Supplementary Materials

Analytical investigation of light intensity independency of GLIE SD

In a Poisson PDF distribution, the variance equals the average. Accordingly concerning photon noise:

$$\sigma^2_{\mathrm{pixel}} = \mathrm{N} \tag{S1}$$

where N is the average number of the detected photons during a unit of time, i.e. the light intensity and $\sigma^2_{\mathrm{pixel}}$ is the variance of light intensities acquired in a single pixel during a few acquisitions (temporal variance). On the other hand, the variance of many intensities acquired, each in a different pixel of the 25 pixels (unit measurement) along the same single Δt, is defined as the spatial variance σ^2_{s25} and must obey the ergodic principle [1]. That is to say:

$$\sigma^2_{\mathrm{s25}} \cong \sigma^2_{\mathrm{pixel}} \tag{S2}$$

In a Poisson PDF distribution, when the detected value, i.e. the K variable, is large enough, the Poisson and the Gaussian distribution become similar enough to justify the determination that the entropy of both distributions is very similar (2).

In keeping with the above, according to Equation 2 and Equations S2 and S1:

$$\mathrm{GLIE}_{25} = 0.5\ln(2\sigma^2_{\mathrm{s25}}\pi\mathrm{e}) \cong 0.5\ln\left(2\sigma^2_{\mathrm{pixel}}\pi\mathrm{e}\right) = 0.5\ln(2\mathrm{N}\pi\mathrm{e}) \tag{S3}$$

Say that one acquires a few 5×5 pixel images, yielding an ensemble of images (samples). Each of the images in the ensemble has a spatial SD, i.e. σ_{s25}. Next, the variance of σ_{s25} in the ensemble of samples is, (Wolfram and Mathworld - Section: "Standard Deviation Distribution"):

$$\text{variance of}(\sigma_{\mathrm{s25}}) = \frac{1}{\mathrm{n}}\left(\mathrm{n}-1-\frac{2\Gamma^2\left(\frac{\mathrm{n}}{2}\right)}{\Gamma^2\left(\frac{\mathrm{n}-1}{2}\right)}\right)\sigma_{\mathrm{pixel}}{}^2 \quad * \tag{S4}$$

In Wolfram and Mathworld $\sigma_{\mathrm{s25}} \equiv \mathrm{s}$, and $\sigma_{\mathrm{pixel}} \equiv \sigma$.

Relation S4 emphasizes a linear relation in which we define $\mathrm{a}^2 = \frac{1}{\mathrm{n}}\left(\mathrm{n}-1-\frac{2\Gamma^2\left(\frac{\mathrm{n}}{2}\right)}{\Gamma^2\left(\frac{\mathrm{n}-1}{2}\right)}\right)$, hence:

$$(\text{SD of}\sigma_{\mathrm{s25}}) = \mathrm{a}\sigma_{\mathrm{pixel}} \tag{S5}$$

n stands for the samples size (in our case 25 pixels), Γ is the gamma function and a is the linear correlation coefficient.

Introducing the number used in our experimental set-up. i.e.: n the sample size that equals 25 (25 pixels of a measurement unit) enables computing a:

$$\mathrm{a} = \sqrt{\frac{1}{25}\left(24-\frac{2\Gamma^2(12.5)}{\Gamma^2(12)}\right)} = 0.140655 \tag{S6}$$

According to the rightest expression of equation S3, for average light intensity $\mathrm{N}_1 = \sigma^2_{\mathrm{pixel1}}$ one gets:

$$\overline{\mathrm{GLIE}}_{25} \cong 0.5\ln(2\overline{\sigma}^2_{\mathrm{s25}\,1}\pi\mathrm{e}) \tag{S7}$$

$\overline{\mathrm{GLIE}}_{25}$ is the average of 200 time measurements of 5×5 pixel GLIE (GLIE25).

The SD of GLIE_{25} in the 200 measurement in that average light intensity (N_1) would be:

$$|\overline{\mathrm{GLIE}}_{25} - \mathrm{GLIE}_{1\ \mathrm{SD}}| \cong 0.5\ln(2(\overline{\sigma}_{s25\ 1} + 1\ \mathrm{SD\ of}\ \overline{\sigma}_{s25\ 1})^2\ \pi e) - 0.5\ln(2\overline{\sigma}_{s25\ 1}^2\pi e) \quad \text{(S8)}$$

where $\mathrm{GLIE}_{1\ \mathrm{SD}}$= GLIE25 value in 1 SD from average GLIE25.

Introducing $a\overline{\sigma}_{\mathrm{pixel1}}$ (Equation S5) into Equation S8 instead of 1 SD of $\overline{\sigma}_{s25\ 1}$ one gets:

$$\mathrm{SD\ of\ GLIE} = |\mathrm{GLIE}_{1\ \mathrm{SD}} - \overline{\mathrm{GLIE}}_{25}| \cong 0.5\ln\big(2(\overline{\sigma}_{s25\ 1} + a\overline{\sigma}_{\mathrm{pixel1}})^2\ \pi e\big) - 0.5\ln(2\overline{\sigma}_{s25\ 1}^2\pi e) \quad \text{(S9)}$$

Next, substituting σ_{pixel1} from Equation S2 into Equation S9 instead of $\sigma_{s25\ 1}$:

$$SD\ of\ GLIE \cong 0.5\ln\big(2(\overline{\sigma}_{pixel1} + a\overline{\sigma}_{pixel1})^2\ \pi e\big) - 0.5\ln\big(2\overline{\sigma}_{pixel1}^2\pi e\big) = 0.5ln\frac{2(\overline{\sigma}_{pixel1} + a\overline{\sigma}_{pixel1})^2\ \pi e}{2\overline{\sigma}_{pixel1}^2\pi e} = 0.5ln\frac{\overline{\sigma}_{pixel1}^2(1+a)^2}{\overline{\sigma}_{pixel1}^2} = 0.5\ln(1+a)^2 = \ln(1+a)$$

Hence, finally

$$\mathrm{SD\ of\ GLIE} \cong \ln(1+a) \quad \text{(S10)}$$

which is in agreement with the horizontal curve shown in Figure 6d (the SD of GLIE mostly depends on coefficient a which reflects the sample size (equation S4) and not on N (light intensity)).

Simulation experiments

A simulation experiment was performed in which, for a given average and SD of source Gaussian distribution ($\mathrm{SD}_{\mathrm{source}}$), a random variable was chosen 25 times, mimicking the "sample" discussed above. This process was repeated 200 times, yielding a "sample ensemble". For each of the sample ensemble components, the $\mathrm{SD}_{\mathrm{sample}}$ was calculated, from which the entire $\mathrm{SD}_{\mathrm{ensemble}}$ of all $\mathrm{SD}_{\mathrm{sample}}$ was calculated as well.

Next, keeping the same average, but with different SDs of Gaussian, the simulation was reproduced. This procedure was repeated 5 times and the graphic presentation of $\mathrm{SD}_{\mathrm{ensemble}}$ versus $\mathrm{SD}_{\mathrm{source}}$ is depicted in Figure S1.

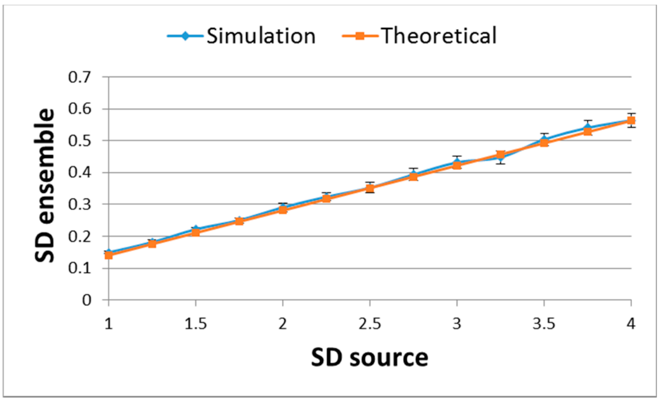

Figure S1. $\mathrm{SD}_{\mathrm{ensemble}}$ versus $\mathrm{SD}_{\mathrm{source}}$, as calculated by simulation (blue curve) and Equation S5 (orange curve).

The graph in Figure S1 clearly supports the linearity claimed in Equation S5.

From the data acquired above, the GLIE of each sample (GLIE25s) and the SD of the ensemble ($SD_{GLIE25-ensemble}$) were calculated, from which the relation $SD_{GLIE25-ensemble}$ versus SD_{source} is depicted in Figure S2.

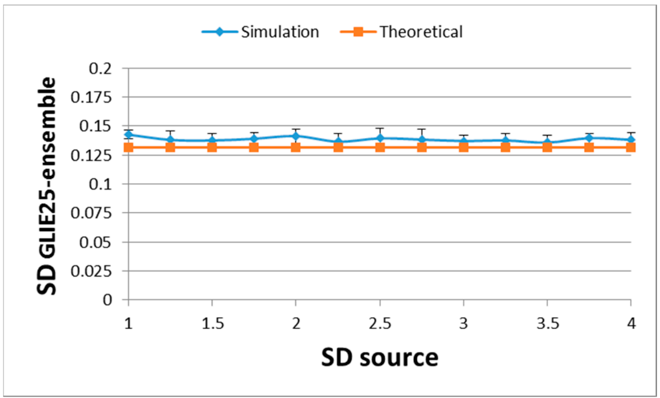

Figure S2. $SD_{GLIE25-ensemble}$ versus SD_{source} as calculated by simulation (blue curve) and Equation S10 (orange curve).

In our simulation SD_{source} represents σ_{pixel}, and relying on Equation S1 ($\sigma_{pixel} = \sqrt{N}$), we can conclude that $SD_{source} = \sigma_{pixel} = \sqrt{N}$. From this conclusion, it follows that, if (according to Figure S2) the $SD_{GLIE25-ensamble}$ is indifferent to SD_{source}, it will be also be indifferent to N (light intensity).

The graph in Figure S2 is clearly in agreement with the results shown in Figure 6d and with the constancy claimed in Equation S10.

References

1. Moore, C.C.; Ergodic theorem, ergodic theory, and statistical mechanics. *PNAS* **2015**, *112*, 197-1911, doi: 10.1073/pnas.1421798112
2. Evans, R.J.; Boersma, J; The Entropy of a Poisson Distribution. *SIAM Rev* **2006**, *30*, 314-317, https://doi.org/10.1137/1030059.