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Article

New Exact Solutions of the New Hamiltonian Amplitude Equation and Fokas-Lenells Equation

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Abstract: In this paper, exact solutions of the new Hamiltonian amplitude equation and Fokas-Lenells equation are successfully obtained. The extended trial equation method (ETEM) and generalized Kudryashov method (GKM) are applied to find several exact solutions of the new Hamiltonian amplitude equation and Fokas-Lenells equation. Primarily, we seek some exact solutions of the new Hamiltonian amplitude equation and Fokas-Lenells equation by using ETEM. Then, we research dark soliton solutions of the new Hamiltonian amplitude equation and Fokas-Lenells equation by using GKM. Lastly, according to the values of some parameters, we draw two and three dimensional graphics of imaginary and real values of certain solutions found by utilizing both methods.

Keywords: new Hamiltonian amplitude equation; Fokas-Lenells equation; extended trial equation method; generalized Kudryashov method; rational function solution; travelling wave solution; hyperbolic function solution; dark soliton solution

1. Introduction

The most important success of exact solutions of nonlinear evolution equations (NLEEs) lies in the fact that they have provided an explanation of some physical phenomena. The variety of solutions of NLEEs has an important function in many sciences such as biology, optical fibers, hydrodynamics, meteorology, elastic media, plasma physics, applied mathematics, computer engineering, chemical kinematics and electromagnetic theory.

Clausius first introduced the concept of entropy in 1865 in order to describe physical phenomena from dynamic information theory to the second law of thermodynamics [1]. Afterwards, some researchers have investigated the vital properties of entropy fields. Carrillo studied entropy solutions for some nonlinear physical problems [2]. Mascia *et al.* have researched uniqueness of entropy solutions for some nonlinear differential equations with nonhomogeneous Dirichlet conditions [3]. Karlsen and Risebro have observed the stability of entropy solutions for nonlinear parabolic equations with rough coefficients [4]. Then, Watanabe has proved that nonlinear parabolic equations with discontinuous coefficients have existence and uniqueness of entropy solutions [5].

In recent years, many mathematicians have proposed a lot of methods to seek exact solutions of NLEEs such as G'/G-expansion method [6,7], exp-function method [8], the tanh method [9], generalized Kudryashov method [10–12], modified Kudryashov method [13] and so on. In this study, GKM [10–12] and ETEM [14–21] will be investigated to find exact solutions of the new Hamiltonian amplitude equation and Fokas-Lenells equation.

The new Hamiltonian amplitude equation was described by Wadati *et al.* in 1992. The authors reported that this equation is apparently not integrable because it does not provide the Painleve property but it is a Hamiltonian analogue of the Kuramoto-Sivashinsky equation which arises in dissipative systems [22]. This equation governs certain instabilities of modulated wave trains [23].

Many authors have tackled numerous methods to find exact solutions of the new Hamiltonian amplitude equation such as the general projective Riccati equation method [24], the sinh-Gordon expansion method [25], the extended F-expansion method [26], the first integral method [27], the G'/G-expansion method [28], the functional variable method [29], Lie symmetry method [30], the simplest equation method [31], He's semi-inverse variational principle method and Ansatz method [32], modified simplest equation method [33].

Firstly, we consider the following new Hamiltonian amplitude equation [22–33]:

$$iu_x + u_{tt} + 2\sigma|u|^2 u - \varepsilon u_{xt} = 0,\ \sigma = \pm 1,\ \varepsilon \ll 1, \tag{1}$$

where $u = u(x,t)$ is the complex function. The subscripts denote partial derivatives. The notation $-\varepsilon u_{xt}$ overcomes the ill-posedness of the unstable nonlinear Schrödinger equation [23].

The Fokas-Lenells equation arises as a pattern which defines nonlinear pulse propagation in optical fibers. This equation is related to the nonlinear Schrödinger (NLS) equation in the same way that the Camassa-Holm equation is associated with the KdV equation [34].The Fokas-Lenells equation is a completely integrable equation which has been derived as an integrable generalization of the NLS equation using bi-Hamiltonian methods [35]. In optics, the FL equation models the propagation of nonlinear light pulses in monomode optical fibers when certain higher order nonlinear effects are taken into consideration [36]. The complete integrability of the FL equation has been exhibited by using the inverse scattering transform (IST) method [37]. Especially, a Lax pair and a few conservation laws related to it have been found clearly using the bi-Hamiltonian structure and the multisoliton solutions have been obtained by using the dressing method [38]. Another important property of the FL equation is that it is the first negative flow of the integrable hierarchy of the derivative NLS equation [39]. The two different statements of the bright *N*-soliton solution of the FL equation have been found by using a direct method [40]. The dark *N*-soliton solution of the FL equation have been obtained by using a direct method [41]. The lattice representation and the dark solitons of the FL equation have been considered in [42], where a relationship is also established between the FL equation and other integrable models. Taylor series expansion for the *n*-order breather solutions obtained by Darboux transformation have been constituted the *n*-order rogue waves of the FL equation [43]. The leading order asymptotic of the solution to the Cauchy problem of the Fokas-Lenells equation have been obtained by using Deift-Zhou method [44]. Besides, the authors have constructed physically relevant classes of solutions for FL hierarchy by studying the reality conditions [45].

Secondly, we investigate the following Fokas-Lenells equation [44,46]:

$$u_{tx}+\alpha\beta^2 u-2i\alpha\beta u_x-\alpha u_{xx}+\sigma i\alpha\beta^2|u|^2u_x=0, \tag{2}$$

where $u=u(x,t)$ demonstrates the complex field. The subscripts indicate partial derivatives [34].

Our purpose in this paper is to submit exact solutions of the new Hamiltonian amplitude equation and Fokas-Lenells equation. In Section 2, we consider exact solutions of the new Hamiltonian amplitude equation by using ETEM and GKM. In Section 3, we investigate exact solutions of Fokas-Lenells equation by using ETEM and GKM.

2. The Investigation of the New Hamiltonian Amplitude Equation

In this section, we consider the exact solutions of the new Hamiltonian amplitude equation [22–33] by using ETEM and GKM.

In order to obtain travelling wave solutions of Equation (1), we take the transformation by using the wave variables:

$$u(x,t)=e^{i\theta}u(\eta),\ \theta=\alpha x-\beta t,\ \eta=k(x-\lambda t), \tag{3}$$

where α,β,k,λ are arbitrary constants.

Substituting the following (4–6) derivatives into Equation (1):

$$iu_x=ie^{i\theta}ku'-e^{i\theta}\alpha u, \tag{4}$$

$$u_{tt}=-\beta^2e^{i\theta}u+2i\beta k\lambda e^{i\theta}u'+k^2\lambda^2e^{i\theta}u'', \tag{5}$$

$$u_{xt}=\alpha\beta e^{i\theta}u-ik\lambda\alpha e^{i\theta}u'-ik\beta e^{i\theta}u'-k^2\lambda e^{i\theta}u'', \tag{6}$$

we get following system:

$$\begin{gathered} i\left[\left(1+2\beta\lambda+\lambda\alpha\varepsilon+\beta\varepsilon\right)u'(\eta)\right]=0,\\ k^2\left(\lambda^2+\lambda\varepsilon\right)u''(\eta)-\left(\alpha+\beta^2+\varepsilon\alpha\beta\right)u(\eta)+2\sigma u^3(\eta)=0, \end{gathered} \tag{7}$$

where the prime demonstrates the derivative with respect to η.

2.1. ETEM for the New Hamiltonian Amplitude Equation

In this section, we will use ETEM to find exact solutions of the new Hamiltonian amplitude equation. Take transformation and trial equation as follows:

$$u=\sum_{i=0}^{\delta}\tau_i\Gamma^i, \tag{8}$$

where:

$$(\Gamma')^2 = \Lambda(\Gamma) = \frac{\phi(\Gamma)}{\psi(\Gamma)} = \frac{\xi_\theta \Gamma^\theta + \ldots + \xi_1 \Gamma + \xi_0}{\zeta_\in \Gamma^\in + \ldots + \zeta_1 \Gamma + \zeta_0}. \tag{9}$$

Taking into consideration Equations (8) and (9), we can get:

$$(u')^2 = \frac{\phi(\Gamma)}{\psi(\Gamma)} \left(\sum_{i=0}^{\delta} i \tau_i \Gamma^{i-1} \right)^2, \tag{10}$$

$$u'' = \frac{\phi'(\Gamma)\psi(\Gamma) - \phi(\Gamma)\psi'(\Gamma)}{2\psi^2(\Gamma)} \left(\sum_{i=0}^{\delta} i \tau_i \Gamma^{i-1} \right) + \frac{\phi(\Gamma)}{\psi(\Gamma)} \left(\sum_{i=0}^{\delta} i(i-1) \tau_i \Gamma^{i-2} \right) \tag{11}$$

where $\phi(\Gamma)$ and $\psi(\Gamma)$ are polynomials. According to the balance principle, we can identify a formula of $\theta, \in$ and δ. We can get some values of $\theta, \in$ and δ.

Simplify Equation (9) to elementary integral form:

$$\pm(\eta - \eta_0) = \int \frac{d\Gamma}{\sqrt{\Lambda(\Gamma)}} = \int \sqrt{\frac{\psi(\Gamma)}{\phi(\Gamma)}} d\Gamma. \tag{12}$$

Applying a complete discrimination system for polynomial to classify the roots of $\phi(\Gamma)$, we solve the infinite integral (12) and categorize the exact solutions to Equation (1) by Wolfram Mathematica 9. Substituting Equations (8) and (11) into Equation (7), and using the balance principle, we get:

$$\theta = 2\delta + \in + 2. \tag{13}$$

In order to find exact solutions of Equation (1), if we choose $\in = 0, \delta = 1$ and $\theta = 4$ in Equation (13), then:

$$(v')^2 = \frac{\tau_1^2 \left(\xi_0 + \Gamma \xi_1 + \Gamma^2 \xi_2 + \Gamma^3 \xi_3 + \Gamma^4 \xi_4 \right)}{\zeta_0}, \; v'' = \frac{\tau_1 \left(\xi_1 + 2\Gamma \xi_2 + 3\Gamma^2 \xi_3 + 4\Gamma^3 \xi_4 \right)}{2\zeta_0}, \tag{14}$$

where $\xi_4 \neq 0, \zeta_0 \neq 0.$ Solving the algebraic equation system (7) supplies:

$$\begin{gathered} \xi_0 = \xi_0,\; \xi_2 = \xi_2,\; \xi_3 = \xi_3,\; \xi_4 = \xi_4, \xi_1 = -\frac{\xi_3^3 - 4\xi_2\xi_3\xi_4}{8\xi_4^2},\; \zeta_0 = \frac{-k^2 \lambda(\varepsilon + \lambda)\xi_4}{\sigma \tau_1^2}, \\ \tau_0 = \frac{\xi_3 \tau_1}{4\xi_4},\; \tau_1 = \tau_1, \beta = \frac{-2\alpha\varepsilon\xi_4^2 + \sqrt{4\alpha\left(-4 + \alpha\varepsilon^2\right)\xi_4^4 + 2\sigma\xi_4^2\left(3\xi_3^2 - 8\xi_2\xi_4\right)\tau_1^2}}{4\xi_4^2}. \end{gathered} \tag{15}$$

Substituting these results into Equations (9) and (12), we get:

$$\pm(\eta - \eta_0) = A \int \frac{d\Gamma}{\sqrt{\frac{\xi_0}{\xi_4} + \frac{\xi_1}{\xi_4}\Gamma + \frac{\xi_2}{\xi_4}\Gamma^2 + \frac{\xi_3}{\xi_4}\Gamma^3 + \Gamma^4}} \tag{16}$$

where $A = \sqrt{\frac{-k^2 \lambda(\varepsilon + \lambda)\xi_4}{\sigma \tau_1^2}}.$

If we integrate Equation (16), we obtain the solutions of Equation (1) as follows:

$$\pm(\eta-\eta_0)=-\frac{A}{\Gamma-\alpha_1}, \tag{17}$$

$$\pm(\eta-\eta_0)=\frac{2A}{\alpha_1-\alpha_2}\sqrt{\frac{\Gamma-\alpha_2}{\Gamma-\alpha_1}},\quad \alpha_2>\alpha_1, \tag{18}$$

$$\pm(\eta-\eta_0)=\frac{A}{\alpha_1-\alpha_2}\ln\left|\frac{\Gamma-\alpha_1}{\Gamma-\alpha_2}\right|,\quad \alpha_1>\alpha_2, \tag{19}$$

$$\pm(\eta-\eta_0)=\frac{2A}{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}\ln\left|\frac{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}-\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}+\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}\right|,\quad \alpha_1>\alpha_2>\alpha_3, \tag{20}$$

$$\pm(\eta-\eta_0)=\frac{2A}{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}F(\varphi,l),\quad \alpha_1>\alpha_2>\alpha_3>\alpha_4, \tag{21}$$

where:

$$F(\varphi,l)=\int_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}},\ \varphi=\arcsin\sqrt{\frac{(\Gamma-\alpha_1)(\alpha_2-\alpha_4)}{(\Gamma-\alpha_2)(\alpha_1-\alpha_4)}},\ l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}. \tag{22}$$

Also, $\alpha_1,\alpha_2,\alpha_3$ and α_4 are the roots of the polynomial equation:

$$\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4}=0. \tag{23}$$

Substituting the solutions (17)–(21) into (8) and using Equation (3), the solutions of Equation (1) are obtained as rational function solutions:

$$u_1(x,t)=e^{i(\alpha x-\beta t)}\left(\pm\frac{A_1}{k(x-\lambda t)}\right) \tag{24}$$

$$u_2(x,t)=e^{i(\alpha x-\beta t)}\left(\frac{4A^2(\alpha_2-\alpha_1)\tau_1}{4A^2-\left[(\alpha_1-\alpha_2)(k(x-\lambda t))\right]^2}\right), \tag{25}$$

travelling wave solution:

$$u_3(x,t)=e^{i(\alpha x-\beta t)}\left(\frac{(\alpha_2-\alpha_1)\tau_1}{2}\left\{1\pm\coth\left[\frac{(\alpha_1-\alpha_2)}{A}(k(x-\lambda t))\right]\right\}\right), \tag{26}$$

soliton solution:

$$u_4(x,t)=e^{i(\alpha x-\beta t)}\frac{A_2}{\left(D+\cosh\left[B(k(x-\lambda t))\right]\right)}, \tag{27}$$

and Jacobi elliptic function solution:

$$u_5(x,t) = e^{i(\alpha x - \beta t)} \frac{A_3}{\left(M + N\, sn^2(\varphi, l)\right)}, \tag{28}$$

where $A_1 = \tau_1 A,\ A_2 = \left(\frac{2\tau_1(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{\alpha_3-\alpha_2}\right), B = \frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A}, D = \frac{2\alpha_1-\alpha_2-\alpha_3}{\alpha_3-\alpha_2},\quad A_3 = \left(2\tau_1(\alpha_1-\alpha_3)(\alpha_4-\alpha_2)\right),$ $M = \alpha_4 - \alpha_2,\ N = \alpha_1 - \alpha_4,\ l^2 = \frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)},\ \varphi = \frac{\pm\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}\left(k(x-\lambda t)\right).$ Here, A_2 is the amplitude of the soliton, and B is the inverse width of the solitons. Therefore, it can be said that the solitons exist for $\tau_1 < 0$.

Remark 1. *If the modulus* $l \to 1$, *then by using Equation (3), the solution (28) can be converted to the hyperbolic function solution:*

$$u_6(x,t) = e^{i(\alpha x - \beta t)} \frac{A_3}{\left(M + N \tanh^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}\left(k(x-\lambda t)\right)\right]\right)}, \tag{29}$$

where $\alpha_3 = \alpha_4$.

Remark 2. *If the modulus* $l \to 0$, *by using Equation (3), the solution (28) can be turned to the periodic wave solution:*

$$u_7(x,t) = e^{i(\alpha x - \beta t)} \frac{A_3}{\left(M + N \sin^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}\left(k(x-\lambda t)\right)\right]\right)}, \tag{30}$$

where $\alpha_2 = \alpha_3$.

2.2. Generalized Kudryashov Method for the New Hamiltonian Amplitude Equation

In this section, we will use the generalized Kudryashov method to find exact solutions of the new Hamiltonian amplitude equation. Recently, some scientists have introduced the Kudryashov method [47–49]. But, in this study, we constitute generalized form of Kudryashov method.

Assume that the exact solutions of Equation (1) can be given as the following form:

$$u(\eta) = \frac{\sum_{i=0}^{N} a_i Q^i(\eta)}{\sum_{j=0}^{M} b_j Q^j(\eta)} = \frac{A[Q(\eta)]}{B[Q(\eta)]} \tag{31}$$

where Q is $\frac{1}{1 \pm e^{\eta}}$. We note that the function Q is solution of the following equation [47]:

$$Q_\eta = Q^2 - Q. \tag{32}$$

Taking into consideration Equation (31), we find:

$$u'(\eta)=\frac{A'Q'B-AB'Q'}{B^2}=Q'\left[\frac{A'B-AB'}{B^2}\right]=\left(Q^2-Q\right)\left[\frac{A'B-AB'}{B^2}\right], \tag{33}$$

$$u''(\eta)=\frac{Q^2-Q}{B^2}\left[(2Q-1)(A'B-AB')+\frac{Q^2-Q}{B}\left[B(A''B-AB'')-2B'A'B+2A(B')^2\right]\right]. \tag{34}$$

To compute the values M and N in Equation (31) that is the pole order for the general solution of Equation (1), we enhance conformably as in the classical Kudryashov method on balancing the highest order nonlinear terms in Equation (1) and we can define a formula of M and N. We can receive some values of M and N.

After Equation (1) has been turned into Equation (7) as in Section 2, substituting Equations (31) and (34) into Equation (7) and balancing the highest order nonlinear terms of u'' and u^3 in Equation (7), then the following relation is obtained:

$$N-M+2=3N-3M \Rightarrow N=M+1. \tag{35}$$

When we choose $M=1$ and $N=2$, then:

$$u(\eta)=\frac{a_0+a_1Q+a_2Q^2}{b_0+b_1Q}, \tag{36}$$

$$u'(\eta)=\left(Q^2-Q\right)\left[\frac{(a_1+2a_2Q)(b_0+b_1Q)-b_1\left(a_0+a_1Q+a_2Q^2\right)}{(b_0+b_1Q)^2}\right], \tag{37}$$

$$\begin{aligned}u''(\eta)=&\frac{Q^2-Q}{(b_0+b_1Q)^2}(2Q-1)\left[(a_1+2a_2Q)(b_0+b_1Q)-b_1\left(a_0+a_1Q+a_2Q^2\right)\right]\\&+\frac{\left(Q^2-Q\right)^2}{(b_0+b_1Q)^3}\left[2a_2(b_0+b_1Q)^2-2b_1(a_1+2a_2Q)(b_0+b_1Q)+2b_1^2\left(a_0+a_1Q+a_2Q^2\right)\right].\end{aligned} \tag{38}$$

The exact solutions of Equation (1) is found as follows:

Case 1

$$\begin{gathered}a_0=\frac{ik\sqrt{\lambda(\varepsilon+\lambda)}\,b_0}{2\sqrt{\sigma}},\ a_1=-\frac{ik\sqrt{\lambda(\varepsilon+\lambda)}\,(2b_0-b_1)}{2\sqrt{\sigma}},a_2=-\frac{ik\sqrt{\lambda(\varepsilon+\lambda)}\,b_1}{\sqrt{\sigma}},\\ \beta=\frac{1}{2}\left[-\alpha\varepsilon-\sqrt{-4\alpha+\alpha^2\varepsilon^2-2k^2\lambda(\varepsilon+\lambda)}\right].\end{gathered} \tag{39}$$

If we substitute Equation (39) into Equation (36), we obtain the following dark soliton solution of Equation (1):

$$u_1(x,t)=e^{i(\alpha x-\beta t)}\left[P\tanh\left(k_1(x-\lambda t)\right)\right], \tag{40}$$

where $P=\dfrac{ik\sqrt{\lambda(\varepsilon+\lambda)}}{2\sqrt{\sigma}}$, and $k_1=\dfrac{k}{2}$.

Case 2

$$a_0 = -\frac{ik\sqrt{\lambda(\varepsilon+\lambda)}\,b_0}{2\sqrt{\sigma}},\ a_1 = 0,\ a_2 = 0,\ b_1 = -2b_0,$$
$$\beta = \frac{1}{2}\left[-\alpha\varepsilon + \sqrt{-4\alpha + \alpha^2\varepsilon^2 - 2k^2\lambda(\varepsilon+\lambda)}\right]. \tag{41}$$

If we substitute Equation (41) into Equation (36), we get the following dark soliton solution of Equation (1):

$$u_2(x,t) = e^{i(\alpha x-\beta t)}\left[-P\coth\left(k_1(x-\lambda t)\right)\right]. \tag{42}$$

Case 3

$$a_0 = \frac{ik\sqrt{\lambda(\varepsilon+\lambda)}\,b_0}{\sqrt{\sigma}},\ a_1 = -\frac{2ik\sqrt{\lambda(\varepsilon+\lambda)}\,b_0}{\sqrt{\sigma}}, a_2 = \frac{2ik\sqrt{\lambda(\varepsilon+\lambda)}\,b_0}{\sqrt{\sigma}},$$
$$b_1 = -2b_0, \beta = \frac{1}{2}\left[-\alpha\varepsilon - \sqrt{-4\alpha + \alpha^2\varepsilon^2 - 8k^2\lambda(\varepsilon+\lambda)}\right]. \tag{43}$$

If we substitute Equation (43) into Equation (36), we find the following dark soliton solution of Equation (1):

$$u_3(x,t) = e^{i(\alpha x-\beta t)}\left[P\left(\coth\left(k_1(x-\lambda t)\right) + \tanh\left(k_1(x-\lambda t)\right)\right)\right]. \tag{44}$$

Case 4

$$a_0 = 0,\ a_1 = \frac{2ik\sqrt{\lambda(\varepsilon+\lambda)}\,b_0}{\sqrt{\sigma}}, a_2 = -\frac{2ik\sqrt{\lambda(\varepsilon+\lambda)}\,b_0}{\sqrt{\sigma}},$$
$$b_1 = -2b_0, \beta = \frac{1}{2}\left[-\alpha\varepsilon - \sqrt{-4\alpha + \alpha^2\varepsilon^2 + 4k^2\lambda(\varepsilon+\lambda)}\right]. \tag{45}$$

If we substitute Equation (45) into Equation (36), we obtain the following dark solution of Equation (1):

$$u_4(x,t) = e^{i(\alpha x-\beta t)}\left[P\left(\coth\left(k_1(x-\lambda t)\right) - \tanh\left(k_1(x-\lambda t)\right)\right)\right]. \tag{46}$$

Remark 3. *The exact solutions of Equation (1) have been found by using ETEM and GKM and have been calculated by the help of Wolfram Mathematica 9. If we compare with the exact solutions of Equation (1) reported by the other authors, we have obtained the similar solution with the solution Equation (58) in [24], the solution Equation (22) in [27], the solution Equation (23) in [31], the solution Equation (53) in [32] and the solution Equation (53) in [33] in this study as the solution Equation (40). Besides, we have found the similar solution with the solution Equation (59) in [24] and the solution Equation (54) in [33] in this study as the solution Equation (42). To our knowledge, other solutions of Equation (1) that we reported here, are new and are not trackable in the previous literature.*

3. The Investigation of Fokas-Lenells Equation

Fokas-Lenells equation which is subfield of the nonlinear Schrödinger equation [34] has a lot of application fields such as quantum mechanics, quantum field theory, complex system theory, telecommunication modals, computational systems, electrical and mechanical structures in Entropy concepts. In this section, we have obtained the exact solutions of nonlinear complex Fokas-Lenells equation [44,46] by using ETEM and GKM.

In order to find travelling wave solutions of the Equation (2), we take the transformation by using the wave variables:

$$u(x,t)=e^{i\mu}u(\rho),\ \ \mu=kx+ct,\ \rho=mx+nt, \tag{47}$$

where k,c,m,n are arbitrary constants.

Substituting the following (48)–(50) derivatives into Equation (2):

$$u_x=ie^{i\mu}ku+e^{i\mu}mu', \tag{48}$$

$$u_{tx}=-cke^{i\mu}u+icme^{i\mu}u'+ikne^{i\mu}u'+nme^{i\mu}u'', \tag{49}$$

$$u_{xx}=-k^2e^{i\mu}u+2ikme^{i\mu}u'+m^2e^{i\mu}u'', \tag{50}$$

we get following system:

$$\begin{gathered}\left(nm-\alpha m^2\right)u''(\rho)+\left(-ck+\alpha\beta^2+2\alpha\beta k+\alpha k^2\right)u(\rho)-\sigma\alpha\beta^2ku^3(\rho)=0,\\ i\left[\left(cm+kn-2\alpha\beta m-2\alpha km\right)u'(\rho)+\left(\sigma\alpha\beta^2m\right)u^2(\rho)u'(\rho)\right]=0,\end{gathered} \tag{51}$$

where the prime demonstrates the derivative with respect to ρ.

3.1. ETEM for Fokas-Lenells Equation

In this section, we will apply ETEM to obtain exact solutions of Fokas-Lenells equation. Substituting Equations (8) and (11) into Equation (51), and using the balance principle, we find:

$$\theta=2\delta+\in+2. \tag{52}$$

In order to obtain exact solutions of Equation (2), when we choose $\in=0,\delta=1$ and $\theta=4$ in Equation (52), then:

$$\left(v'\right)^2=\frac{\tau_1^2\left(\xi_0+\Gamma\xi_1+\Gamma^2\xi_2+\Gamma^3\xi_3+\Gamma^4\xi_4\right)}{\zeta_0},\ v''=\frac{\tau_1\left(\xi_1+2\Gamma\xi_2+3\Gamma^2\xi_3+4\Gamma^3\xi_4\right)}{2\zeta_0}, \tag{53}$$

where $\xi_4\neq0,\zeta_0\neq0.$ Respectively, solving Equation (51) yields:

$$\begin{gathered}\xi_0=\xi_0,\ \xi_2=\xi_2,\ \xi_3=\xi_3,\ \xi_4=\xi_4,\xi_1=-\frac{\xi_3^3-4\xi_2\xi_3\xi_4}{8\xi_4^2},\\ \zeta_0=\frac{m}{2k}\left(1+\frac{4(m\alpha-n)\xi_4}{\alpha\beta^2\sigma\tau_1^2}\right),\tau_0=\frac{\xi_3\tau_1}{4\xi_4},\ \tau_1=\tau_1,\\ c=-\frac{X+\alpha\beta^2\sigma\left[3km(n-m\alpha)\xi_3^2+4Y\xi_4\right]\tau_1^2}{8km\xi_4\left[2(n-m\alpha)\xi_4+\alpha\beta^2\sigma\tau_1^2\right]},\end{gathered} \tag{54}$$

where $X = 16m\alpha(-n+m\alpha)(k+\beta)^2\xi_4^2, Y = k^2(3n-5m\alpha)-4km\alpha\beta+m\alpha\beta^2+2km(-n+m\alpha)\xi_2$.

Substituting these results into Equations (9) and (12), we get:

$$\pm(\eta-\eta_0) = A\int\frac{d\Gamma}{\sqrt{\frac{\xi_0}{\xi_4}+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_3}{\xi_4}\Gamma^3+\Gamma^4}} \tag{55}$$

where $A = \sqrt{\frac{m}{2k}\left(1+\frac{4(m\alpha-n)\xi_4}{\alpha\beta^2\sigma\tau_1^2}\right)}$.

If we integrate Equation (55), we find the solutions of Equation (2) as following:

$$\pm(\eta-\eta_0) = -\frac{A}{\Gamma-\alpha_1}, \tag{56}$$

$$\pm(\eta-\eta_0) = \frac{2A}{\alpha_1-\alpha_2}\sqrt{\frac{\Gamma-\alpha_2}{\Gamma-\alpha_1}}, \quad \alpha_2 > \alpha_1, \tag{57}$$

$$\pm(\eta-\eta_0) = \frac{A}{\alpha_1-\alpha_2}\ln\left|\frac{\Gamma-\alpha_1}{\Gamma-\alpha_2}\right|, \quad \alpha_1 > \alpha_2, \tag{58}$$

$$\pm(\eta-\eta_0) = \frac{2A}{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}\ln\left|\frac{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}-\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}+\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}\right|, \quad \alpha_1 > \alpha_2 > \alpha_3, \tag{59}$$

$$\pm(\eta-\eta_0) = \frac{2A}{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}F(\varphi, l), \quad \alpha_1 > \alpha_2 > \alpha_3 > \alpha_4, \tag{60}$$

where:

$F(\varphi, l) = \int_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}}$, $\varphi = \arcsin\sqrt{\frac{(\Gamma-\alpha_1)(\alpha_2-\alpha_4)}{(\Gamma-\alpha_2)(\alpha_1-\alpha_4)}}$, $l^2 = \frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}$. Also, $\alpha_1, \alpha_2, \alpha_3$ and α_4 are the roots of the polynomial equation

$$\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4} = 0. \tag{61}$$

Substituting the solutions (56)–(60) into (8) and using Equation (47), the solutions of Equation (2) are found rational function solutions,

$$u_1(x,t) = e^{i(kx+ct)}\left(\pm\frac{A_1}{mx+nt}\right), \tag{62}$$

$$u_2(x,t) = e^{i(kx+ct)}\left(\frac{4A^2(\alpha_2-\alpha_1)\tau_1}{4A^2-\left[(\alpha_1-\alpha_2)(mx+nt)\right]^2}\right), \tag{63}$$

travelling wave solution:

$$u_3(x,t) = e^{i(kx+ct)}\left(\frac{(\alpha_2-\alpha_1)\tau_1}{2}\left\{1\pm\coth\left[\frac{(\alpha_1-\alpha_2)}{A}(mx+nt)\right]\right\}\right), \tag{64}$$

soliton solution:

$$u_4(x,t)=e^{i(kx+ct)}\frac{A_2}{\left(D+\cosh\left[B\left(mx+nt\right)\right]\right)},\tag{65}$$

and Jacobi elliptic function solution:

$$u_5(x,t)=e^{i(kx+ct)}\frac{A_3}{\left(M+N\,sn^2\left(\varphi,l\right)\right)},\tag{66}$$

where $A_1=\tau_1 A,\ A_2=\left(\dfrac{2\tau_1(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{\alpha_3-\alpha_2}\right), B=\dfrac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A}, D=\dfrac{2\alpha_1-\alpha_2-\alpha_3}{\alpha_3-\alpha_2},\quad A_3=\left(2\tau_1\left(\alpha_1-\alpha_3\right)\left(\alpha_4-\alpha_2\right)\right),$ $M=\alpha_4-\alpha_2,\ N=\alpha_1-\alpha_4,\ l^2=\dfrac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)},\ \varphi=\dfrac{\pm\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(mx+nt).$ Here, A_2 is the amplitude of the soliton, and B is the inverse width of the solitons. Therefore, it can be said that the solitons exist for $\tau_1<0$.

Remark 4. *If the modulus* $l\to 1,$ *then by using Equation (47), the solution (66) can be turned to the hyperbolic function solution:*

$$u_6(x,t)=e^{i(kx+ct)}\frac{A_3}{\left(M+N\tanh^2\left[\dfrac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(mx+nt)\right]\right)},\tag{67}$$

where $\alpha_3=\alpha_4.$

Remark 5. *If the modulus* $l\to 0,$ *then by using Equation (47), the solution (66) can be converted to the periodic wave solution:*

$$u_7(x,t)=e^{i(kx+ct)}\frac{A_3}{\left(M+N\sin^2\left[\dfrac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(mx+nt)\right]\right)},\tag{68}$$

where $\alpha_2=\alpha_3.$

3.2. GKM for Fokas-Lenells Equation

In this section, we will use generalized Kudryashov method to get exact solutions of Fokas-Lenells equation.

After Equation (2) has been turned into Equation (51) as in Section 3, substituting Equations (31) and (34) into Equation (51) and balancing the highest order nonlinear terms of u'' and u^3 in Equation (51), then the following relation is found:

$$N-M+2=3N-3M\Rightarrow N=M+1.\tag{69}$$

When we choose $M=1$ and $N=2$, by using Equations (36)–(38), the exact solution of Equation (2) is obtained as the following:

$$a_0=-\frac{i\sqrt{m(n-m\alpha)}\,b_0}{\sqrt{2}\sqrt{k-m}\sqrt{\alpha}\,\beta\sqrt{\sigma}},a_1=-\frac{a_2}{2}+\frac{i\sqrt{2}\sqrt{m(n-m\alpha)}\,b_0}{\sqrt{k-m}\sqrt{\alpha}\,\beta\sqrt{\sigma}},b_1=-\frac{i\sqrt{k-m}\sqrt{\alpha}\,\beta\sqrt{\sigma}\,a_2}{\sqrt{2mn-2m^2\alpha}},$$
$$c=\frac{6kn+mn+2k^2\alpha-12km\alpha-m^2\alpha+4(k-3m)\alpha\beta+2\alpha\beta^2}{2(k-3m)}, \tag{70}$$

If we substitute Equation (70) into Equation (36), we find the following dark soliton solution of Equation (2):

$$u(x,t)=e^{i(kx+ct)}\left[R\tanh(m_1x+n_1t)\right], \tag{71}$$

where $m_1=\frac{m}{2}$, $n_1=\frac{n}{2}$, and $R=-\frac{i\sqrt{m(n-m\alpha)}}{\sqrt{2}\sqrt{k-m}\sqrt{\alpha}\,\beta\sqrt{\sigma}}$.

In Figures 1 and 2, we plot two and three dimensional graphics of imaginary and real values of Equation (26), which denote the dynamics of solutions with appropriate parametric selections. Also, in Figures 3 and 4, we draw two and three dimensional graphics of imaginary and real values of Equation (27), which demonstrate the dynamics of solutions with convenient parametric choices. In Figures 5 and 6, we plot two and three dimensional graphics of Equation (44), which represent the dynamics of solutions with proper parametric values.

Moreover, in Figures 7 and 8, we draw two and three dimensional graphics of imaginary and real values of Equation (66), which display the dynamics of solutions with suitable parametric selections. Finally, in Figures 9 and 10, we plot two and three dimensional graphics of Equation (71), which indicate the dynamics of solutions with convenient parametric choices.

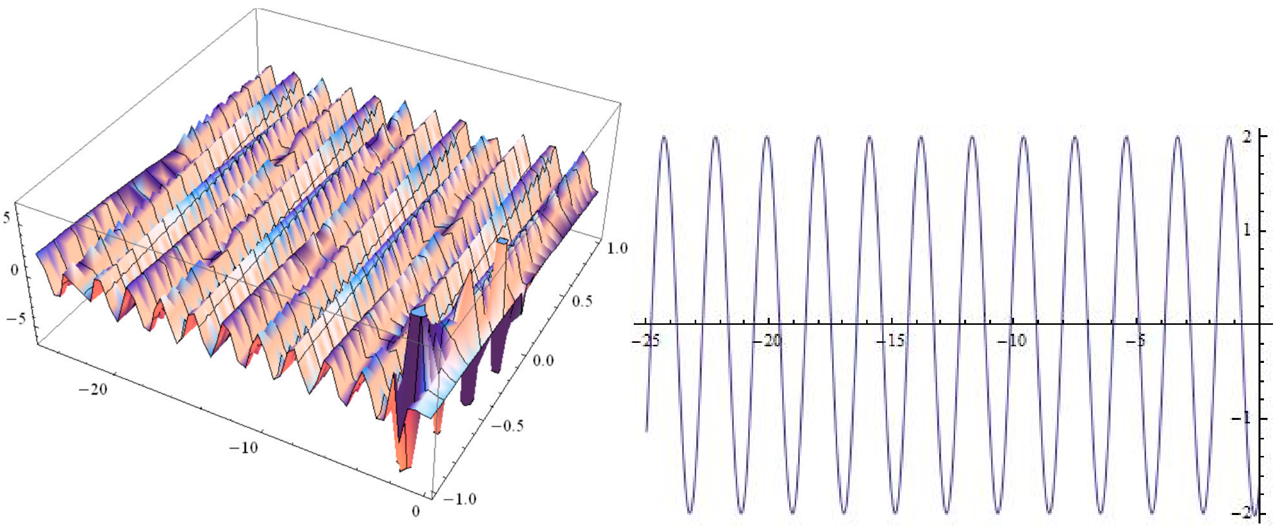

Figure 1. Graph of imaginary values of Equation (26) is demonstrated at $k=1,\ \lambda=2,\ \alpha=3,\ \beta=1,\ \sigma=1, \varepsilon=-3,\ \ \tau_1=1, \xi_4=2, \alpha_1=2, \alpha_2=4, -25<x<0, -1<t<1$ and the second graph indicates imaginary values of Equation (26) for $-25<x<0,\ t=1$.

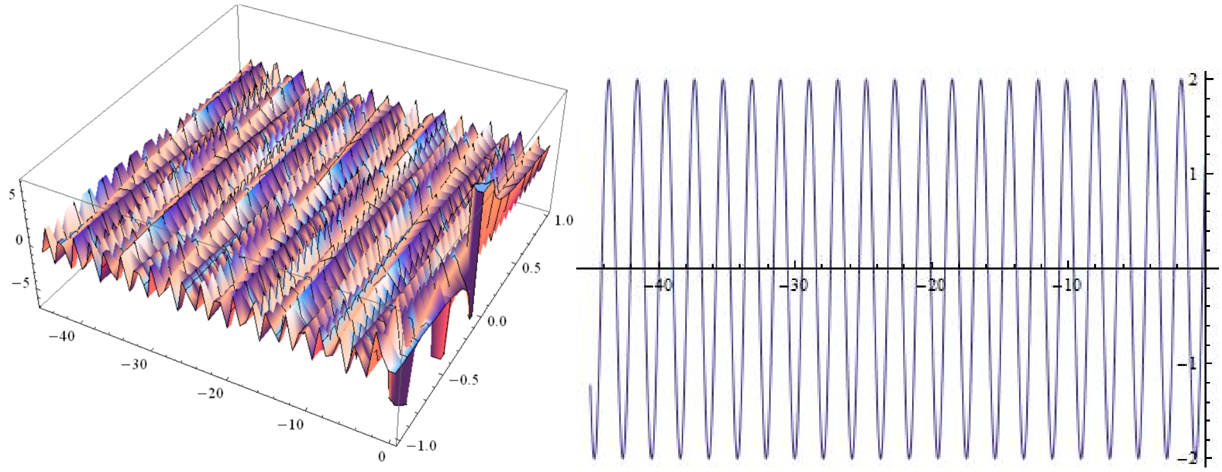

Figure 2. Graph of real values of Equation (26) is demonstrated at $k=1,\ \lambda=2,\ \alpha=3,\ \beta=1,\ \sigma=1, \varepsilon=-3,\ \ \tau_1=1, \xi_4=2, \alpha_1=2, \alpha_2=4, -45<x<0, -1<t<1$ and the second graph indicates real values of Equation (26) for $-45<x<0,\ t=1$.

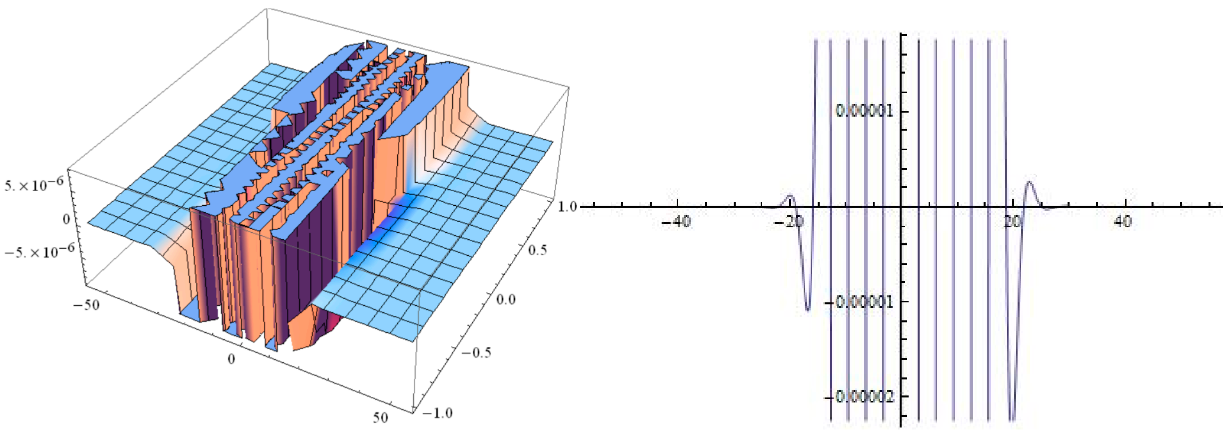

Figure 3. Graph of imaginary values of Equation (27) is demonstrated at $k=1,\ \lambda=2,\ \alpha=1,\ \beta=3,\ \sigma=1, \varepsilon=-3,\ \ \tau_1=1, \xi_4=2, \alpha_1=4, \alpha_2=2, \alpha_3=3, -55<x<55, -1<t<1$ and the second graph indicates imaginary values of Equation (27) for $-55<x<55,\ t=1$.

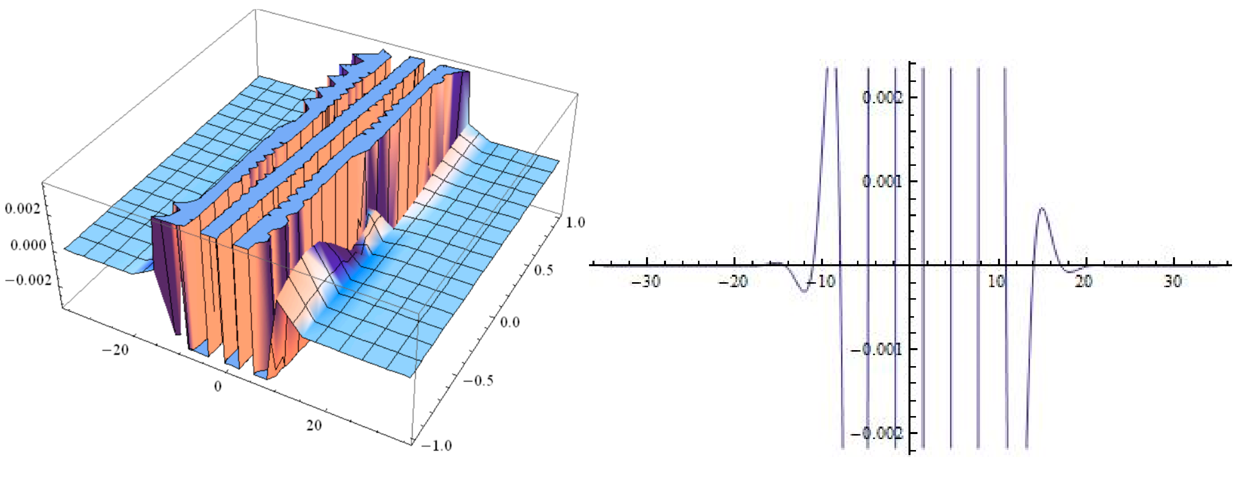

Figure 4. Graph of real values of Equation (27) is demonstrated at $k=1,\ \lambda=2,\ \alpha=1,\ \beta=3,\ \sigma=1, \varepsilon=-3,\ \ \tau_1=1, \xi_4=2, \alpha_1=4, \alpha_2=2, \alpha_3=3, -35<x<35, -1<t<1$ and the second graph indicates real values of Equation (27) for $-35<x<35,\ t=1$.

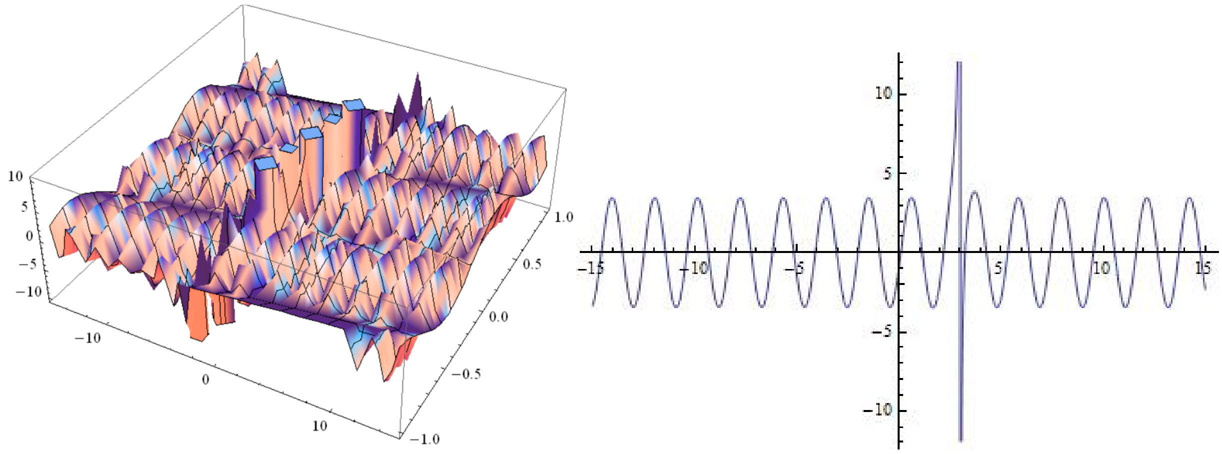

Figure 5. Graph of imaginary values of Equation (44) is demonstrated at $k = 2,\ \lambda = 3,\ \alpha = 3,\ \beta = 5,\ \sigma = 1, \varepsilon = -2,\quad -15 < x < 15, -1 < t < 1$ and the second graph indicates imaginary values of Equation (44) for $-15 < x < 15,\ t = 1$.

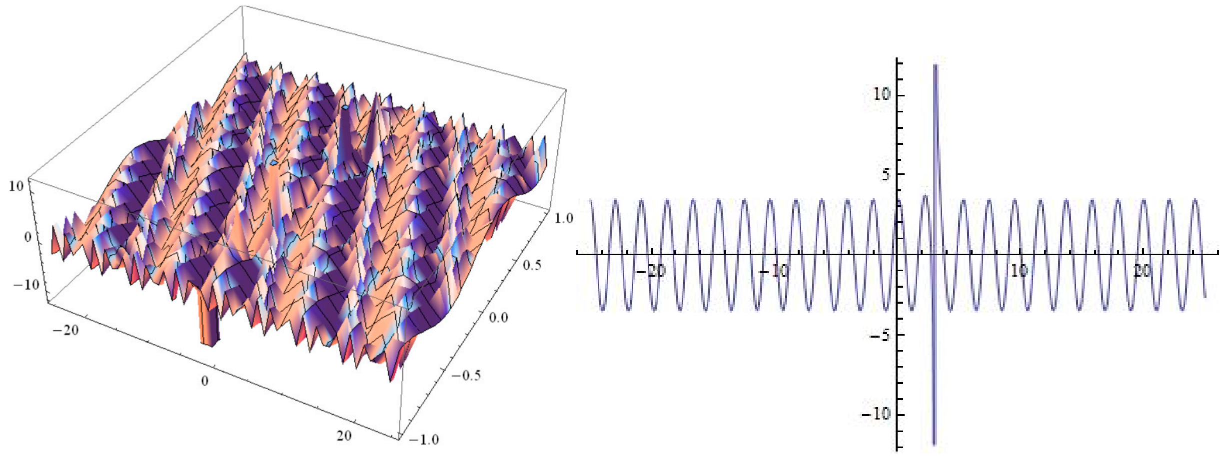

Figure 6. Graph of real values of Equation (44) is demonstrated at $k = 2,\ \lambda = 3,\ \alpha = 3,\ \beta = 5,\ \sigma = 1, \varepsilon = -2,\quad -25 < x < 25, -1 < t < 1$ and the second graph indicates real values of Equation (44) for $-25 < x < 25,\ t = 1$.

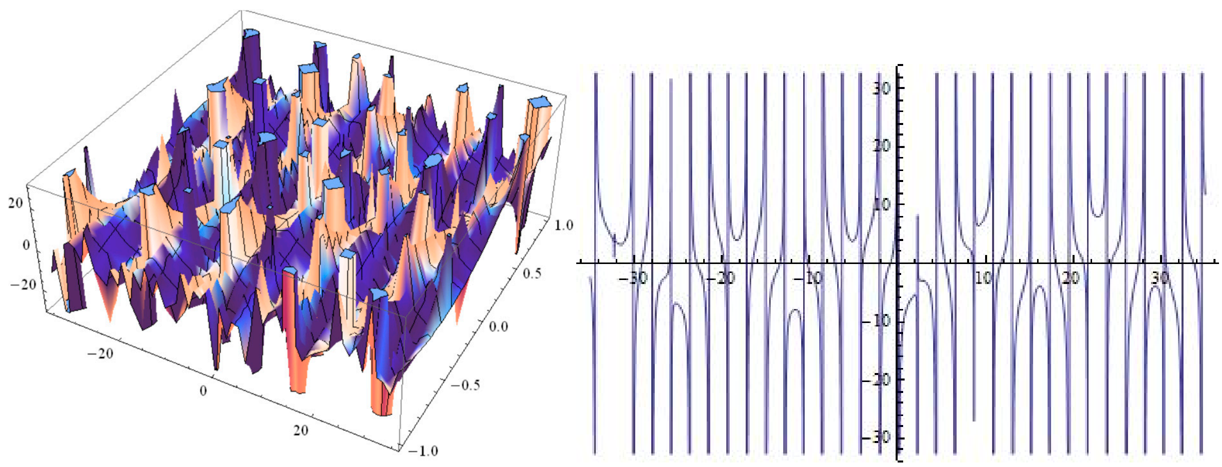

Figure 7. Graph of imaginary values of Equation (66) is demonstrated at $k = 1,\ c = 4,\ m = 2, \alpha = 4, \beta = 2, n = 2, \sigma = 3,\quad \tau_1 = 1, \xi_4 = 6, \alpha_1 = 2, \alpha_2 = 3, \alpha_3 = 4, \alpha_4 = 5, -35 < x < 35, -1 < t < 1$ and the second graph indicates imaginary values of Equation (66) for $-35 < x < 35,\ t = 1$.

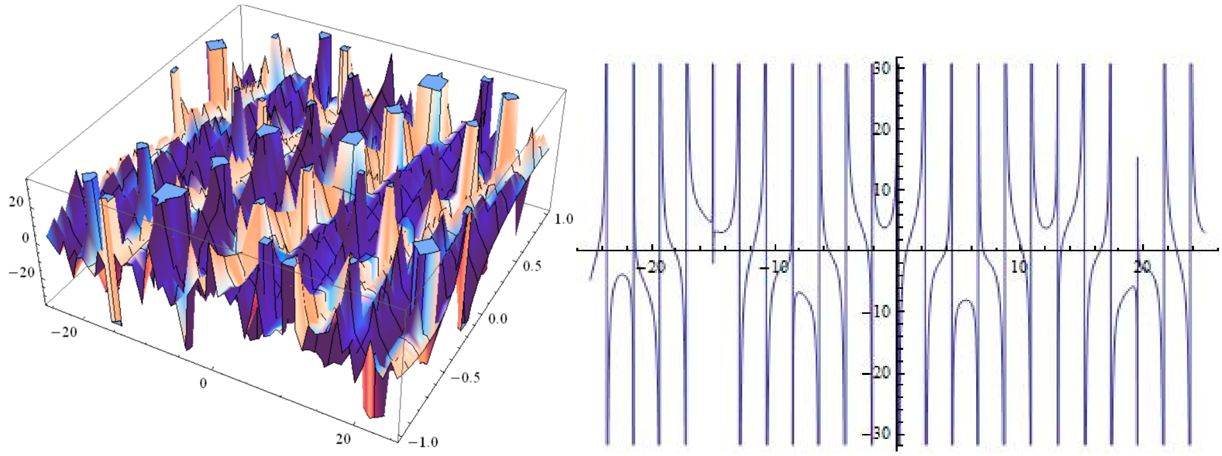

Figure 8. Graph of real values of Equation (66) is demonstrated at $k=1,\ c=4,\ m=2, \alpha=4, \beta=2, n=2, \sigma=3,\quad \tau_1=1, \xi_4=6, \alpha_1=2, \alpha_2=3, \alpha_3=4, \alpha_4=5, -25<x<25, -1<t<1$ and the second graph indicates real values of Equation (66) for $-25<x<25,\ t=1$.

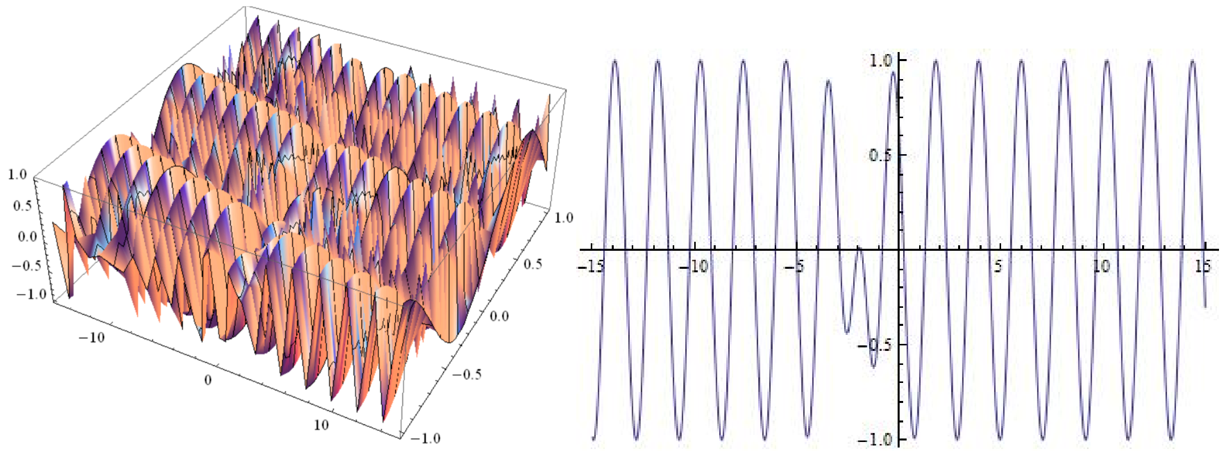

Figure 9. Graph of imaginary values of Equation (71) is demonstrated at $k=3,\ c=4,\ \alpha=1,\ m=2,\ n=4,\ \beta=1,\ \sigma=2,\ -15<x<15, -1<t<1$ and the second graph indicates imaginary values of Equation (71) for $-15<x<15,\ t=1$.

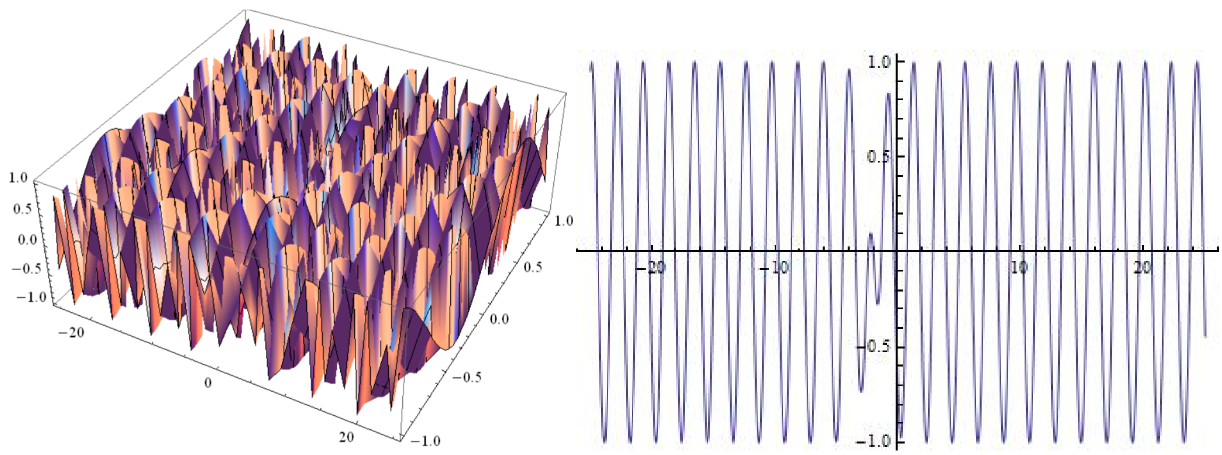

Figure 10. Graph of real values of Equation (71) is demonstrated at $k=3,\ c=4,\ \alpha=1,\ m=2,\ n=4,\ \beta=1,\ \sigma=2,\ -15<x<15, -1<t<1$ and the second graph indicates real values of Equation (71) for $-15<x<15,\ t=1$.

Remark 6. *The exact solutions of Equation (2) have been found by using ETEM and GKM and have been computed by the help of Mathematica Release 9. As far as we know, all solutions of Equation (2) that we reported here, are new and are not reported in the previous literature.*

4. Conclusions

In this paper, we seek exact solutions of the new Hamiltonian amplitude equation and Fokas-Lenells equation by using an extended trial equation method and generalized Kudryashov method. Firstly, we find exact solutions including rational function solution, travelling wave solution, soliton solution, Jacobi elliptic function solution, hyperbolic function solution and periodic wave solution of the new Hamiltonian amplitude equation and Fokas-Lenells equation by using the extended trial equation method. Secondly, we obtain dark soliton solutions of the new Hamiltonian amplitude equation and Fokas-Lenells equation by using the generalized Kudryashov method.

From these results, it is necessary to note that the extended trial equation method and generalized Kudryashov method provide strong mathematical tools for finding the analytical solutions of the new Hamiltonian amplitude equation and Fokas-Lenells equation and both methods are highly influential in terms of giving new solutions such as rational function solution, travelling wave solution, soliton solution, Jacobi elliptic function solution, hyperbolic function solution and periodic wave solution. The most important accomplishment of this study lies in the fact that we have succeeded in creating an appropriate algorithm in order to find exact solutions of these equations. Thus, we can emphasize that not only do the extended trial equation method and generalized Kudryashov method play important roles in researching nonlinear complex equations but also they are highly powerful in providing analytical solutions of nonlinear complex equations. We believe that both methods can also be applied to other nonlinear complex equations which arise in soliton theory.

Author Contributions

All authors have read and approved the final manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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