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Modeling and Analyzing the Interaction between Network Rumors and Authoritative Information

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Abstract: In this paper, we propose a novel two-stage rumor spreading Susceptible-Infected-Authoritative-Removed (SIAR) model for complex homogeneous and heterogeneous networks. The interaction Markov chains (IMC) mean-field equations based on the SIAR model are derived to describe the dynamic interaction between the rumors and authoritative information. We use a Monte Carlo simulation method to characterize the dynamics of the Susceptible-Infected-Removed (SIR) and SIAR models, showing that the SIAR model with consideration of authoritative information gives a more realistic description of propagation features of rumors than the SIR model. The simulation results demonstrate that the critical threshold λ_c of the SIAR model has the tiniest increase than the threshold of SIR model. The sooner the authoritative information is introduced, the less negative impact the rumors will bring. We also get the result that heterogeneous networks are more prone to the spreading of rumors. Additionally, the inhibition of rumor spreading, as one of the characteristics of the new SIAR model itself, is instructive for later studies on the rumor spreading models and the controlling strategies.

Keywords: rumor spreading; homogeneous networks; heterogeneous networks; authoritative information; SIAR model; threshold

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1. Introduction

Rumors, in itself neither true nor false, have great impact on society and people's life. As is known to all, there are many rumors in real life. The rise and use of the Internet today, has changed communication and made rumors disseminate more quickly. All kinds of online social networks (*i.e.*, Facebook and Twitter) and instant messaging tools (*i.e.*, Skype and MSN) will inevitably generate a lot of rumors every day. In this paper, rumors can be considered as unconfirmed information whose content can range from simple gossip to health knowledge or stock information, and cannot be involved in political affairs. Authoritative information can be considered as confirmed information, and may come from authoritative organizations or media such as World Health Organization, BBC, People's Daily, and so on. After the rumors spread around the real world, the authoritative organizations or media will immediately announce authoritative information to the public, which decreases the negative influence of rumors.

In recent years, more and more rumors have swept over China. On 11 March 2011, an earthquake that happened in Japan led to nuclear leakage. At the same time, the rumors that iodized salt can protect people against the nuclear radiation spread among the public. On 16 March 2011, many people started to buy and hoard iodized salt, and this irrational behavior of storing up iodized salt swept the whole country [1]. Moreover, the salt market order was broken. In the noon of 17 March 2011, the National Development and Reform Commission (NDRC) released an urgent announcement, appealing to people to resist the rumors. On 18 March 2011, the salt price of each region gradually returned to normal, and the rumors gradually ended. Another case was the 2012 doomsday rumor, which puzzled people for a whole year. The event of Malaysia Airlines losing contact happened on 8 March 2014, and affected people all over the world. Then a series of reports about the location of the plane spread on the Internet, but people cannot confirm which one is correct. The Malaysia government released timely information about the event, but the information content was inconsistent. What is more, a variety of sources through different channels were also different. Some news media reported news without verification because they were afraid of missing the latest news. From the above we can see that the absence of authoritative sources led to rumors.

When the network rumors appear in our real life, when to release the timely and effectively confirmed content for controlling the propagation of rumors? How will people's attitudes be changed when they face the rumors and authoritative information? What will happen when two different kinds of information spread on the same network? Furthermore, we can still ask, is there a critical threshold in the novel rumor spreading model when authoritative information is taken into account? These problems will be considered in this paper.

Many existing works about propagation dynamics [2–5] have focused on the single infected source. However, in the real world, there are many coexisting information dissemination phenomena, such as the contagion of two contrary computer viruses in the same computer network, the diffusion of many different infectious diseases among people, and the propagation of two different rumors in the same social network. However now, the study on the interaction between two information sources is still at early stage. Rumor spreading has shown an interesting similarity to computer virus propagation [2] and disease spreading [3,4], so we can refer to the study of diseases and computer viruses spreading models with two species [6–9]. In [7], a mathematical model using ordinary differential equations was

created to describe the interaction of worms and antiworms. Some researchers [8] studied the process of hybrid structured benign worms countering against worms based on infectious model. In [9], the authors studied the nonequilibrium phase transition in an epidemic spreading model considering two particle species on scale-free networks. Trpevski *et al*. [10] suggested an alternate model, where two rumors with different probabilities of acceptance spread among nodes. Some scholars [11] further investigated the rumor spreading model with skepticism mechanism and analyzed the stability of corresponding mean-field equations. A model combining the spreading stage and the interaction stage for opinions [12] was proposed to illustrate the process of dispelling a rumor.

Currently, the authoritative organizations or media often clarify the fact and release the authoritative information to confirm or refute the content of network rumors. Thus, rumors and authoritative information spread in the same network. However, the above studies [10–12] did not provide a realistic description of this situation, and many other works [13–21] are mainly involved the diffusion of a single rumor. In order to provide a relatively realistic description of the interaction between rumors and authoritative information, we introduce a new SIAR model of rumor-spreading on complex networks considering the hysteresis phenomenon of authoritative information which lags behind the rumor. We should analyze in depth the dynamic interaction process of network rumors and authoritative information, and understand the characteristics of rumor diffusion, which has an important scientific significance and application value.

The rest of the paper is organized as follows: we introduce the concepts of our new SIAR model and derive the corresponding mean-field equations in Section 2. We perform a Monte Carlo simulation on the dynamics of the SIR model and SIAR model, and analyze the simulation results in Section 3, and the conclusions are given in Section 4.

2. SIAR Rumor-Spreading Model

In the previous SIR models [13–17] of rumor spreading based on complex networks, individuals are always divided into three categories which are ignorants (S), spreaders (I), and stiflers (R). Compared with the previous SIR models, in this paper we introduce a new authoritative (A) state into the SIAR model, which is defined in the following way: S, I, A, R stand for the people who never heard rumor (susceptible, similar to ignorants), the people who spread the rumor (infected, similar to spreaders), the people who spread the authoritative information (authoritative), and the people who heard the rumor but had no interest in spreading it (removed, similar to stiflers), respectively. At the first stage $0 \le t < \tau$, each individual in network adopts one of the three states: S, I, and R. At the second stage $t \ge \tau$, each individual in network adopts one of the four states: S, I, A, and R. We consider a population consisting of N users on the social network where users are nodes and contacts between users are edges. We assume that the total number of social users is N, and all social users are equal and can be considered as ordinary people rather than organizations. That is, the authoritative organizations or media are not regarded as the members of social network. All the authoritative nodes are ordinary social users, and the first authoritative node may be the staff of authoritative organizations. When the authoritative individual starts to spread in the social network, the first authoritative individual is an individual randomly chosen from susceptible individuals (*i.e.*, ignorants). In the real world, although the authoritative user may sometimes come from the spreaders,

this situation will not be considered in the model because the situation rarely happens. In the initial network $t = 0$, we consider that there is an infected node (*i.e.*, spreader) chosen randomly, while all the others start from the susceptible state. At the time $t = \tau(\tau > 0)$, an authoritative node chosen randomly from the susceptible nodes starts to diffuse authoritative information in the network.

The contacts between two different people are governed by the following set of rules: (1) whenever a spreader (infected node) contacts an ignorant (susceptible node), the ignorant becomes a spreader at a spreading rate $\lambda(0 < \lambda \leq 1)$; (2) whenever an authoritative individual contacts an ignorant, the ignorant becomes an authoritative one at a diffusion rate $\beta(0 < \beta \leq 1)$; (3) when a spreader contacts with another spreader or authoritative individual or stifler (removed node), only the initiating spreader becomes a stifler at a rate α. Moreover, the initiating spreader itself without any contacts becomes the stifler at a rate δ, where $\delta(0 < \delta \leq 1)$ is called forgetting rate. The process of SIAR rumor-spreading is shown in Figure 1. The third rule conforms to the hypothesis [1,13–21] that an active spreader (*i.e.*, infected individual) stops spreading the rumor because he learns that it has lost its "news value"; if this happens as soon as he meets another individual (that is, spreader or stifler or authoritative individual) knowing or refuting the rumor, then transitions from infected state (I) to removed state (R) occur as a result of II, IA, and IR encounters. The above assumptions are derived from the rumor diffusion mechanism of online social network and different people's attitudes to rumors.
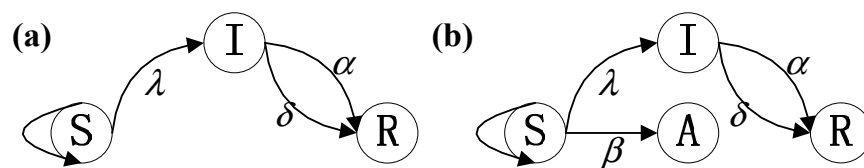

Figure 1. Structure of two-stage SIAR rumor spreading process. (**a**) $0 \leq t < \tau$, there are three kinds of states in the network; (**b**) $t \geq \tau$, there are four kinds of states in the network, where the authoritative node starts to spread authoritative information at time $t = \tau$.

We describe the dynamics of the above SIAR model on a network within the framework of the IMC. The IMC was originally introduced as a method of modeling social processes involving many interacting actors [22]. It has always been applied to the dynamics of cascading failures in electrical power networks [23], and the propagation of malicious software on the Internet [24]. Recently, some researchers have used this method in the dynamics of rumor diffusion [14] and epidemic spreading on complex networks [25]. Unlike conventional Markov chains, these internal transition probabilities of IMC depend not only on the current state of a node itself, but also on the states of all the direct neighboring nodes. In our SIAR model, each internal Markov chain can be one of four states: susceptible, infected, authoritative, and removed. A description of susceptible node j connection in the online social network is shown in Figure 2.
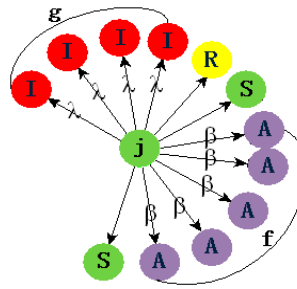
Figure 2. A description of susceptible node j connection.

We derive mean-field equations as follows:

$$P_{si}^j = (1-(1-\Delta t\lambda)^g)(1-\Delta t\beta)^f \tag{1}$$

$$P_{sa}^j = (1-\Delta t\lambda)^g (1-(1-\Delta t\beta)^f) \tag{2}$$

$$P_{ss}^j = 1 - P_{si}^j - P_{sa}^j = (1-\Delta t\lambda)^g(1-\Delta t\beta)^f + (1-(1-\Delta t\lambda)^g)(1-(1-\Delta t\beta)^f) \tag{3}$$

P_{ss}^j is the probability that susceptible node j stays in the susceptible state in the time interval $[t, t+\Delta t]$, and P_{si}^j is the probability that susceptible node j makes a transition to the infected state. P_{sa}^j is the probability that the susceptible node makes a transition to the authoritative state. Referring to reference [14], we consider the simple situation that a susceptible node, infected node and another infected node contact simultaneously in (1). $g = g(t)$ is the number of neighbors of susceptible node j which are in the infected state at time t. In (2), we consider the situation that susceptible node, authoritative node and another authoritative node contact simultaneously. $f = f(t)$ is the number of neighbors of susceptible node j that are in the authoritative state at time t. In the real world, the situation that a susceptible individual contacts infected individual and authoritative individual at the same time is rarely happened, and the corresponding process of state transition of susceptible node is very complicated, so we ignore this situation. From Equations (1) and (2), we can obtain (3). We coarse-grain the micro-dynamics of the system by classifying nodes in our network according to their degree and taking statistical average of the above transition probability over degree classes.

If a node j has k links, g and f can be considered as two stochastic variables that have the following binomial distribution:

$$\Pi(g,t) = \binom{k}{g}\theta(k,t)^g(1-\theta(k,t))^{k-g} \tag{4}$$

$$\Pi(f,t) = \binom{k-g}{f}\omega(k,t)^f(1-\omega(k,t))^{k-g-f} \tag{5}$$

where $\theta(k,t)$ is the probability at time t that an edge emanating from a susceptible node with k links points to an infected node. $\omega(k,t)$ is the probability at time t that an edge emanating from a susceptible node with k links points to an authoritative node, and the largest number of authoritative nodes is $k-g$. They can be written as:

$$\theta(k,t) = \sum_{k'} P(k'/k)P(i_{k'}/s_k) \approx \sum_{k'} P(k'/k)\rho^i(k',t) \tag{6}$$

$$\omega(k,t) = \sum_{k'} P(k'/k)P(a_{k'}/s_k) \approx \sum_{k'} P(k'/k)\rho^a(k',t) \tag{7}$$

where $P(k'/k)$ represents the conditional probability that a node with degree k is connected to a node with degree k'. $P(i_{k'}/s_k)$ is the conditional probability that an infected node with degree k' is connected to a susceptible node with degree k, and the $\rho^i(k',t)$ is the density of infected nodes at time t which belong to connectivity class k'. $P(a_{k'}/s_k)$ is the conditional probability that an authoritative node with degree k' is connected to a susceptible node with degree k. $\rho^a(k',t)$ is the density of authoritative nodes at time t which belong to connectivity class k'. By ignoring dynamic correlations between the states of neighboring nodes, we can obtain the final approximation in Equations (6) and (7).

In this network, we assume that the stochastic variables g and f are identically distributed but not independent. The transition probability $\overline{P}_{ss}(k,t)$ is averaged over all possible values of g and f:

$$
\begin{aligned}
\overline{P}_{ss}(k,t) &= \sum_{g=0}^{g=k} \binom{k}{g} \theta(k,t)^g (1-\theta(k,t))^{k-g} \bullet P_{ss}^j \bullet \sum_{f=0}^{f=k-g} \binom{k-g}{f} \omega(k,t)^f (1-\omega(k,t))^{k-g-f} \\
&= 1 - (1-\lambda\Delta t\theta(k,t))^k - (1-\beta\Delta t\omega(k,t) + \theta(k,t)\omega(k,t)\Delta t\beta)^k \\
&\quad + 2(1-\lambda\Delta t\theta(k,t) - \beta\Delta t\omega(k,t) + \theta(k,t)\omega(k,t)\Delta t\beta)^k
\end{aligned}
\tag{8}
$$

Similarly, we can obtain:

$$
\begin{aligned}
\overline{P}_{si}(k,t) &= (1-\Delta t\beta\omega(k,t) + \Delta t\beta\theta(k,t)\omega(k,t))^k \\
&\quad - (1-\lambda\Delta t\theta(k,t) - \beta\Delta t\omega(k,t) + \theta(k,t)\omega(k,t)\Delta t\beta)^k
\end{aligned}
\tag{9}
$$

$$\overline{P}_{sa}(k,t) = (1-\Delta t\lambda\theta(k,t))^k - (1-\lambda\Delta t\theta(k,t) - \beta\Delta t\omega(k,t) + \theta(k,t)\omega(k,t)\Delta t\beta)^k \tag{10}$$

Similarly, we can derive an expression for the probability $\overline{P}_{ii}(k,t)$ that an infected node with degree k stays in this state in the interval $[t, t+\Delta t]$. The probability for a transition from the infected node to the removed one, $\overline{P}_{ir}(k,t)$ is given by $\overline{P}_{ir}(k,t) = 1 - \overline{P}_{ii}(k,t)$. Following steps similar to the above paragraphs, we obtain:

$$\overline{P}_{ii}(k,t) = (1-\alpha\Delta t\sigma(k,t))^k (1-\delta\Delta t) \tag{11}$$

where $P_{ii}^j = (1-\Delta t\alpha)^\eta (1-\Delta t\delta)$, and η can be regarded as a stochastic variable which has the following binomial distribution:

$$\Pi(\eta,t) = \binom{k}{\eta} \sigma(k,t)^\eta (1-\sigma(k,t))^{k-\eta} \tag{12}$$

$$
\begin{aligned}
\sigma(k,t) &= \sum_{k'} P(k'/k)(P(i_{k'}/s_k) + P(a_{k'}/s_k) + P(r_{k'}/s_k)) \\
&\approx \sum_{k'} P(k'/k)(\rho^i(k',t) + \rho^a(k',t) + \rho^r(k',t))
\end{aligned}
\tag{13}
$$

where $\sigma(k,t)$ is the probability that an edge of a node with degree k points to an infected, authoritative or removed node at time t.

We use $S(k,t)$, $I(k,t)$, $A(k,t)$, $R(k,t)$ to represent the expected values of the number of nodes belonging to connectivity class k which at time t are in the susceptible, infected, authoritative, or

removed state, respectively. The event that a susceptible node in class k will make a transition to the other state (infected state or authoritative state) during $[t, t+\Delta t]$ is a Bernoulli random variable with probability $(1 - \overline{P}_{ss}(k,t))$ of success. We can obtain the number of change in the expected value of the population of susceptible nodes belonging to class k:

$$S(k, t+\Delta t) = S(k,t) - S(k,t)(1 - \overline{P}_{ss}(k,t)) \tag{14}$$

In a similar way, we can get the corresponding number of change in the population of infected, authoritative, and removed nodes as follows:

$$I(k, t+\Delta t) = I(k,t) + S(k,t)\overline{P}_{si}(k,t) - I(k,t)(1 - \overline{P}_{ii}(k,t)) \tag{15}$$

$$A(k, t+\Delta t) = A(k,t) + A(k,t)\overline{P}_{sa}(k,t) \tag{16}$$

$$R(k, t+\Delta t) = R(k,t) + I(k,t)(1 - \overline{P}_{ii}(k,t)) \tag{17}$$

Denote with $\rho^s(k,t)$, $\rho^i(k,t)$, $\rho^a(k,t)$, $\rho^r(k,t)$ the fraction of nodes belonging to class k which are in the susceptible, infected, authoritative, or removed state, respectively. These quantities satisfy the normalization condition $\rho^s(k,t) + \rho^i(k,t) + \rho^a(k,t) + \rho^r(k,t) = 1$. In the limit $\Delta t \to 0$, we obtain:

$$\frac{\partial \rho^s(k,t)}{\partial t} = -k\lambda\rho^s(k,t)\theta(k,t) - k\beta\rho^s(k,t)\omega(k,t) + k\beta\rho^s(k,t)\theta(k,t)\omega(k,t) \tag{18}$$

$$\frac{\partial \rho^i(k,t)}{\partial t} = k\lambda\rho^i(k,t)\theta(k,t) - k\alpha\rho^i(k,t)\sigma(k,t) - \delta\rho^i(k,t) \tag{19}$$

$$\frac{\partial \rho^a(k,t)}{\partial t} = k\beta\rho^s(k,t)\omega(k,t) - k\beta\rho^s(k,t)\theta(k,t)\omega(k,t) \tag{20}$$

$$\frac{\partial \rho^r(k,t)}{\partial t} = k\alpha\rho^i(k,t)\sigma(k,t) + \delta\rho^i(k,t) \tag{21}$$

3. Simulation Results

In this section, we perform Monte Carlo simulations to verify the above analytical predictions and further investigate the properties of the SIAR rumor spreading model. All the simulation results are averaged over two networks (*i.e.*, two Watts-Strogatz (WS) networks [26] and two uncorrelated Barabási-Albert (BA) networks [27]) to obtain better statistics. The first WS network size is $N = 3000$, the rewiring probability is $p = 0.2$, and the average degree is $\overline{k} = 6$; The second WS network size is $N = 5000$, the rewiring probability is $p = 0.3$, and the average degree is $\overline{k} = 6$. The size of the BA networks is $N = 3000$ and 5000, the initial number of nodes is $m_0 = 3$, and the average degree is fixed at $\overline{k} = 6$. For each network, the simulations are performed by starting the rumor from a randomly chosen initial spreader, and the corresponding results are averaged over 500 runs with different rumormongers. At the first stage $0 \le t < \tau$, we use SIR model [14] which has only three states, and each node in network adopts one of the three states: S, I, and R. When $t = \tau$, the authoritative node chosen randomly from the susceptible nodes starts to diffuse authoritative information in the network. At the second stage $t \ge \tau$, each node in network adopts one of four states: S, I, A, and R. The whole spreading process characterizes the dynamics of two-stage SIAR model.
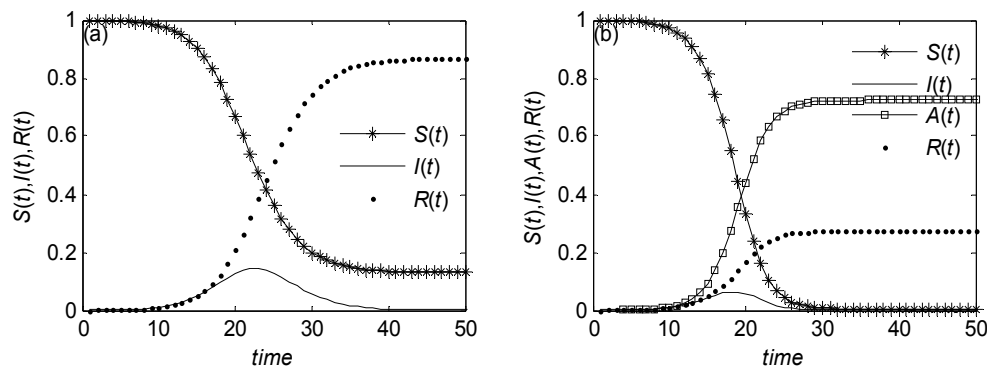
Figure 3. The evolution of rumor propagation with $\lambda = 0.2$, $\beta = 0.2$, $\alpha = 0.1$, $\delta = 0.2$, $\tau = 4$. (**a**) SIR model in the WS network; (**b**) SIAR model in the WS network.

Figure 3 displays the evolution of rumor propagation in the WS network. During the simulation, we use SIR model with the forgetting mechanism [14] and modified two-stage SIAR model to characterize the evolution process of rumor spreading.

We can see that SIAR model is better than SIR model because the former can describe more precisely the actual propagation of the rumor. The curves in Figure 3a show that each individual will be either susceptible or a removed person in the steady state. However, in reality, when the rumor spreading process reaches the steady state, there are always some people who hold authoritative information in the network. Figure 3b shows that there exist authoritative nodes in the WS network when the spreading process reaches the steady state. Thus, it is necessary to introduce the authoritative state into the rumor spreading model. Additionally, the final rumor size R of SIAR model is less than the corresponding R of SIR model. This result is also aligned with the simulation results in Figure 4. The model reveals that the inhibition of rumor spreading can be considered as one of the characteristics of the new SIAR model itself.
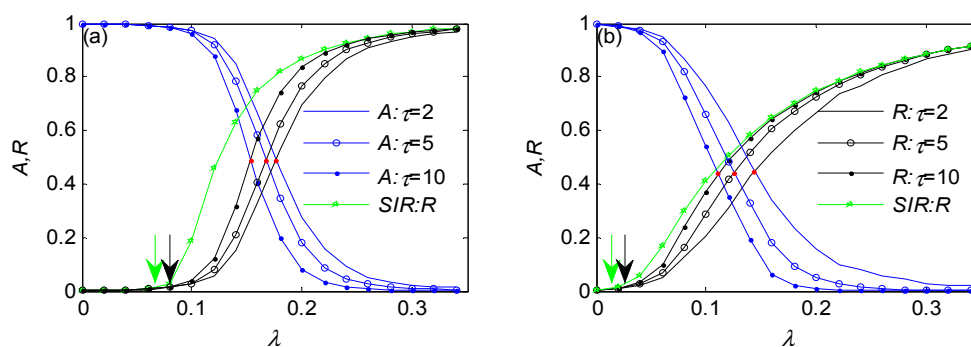

Figure 4. The final size A and R *versus* the rumor spreading rate λ for several values of delay τ. (**a**) The final rumor size R (green curve) of SIR model and the final size A (blue curves) and R (black curves) of SIAR model in the WS network; (**b**) The final rumor size R (green curve) of SIR model and the final size A (blue curves) and R (black curves) of SIAR model in the BA network.

Figure 4 shows the results for the final authoritative information size A (blue curves) and the final rumor size R (green curve and black curves) based on the SIR and SIAR models respectively in the WS network and in the BA network, where we set $\beta = 0.1$, $\alpha = 0.1$, $\delta = 0.2$ and $\tau = 2$, 5, and 10. We

use green arrows to denote the critical thresholds λ_c of SIR model below which a rumor cannot spread. The black arrows denote the critical thresholds λ_c of SIAR model below which all the people hold authoritative information. It means that rumor cannot spread and influence people in the network when the spreading rate λ is less than critical threshold λ_c of SIAR model. The spreading critical threshold λ_c of the SIAR model (black arrow) on both WS network and BA network is independent of lag time τ. The ultimate rumor influence R decreases as lag time τ is decreased. In Figure 4, it can be noticed that the critical threshold λ_c of SIAR model has a tinier increase than the corresponding threshold λ_c of SIR model on both WS network and BA network. Moreover, we observe that the critical threshold λ_c on the BA network is smaller than the threshold λ_c on WS network for both SIR model and SIAR model. Our results above are similar to the results of [14], which reveal that the critical threshold λ_c of rumor spreading model obeys the relation "WS network > BA network".

From Figure 4, we also can see that the intersection point (red symbols) of two curves of A (blue curves) and R (black curves) with the same value of τ moves to the left as τ is increased. Further, one can easily find that the ordinate values of three intersection points are almost same, and the areas on the left of intersection points have the features of $A > R$. When $A > R$, it means that the final number of people who hold authoritative information is greater than the final rumor size. If we want to reduce the final rumor size R, we should enlarge the areas of $A > R$. The abscissa value of intersection point is the spreading rate λ under the condition that A equals R. To further investigate the propagation features of SIAR model on the complex networks, we give a new definition of critical value λ_c' which has a extremely different meaning with the traditional threshold λ_c. The value of λ under the condition of $A = R$ in the SIAR model is termed as the critical value λ_c'. When $\lambda > \lambda_c'$, $A < R$. Otherwise, $A \geq R$. With the decrease of τ, the λ_c' increases. Correspondingly, when there is a rumor spreading in real world, the sooner the authoritative organizations or media releases the authoritative information, the less negative impact the rumors will bring. That means the authoritative organizations or media should give the authoritative information as soon as possible.

Then, we investigate the impact of the authoritative information diffusion rate β on the SIAR rumor spreading process.
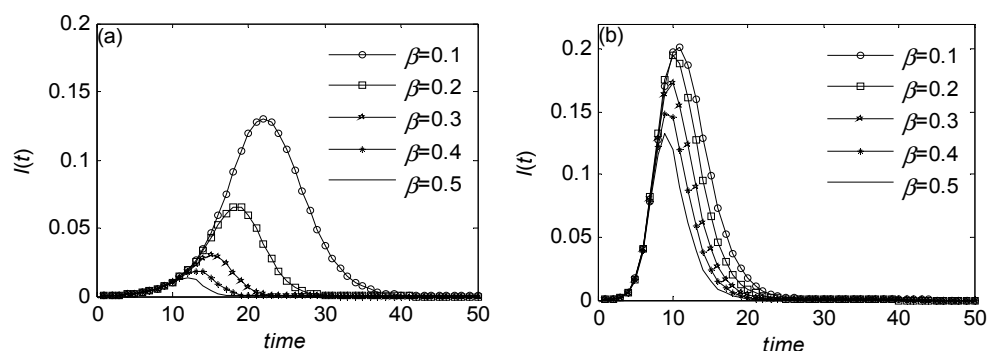

Figure 5. The density of the infected nodes $I(t)$ as a function of time t for diffusion rate β. (**a**) SIAR model in the WS network; (**b**) SIAR model in the BA network.

Figure 5 illustrates the density of the infected nodes $I(t)$ as a function of time t for different values of β in the WS and BA networks, where we set $\lambda = 0.2$, $\alpha = 0.1$, $\delta = 0.2$, $\tau = 4$, $\beta = 0.1$, 0.2, 0.3, 0.4, and 0.5. We use the peak value of $I(t)$ (the highest density of the infected nodes) to measure the

maximum rumor influence, and $I(t)_{max}$ denotes the peak value of $I(t)$. As can be seen in Figure 5, the initial spreading speed of a rumor on the BA network is much faster than that on the WS network, and the highest density of the infected nodes on the BA network is also much larger than that on the WS network. Furthermore, we find that the time required for the rumor to reach the $I(t)_{max}$ on the WS network is nearly twice as long as the corresponding time on the BA network. Thus, due to the propagation rules of SIAR model, BA network is more prone to the spreading of rumors than the WS network, which is in good agreement with the results in [14] and [20].
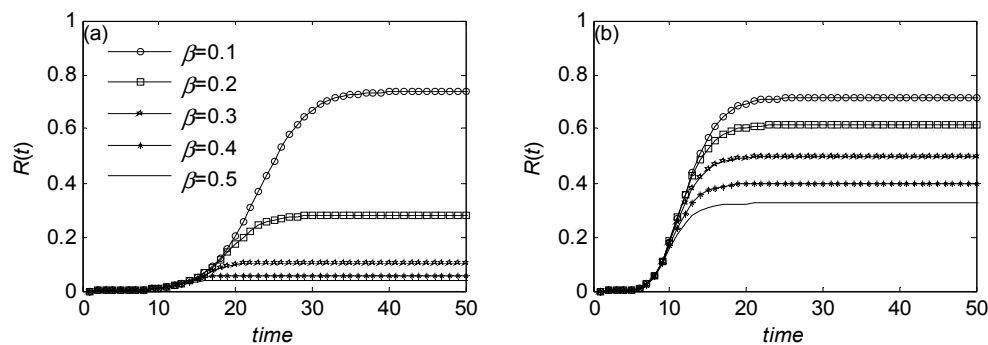

Figure 6. The density of the removed nodes $R(t)$ as a function of time t for diffusion rate β. (**a**) SIAR model in the WS network; (**b**) SIAR model in the BA network.

It also can be seen from Figures 5 and 6 that with the increase of β, the peak value $I(t)_{max}$ and the final rumor size denoted with $R(\infty)$ will decrease. In the WS network, they both decrease apparently as β is increased. In the BA network, they both show a small decrease as β is increased. This behavior results from the conflicting roles hubs play when the stifling mechanism (the third propagation rule) is switched on. Initially, the existing of hubs speeds up the spreading but once they transform into removed states they also effectively impede further spreading of the rumor. Thus in BA network (heterogeneous network) with some social hubs, the time required for the rumor to reach $I(t)_{max}$ is short, which leaves little time for diffusion of authoritative information. With the increase of β, the number of authoritative nodes that has a time delay that is increasing slower than the number of infected nodes in unit time. These factors can lead to a small change in the $I(t)_{max}$ or $R(\infty)$ for a BA network. Unlike the BA network, the initial spreading speed of a rumor on the WS network (homogeneous network) is slow, which leaves relatively enough time for diffusion of authoritative information. Then with the increase of β, the number of authoritative nodes grows faster than the number of infected nodes in the WS network. These factors lead to an obvious change in the $I(t)_{max}$ or $R(\infty)$ for WS network. This allows us to conclude that the increase of the diffusion rate of authoritative information in homogeneous networks is able to decrease the final rumor influence more effectively than that in heterogeneous networks.

4. Conclusions

The dynamics of rumor spreading on complex networks has been studied in this paper. A modified two-stage SIAR rumor-spreading model is proposed and simulated. Different from the standard or existing rumor spreading models, this model takes authoritative individuals who can diffuse

authoritative information into account. The simulation results show that SIAR model can realistically characterize the evolution of the rumor propagation, and the spreading critical threshold λ_c of the SIAR model on both WS network and BA network is independent of lag time τ. We find that BA network yields the most effective rumor spreading, and authoritative information should be released early to decrease the number of people infected by rumors.

Acknowledgments

Author Contributions

Lingling Xia and Guoping Jiang proposed and designed the research; Lingling Xia performed the simulations and wrote the paper; Guoping Jiang, Yurong Song and Bo Song analyzed the simulation results. All authors have read and approved the final manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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