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Disproportionate Allocation of Indirect Costs at Individual-Farm Level Using Maximum Entropy

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Abstract: This paper addresses the allocation of indirect or joint costs among farm enterprises, and elaborates two maximum entropy models, the basic *CoreModel* and the *InequalityModel*, which additionally includes inequality restrictions in order to incorporate knowledge from production technology. Representing the indirect costing approach, both models address the individual-farm level and use standard costs from farm-management literature as allocation bases. They provide a disproportionate allocation, with the distinctive feature that enterprises with large allocation bases face stronger adjustments than enterprises with small ones, approximating indirect costing with reality. Based on crop-farm observations from the Swiss Farm Accountancy Data Network (FADN), including up to 36 observations per enterprise, both models are compared with a proportional allocation as reference base. The mean differences of the enterprise's allocated labour inputs and machinery costs are in a range of up to ±35% and ±20% for the *CoreModel* and *InequalityModel*, respectively. We conclude that the choice of allocation methods has a strong influence on the resulting indirect costs. Furthermore, the application of inequality restrictions is a precondition to make the merits of the maximum entropy principle accessible for the allocation of indirect costs.

Keywords: cost allocation; absorption costing; full costing; cost analysis; agriculture

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1. Introduction

The total cost of production—also called full cost, unit cost, or average cost—is a key item of information for managers for several reasons. Firstly, profitability is determined by examining the difference between output price and the total cost of production. Secondly, information of a complete cost structure is of great importance where costs must be reduced. Finally, total cost of production is a useful base for the benchmarking of companies or farms, as is the case for dairy farms in the International Farm Comparison Network (IFCN [1]).

Total cost of production should be known at decision level—i.e., enterprise level, also called "production branches" or "activities". Given that most Swiss farms have several enterprises (e.g., *Wheat*, *Potatoes* or *Grassland*), two components—direct and indirect costs—need to be distinguished. While direct or variable costs such as seeds or pesticides are available at an enterprise level for farms of the Swiss Farm Accountancy Data Network (FADN), indirect or joint costs such as labour or machinery costs are only available at a farm level. Accordingly, the allocation of indirect costs is necessary.

The literature provides an approach called indirect costing [2], which allocates indirect costs for all enterprises by means of allocation bases, also denoted as cost drivers (e.g., [3,4] or [5]). Drury [3] points out that volume-based allocation bases are frequently used, and distinguishes between physical measures (e.g., number of units) and measures indicating the ability to absorb indirect costs, such as market values of products. In the case of agriculture, the allocation bases used should reflect the marginal factor cost such as acreage, (working) hours, or number of (output) units [6]. For example, if available, Agri Benchmark prefers machine runtime-hours as an allocation base for machinery costs of crop enterprises [7].

As a specific form of indirect costing, activity-based costing (ABC) focuses on activities (e.g., machine services), rather than enterprises. In ABC, the cost-allocation base is a measure of the activity performed [4]. Drury [3] emphasises that ABC systems use both volume-based (e.g., units of output) and non-volume-based (e.g., number of set-ups or batches in the industrial manufacturing) cost drivers. For example, Carli et al. [8] apply ABC for arable crops, while Wouters and Stecher [9] provide an application for industrial manufacturing.

In indirect costing, the allocation itself is usually performed in a proportional manner. Although widely applied and sometimes not even mentioned, it implies a fairly strong assumption because all enterprises or activities are treated identically. In practice, the higher the allocation base, the greater are the opportunities to alter costs. Furthermore, when analysing several farms with the same allocation bases, proportional allocation implies that the ratio of a specific indirect cost item between enterprises is identical for all farms (e.g., the machinery costs of *Sugar Beet* will be 1.8 times the machinery costs of *Wheat*).

Basically, the challenge of indirect costing is to deal with the missing information that would be needed for the true cost allocation, meaning the exact documentation of all indirect cost items. For example, the working hours of all labour forces must be documented separately for all enterprises, which would require an excessive effort. To our knowledge, maximum entropy as a method to overcome data gaps has never been applied for this challenge at the individual-company or the individual-farm level. This paper contributes to the literature by elaborating two maximum entropy models grounded on allocation bases as alternatives to indirect costing. Furthermore, the results of both models are compared with a *Proportional* allocation as reference base.

It is important to mention that the literature provides numerous contributions for a topic related to cost allocation at the individual-farm level, namely the estimation of indirect costs of a composite of farm observations. Typically, these analyses are designed for agricultural policy modelling purposes, and do not address the single farm but rather the average farm. They interpret the deviation of an individual farm from the estimated indirect costs as an error term, while analyses at individual-farm level allocate all indirect costs completely among enterprises. From a methodological point of view, the analyses of a composite of farms can be divided into three approaches, with maximum entropy being the most prominent. Léony et al. [10], Peeters and Surry [11,12], as well as Fragoso and da Silva Carvalho [13,14] use generalised maximum entropy (GME) to derive input-output coefficients, also called cost-allocation coefficients or production coefficients, indicating the costs of inputs per unit of output value (e.g., seeds equals 7% of the output value of *Wheat*). Garvey and Britz [15] estimate more than twenty cost items per hectare or animal head (as enterprises' activity levels) by means of GME. Lence and Miller [16] present a generalised cross-entropy approach to estimate enterprise-specific input allocations. Apart from the applications of maximum entropy, a second approach consists of regression techniques. Using the general cost of production model (GECOM [17]), Kleinhanss et al. [18] and Tiberti [19]) estimates input-output coefficients. Moxey and Tiffin [20], and Griffiths et al. [21] use constraint estimations for input-output coefficients and cost shares of inputs, respectively. Desbois et al. [22] apply quantile regressions to estimate the cost of plant protection per hectare and per Euro 1000.- of turnover for crop enterprises. Finally, as a third approach to address cost allocation, Arfini et al. [23] estimate the total variable costs per tonne of crop outputs by means of positive mathematical programming (PMP).

The paper is organised as follows: the Methods section starts with the usual *Proportional* indirect-cost allocation at the individual-farm level, which represents the widely used approach of indirect costing and builds the reference base for comparisons (Section 2.1). Furthermore, it outlines the maximum entropy *CoreModel* (Section 2.2), as well as an extension, including inequality restrictions (*InequalityModel*, Section 2.3). Finally, to assess the performance of both maximum entropy models, a specific comparison and an *intermediate test* are outlined (Section 2.4). Section 3 describes the data used for the illustrative application from the Swiss Farm Accountancy Data Networks (FADN). Section 4 presents the resultant indirect costs of both the maximum entropy models and the *Proportional* cost allocation, as well as the *intermediate test* results, and Section 5 sets out our discussion and conclusions.

2. Methods

For simplicity, this section focuses on one cost item (e.g., machinery costs) on one particular farm observation (one year's accountancy). For the following empirical illustration (Sections 3 and 4), the model needs to be solved for each cost item and every farm observation separately.

2.1. Proportional Allocation

Let us assume that the farm under consideration produces i arable crops ($i = 1, 2,..., I$). An arable crop is considered as an enterprise (e.g., *Wheat*). Each crop i is grown on an area x_i, measured in hectares. From the accountancy figures (actual costs), we know the observed total indirect costs c, i.e., and the total machinery costs at a farm level in Swiss Francs (CHF). As a result of the indirect-cost allocation, we are looking for β_i, the (machinery) cost in CHF per hectare of crop i. Accordingly, c can be formulated as the sum of all enterprises' costs:

$$c = \sum_{i=1}^{I} x_i \beta_i \tag{1}$$

For the allocation of indirect costs, we use the standard costs μ_i as allocation bases. Standard costs indicate the costs of the usual production techniques applied in an efficient manner (e.g., machinery cost per hectare of *Wheat*) representing a normative perspective, and are normally supplied by the farm-management literature as planning data. We have opted for standard costs over other potential allocation bases such as acreage or machine runtime-hours because they represent indirect costs in a comprehensive manner. Physical allocation bases as an alternative bear the risk of an incomplete representation of indirect costs. For instance, for machinery costs, the machine runtime-hours as an allocation base do not differentiate between the sizes in terms of investment of the used machinery (e.g., harrow or potato harvester).

Based on the allocation bases μ_i and the areas x_i, the farm-wide standard costs s can be calculated, representing the costs we would expect from a farm, which is perfectly in line with management literature.

$$s = \sum_{i=1}^{I} x_i \mu_i \tag{2}$$

The coefficient alpha (α) is defined as the relation between actual costs (c) and standard costs (s):

$$\alpha = \frac{c}{s} \tag{3}$$

Because c and s are denoted in the same unit (e.g., CHF), α lacks a dimension. Furthermore, α can be used as an aggregated cost indicator. A value above/below 1 would indicate exaggerated/reduced costs. Accordingly, the difference between α and 1 indicates the deviation of a particular farm observation from the suggested standard costs in farm-management literature.

Under a *Proportional* indirect-cost allocation, α serves as adjustment factor, transforming all allocation bases μ_i equally:

$$\beta_i = \alpha \mu_i \quad \forall i \tag{4}$$

The relation between β_i and μ_i is equal to α for all enterprises (i), highlighting the uniform treatment of all enterprises under a *Proportional* allocation.

Figure 1 graphically illustrates a *Proportional* indirect-cost allocation and underscores the transformation of allocation bases. Specifically, the horizontal axis depicts the size of the allocation base per hectare. For two crops—*Grassland* (G) and *Potatoes* (P)—the cost allocation is depicted with *Potatoes,* representing a crop with a markedly higher allocation base. The resulting or allocated indirect costs per hectare are reported on the vertical axis. If we assume that the farm as a whole has lower costs than suggested by the farm-management literature ($\alpha < 1$), the *Proportional* line, whose slope is equal to

α, lies below the angle bisector (45°). According to Equation (4), the allocation bases $\mu_{Grassland}$ and $\mu_{Potatoes}$ are transformed to β_{G_Prop} and β_{P_Prop}, respectively.

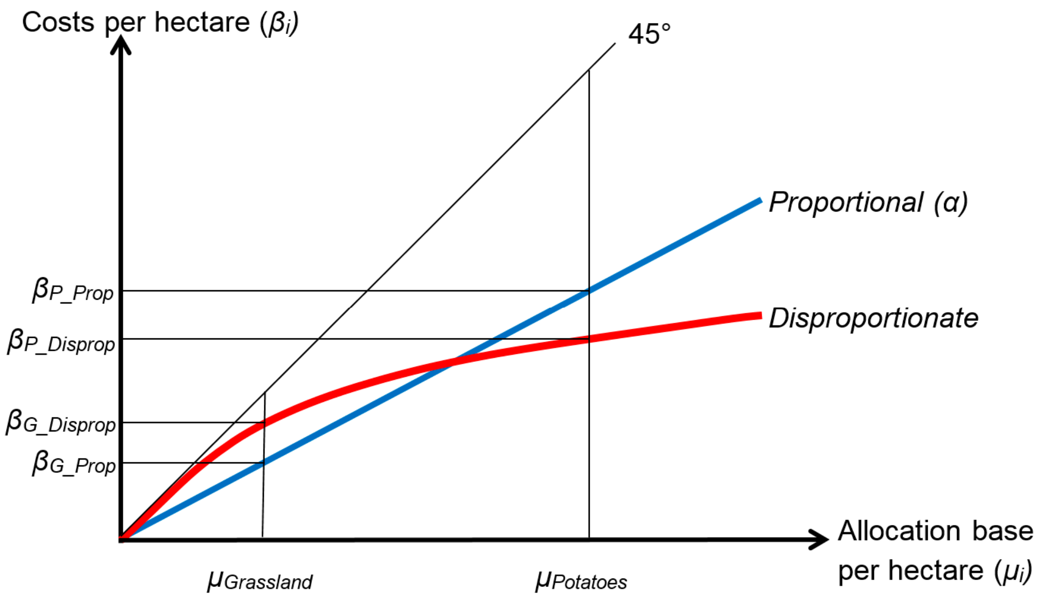

Figure 1. *Proportional* and disproportionate indirect cost allocation. G = Grassland; P = Potatoes; Prop = *Proportional*; Disprop = disproportionate.

2.2. Core Maximum Entropy Model

For the formulation of the core maximum entropy model (*CoreModel*), we follow the basic maximum entropy principle outlined by Golan et al. [24] (Chapters 3 and 5). The *CoreModel*'s purpose is to derive the indirect cost of enterprise i (β_i) for one cost item (i.e., machinery). Because allocation bases (μ_i) refer to the hectare level, the model focuses on the individual-hectare level. As a consequence, for the model specification, each individual hectare of crop i is treated as an independent activity. Accordingly, the number of hectares is crop specific and denoted as $N(i)$, while j refers to the individual hectare's number ($j = 1, 2, \ldots, N(i)$). Assuming that $N(i)$ is an integer, the individual hectares of crop i are denoted as $x_{i,j}$:

$$x_i = \sum_{j=1}^{N(i)} x_{i,j} \tag{5}$$

β_i, the resulting cost per hectare, is provisionally defined for each hectare individually as $\beta_{i,j}$. Accordingly, Equation (1) for the farm-wide costs c needs to be reformulated:

$$c = \sum_{i=1}^{I} \sum_{j=1}^{N(i)} \beta_{i,j} \tag{6}$$

We assume that $\beta_{i,j}$ lies in a range of $\mu_i \pm \mu_i$, or in an interval between 0 and $2\mu_i$. Because costs must be positive, we restrict the range to positive values. As suggested by Golan et al. [24] (Chapter 8), five support points ($z_{i,k}$; $K = 5$) are defined within the range [0, $0.5\mu_i$, μ_i, $1.5\mu_i$, $2\mu_i$]. Because we have no prior information about the probabilities of the support points, we apply this uniform distribution. In a cross-entropy application, the prior probabilities of support points may vary. As a consequence, the presented application belongs to maximum entropy, a special case of the cross-entropy formulation [24].

Because support points ($z_{i,k}$) do not differ between the different hectares of crop i, the hectare-wise distinctions need not be considered. Each support point k of all hectares j of all enterprises i has a probability denoted as $p_{i,j,k}$. Each $\beta_{i,j}$ is defined as a weighted sum of its support points times probabilities:

$$\beta_{i,j} = \sum_{k=1}^{K} p_{i,j,k} z_{i,k} \quad (7)$$

For each hectare *j*, the probabilities must add up to 1:

$$\sum_{k=1}^{K} p_{i,j,k} = 1 \quad \forall i,j \quad (8)$$

Maximising the Shannon Entropy measure *H* allows us to determine the probabilities $p_{i,j,k}$:

$$\max H = -\sum_{i=1}^{I} \sum_{j=1}^{N(i)} \sum_{k=1}^{K} p_{i,j,k} \ln p_{i,j,k} \quad (9)$$

Given that the support points within the different hectares of a specific crop are identical, the resultant probabilities must also be identical for all hectares *j*:

$$p_{i,j,k} = p_{i,k} \quad \forall j \quad (10)$$

Consequently, Equation (8) can be reformulated, which facilitates the subsequent model formulation:

$$\sum_{k=1}^{K} p_{i,k} = 1 \quad \forall i \quad (11)$$

The fact that the resulting probabilities of all hectares of a particular crop are equal leads to a further simplification:

$$\beta_{i,j} = \beta_i \quad \forall j \quad (12)$$

Thus, Equation (7) is reformulated as follows:

$$\beta_i = \sum_{k=1}^{K} p_{i,k} z_{i,k} \quad (13)$$

Equation (6) can also be formulated differently, using Equations (5) and (12), which leads again to Equation (1). Finally, the Shannon Entropy measure *H* (Equation (9)) is reformulated, making use of Equation (5):

$$\max H = -\sum_{i=1}^{I} x_i \sum_{k=1}^{K} p_{i,k} \ln p_{i,k} \quad (14)$$

The variable x_i serves in Equation (14) as a weighting factor that takes account of the differing numbers of hectares of the farm's enterprises.

In terms of support points, we must bear in mind that some farms may have actual costs far exceeding standard costs, as indicated by the farm-management literature. In other words, the above-mentioned interval may not be wide enough to allow for a feasible solution. Therefore, the expansion coefficient theta (θ) is defined, which depends on the coefficient alpha (α, Equation (3)):

$$\theta = \begin{cases} 2 \; if \; \alpha \le 1 \\ 1 + \alpha \; if \; \alpha > 1 \end{cases} \quad (15)$$

If the actual costs are lower than, or equal to standard costs ($\alpha \le 1$), the support points are in an interval between 0 and $2\mu_i$. For farms with higher costs, the range is extended to $(1 + \alpha)\mu_i$. Keeping the symmetrical structure, the support points are 0, $0.25\theta\mu_i$, $0.5\theta\mu_i$, $0.75\theta\mu_i$, and $\theta\mu_i$, respectively. The case differentiation for θ allows for a feasible solution for all farms. Furthermore, it ensures a continuous treatment for farm observations with a value of α around 1.

Equations (1), (11), (13)–(15) constitute the foundation of the maximum entropy cost-allocation model, even allowing the use of non-integer values for crop areas x_i. Together they form the *CoreModel* and are summarised in the Appendix A.

Because the model is solved for all indirect cost items and all farms, all equations must be expanded by both dimensions—that is, cost items and farm observations. The *CoreModel* is formulated in the General Algebraic Modeling System (GAMS [25]) and relies on the model code of a cross-entropy application for input-output tables by Robinson and El-Said [26]. The sequential (farm observation-wise) solving procedure is organised by the *loop* command of GAMS.

Returning to the indirect-cost allocation issue, the question arises as to how the *CoreModel* differs from a *Proportional* cost allocation. The outcome of a maximum entropy model is the optimal solution in terms of the probability distribution. Accordingly, the Shannon Entropy measure H (Equation (14)) reaches its maximum value when the distribution of all probabilities $p_{i,k}$ is uniform. Thus, the maximum is attained when each of the support points of all crops is assigned the probability of $1/K$, i.e., if β_i is equal to the allocation base μ_i. In the case that α exceeds 1, β_i would be equal to $0.5\theta\mu_i$. The approach therefore minimises the deviation from these values. For instance, if total costs c are smaller than what is suggested by the allocation bases ($\alpha < 1$), the model must cause a reduction of the allocation bases. In absolute terms, a 1% probability shift has a stronger impact on a crop with a large allocation base (such as *Potatoes*) than on one with low costs (*Grassland*). In Figure 1, the adjustment by means of the *CoreModel* is roughly approximated by the line *Disproportionate.* Starting with the same allocation bases as used for the *Proportional* allocation, the results for *Grassland* and *Potatoes* are $\beta_{G\_Disprop}$ and $\beta_{P\_Disprop}$, respectively. The resulting probability distribution leads to a disproportionate adjustment of allocation bases. Regarding the allocation of one cost item of one farm observation, all allocation bases are adjusted in either an increasing or diminishing direction according to the model specification. In other words, the line *Disproportionate* in Figure 1 never intersects the angle bisector (45°).

From a production technology perspective, a disproportionate adjustment addresses the allocation of indirect costs better than a *Proportional* adjustment for two reasons. Firstly, the higher the allocation base μ_i, the greater are the opportunities for altering costs in practice. *Potatoes*, for instance, incur much higher machinery costs than *Grassland*. If farm-wide machinery costs differ greatly from standard costs from farm-management literature, there are more possibilities on the field for adjusting machinery costs for *Potatoes*, because more operational steps are necessary (e.g., for plant protection and fertilisation). Secondly, there might be restrictions in terms of production technology, casting doubt on the reduction of small allocation bases in particular. For example, machinery cost of *Fallow Land* includes a once-a-year cutting or mulching. These minimal costs can hardly be undercut.

2.3. Inequality Restrictions

The disproportionate adjustment of the *CoreModel* can lead to a situation in which crops with large allocation bases (e.g., *Potatoes*) are so strongly reduced that they even undercut crops with small allocation bases (e.g., *Grassland*). In other words, the slope of the line *Disproportionate* in Figure 1 can become negative towards the far right. Such a result, which is only possible if the coefficient alpha has a low value, is useless from a production technology point of view. Generally speaking, there are fundamental relationships between crops, which should be reflected in the results at the farm level. The approach of imposing parameter inequality restrictions for maximum entropy models by Campbell and Hill [27] addresses such relationships and enables an extension of the *CoreModel*. For example, to ensure that the above mentioned inequality restriction $\beta_{Potatoes} > \beta_{Grassland}$ is maintained, Equation (13) must be reformulated as follows:

$$\beta_{Potatoes} = \beta_{Grassland} + \sum_{k=1}^{K} p_{(Potatoes-Grassland),k}\, y_{(Potatoes-Grassland),k} \quad (16)$$

Accordingly, $\beta_{Potatoes}$ consists of $\beta_{Grassland}$ and the sum of K support points, each multiplied by its probability. The support points y are related to the difference between allocation bases of the two crops ($y_{(Potatoes-Grassland),k} = z_{Potatoes,k} - z_{Grassland,k}$), and must be greater than or equal to 0. To summarise, by introducing inequality restrictions, we basically break allocation bases down into several components.

Although it would be possible to define an inequality restriction for each crop, such a procedure would be inappropriate for two reasons. Firstly, the application would become very complex, because an inequality restriction would be required for each crop and each indirect cost item. Secondly, the rank order of crops is not necessarily identical for all farms. As an alternative to the single-crop treatment, we define groups of crops. The Campbell–Hill approach allows us to maintain the rank order between groups completely, while the rank order within groups may change. To form groups, we look for similar crops in terms of production technology and agronomical considerations (e.g., cereals or root crops). Consequently, group assignments are reflected in similar allocation bases. Generally speaking, differences within groups should be smaller than between groups. Within each group, one crop is singled out. The crop with the largest allocation base is termed the boundary of the group, and plays a crucial role in organising the allocation base components. All support points y refer to the boundary of the downward or lower group. All crops of the first group refer to 0, which is interpreted as the very first boundary. Accordingly, z and y are identical for the crops of the first group.

Linking the groups' boundaries enables a clear upward/downward structure among groups, as depicted in Figure 2 for four groups. The first group comprises of *Forest* and *Fallow Land*, the crops with clearly smaller allocation bases (e.g., for machinery costs) than *Oilseeds*, *Wheat*, and *Peas*, which constitute Group 2. The latter is surpassed by the allocation bases of *Grassland* and *Sugar Beet*, which build Group 3. Finally, Group 4 includes *Potatoes* and *Other Crops*. However, the grouping might be different for other indirect cost items, such as labour.

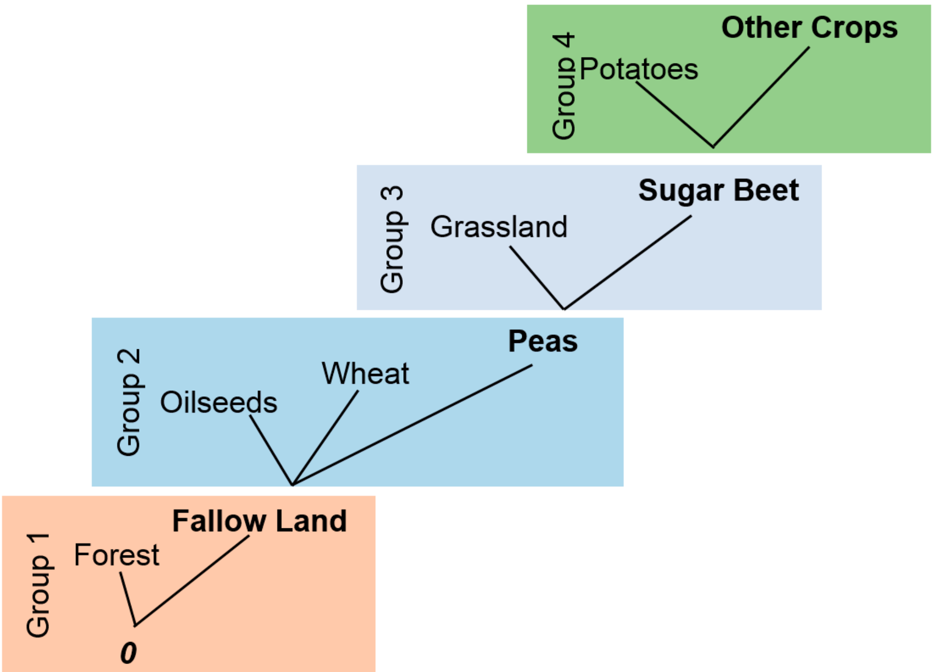

Figure 2. Upward/downward structure of four groups. The boundaries are indicated in bold text. The depicted upward/downward structure corresponds to the allocation bases of machinery costs outlined in Table 1. For simplicity reasons only a selection of crops of Table 1 is depicted.

How does the upward/downward group structure contribute to our basic intention of calculating the indirect costs (β_i) for *crop i*? β_i consists of components coming from at least one group g ($g = 1,\ldots, G(i)$). As an example, we focus on machinery costs of *Sugar Beet*, found in Group 3 ($G = 3$) of Figure 2. $\beta_{Sugar\ Beet}$ will consist of three components: firstly, the indirect costs of the boundary of the first group (*Fallow Land*); secondly, the difference between the boundaries of Groups 2 and 1, i.e., *Peas* and *Fallow Land*, respectively; and finally, the difference between *Sugar Beet* and the boundary of Group 2, *Peas*. For *Grassland*, another crop of Group 3, the calculation is identical except the third component, which is related to the difference between *Grassland* and *Peas*. To summarise we are looking at all components g of crop i. Continuing our example, all three components of $\beta_{Sugar\ Beet}$ (and similarly $\beta_{Grassland}$) are based on five support points $y_{i,g,k}$ and their resulting probabilities $p_{i,g,k}$. In total, we are looking for the probabilities of 15 support points for $\beta_{Sugar\ Beet}$. Generally speaking, we have to reformulate Equation (13) as follows:

$$\beta_i = \sum_{g=1}^{G(i)} \sum_{k=1}^{K} p_{i,g,k} y_{i,g,k} \tag{17}$$

At the same time, Equation (11) needs to be reformulated to hold for all crops and all groups:

$$\sum_{k=1}^{K} p_{i,g,k} = 1 \quad \forall i,g \tag{18}$$

Similarly to the reformulation of β_i (Equation (17)), we have to consider all of the groups for the Shannon Entropy measure *H*:

$$\max H = -\sum_{i=1}^{I} x_i \sum_{g=1}^{G(i)} \sum_{k=1}^{K} p_{i,g,k} \ln p_{i,g,k} \tag{19}$$

The Campbell–Hill approach is represented by Equations (17)–(19). Together with Equations (1) and (15) from the *CoreModel*, they build the *InequalityModel.*

Although being complete, the model description is inconvenient for the data processing because the same calculations are carried out several times. For example, as illustrated in Figure 2, the probabilities of *Fallow Land* in Group 1 are relevant for all crops belonging to 'upward' groups, because *Fallow Land* is the boundary of the very first group. Similarly, the probabilities of *Peas* and *Fallow Land* (boundaries of Groups 2 and 1, respectively) are included in the β's of all crops belonging to Groups 3 and 4. Generally speaking, the calculation of the boundaries' probabilities can be summarised or pooled, taking into account that the same allocation base *z*, or difference of allocation bases *y* always comes up with the same result independent of the enterprise for which they are calculated.

As a consequence, we reformulate Equation (17) by distinguishing between the boundaries *b* of the downward groups ($\beta_{b,G(i)-1}$), and the probabilities and support points of the last group *G*(*i*):

$$\beta_i = \beta_{b,G(i)-1} + \sum_{k=1}^{K} p_{i,G(i),k} y_{i,G(i),k} \tag{20}$$

For example, the enterprise *Peas* in Figure 2 is involved in two groups ($G_{Peas} = 2$). β_{Peas} consists of the beta of the boundary of the downward group ($G_{Peas} - 1 = 1$), e.g., $\beta_{Fallow\ Land}$. As a component of the second group, the probabilities and support points related to the difference of allocation bases between *Peas* and *Fallow Land* are considered. Please note that *Peas* is a boundary itself, the boundary of Group 2. Accordingly, β_{Peas} equals $\beta_{b,G(i)-1}$ for the enterprises *Grassland* and *Sugar Beet* in the equations for their β_i's.

Using the boundaries of downward groups as components of β_i is also relevant for the Shannon Entropy measure. Instead of maximising all probabilities of all groups as it is the case in Equation (19), we can pool them by making two changes: Firstly, we concentrate on the probabilities of the last group *G*(*i*) for all enterprises. Secondly, we expand the weights or areas of the enterprises serving as boundaries. All boundaries incorporate the surfaces of themselves plus the surfaces of all enterprises that belong to 'upward' groups. Accordingly, the weighting factor x_i in Equation (19) is replaced by u_i. For boundaries, u_i also comprises all 'upward' groups. For all non-boundary enterprises, u_i equals x_i. The necessary reformulation of Equation (19) is similar to the reformulation of Equation (9) to Equation (14) and results in:

$$\max H = -\sum_{i=1}^{I} u_i \sum_{k=1}^{K} p_{i,G(i),k} \ln p_{i,G(i),k} \tag{21}$$

The applied formulation of the *InequalityModel* consists of Equations (1), (15), (18), (20), and (21), and is summarised in the Appendix A. Because of the pooling process, Equation (18) needs to be considered only for the last group $G(i)$ of enterprises.

2.4. Comparison of Applications

As the 'true' allocation of indirect costs is unknown, the results of the proposed maximum entropy applications *CoreModel* and *InequalityModel* are compared with the *Proportional* cost allocation, outlined in Equations (1)–(4) as a reference base. Furthermore, the performance of the two models is assessed by comparing just two crops, *Potatoes* and *Grassland* as illustrated in Figure 1. We check whether the results for machinery costs are greater for *Potatoes* than for *Grassland*, which would be in line with considerations based on the production technology. In addition, the '*intermediate test*' systematically examines whether the rank order of crops according to their allocation bases is maintained during the indirect-cost allocation. In detail, the test compares the sizes of the resulting indirect costs (β_i), as illustrated in Figure 3. The initial rank order given by the allocation bases (μ_i) is depicted on the horizontal axis, while the vertical axis indicates the resulting indirect costs (β_i). *Grassland*—a crop with a relatively small (machinery) allocation base—is assigned the indirect costs $\beta_{Grassland}$. Furthermore, $\beta_{Potatoes}$ is the resultant cost for *Potatoes*, a crop with a large allocation base. For every crop with an allocation base between *Grassland* and *Potatoes*, such as *Sugar Beet* ($\beta_{Sugar\ Beet}$), we expect an intermediate value between those for *Grassland* and *Potatoes* ($\beta_{Grassland} < \beta_{Sugar\ Beet} < \beta_{Potatoes}$). The *intermediate test* checks whether the β_i's are in a strict upward order for all triplet ratios fulfilling two preconditions: firstly, the farm must grow all three crops involved ($x_i > 0$). Secondly, the allocation bases (μ_i) must be in a strict upward order ($\mu_1 < \mu_2 < \mu_3$). Given i crops, the number of analysed triplets per farm is $i(i-1)(i-2)/(3!)$. After performing the test, we count the failed triplets. In addition, a case differentiation is necessary depending on whether knowledge from production technology is challenged. A failure due to the crops belonging to the same group, the first case, would not matter because the rank order within groups may change, as discussed in Section 2.3. The second case, a failure due to crops belonging to different groups, is a serious issue because results are contradicting production technology. By definition, the second case cannot appear in the *InequalityModel* while failures are expected in the *CoreModel*.

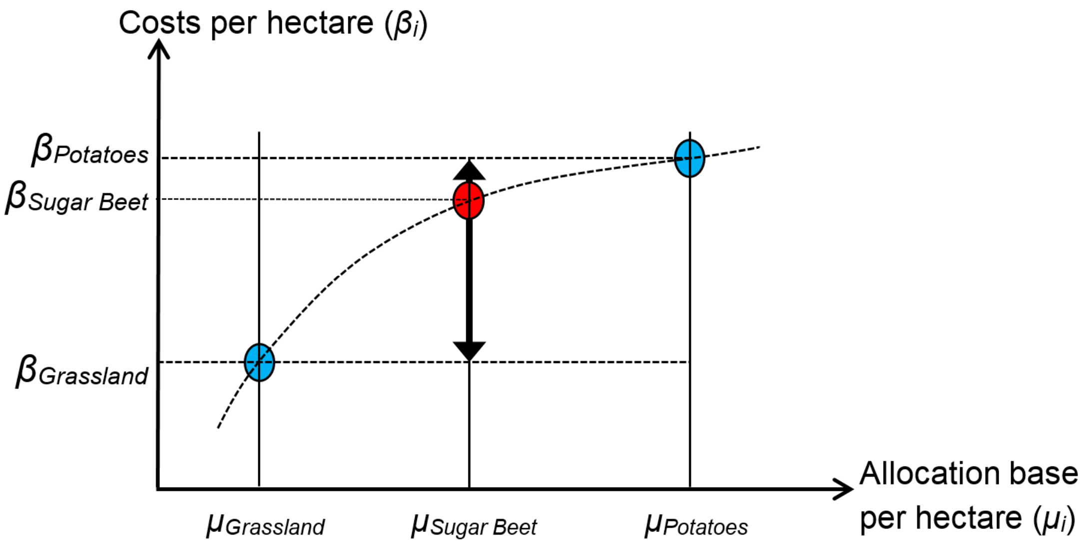

Figure 3. Intermediate test.

3. Data

To compare the different indirect-cost allocations, we focus on the arable-crop farm type from the Swiss FADN [28]. Excluding farms with even a very limited level of animal husbandry, as well as farms markedly involved in agriculture-related activities such as direct sales, 36 farm observations from the accountancy years of 2007 and 2008 are available, while twelve farms contributed data for both years. We aggregate farm production activities into 12 enterprises, resulting in 235 available enterprise cases. There are between 7 and 36 cases available for each enterprise: *Wheat, Barley*, *Maize* (*grain*), *Silage Maize*, *Potatoes*, *Sugar Beet*, *Oilseeds*, *Peas*, *Grassland*, *Fallow Land*, *Forest*, *and Other Crops* including highly labour-intensive activities such as fruit production. Two indirect cost items are analysed:

- *Labour input* is reported in normal working days (NWD), including all work forces (family members and employed workers).
- *Machinery costs* include all costs in CHF related to farm-owned machines, as well as machinery services contracted from other farmers or companies.

For the allocation bases (μ_i), standard costs are taken from farm-management literature [29–33]. Table 1 presents the allocation bases of all enterprises and both indirect cost items. As is essential for the *InequalityModel*, crops are depicted in a strictly increasing order based on their allocation bases. Furthermore, the crops are assigned to four groups based on production technology considerations, taking account of the fact, that they differ markedly in costs of labour and machinery. For example, *Forest* and *Fallow Land* show relatively small costs for machinery, resulting in the same group assignment (Group 1). Conversely, *Fallow Land* needs over three times as much of the labour input as *Forest*, requiring an assignment to different groups. As a consequence, the upward/downward structure is implemented separately for each indirect cost item. The boundaries of the groups are shown in bold and italicised letters. Another column gives the differences of each crop to the boundary of the lower group.

Table 1. Allocation bases for both indirect cost items.

Labour Input			
Group No.	**Enterprise**	**μ NWD**	**Δ NWD**
Group 4	**Other Crops**	64.00	57.59
	Potatoes	18.30	11.89
Group 3	**Sugar Beet**	6.41	2.11
Group 2	**Peas**	4.30	3.30
	Grassland	3.75	2.75
	Silage Maize	3.70	2.70
	Maize	3.60	2.60
	Fallow Land	3.33	2.33
	Wheat	3.30	2.30
	Barley	3.24	2.24
	Oilseeds	2.78	1.78
Group 1	**Forest**	1.00	1.00

Machinery Costs			
Group No.	**Enterprise**	**μ CHF**	**Δ CHF**
Group 4	**Other Crops**	5000	2161
	Potatoes	4553	1714
Group 3	**Sugar Beet**	2839	1187
	Silage Maize	2629	977
	Grassland	2221	569
Group 2	**Peas**	1652	1110
	Wheat	1591	1049
	Barley	1506	964
	Oilseeds	1366	824
	Maize	1338	796
Group 1	**Fallow Land**	542	542
	Forest	352	352

Sources: [29–33] and own calculations. The boundary of the group is indicated in bold text. NWD = Normal working days; CHF = Swiss Francs; μ (Mu) = Allocation bases from farm-management literature; Δ = Differences between the crops and the boundaries of the lower group (crops with largest μ in the lower group); Average exchange rates July 2017; 1 CHF = 0.90 Euro = 1.04 USD (https://data.snb.ch, accessed 29 August 2017).

4. Results

As a first result, farm-specific coefficients alpha (Equation (3)) are calculated for both indirect cost items on the basis of all 36 farm observations and allocation bases (Table 2). The mean value of labour input (measured in NWD) indicates that far more labour is used than suggested by the standard costs from the farm-management literature (α = 2.52). As regards machinery costs, the observed costs are 15% lower than the suggested. Looking at the extreme values, a marked variability can be observed.

There are farms with very low machinery costs, while for labour the most extreme value for the coefficient alpha is beyond 9.

Table 2. Coefficient alpha for both indirect cost items of 36 farm observations.

	Labour Input	Machinery Costs
Mean value of α	2.52	0.85
Standard error of α	1.86	0.34
Median of α	1.99	0.80
Minimum value of α	1.05	0.40
Maximum value of α	9.55	1.73

α = Coefficient alpha (see Equation (3)).

The results for the allocated indirect costs at the enterprise level are reported for each cost item separately, including the number of enterprise cases involved (Table 3). The table reports the resulting absolute mean values for all three allocation methods. In addition, the results of the two maximum entropy applications *CoreModel* and *InequalityModel* are also reported as deviations from a *Proportional* allocation in percent. Besides the mean deviation of all cases involved, the median, as well as the standard error of the deviations are indicated.

Table 3. (**a**) Results for labour input measured in normal working days (NWD) per hectare; (**b**) Results for machinery costs in Swiss Francs (CHF) per hectare.

Enterprise	No. of Cases	*Proportional*	*CoreModel*				*InequalityModel*			
		Mean in NWD/CHF	Mean in NWD/CHF	Deviations from *Proportional* in %			Mean in NWD/CHF	Deviations from *Proportional* in %		
				Mean	SE	Median		Mean	SE	Median
(**a**)										
Wheat	33	8.7	8.2	−7.1	7.0	−5.1	8.3	−5.8	7.0	−2.4
Barley	22	7.7	7.4	−4.6	5.3	−2.3	7.5	−3.8	4.7	−1.6
Maize	15	9.3	8.7	−7.6	9.0	−2.9	8.9	−5.7	9.6	0.1
Silage Maize	15	8.8	8.6	−2.8	5.5	−1.3	8.7	−0.9	4.6	0.8
Potatoes	7	37.5	44.4	16.5	10.7	18.2	42.8	12.5	8.9	15.1
Sugar Beet	23	17.0	18.1	4.9	10.9	9.5	17.1	−0.2	8.8	3.7
Oilseeds	31	7.4	6.8	−8.8	6.3	−7.9	6.8	−8.4	6.2	−6.8
Peas	13	8.2	7.7	−4.3	7.6	−7.3	8.0	−1.4	6.9	−4.1
Grassland	36	9.4	9.1	−4.9	7.4	−1.9	9.3	−2.9	7.6	1.1
Fallow Land	13	6.8	6.4	−5.4	6.0	−4.8	6.5	−4.0	5.0	−2.0
Forest	20	2.6	2.0	−18.9	8.5	−22.5	2.2	−14.0	7.2	−14.6
Other Crops	7	117.1	138.0	14.3	10.3	9.2	137.6	14.3	9.4	9.5
(**b**)										
Wheat	33	1275	1337	7.5	11.4	3.7	1282	0.8	4.0	0.0
Barley	22	1367	1409	6.2	13.3	2.7	1382	2.1	5.6	0.5
Maize	15	1266	1310	6.2	9.7	5.6	1295	4.0	6.0	3.7
Silage Maize	15	2217	2134	−7.1	14.5	−5.0	2203	−1.3	5.3	−0.3
Potatoes	7	3345	2611	−31.7	22.1	−39.4	3039	−13.3	9.6	−15.8
Sugar Beet	23	2376	2228	−9.5	14.8	−2.8	2317	−3.8	6.0	−1.2
Oilseeds	31	1124	1199	10.0	11.0	7.8	1170	6.1	6.6	4.7
Peas	13	1080	1191	13.1	14.2	11.5	1079	−0.4	5.8	−0.2
Grassland	36	1884	1851	−3.1	9.6	−0.3	1914	2.5	4.8	1.0
Fallow Land	13	449	535	36.3	41.0	37.5	489	17.2	19.6	16.6
Forest	20	312	361	29.4	38.5	20.3	346	20.5	26.7	14.4
Other Crops	7	3508	2518	−34.8	26.7	−38.5	2992	−18.0	13.1	−22.2

NWD = Normal working days; CHF = Swiss Francs; SE = standard error.

Considering that the farms in question use much more labour input (measured in NWD) than suggested by the farm-management literature, the allocated indirect costs (β_i) exceed the allocation bases for all three allocation methods (Table 3a). For example, for *Wheat* the absolute mean result of the 33 farm observations involved for labour is 8.7 NWD for the *Proportional* allocation, which clearly exceeds the standard costs of 3.3 NWD (Table 1). The *CoreModel* shows an average of 8.2 NWD, with a mean deviation from the *Proportional* allocation of −7.1%. Generally speaking, the deviations from *Proportional* of *CoreModel* and *InequalityModel* are similar, although the deviations of the latter are typically smaller.

Looking at all of the deviations from the *Proportional* allocation, the resulting distinction between small and large allocation bases is obvious. For crops with small allocation bases such as *Forest* and *Fallow Land*, the mean deviations from *Proportional* are negative, indicating that the resulting labour inputs are smaller under the maximum entropy models. Conversely, *Potatoes* and *Other Crops* exhibit positive mean deviations, indicating larger resulting labour inputs than under the *Proportional* allocation. The mean deviations of enterprises' labour inputs in the *CoreModel* allocation fall within a range of −18.9% (*Forest*) to +16.5% (*Potatoes*), and in the *InequalityModel*, within a range of −14.0% (*Forest*) to +14.3% (*Other Crops*). In respect of the mean differences in absolute numbers, the enterprises *Potatoes* and *Other Crops* are most strongly affected, with +6.9 NWD/hectare (44.4 NWD instead of 37.5 NWD) and +20.9 NWD/hectare (*CoreModel*) and +5.3 NWD/hectare and +20.5 NWD/hectare (*InequalityModel*), respectively.

Given an average coefficient alpha for machinery costs of 0.85 (Table 2), the allocated costs must be below the allocation bases, which correspond to the illustrated situation in Figure 1. For crops with small allocation bases (e.g., *Forest*, *Fallow Land*), both maximum entropy models lead to higher machinery costs than the *Proportional* allocation, expressed as positive deviations (Table 3b). Conversely, they show a more substantial reduction for *Potatoes* and *Other Crops*, the crops with the largest allocation bases, leading to negative deviations from the *Proportional* allocation. The mean deviations of *CoreModel* from the *Proportional* range between −34.8% (*Other Crops*) and +36.3% (*Fallow Land*), while those of *InequalityModel* range between −18.0% (*Other Crops*) and +20.5% (*Forest*). The largest absolute mean differences are presented again by the enterprises *Potatoes* and *Other Crops*, with CHF −734.-/hectare and CHF −990.-/hectare (*CoreModel*) and CHF −306.-/hectare and CHF −516.-/hectare (*InequalityModel*), respectively.

To examine the models' performance, the comparison of machinery costs for *Grassland* and *Potatoes* is carried out for seven farm observations, growing both crops. The *CoreModel* allocation shows in two observations a failure, i.e., larger machinery costs for *Grassland* than *Potatoes*. Accordingly, the disproportionate adjustment leads to a severe reduction of the fairly large allocation bases for the machinery costs of *Potatoes*. Both farm observations that fail show low machinery costs for the entire farm; in fact, their corresponding coefficients alpha are less than or equal to 0.51. For the *InequalityModel*, *Potatoes* and *Grassland* are assigned to different groups for machinery costs (Table 1), ensuring that the rank order is respected. Hence, the comparison shows no failures at all.

The *intermediate test* is conducted for all 1189 triplet relationships, and examines whether the strict rank order as dictated by the allocation bases is guaranteed for the allocated indirect costs per hectare (β_i). For the labour input, both models pass the test for all triplets. *CoreModel* fails for machinery costs for 149 triplets or 13%. Although there is no group assignment in *CoreModel*, we analyse how many of the failed triplets show a not-strict upward rank order between crops belonging to different groups according to the *InequalityModel*. This is the case for 127 triplets, or 11%. They also include all triplets that comprise *Grassland* and *Potatoes* from the above-mentioned comparison (e.g., the triplet *Grassland-Sugar Beet-Potatoes;* totally 13 triplets). The *InequalityModel* shows failures for 67 triplets (6%) for machinery costs. All of these triplets fail due to changes of the rank order of crops assigned to the same group, which is in line with the applied group-building concept.

5. Discussion and Conclusions

This paper elaborates on two maximum entropy applications—*CoreModel* and *InequalityModel*—for the allocation of indirect costs among enterprises at individual-farm level, using standard costs from farm-management literature as allocation bases.

Compared with the *Proportional* indirect-cost allocation, which is widely applied in the literature, the application of the maximum entropy principle leads to a disproportionate adjustment of allocation bases, reflecting a probability distribution in which large allocation bases face an over-proportional adjustment. The larger the allocation bases, the greater are the opportunities to adjust them on the field. As a result, the maximum entropy approach approximates indirect costing with reality.

We observe that the *CoreModel* tends to reduce large allocation bases excessively when indirect costs are appreciably lower than the standard costs from farm-management literature. According to the *intermediate test*, 11% of the analysed triplets call into question knowledge from production technology (e.g., higher machinery costs for *Potatoes* than for *Grassland*). As a consequence, the *CoreModel* cannot be considered for an empirical application. Adding inequality restrictions ensures that the rank order between groups of enterprises is maintained, and helps to overcome these shortcomings. The *InequalityModel* therefore allows the knowledge from production technology to be respected and enables the merits of the maximum entropy principle to be used for indirect costing at individual-farm level. Accordingly, the *InequalityModel* is recommended to provide indirect-cost at the enterprise level.

As indicated by the coefficient alpha, the data sample used represents a highly heterogeneous group of crop-farm observations from the Swiss FADN, and is ideal for test purposes. Both possibilities for the adjustment of allocation bases, i.e., to increase (for labour) and to reduce (for machinery cost) can be illustrated. Furthermore, the data sample used also enables us to show the different effects of a disproportionate cost allocation on crops with small (e.g., *Forest*) or large (e.g., *Potatoes*) allocation bases. Compared with the *Proportional* allocation, i.e., the reference base, the *CoreModel* and *InequalityModel* show deviations in a range of up to ±35% and ±20%, respectively. The standard errors and median values indicate that most enterprise cases are clearly influenced by the allocation method applied. In respect of absolute deviations measured in NWD/hectare or CHF/hectare, enterprises with large allocation bases are strongly affected (e.g., *Potatoes* and *Other Crops*). As a core conclusion, our results highlight that the choice of allocation method has a strong influence on the resulting indirect costs. Accordingly, the comparison of the *InequalityModel* as one potential alternative allocation method to the widely applied application of a *Proportional* allocation indicates that the latter constitutes an influential assumption. The proposed *InequalityModel* allows for it to be discarded and overcomes a uniform treatment of all enterprises in the indirect-cost allocation process. Based on the substantial differences between a *Proportional* allocation and the *InequalityModel*, we recommend using a disproportionate cost allocation for making (farm) management decisions and drawing policy conclusions.

By applying the same allocation bases to all farm observations in the presented analysis, the heterogeneity of farms and enterprises in terms of size and type of mechanisation is not addressed. Further work is needed to differentiate allocation bases of the individual enterprises. The presented analysis is restricted to crop enterprises. Given the relevance of animal production in Swiss agriculture, an expansion to enterprises of animal husbandry or agriculture-related activities such as direct sale is necessary. As a first application of the outlined method, full costs for all enterprises of Swiss dairy farms in the mountain region, including, among others, dairy-cow husbandry and sheep and goat farming, were derived [34]. Given that the FADN of some countries report direct costs such as plant protection costs on the farm level but not on the enterprise level, the allocation of direct cost items is a relevant topic (e.g., [18,19,22]). The *InequalityModel* could also be applied for these cost items because the standard costs of direct cost items are typically reported in gross margins calculations, and consequently widely available (e.g., [35,36]).

Compared with a *Proportional* allocation, the suggested approach is more costly and hence less suited to manual computing by farm managers. There are at least two ways to make the potential of the maximum entropy principle accessible for practice. Firstly, the *InequalityModel* could be incorporated into farm-management analyses based on FADN data carried out by research institutions. Secondly, the model could be a part of accounting software packages providing additional analytical tools besides the accounting for the tax declaration.

As far as we are aware, the presented work is the first analysis of different indirect-cost allocations methods at individual-farm or individual-company level. Given the importance of cost analysis in daily farm management or even general management, we expect the allocation of indirect costs to receive more attention in management research in future.

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Conflicts of Interest: The author declares no conflict of interest.

Appendix A

The following overview includes all equations with their numbers of both models, *CoreModel* and *InequalityModel*. The equations refer to one indirect-cost item (e.g., machinery costs) of one farm.

CoreModel	*InequalityModel*
(1) $c = \sum_{i=1}^{I} x_i \beta_i$	(1) $c = \sum_{i=1}^{I} x_i \beta_i$
(11) $\sum_{k=1}^{K} p_{i,k} = 1 \quad \forall i$	(18) $\sum_{k=1}^{K} p_{i,G(i),k} = 1 \quad \forall i$
(13) $\beta_i = \sum_{k=1}^{K} p_{i,k} z_{i,k}$	(20) $\beta_i = \beta_{b,G(i)-1} + \sum_{k=1}^{K} p_{i,G(i),k} y_{i,G(i),k}$
(14) $\max H = -\sum_{i=1}^{I} x_i \sum_{k=1}^{K} p_{i,k} \ln p_{i,k}$	(21) $\max H = -\sum_{i=1}^{I} u_i \sum_{k=1}^{K} p_{i,G(i),k} \ln p_{i,G(i),k}$
(15) $\theta = \begin{cases} 2 \; if \; \alpha \leq 1 \\ 1 + \alpha \; if \; \alpha > 1 \end{cases}$	(15) $\theta = \begin{cases} 2 \; if \; \alpha \leq 1 \\ 1 + \alpha \; if \; \alpha > 1 \end{cases}$

Remark for the *InequalityModel:* Because of the pooling process, Equation (18) needs to be considered only for the last group $G(i)$ of enterprise i.

α coefficient alpha (Equation (3))
b boundary
β indirect cost item, e.g., machinery costs, in CHF per hectare
c total indirect costs, e.g., machinery, in CHF
g group of the upward/downward structure, (g = 1,..., $G(i)$)
$G(i)$ the group of the last component of enterprise i
H Shannon Entropy measure
i arable crop, (i = 1, 2,..., I)
k support point, (k = 1, 2,..., K)
p probability of support point
u area in hectares, corrected for boundaries, equal to x for all non-boundary enterprises
x area in hectares
θ expansion coefficient theta (Equation (15))
y support point (differences between z's)
z support point

References

1. Hemme, T. (Ed.) *Dairy Report 2016 for a Better Understanding of the Dairy World*; International Farm Comparison Network: Kiel, Germany, 2016.
2. Ciaian, P.; Langrell, S.; y Paloma, S.G. (Eds.) Chapter 1. Introduction to Production Costs. In *Sustainability and Production Costs in the Global Farming Sector: Comparative Analysis and Methodologies, Scientific and Policy Reports*; Report EUR 25436EN; European Commission Joint Research Centre: Seville, Spain, 2012.
3. Drury, C. *Management and Cost Accounting*, 6th ed.; Thomson: London, UK, 2004.
4. Horngren, C.T.; Bhimani, A.; Datar, S.M.; Foster, G. *Management and Cost Accounting*, 3rd ed.; Prentice Hall: Harlow, UK, 2005.
5. McLaney, E.; Atrill, P. *Accounting an Introduction*, 4th ed.; Prentice Hall: Harlow, UK, 2008.
6. AAEA. *Commodity Costs and Returns Estimation Handbook*; A Report of the AAEA Task Force on Commodity Costs and Returns; American Agricultural Economics Association: Ames, IA, USA, 2000.

7. Agri Benchmark. *Cash Crop Report 2011*; Agri Benchmark Cash Crop Network; Johann Heinrich von Thünen-Institute: Braunschweig, Germany, 2011.
8. Carli, G.; Canavari, M.; Grandi, A. Introducing Activity-Based Costing in Farm Management: The Design of the FarmBO System. *Int. J. Agric. Environ. Inf. Syst.* **2014**, *5*, 69–84.
9. Wouters, M.; Stecher, J. Development of real-time product cost measurement: A case study in a medium-sized manufacturing company. *Int. J. Prod. Econ.* **2017**, *183*, 235–244.
10. Léony, Y.; Peeters, L.; Quinqu, M.; Surry, Y. The Use of Maximum Entropy to Estimate Input-Output Coefficients from Regional Farm Accounting Data. *J. Agric. Econ.* **1999**, *50*, 425–439.
11. Peeters, L.; Surry, Y.R. *Farm Cost Allocation Based on the Maximum Entropy Methodology—The Case of Saskatchewan Crop Farms*; Agriculture and Agri-Food Canada: Ottawa, ON, Canada, 2003.
12. Peeters, L.; Surry, Y. Estimation d'un modèle à paramètres variables par la méthode d'entropie croisée généralisée et application à la répartition des coûts de production en agriculture. *Insee—Actes des Journées de Méthodologie Statistique* **2005**. Available online: http://jms.insee.fr/index.php?php_action=HISTO&id_histo=8&liste=histo (accessed on 29 August 2017).
13. Fragoso, R.M.; da Silva Carvalho, M.L. Estimation of joint costs allocation coefficients using the maximum entropy: A case of Mediterranean farms. *J. Quant. Econ.* **2012**, *10*, 91–111.
14. Fragoso, R.M.; da Silva Carvalho, M.L. Estimation of cost allocation coefficients at the farm level using an entropy approach. *J. Appl. Stat.* **2013**, *40*, 1893–1906.
15. Garvey, E.; Britz, W. *Estimation of Input Allocation from EU Farm Accounting Data Using Generalized Maximum Entropy*; Working Paper 02-01; University of Ireland: Dublin, Ireland; Universität Bonn: Bonn, Germany, 2002.
16. Lence, S.H.; Miller, D.J. Recovering Output-Specific Inputs from Aggregate Input Data: A Generalized Cross-Entropy Approach. *Am. J. Agric. Econ.* **1998**, *80*, 852–867.
17. Butault, J.-P. Chapter 2, The GECOM model. In *Implementation, Validation and Results of the Cost of Production Model Using the EU FADN, FACEPA Deliverable No. 3*; Kleinhanss, W., Ed.; vTI: Braunschweig, Germany, 2011.
18. Kleinhanss, W.; Offermann, F.; Butault, J.-P.; Surry, Y. *Cost of Production Estimates for Wheat, Milk and Pigs in Selected EU Member States*; Arbeitsberichte aus der vTI-Agrarökonomie; No. 07/2011; 2011. Available online: http://www.econstor.eu/handle/10419/53152 (accessed on 22 June 2017).
19. Tiberti, M. Production costs of Soft Wheat in Italy. In Proceedings of the Second Conference of the Italian Association of Agricultural and Applied Economics, Parma, Italy, 6–7 June 2013.
20. Moxey, A.; Tiffin, R. Estimating linear production coefficients from farm business survey data: A note. *J. Agric. Econ.* **1994**, *45*, 381–385.
21. Griffiths, W.E.; O'Donnell, C.J.; Tan Cruz, A. Imposing regularity conditions on a system of cost and factor share equations. *Aust. J. Agric. Resour. Econ.* **2000**, *44*, 107–127.
22. Desbois, D.; Butault, J.-P.; Surry, Y. Estimation des coûts de production en phytosanitaires pour les grandes cultures. Une approche par la régression quantile. *Écon. Rural.* **2013**, *333*, 27–49.
23. Arfini, F.; Donati, M.; Veneziani, M. Assessing farm production costs using PMP: Theoretical foundation and empirical validation. In Proceedings of the 29th International Conference of Agricultural Economists, Milan, Italy, 8–14 August 2015.
24. Golan, A.; Judge, G.; Miller, D. *Maximum Entropy Econometrics: Robust Estimation with Limited Data*; John Wiley & Sons: Chichester, UK, 1996.
25. Rosenthal, R. *GAMS—A User's Guide, Tutorial*; GAMS Development Corporation: Washington, DC, USA, 2013.
26. Robinson, S.; El-Said, M. *Estimating a Social Accounting Matrix Using Entropy Difference Methods*; TMD Discussion Paper No. 21; International Food Policy Research Institute, Washington, DC, USA, 1997.
27. Campbell, R.C.; Hill, R.C. Imposing Parameter Inequality Restrictions using the Principle of Maximum Entropy. *J. Stat. Comput. Simul.* **2006**, *76*, 985–1000.
28. Roesch, A.; Hausheer Schnider, J. *Grundlagenbericht 2008*; Agroscope Reckenholz-Tänikon Research Station: Ettenhausen, Switzerland, 2009.
29. Agridea; FiBL. *Deckungsbeiträge Ausgabe 2008*; Agridea and Forschungsinstitut für Biologischen Landbau (FiBL): Lindau, Switzerland, 2008.
30. Albisser, G.; Gazzarin, C. *Vollkostenkalkulationen für Betriebszweige*; Agroscope Reckenholz-Tänikon Research Station: Ettenhausen, Switzerland, 2010.
31. Lips, M.; Ammann, H. Vollkostenkalkulationen für Ackerkulturen. *Agrarforschung* **2006**, *13*, 210–214.

32. Alder, T. *Vollkostenkalkulation für die Mostobstproduktion, Vergleich der Produktionskosten von Mostobst zwischen der Ostschweiz und Baden-Württemberg*; ART-Bericht No. 691; Agroscope Reckenholz-Tänikon Research Station: Tänikon, Switzerland, 2007.
33. Bundesamt für Landwirtschaft. *Agrarbericht 2009 des Bundesamtes für Landwirtschaft*; Bundesamt für Landwirtschaft: Bern, Switzerland, 2009.
34. Lips, M. Calculating full costs for Swiss dairy farms in the mountain region using a maximum entropy approach for joint-cost allocation. *Int. J. Agric. Manag.* **2014**, *3*, 145–153.
35. Craig, K. (Ed.) *The Farm Management Handbook 2016/17*, 37th ed.; The UK Reference for Farm Business Management; SAC Consulting: Midlothian, UK, 2016.
36. Redman, G. (Ed.) *John Nix Farm Management Pocketbook for 2016*, 46th ed.; The Pocketbook: Melton Mowbray, UK, 2015.
37. Lips, M. Joint Cost Allocation by Means of Maximum Entropy. In Proceedings of the 28th International Conference of Agricultural Economists, Foz do Iguaçu, Brazil, 18–24 August 2012.
38. Lips, M. *Disproportionate Joint Cost Allocation at Individual-Farm Level Using Maximum Entropy*; Working Paper; Institute for Sustainability Sciences: Ettenhausen, Switzerland, 2014.