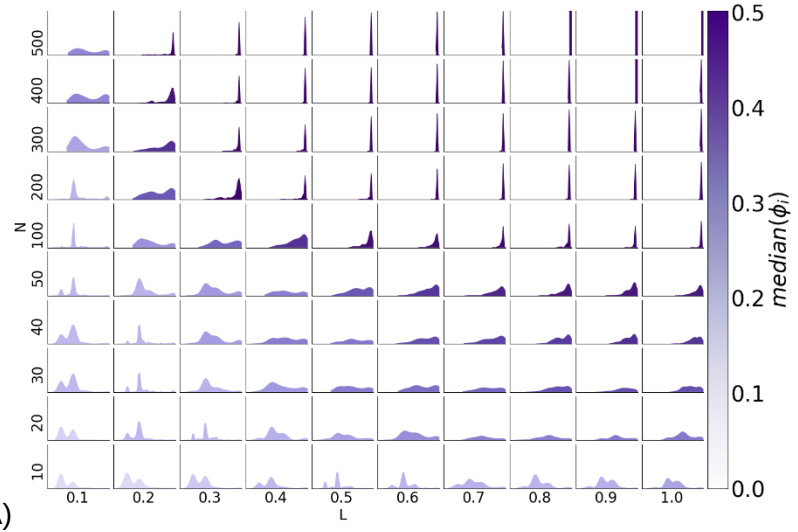
(P)



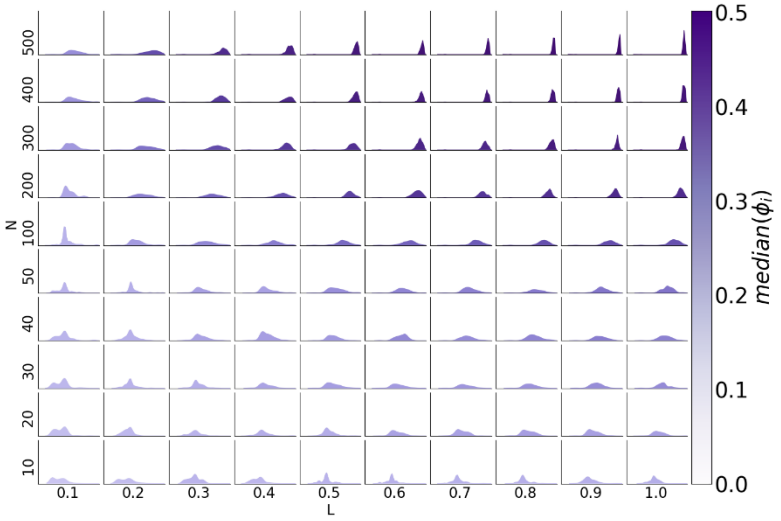
(Q) (R)

Figure S3. Mean accuracy, $\bar{a}_j$, vs. L for the 3-bit recursive median(Left Column) and parity(Right Column) functions for different $\bar{K}$-valued reservoirs with N= 10(A,B), 20(C,D), 30(E,F), 40(G,H), 50(I,J), 100(K,L), 200(M,N), 300(O,P), 400(Q,R).

Figure S4



(A)



(B)

Figure S4. Histogram of Φ_i across all 100 reservoirs for each N, L with $\bar{K}$ = 1(A), $\bar{K}$ = 3(B) for 3-bit functions. Each subplot represents the density for all the reservoirs with one N and L, with the x-axis being Φ and the y-axis being number of reservoirs[0,256].

Figure S5

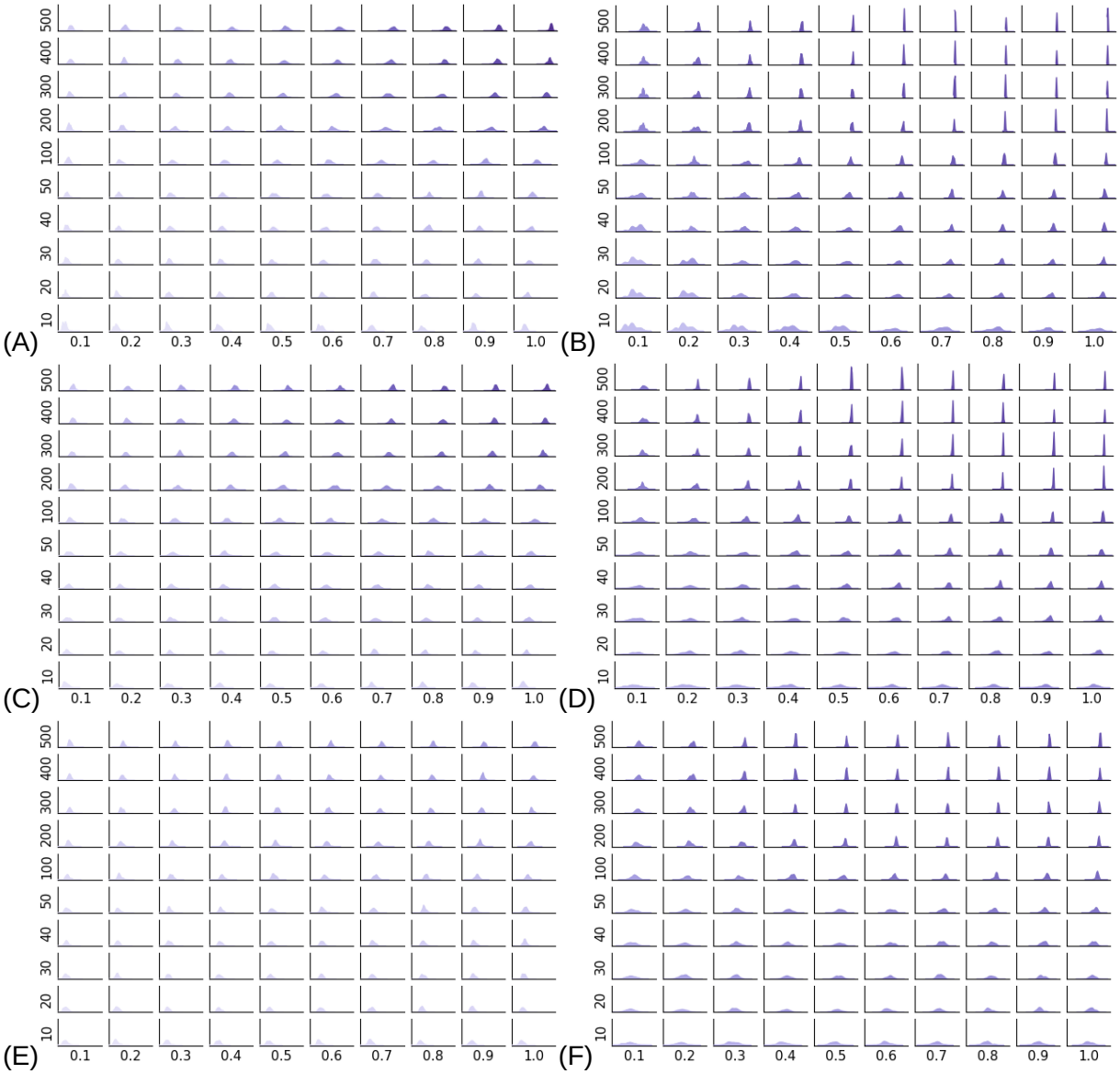
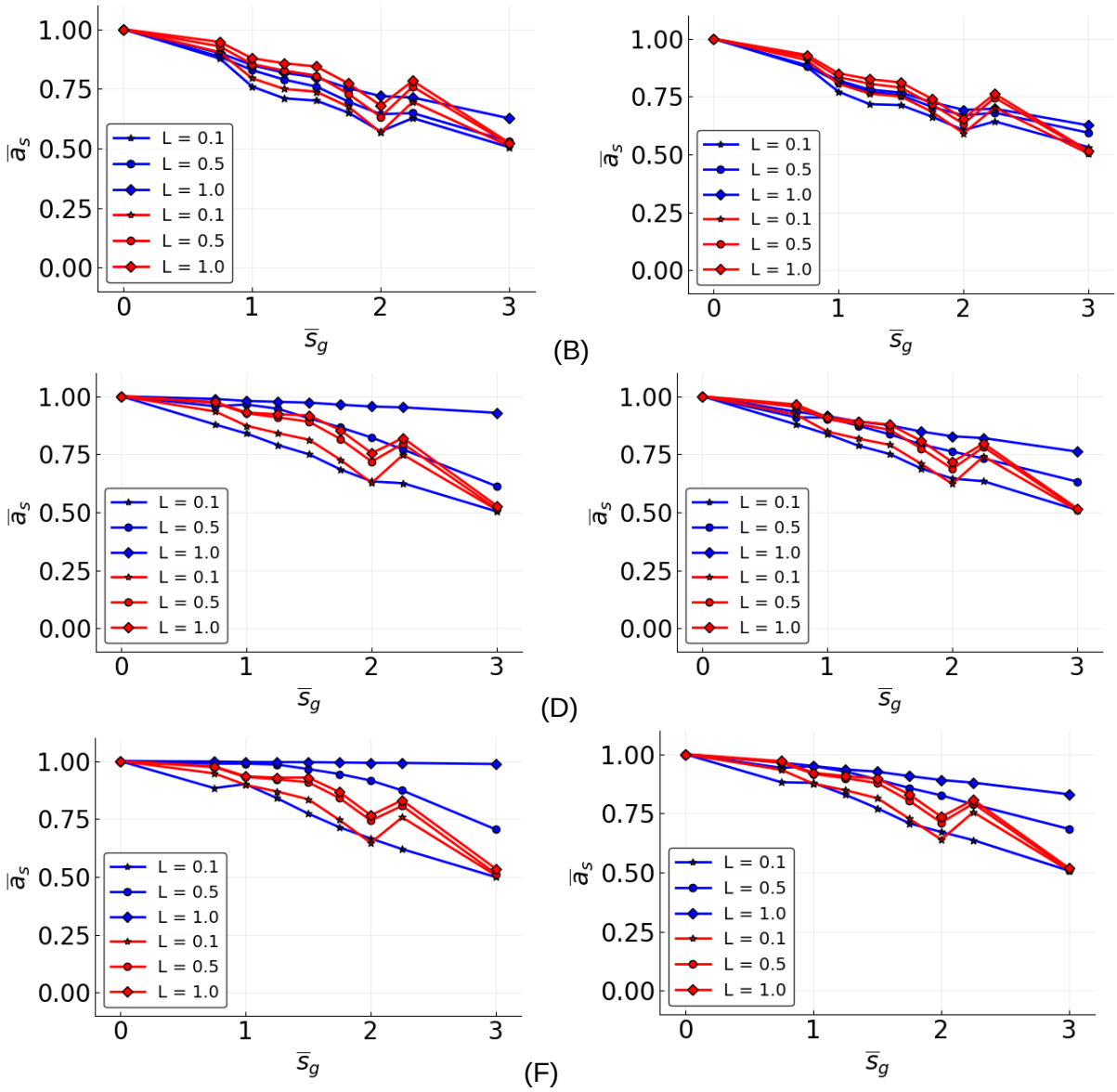
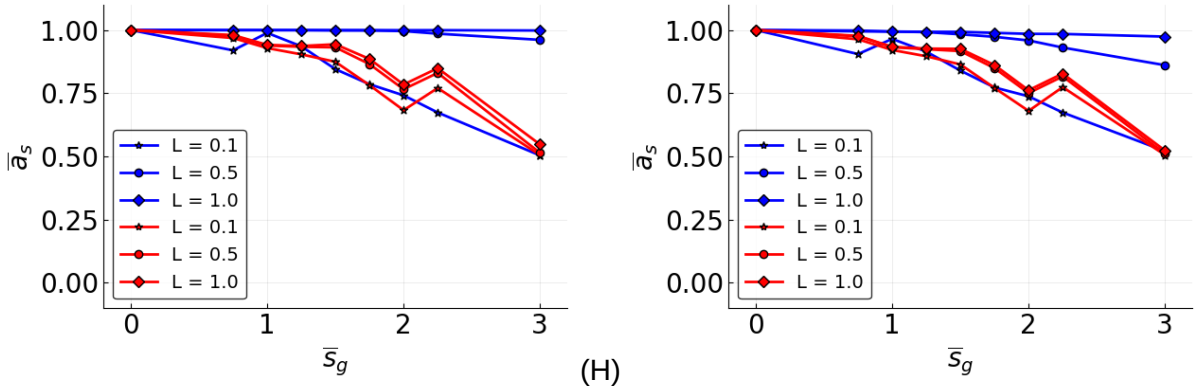


Figure S5. Histogram of Φ_i across all 100 reservoirs for each N, L for 5-bit functions (Left column) and recursive 3-bit functions(Right column). Reservoirs with the three values of $\bar{K}$ are shown: $\bar{K}$ = 1 (A,B), $\bar{K}$ = 2 (C,D), and $\bar{K}$ = 3 (E,F). Each subplot represents the density for all the reservoirs with one N and L, with the x-axis being Φ and the y-axis being number of reservoirs.

Figure S6

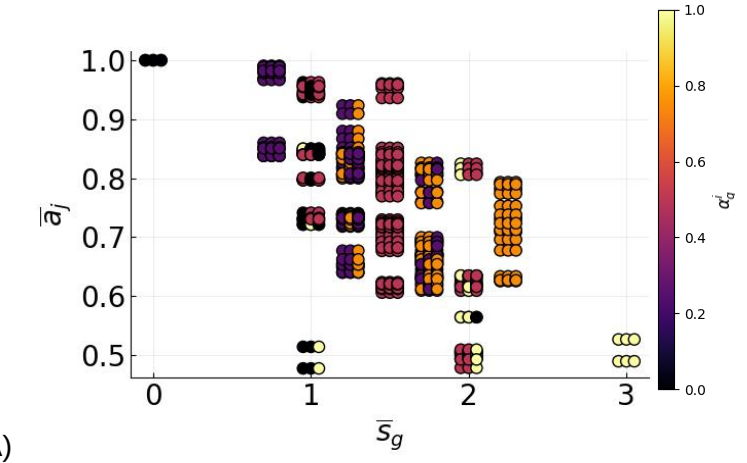

(A)
(B)
(C)
(D)
(E)
(F)
$\overline{a}_s$
$\overline{s}_g$
1.00
0.75
0.50
0.25
0.00
0
1
2
3
L = 0.1
L = 0.5
L = 1.0
L = 0.1
L = 0.5
L = 1.0



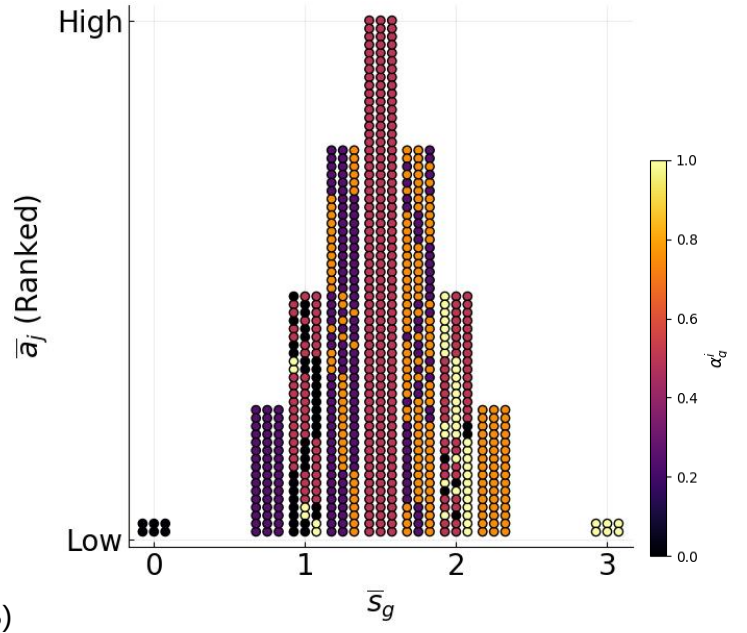
(G) (H)

Figure S6. Mean accuracy, $\bar{a}_s$, vs. function average sensitivity, $\bar{s}_g$. 3-bit functions shown in blue and recursive 3-bit functions shown in red with L = 0.1 (stars), 0.5(circles), 1(diamonds). Each row shows data from reservoirs with different values of N, from top to bottom N= 10 (A,B), N = 50 (C,D), N = 100 (E,F), N = 500 (G,H). Columns show $\bar{K}$ = 1 (Left) and $\bar{K}$ = 3 (Right).

Figure S7



(A)



(B)

Figure S7. Example of mean function accuracy vs. average sensitivity with activities of each variable displayed. Data shown is for recursive 3-bit functions- N = 10, L= 0.1, and $\bar{K}$ = 2. (A) Each function is visualized as a horizontal triplet of circles, where each circle corresponds to a variable (left to right,u^{t-tau},$u^{t-tau-1}$,y^{t-1}), colored by its activity. (B) In order to more clearly see the relationship between distribution of activity and accuracy, functions are plotted by ranked accuracy rather than absolute accuracy.