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Distributed Consensus of Nonlinear Multi-Agent Systems on State-Controlled Switching Topologies

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Abstract: This paper considers the consensus problem of nonlinear multi-agent systems under switching directed topologies. Specifically, the dynamics of each agent incorporates an intrinsic nonlinear term and the interaction topology may not contain a spanning tree at any time. By designing a state-controlled switching law, we show that the multi-agent system with the neighbor-based protocol can achieve consensus if the switching topologies jointly contain a spanning tree. Moreover, an easily manageable algebraic criterion is deduced to unravel the underlying mechanisms in reaching consensus. Finally, a numerical example is exploited to illustrate the effectiveness of the developed theoretical results.

Keywords: multi-agent systems; consensus; nonlinear systems; switching topologies; spanning tree

1. Introduction

Recent years have witnessed a growing interest in the consensus problem of multi-agent systems in system and control community. A lot of effort has been made to design distributed control law for each agent such that the system as a whole can perform complex tasks in a cooperative manner. The distinguishing feature of such control law lies in its lack of global information while aiming to cooperate with all agents. In this paper, we deal with the consensus problem of multi-agent systems with nonlinear dynamics and switching topologies jointly containing a spanning tree.

Over the past decade, many well-known results on consensus have been reported in [1–7], to name just a few. Based on algebraic graph theory, Olfati-Saber and Murray [1] discussed the consensus problem for networked single-integrator agents over directed fixed and switching topologies with communication time-delays. Following this work, the consensus problem has been recently investigated from various perspectives, for example, system with second-order dynamics [8,9], nonlinear agent dynamics [10,11], time-delays [12,13], quantization [14], saturation [15], *etc*. However, most of the aforementioned works were predominantly concerned with the multi-agent systems under fixed communication topologies.

In practical application, the interaction topology among agents may change dynamically due to the limited sensing regions of sensors or effect of obstacles. Different assumptions on the switching topologies for multi-agent systems have been explored in recent years [16–18]. By assuming that the switching topologies keep connected or contain a spanning tree at every time, a heap of results have been reported [19–23]. However, it is impractical to impose the connectivity condition on all possible topologies. Thus, seeking feasible while less restrictive condition on the switching topologies becomes a mix of diverse challenging, yet interesting topic. In discrete-time setting, Jadbabaie *et al.* [24] provided a simple consensus protocol for Vicsek's model [25], which was analyzed theoretically by exploiting properties of products of stochastic matrices under jointly connected topologies. The result was later extended in [26] to the case of directed graphs where conditions for consensus under switching interaction topologies were presented. For continuous-time systems, Hong *et al.* [27] proposed a local control strategy for multi-agent systems with jointly connected topologies. In [28–30], the switching communication topologies were assumed to be governed by continuous-time homogeneous Markov processes, whose state space corresponds to the communication patterns. The authors of [31–33] considered continuous-time multi-agent systems under jointly connected topologies, which had less constraints on each possible topology. However, these results are quite conservative in the sense that the underlying topology of the system switches without concerning the current states of the multi-agent systems.

Inspired by the above discussion, this paper aims to investigate the leadless consensus problem of multi-agent systems with Lipschitz nonlinear dynamics over state-controlled switching topologies. Relevant work can be found in [34], where the author studied leader-following consensus for double-integrator-based multi-agent systems under jointly connected topologies. In this paper, all possible topologies are allowed to be disconnected, and only jointly contains a spanning tree is required for the system to achieve consensus. By using the state transformation method, the consensus problem becomes a stability problem of a nonlinear switched system. Then, based on the Lyapunov stability approach, the consensus of the considered system is proved to be achieved with a prescribed consensus error. The contribution of this paper can be ascribed as follows: (1) Inspired by the stabilizing switching theory [35], we design a state-controlled switching law for the considered switching topologies. To avoid the switching signal from chattering, a new mechanism is introduced, and then, the low bound of dwell time of switching topologies is explicitly calculated. This is neglected in [34]; therefore, the controllers therein may suffer from chattering. (2) The dynamics of multi-agent system incorporates nonlinearities, which have been less reported in the literature, especially when the topologies are assumed to jointly contain a spanning tree. This paper attempts to explore the consensus of multi-agent systems with both nonlinear dynamics and state-controlled switching topologies, which thus constitutes a necessary complement to the existing literature.

The rest of the paper is organized as follows. In Section 2, some preliminaries on algebraic graph theory and model formulation are given. Sufficient conditions are given to ensure consensus of first-order multi-agent systems in Section 3. In Section 4, we give a numerical example to illustrate the proposed protocol. Conclusions are drawn in Section 5.

Throughout the paper, the following notations are adopted for the ease of presentation. $\mathbb{R}^n$ is the n-dimensional Euclidean space and $\mathbb{R}^{n\times n}$ stands for the set of $n \times n$ real matrix. I_n and O_n are $n \times n$ identity and zero matrices, respectively. $\mathrm{diag}\{x_1, x_2, \ldots, x_m\}$ denotes the diagonal matrix with diagonal elements x_1 to x_m. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm.

2. Preliminaries and Problem Statement

A weighted digraph (or directed graph) $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ of order n consists of a set of nodes $\mathcal{V} = \{1, \ldots, n\}$, a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and a weighted adjacency matrix $\mathcal{A} = [\alpha_{ij}] \in \mathbb{R}^{n\times n}$. A directed edge in $\mathcal{E}$ is denoted by $e_{ij} = (i, j) \in \mathcal{E}$, which means node i has access to the information of j. The element α_{ij} in $\mathcal{A}$ is decided by the edge between i and j, *i.e.*, $e_{ij} \in \mathcal{E} \Leftrightarrow \alpha_{ij} > 0$; otherwise $\alpha_{ij} = 0$. The set of neighbors of node i is denoted by $N_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$. The Laplacian matrix L of graph G is defined by $L = D - \mathcal{A}$, where $D = \mathrm{diag}\{d_1, d_2, \ldots, d_n\}$, and $d_i = \sum_{j\in N_i}^{n} \alpha_{ij}$ is the in-degree of node i. A sequence of edges $(i_1, i_2), (i_2, i_3), \ldots, (i_{s-1}, i_s)$ is called a directed path from node i_s to node i_1. If there exists at least one node (called the root) having directed path to any other nodes, the digraph is said to have a spanning tree.

To depict the varying topologies, let $\bar{\mathcal{G}} = \{\mathcal{G}_m = (\mathcal{V}, \mathcal{E}_m, \mathcal{A}_m) | m \in \mathcal{M}\}$ denote the collection of all possible digraphs on the same node set $\mathcal{V} = \{1, \ldots, n\}$, and $\mathcal{M} = \{1, \ldots, M\}$ be the index set of possible topologies, where M is the number of possible topologies. Then, the underlying graph at time t can be denoted by $\mathcal{G}_{\sigma(t)}$, where $\sigma(t)$ is a piecewise constant switching function defined as

$\sigma(t) : [0, +\infty) \to \mathcal{M}$. It is assumed that $\sigma(t)$ switches finite times in any bounded time interval. For a collection $\bar{\mathcal{G}}$ of digraphs, its union digraph is defined as $\mathcal{G}_u = (\mathcal{V}, \cup_{m=1}^{M} \mathcal{E}_m, \sum_{m=1}^{M} \mathcal{A}_m)$. Moreover, we say that the collection $\bar{\mathcal{G}}$ jointly contains a spanning tree if its union digraph $\mathcal{G}_u$ has a spanning tree.

Consider a multi-agent system consisting of n agents. The dynamics of each agent is

$$\dot{x}_i = f(x_i, t) + u_i, i = 1, 2, \ldots, n, \tag{1}$$

where $x_i \in \mathbb{R}$ is the state of agent i, $f(x_i, t)$ is a nonlinear function describing the self-dynamics of agent i, and u_i is the control input.

Assumption 1. The nonlinear function $f(x, t)$ satisfies the Lipschitz condition with the Lipschitz constant ρ, *i.e.*,

$$|f(x_2, t) - f(x_1, t)| \leq \sqrt{\rho}|x_2 - x_1|, \ \forall x_1, x_2 \in \mathbb{R}, t \geq 0.$$

Assumption 2. The switching topologies $\mathcal{G}_{\sigma(t)}$ jointly contain a spanning tree.

For system Equation (1), we consider the following control input for the ith agent:

$$u_i(t) = -k \sum_{j \in N_i} \alpha_{ij}^{\sigma(t)} (x_i - x_j). \tag{2}$$

Hence, the closed-loop system can be rewritten in compact form as

$$\dot{x}(t) = -kL_{\sigma(t)} x(t) + f(x, t), \tag{3}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$ and $f(x, t) = [f(x_1, t), f(x_2, t), \ldots, f(x_n, t)]^T$.

Here, we introduce a state transformation for system Equation (3)

$$\xi = Ex,$$

where $E = [-\mathbf{1}_{n-1} \ I_{n-1}]$ so that system Equation (3) can be rewritten in the following reduced-order form with respect to ξ

$$\dot{\xi}(t) = -k\bar{L}_{\sigma(t)} \xi(t) + \bar{f} \tag{4}$$

where $\bar{L}_{\sigma(t)} = EL_{\sigma(t)}F$, $F = [\mathbf{0}_{n-1} \ I_{n-1}]$ and $\bar{f} = [f(x_2, t) - f(x_1, t), f(x_3, t) - f(x_1, t), \ldots, f(x_N, t) - f(x_1, t)]^T$.

Definition 1. *The consensus error $\xi(t) \in \mathbb{R}^{n-1}$ is uniformly ultimately bounded (UUB) if there exists a bound B and a time $t_f(B, \xi(t_0))$, which are independent of $t_0 \geq 0$, such that $||\xi(t)|| \leq B$ for $\forall t \geq t_0 + t_f$.*

Remark 1. *From the structure of transformation matrix E, we know that ξ is the indicator of the consensus performance of multi-agent system Equation* (1)*. That is, the system Equation* (1) *achieves consensus if and only if $\xi(t) = 0$ of Equation* (4) *is asymptotically stable. In what follows, ξ is called the consensus error of the system. When $\xi(t)$ is UUB, $x_i(t)$ is bounded within a bounded neighborhood of $x_1(t)$ for $i = 2, 3, \ldots, n$ and $t \geq t_0 + t_f$. Thus, this depicts an intuitive notion of "close enough" consensus.*

Lemma 1. [22] *Let $L_1, L_2, \ldots, L_M$ be the Laplacian matrices associated with the digraphs $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_M$, respectively, then $-\sum_{m=1}^{M} \bar{L}_m$ is Hurwitz stable if and only if the union of digraph $\mathcal{G}_u$ of these graphs contains a spanning tree.*

Lemma 2. [36] *For any two real vectors $x, \ y \in \mathbb{R}^n$ and positive definite matrix $\Phi \in \mathbb{R}^{n \times n}$, we have*

$$2x^T y \leq x^T \Phi x + y^T \Phi^{-1} y.$$

3. Main Results

In this section, we first design a stabilizing switching law for multi-agent system Equation (1). Then, the main result of this paper will be presented with the help of the above preliminary knowledge.

3.1. Switching Law Design

Define average matrix

$$\bar{L}_0 = \frac{1}{M}\sum_{m=1}^{M}\bar{L}_m,$$

and $-\bar{L}_0$ is Hurwitz. As a result, the following Lyapunov equation

$$\bar{L}_0^T Q + Q\bar{L}_0 = I_{n-1} \tag{5}$$

has a positive definite solution Q.

Define auxiliary matrices $\bar{\mathfrak{L}}_m$ $(m \in \mathcal{M})$ as follows:

$$\bar{\mathfrak{L}}_m = \bar{L}_m^T Q + Q\bar{L}_m$$

For initial state $\xi(t_0) = \xi_0$, let

$$\sigma(t_0) = \text{argmax}\{\xi_0^T\bar{\mathfrak{L}}_1\xi_0, \ldots, \xi_0^T\bar{\mathfrak{L}}_M\xi_0\}, \tag{6}$$

where argmax stands for the index which reaches the maximum among $\mathcal{M}$. If there is more than one index, we choose the minimum index.

Then, we define the switching instant and index sequences recursively by

$$\begin{aligned} &t_{c+1} = \inf\{t > t_c : \xi^T(t)\bar{\mathfrak{L}}_{\sigma(t_c)}\xi(t) < r_{\sigma(t_c)}\xi^T(t)\xi(t), ||\xi(t)|| > \varpi\} \\ &\sigma(t_{c+1}) = \text{argmax}\{\xi(t_{c+1})^T\bar{\mathfrak{L}}_1\xi(t_{c+1}), \ldots, \xi(t_{c+1})^T\bar{\mathfrak{L}}_M\xi(t_{c+1})\},\ c = 1, 2, \ldots, \end{aligned} \tag{7}$$

where $r_m \in (0, 1)$.

Lemma 3. *The switching signal $\sigma(t)$ is well-defined, i.e., $t_{c+1} - t_c > 0$.*

Proof. Assume t_c and t_{c+1} are two consecutive switching time instants. By the property of the protocol that we design the switching instants, we have

(1) $\xi^T(t_c)\bar{\mathfrak{L}}_{\sigma(t_c)}\xi(t_c) = \max_{i\in\Lambda}\{\xi^T(t_c)\bar{\mathfrak{L}}_i\xi(t_c)\}$.
(2) $\xi^T(t_{c+1})\bar{\mathfrak{L}}_{\sigma(t_c)}\xi(t_{c+1}) \leq r_{\sigma(t_c)}\xi^T(t_{c+1})\xi(t_{c+1})$.
(3) $\|\xi(t_{c+1})\| \geq \varpi$.

As $1/M\sum_p^M \bar{\mathfrak{L}}_p = I_{n-1}$, item (1) also implies that

(4) $\xi^T(t_c)\bar{\mathfrak{L}}_{\sigma(t_c)}\xi(t_c) \geq \xi^T(t_c)\xi(t_c)$.

Firstly, let us consider the case

$$\|\xi(t)\| \leq \vartheta\|\xi(t_{c+1})\| \quad \vartheta > 1 \text{ and } \forall t \in [t_c, t_{c+1}]. \tag{8}$$

Here, we define an auxiliary function

$$w(t) = -\xi^T(t)\bar{\mathfrak{L}}_{\sigma(t_c)}\xi(t) + \xi^T(t)\xi(t). \tag{9}$$

It follows from (2) and (4) that

$$w(t_c) \leq 0 \text{ and } w(t_{c+1}) \geq (1 - r_{\sigma(t_c)})\xi^T(t_{c+1})\xi(t_{c+1}).$$

Calculating the derivative of $w(t)$ along time, we get

$$\begin{aligned} \frac{dw(t)}{dt} =& -k\xi^T[\bar{L}^T_{\sigma(t_c)}(-\bar{\mathfrak{L}}_{\sigma(t_c)} + I_n) + (-\bar{\mathfrak{L}}_{\sigma(t_c)} + I_n)\bar{L}_{\sigma(t_c)}]\xi \\ &+ 2\bar{f}^T(-\bar{\mathfrak{L}}_{\sigma(t_c)} + I_n)\xi. \end{aligned} \tag{10}$$

Now, we denote

$$\nu_1 = \|L^T_{\sigma(t_c)}(-\bar{\mathfrak{L}}_{\sigma(t_c)} + I_n) + (-\bar{\mathfrak{L}}_{\sigma(t_c)} + I_n)L_{\sigma(t_c)}\|,$$

and

$$\nu_2 = \sup_{t\in[t_c,t_{c+1}]} \|\bar{f}\| \text{ and } \nu_3 = \| - \bar{\mathfrak{L}}_{\sigma(t_c)} + I_n\|.$$

Combining with (3) yields

$$\|\frac{dw(t)}{dt}\| \leq (k\vartheta^2\nu_1 + \frac{2\vartheta^2\nu_2\nu_3}{\varpi})\|\xi(t_{c+1})\|^2,$$

which together with the fact that $(k\vartheta^2\nu_1 + \frac{2\vartheta^2\nu_2\nu_3}{\varpi})\|\xi(t_{c+1})\|^2(t_{c+1} - t_c) \geq \omega(t_{c+1}) - \omega(t_c) \geq (1 - r_{\sigma(t_c)})\|\xi(t_{c+1})\|^2$ implies that

$$(k\vartheta^2\nu_1 + \frac{\vartheta^2\nu_2\nu_3}{\varpi})(t_{c+1} - t_c) \geq 1 - r_{\sigma(t_c)},$$

thus

$$t_{c+1} - t_c \geq \frac{1 - r_{\sigma(t_c)}}{k\vartheta^2\nu_1 + \frac{2\vartheta^2\nu_2\nu_3}{\varpi}}.$$

Next, suppose that Equation (8) does not hold, which means that there is a $t^* \in [t_c, t_{c+1})$ satisfying

$$\|\xi(t^*)\| > \vartheta\|\xi(t_{c+1})\|. \tag{11}$$

From the system Equation (5), we have

$$\xi(t_{c+1}) = \xi(t^*)e^{L_{\sigma(t_c)}(t_{c+1}-t_c)} + \int_{t^*}^{t_{c+1}} e^{L_{\sigma(t_c)}(t_{c+1}-s)}\bar{f}(s)ds, \tag{12}$$

which is equivalent to

$$\xi(t^*) = \xi(t_{c+1})e^{L_{\sigma(t_c)}(t^*-t_{c+1})} - \int_{t^*}^{t_{c+1}} e^{L_{\sigma(t_c)}(t^*-s)}\bar{f}(s)ds. \tag{13}$$

Based on the property of exponential function and norm, there is a positive number ν_4 such that

$$\|e^{L_{\sigma(t_c)}t}\| \leq \vartheta - \frac{1}{2} \ \ \forall t \in [-\nu_4, 0].$$

Suppose that

$$t_{c+1} - t^* = \min(\nu_4, \frac{\varpi}{(2\vartheta - 1)\nu_2}).$$

As a result,

$$\|\xi(t^*)\| \leq (\vartheta - \frac{1}{2})\|\xi(t_{c+1})\| + (\vartheta - \frac{1}{2})(t_{c+1} - t^*)\nu_2 \leq \vartheta\|\xi(t_{c+1})\|,$$

which contradicts the inequality Equation (11). Hence, we get

$$\|t_{c+1} - t_c\| \geq t_{c+1} - t^* > \min(\nu_4, \frac{\varpi}{(2\vartheta - 1)\nu_2}).$$

Combining the above discussions shows that for any consecutive switching time instants t_c and t_{c+1}, we have

$$t_{c+1} - t_c \geq \min(\frac{1 - r_{\sigma(t_c)}}{k\vartheta^2\nu_1 + \frac{2\vartheta^2\nu_2\nu_3}{\varpi}}, \nu_4, \frac{\varpi}{(2\vartheta - 1)\nu_2}) > 0, \tag{14}$$

which imposes a lower bound for the dwell time of switching signal. This means the switching signals are well-defined. □

Remark 2. *According to [35], a "good" switching signal should guarantee a positive dwell time and avoid fast switching. In the switching law Equations (6) and (7), we fix a threshold value ϖ for switching, which can prevent the switching signal $\sigma(t)$ from chattering. However, there is a trade-off between the precise of consensus and frequency of switching due to such a threshold value. Specifically, a smaller ϖ may lead to a smaller dwell time, which implies high frequency switching, while a larger ϖ may bring about a larger consensus error which is undesirable. Similarly , there is also a trade-off between the control gain and frequency of switching due to parameters r_m in Equations (6) and (7). Specifically, smaller r_m may result in larger control gain, which can be seen in the upcoming Theorem 1, while larger r_m may bring about high frequency switching. The existence of these two trade-offs suggests that the choice of these parameters should achieve a balanced interplay between consensus performance and feasibility of control protocols.*

3.2. Consensus Analysis

Theorem 1. *Consider the multi-agent system Equation (1) under Assumptions 1 and 2. Adopt the designed switching law in Equations (6) and (7). Then, by employing control protocol Equation (2) and selecting control gain such that*

$$k > \frac{\bar{\lambda}(Q)\rho + \bar{\lambda}(Q)}{r} \tag{15}$$

where $\bar{\lambda}(Q)$ denotes the maximum eigenvalue of Q and $r = \min\{r_1, r_2, \ldots, r_M\}$, the consensus error $\xi(t)$ is UUB. That is, all agents reach consensus with a bounded error ϖ.

Proof. As the switching topologies jointly contain a spanning tree, $-\bar{L}$ is Hurwitz stable by Lemma 1, and the switching law is well-defined in Equations (6) and (7). Here, we consider the Lyapunov function candidate as $V(t) = \xi^T Q\xi$. In case of $||\xi|| > \varpi$, by calculating the derivative of $V(t)$ along the trajectory of system Equation (4), we have

$$\begin{aligned} \frac{dV(t)}{dt} &= \dot{\xi}^T Q\xi + \xi^T Q\dot{\xi} \\ &= -k\xi^T(\bar{L}^T_{\sigma(t)}Q + Q\bar{L}_{\sigma(t)})\xi + 2\xi^T Q\bar{f} \\ &\leq -kr_{\sigma(t)}\xi^T\xi + \xi^T Q\xi + \bar{f}Q\bar{f} \\ &\leq -kr\xi^T\xi + \xi^T Q\xi + \bar{\lambda}(Q)\rho\xi^T\xi \\ &\leq -(kr - \bar{\lambda}(Q)\rho - \bar{\lambda}(Q))\xi^T\xi \\ &< 0. \end{aligned} \tag{16}$$

Therefore, $V(t)$ is strictly decreasing during each time interval. This, together with the fact that $V(t)$ is continuous, implies the consensus error satisfies $\lim_{t\to\infty} ||\xi(t)|| \leq \varpi$, which means the consensus error $||\xi(t)||$ is UUB. This completes the proof. □

Remark 3. *In* [21]*, first-order nonlinear multi-agent system was investigated, where general algebraic connectivity needs to be calculated to design the control parameter. However, the general algebraic connectivity of a graph is not easy to obtain, especially when the network size is large. Here, we provide a novel method to design the control parameter to realize consensus. In addition, we allow the underlying topology to be disconnected all the time, which cannot be analyzed by the technique in* [21]*.*

Remark 4. *In* [18,22,23]*, consensus problems of multi-agent systems with nonlinear dynamics under switching topologies were considered. However, a common assumption of these works on the switching topologies is all the topologies are required to be connected or having a spanning tree. In* [17]*, this assumption is relaxed where consensus of multi-agent systems was achieved without requiring the topology having a spanning tree all the time. However, these results are quite conservative in the sense that the underlying topology of the system switches without concerning the current states of the multi-agent systems. In this paper, another perspective to solve the consensus problem without requiring each possible topology containing a spanning tree is provided. The designed topology switching law arranges the underlying topology by taking states of agents into consideration, which is efficient.*

Remark 5. *When the consensus of the systems is achieved, the state of agents in the system is determined by the nonlinear function. Therefore, denoting the consensus state of the system by* $s(t)$ *(i.e., the trajectory of* $\dot{s}(t) = f(s(t), t)$*), which can be any desired state: an equilibrium point, a nontrivial periodic orbit, or even a chaotic orbit in some applications.*

4. Numerical Simulations

In this section, we present numerical simulations to demonstrate the effectiveness of theoretical results. For simplicity, we only consider the multi-agent systems consisting of ten agents labeled 1 through 10 and assume all weights of edges between agents are 0 or 1.

Consider the consensus of multi-agent system Equation (1) with the communication topology switching in a collection $\bar{G} = \{G_1, G_2, G_3\}$, as shown in Figure 1. Note that each digraph in Figure 1 does not contain a spanning tree, but the union digraph of them contains a spanning tree. The inherent nonlinear dynamics is given as $f(x_i, t) = 0.1x_i \cos(t)$. By Theorem 1, when the feedback gain $k > 20.5456$, the consensus of the system is achieved uniformly ultimately bounded under the designed state-controlled switching topologies. Figure 2 shows the states of the closed-loop system with $k = 21$ and $\varpi = 0.1$. We can see that the ten agents achieve convergence with bounded error although none of digraph G_m contains a spanning tree. Figures 3 and 4 present the switching signal and the consensus error $||\xi(t)||$, respectively.

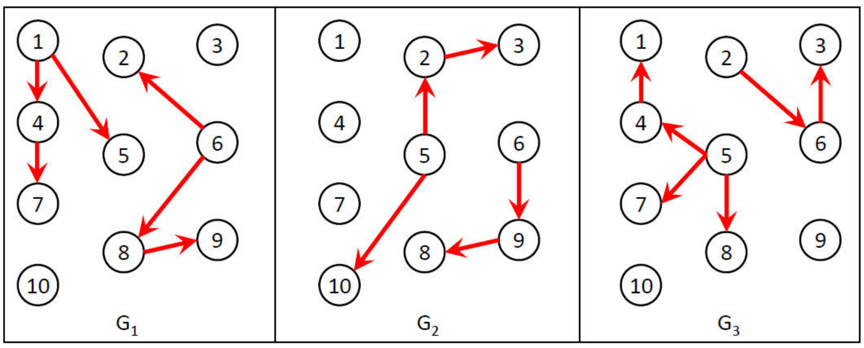

Figure 1. Possible interaction topologies between agents.

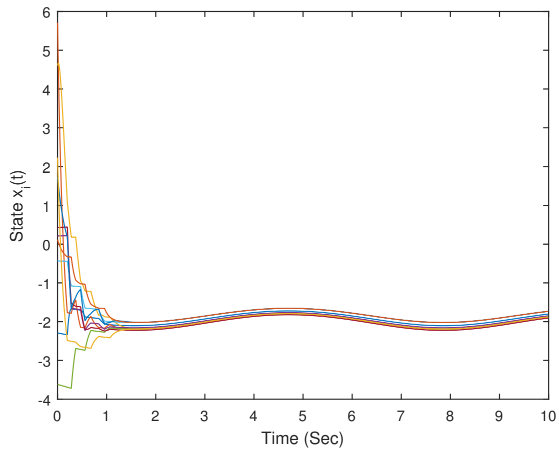

Figure 2. State $x_i(t)$ under the state-controlled switching topologies with a threshold $\varpi = 0.1$.

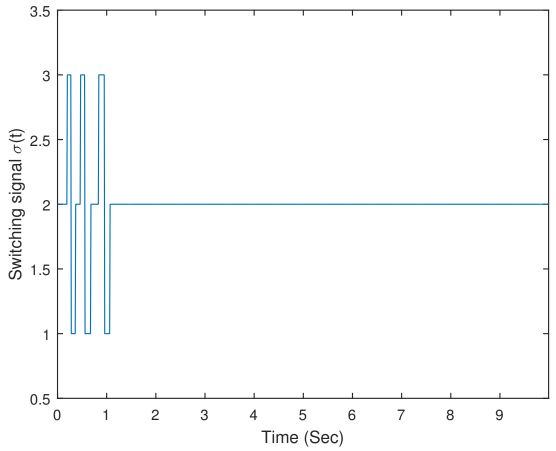

Figure 3. The state-controlled switching signal $\sigma(t)$ with a threshold $\varpi = 0.1$.

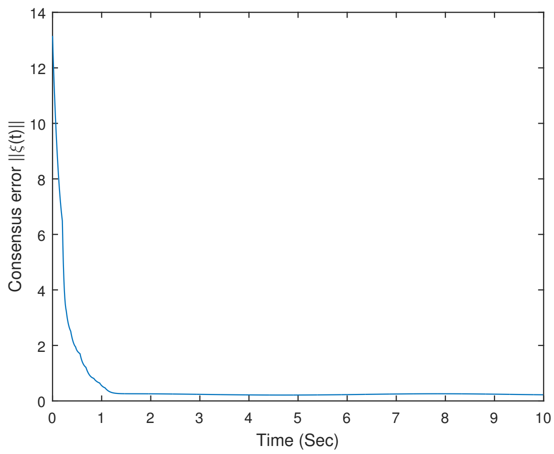

Figure 4. Consensus error $||\xi(t)||$ under the state-controlled switching topologies with a threshold $\varpi = 0.1$.

To demonstrate the merit of fixing a threshold for switching, we also consider the case that $\varpi = 0$ under the same setting as stated above. The states of agents, switching signal and consensus error are shown in Figures 5–7, respectively. From this comparison simulation, we can find that the multi-agent system reaches consensus precisely, while the interaction topology switches much more times than the case with $\varpi = 0.1$.

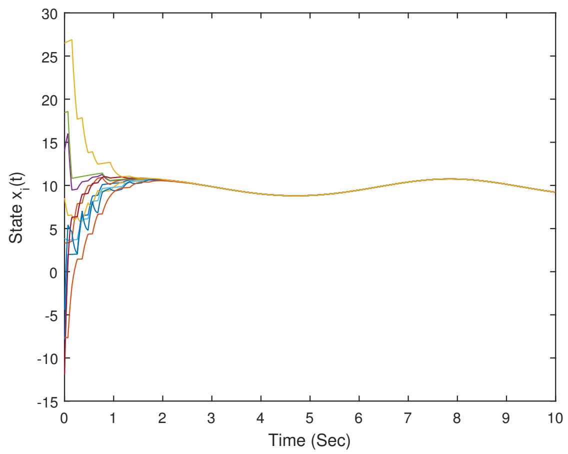

Figure 5. State $x_i(t)$ under the state-controlled switching topologies without a threshold.

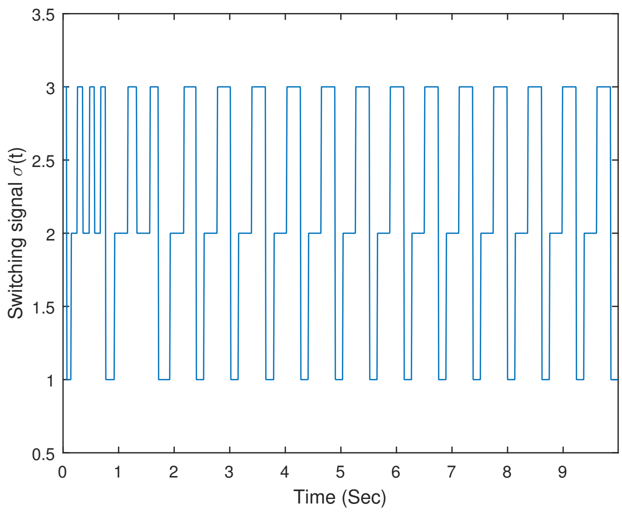

Figure 6. The state-controlled switching signal $\sigma(t)$ without a threshold.

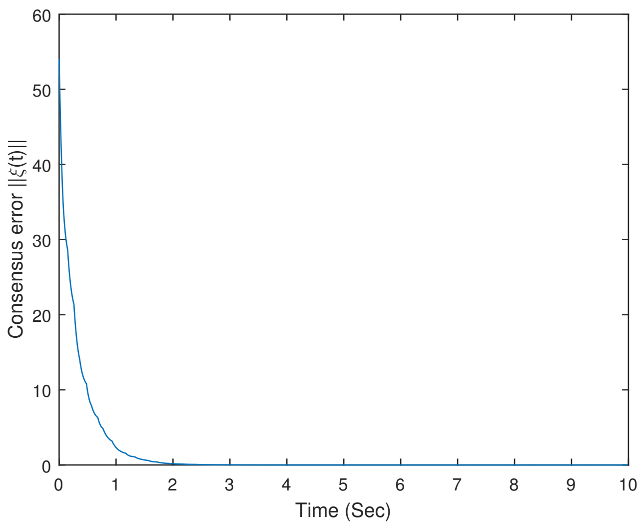

Figure 7. Consensus error $||\xi(t)||$ under the state-controlled switching topologies without a threshold.

5. Conclusions

In this paper, we have investigated the leaderless consensus problem with two practical constraints: (i) The system includes intrinsic nonlinear dynamics; (ii) The switching topology may not contain a spanning tree at any time. We introduced a variable transformation to facilitate the consensus analysis, which shows great potential in solving the considered consensus problem. By designing a state-controlled switching law, the consensus problem has been solved under the assumption that the switching topologies jointly contain a spanning tree. The choice of parameters in the switching law allows us to balance the consensus performance with the feasibility of control protocols. Nevertheless, the nonlinearities in this work are assumed to be Lipschitz-type, which bring about some conservations. In addition, another drawback of this work is that some global information is used in the designed switching law. We believe that the results of this paper could be largely improved if general nonlinearities are considered and the topology switching law depends only on local information, which is still an open issue and will be the object of our future work.

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Author Contributions: Kairui Chen, Junwei Wang and Yun Zhang designed research; Kairui Chen conducted the simulation and analyzed the data; Kairui Chen and Junwei Wang performed research and wrote the paper. All authors have read and approved the final manuscript.

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