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Article

Historical and Physical Account on Entropy and Perspectives on the Second Law of Thermodynamics for Astrophysical and Cosmological Systems

Jeroen Schoenmaker

Centro de Engenharia, Modelagem e Ciências Sociais Aplicadas, Universidade Federal do ABC (UFABC), Santo André, SP CEP 09210-580, Brazil; E-Mail: jschoenmaker@gmail.com or jeroen.schoenmaker@ufabc.edu.br; Tel.: +55-11-4996-8256

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Abstract: We performed an in depth analysis of the subjects of entropy and the second law of thermodynamics and how they are treated in astrophysical systems. These subjects are retraced historically from the early works on thermodynamics to the modern statistical mechanical approach and analyzed in view of specific practices within the field of astrophysics. As often happens in discussions regarding cosmology, the implications of this analysis range from physics to philosophy of science. We argue that the difficult question regarding entropy and the second law in the scope of cosmology is a consequence of the dominating paradigm. We further demonstrate this point by assuming an alternative paradigm, not related to thermodynamics of horizons, and successfully describing entropic behavior of astrophysical systems.

Keywords: second law of thermodynamics; gravity; entropy; paradigm; scientific realism

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1. Introduction

The role of paradigm in the evolution of scientific theories has been extensively studied by philosophers of science. Notorious examples are easily found, such as the atomistic approach, that led to the rapid development and aggregation of the kinetic theory heat, Brownian motion, electronic conduction, the periodic table, molecular structure and quantum mechanics into successful research programs [1]. On the other hand, it is also well known how the theory of relativity and thermodynamics thrived only when the "*caloric*" and "*absolute space*" concepts were abandoned.

Although the role of such Kuhnian aspects seems to be well known to philosophers of science, the same cannot be said about scientists, as the acceptance of good models still faces strong resistance by the scientific community due to paradigmatic issues. It is worth noticing that, in association with the concept of paradigm, we have to take into account that most scientists are tied to some kind of scientific realism, a case is similar to that of philosophers of science (for a survey among philosophers see the article by Chalmers and Bourget [2]). One recent and remarkable example of this issue is the concept of quasicrystals, which led to the Nobel Prize in Chemistry awarded to Shechtman in 2011 [3]. The first paper by Shechtman on quasicrystals was published in 1984. Despite the high pace of contemporary scientific progress, the recognition of his work leading to the Nobel Prize took almost 30 years. In the meantime, Shechtman suffered scorn from fellow scientists and was forced to leave his research group, so strong was the attachment of the scientific community to the theory of crystals based on Bravais lattices. The latter theory is so neat and successful, separating solids in a clear dichotomy between crystalline and amorphous states that condensed matter scientists assumed somehow that the elements behind such theory must represent true aspects of Nature. The advent of quasicrystals forced them to change such a view.

It is not the objective of this manuscript to survey paradigmatic cases along the history of science, but rather to make an in depth analysis of the thermodynamic concepts in the current status of astrophysical thermodynamics. The difficult question regarding entropy and the second law applied to astrophysical systems is well known and a rather recent article entitled "*Is our Universe natural*?" by cosmologist Sean Carroll clearly exhibits this situation [4].

In Section 2 we discuss the historical context in which key thermodynamic concepts and traditional approaches emerged. These concepts and approaches are reanalyzed in the context of astrophysical thermodynamics in Section 3, where we highlight some interesting issues. In Section 4, we propose, conceptually and qualitatively, a new approach to the treatment of entropy in systems where gravity plays a dominant role. A formal model to the approach proposed in Section 4 is presented in Section 5.

2. Key Aspects of the Historical Development of the Concepts of Entropy and the Second Law

Physicists seek unification, and one of the most celebrated unifying theories is statistical mechanics, which enabled phenomenological aspects of thermodynamics to be explained through a constructive model based on the microscopic behavior of atoms and molecules. But this unification came with the cost of having, according to the macro or micro approach, two fundamentally distinct interpretations of entropy which imply different grasps of the second law [5]. This issue is particularly interesting when we note that entropy is not an intuitive concept such as temperature, contributing to the fact that the second law is considered to be more controversial than the other laws of thermodynamics [6,7].

The birth of thermodynamics as an established branch of science is usually attributed to the publication of the book "*Reflexions sur la Puissance Motrice du Feu*" (Reflections on the Motive Power of Heat), by the French engineer Nicolas Sadi Carnot in 1824 [8]. At the time, the Industrial Revolution was already transforming the economic life in Europe, and was strongly based on the application of steam engines developed by individuals such as Thomas Savery, Thomas Newcomen and James Watt. At first, steam engines were used to pump water from mines, but shortly after, they were applied in manufacturing (textile) and transportation (locomotives and steamships). Aware of the economic impact of steam engines, Carnot sought to determine the relevant aspects related to the efficiency of such machines. The result of his work strongly pervades engine technology to this day.

The most notorious contributions of his work are known as Carnot thermodynamic cycle and "Carnot efficiency" (known to be the maximum efficiency of a heat engine). Furthermore, many of the concepts employed in current thermodynamic discussions can already be found in his book. There, Carnot generalized steam engines into the broader realm of heat engines. He defined the concepts of "heat source" along with "heat sink". His assertion regarding the necessity of two heat reservoirs was considered by Rudolf Clausius as an early form of the second law. Before Carnot, engineers strongly focused on heat sources when developing their engines and Carnot draw attention to the fact that a steam engine loses heat to the environment.

Carnot sought to understand the fundamental aspects of a steam engine. To accomplish this task he analyzed a representative system consisting of a gas confined in a cylindrical vessel with a moving embolus subjected to a cyclic sequence of thermodynamic processes (the Carnot cycle). The gas is subjected to two adiabatic (without input or release of heat within the vessel) and two isothermal processes. Each isothermal process is performed with the vessel in thermal contact with a different reservoir, kept at a fixed but distinct temperature, hence, the hot and cold reservoirs. From this study his conclusion that the maximum efficiency of a heat engine depends solely on the temperatures of the heat source and the heat sink follows. In his analysis, Carnot did not use the ideal gas equation, which would be stated by Émile Clapeyron in 1834, but Boyle's law (that he referred to as Mariotte's law) which relates the pressure and the volume of a gas, and Gay-Lussac's law, which in turn relates the temperature and the volume of a gas. It is important to notice that Carnot strongly contributed to the contemporary paradigm of thermodynamics, establishing concepts such as heat reservoir, adiabatic process, thermodynamic cycle, and introducing the representative system consisting of a gas inside a container of a known and controllable volume. Most importantly, Carnot drew attention for considering the system together with its surroundings (which could simply consist of a water bath maintained at constant temperature; in this scenario, the system and its surroundings make up the "Universe" ([8] (p. 52); [9] (p. 62)); thereof the current saying that the "entropy of the universe" is always increasing).

Despite his great achievements, Carnot falls short in some aspects. His analysis is still tied to the view that heat is a substance called "*caloric*". This is especially evident when he compares the motive power of heat with a waterfall. Basically, he describes the workings of a steam engine as the fall (passage) of caloric (heat) from the hot source to the cold sink accompanied by useful work resulting from the process. The resemblance of this process with the image of a watermill is compelling. In the same manner there is no loss of water in the waterfall, Carnot viewed the passage of heat from the hot to the cold reservoir in the steam engine with no loss of "*caloric*". On the conservation of heat, he stated [8] (p. 67): "*This fact has never been called in question. It was first admitted without reflection, and verified afterwards in many cases by experiments with the calorimeter. To deny it would be to overthrow the whole theory of heat to which it serves as a basis.*" At that time, the notion of conservation of heat had an analogous version related to kinetic energy known as "*vis viva*". The concepts of "*caloric*" and "*vis viva*" are inconsistent with the current understanding of conservation of energy and the possibility of transformations between heat and work. This issue is central to the establishment of the second law. Furthermore, Carnot's statement on the conservation of heat is a sample of how some concepts may paradigmatically pervade through a long period of time in the history of science, sometimes eluding even the greatest scientists. Carnot extrapolated conclusions drawn from experiments dealing solely with heat (without interaction with work) to the realm of steam engines, compromising his understanding of the connection between heat and work. His view would later be questioned by individuals such as Rudolf Clausius and James Prescott Joule.

In his first memoir published in 1850 entitled "*On the Moving Force of Heat and the Laws of Heat Which may be Deduced Therefrom*" [10], Clausius confronted Carnot's view on the conservation of heat and defined a broader understanding considering the transformation of heat into work (and *vice versa*)—what is now known as the first law of thermodynamics.

In light of the experimental results obtained by James Joule on the heating manifested by conductors subjected to electric currents, William Thomson (Lord Kelvin) conceded the inaccuracy of the conservation of heat in his "*An account of Carnot's Theory*" published in 1852 [8]. Notice that the change in the view on the conservation of heat was possible due to perspicacious experimental evaluations. In this account, Kelvin reaffirmed Carnot's main statements regarding the efficiency of heat engines and reworked the processes described by Carnot resorting to new developments of thermodynamics such as Clapeyron's graphic representations (nowadays known as pressure *vs.* volume diagrams). From Kelvin's account of Carnot's theory what is now known as the Kelvin-Plank statement of the second law can be derived: "*No process is possible whose sole result is the absorption of heat from a reservoir and the conversion of this heat into work*".

The concept of entropy and its connection to the second law was proposed by Clausius in his fourth memoir entitled: "*On a Modified Form of the Second Fundamental Theorem in the Mechanical Theory of Heat*" published in 1854. This manuscript includes Clausius' statement of the second law [10] (p. 118): "*An uncompensated transmission of heat from a colder body to a warmer body can never occur*". Clausius devised a thermodynamic cycle consisted of three isothermal processes and three adiabatic processes. Afterwards, he proceeded to demonstrate that this cycle is equivalent to Carnot's. In his analysis he defined a quantity that he called "equivalence-value" of a system which is now known as the variation of entropy (ΔS) [10] (p. 126):

$$\Delta S = \frac{\Delta Q}{T} \tag{1}$$

where ΔQ is the heat added to the system and T is the system's temperature. By definition, the adiabatic processes in his cycle presented null "equivalence-value" and the analysis could be focused on the isothermal processes. Hence, in this context it is clear how a system can receive heat and remain at the same temperature. This makes sense once Clausius focused on the transformation between heat and work. In the isothermal processes, heat is supplied/extracted from the system while the volume in increased/decreased resulting in the correspondent amount of work. This analysis connecting the change in entropy with addiction of heat associated with variation of volume is the prelude of the canonical coordinates inherent of the Boltzmannian approach to entropy: "position" and "momentum" which are correlated to the thermodynamic parameters "volume" and "pressure" respectively. In this context, Clausius defined that a reversible thermodynamic cycle would have no net entropy variation, and, accounting that thermodynamic processes are essentially irreversible, he could state the second law with regard of entropy: "*The algebraical sum of all transformations occurring in a cyclical process can only be positive*".

In the turn of the 19th to the 20th century, scientists witnessed great advancement of the corpuscular approach to model physical realm [1]. As mentioned in the introduction, the atomistic approach has been highly successful for modelling Brownian motion, electronic conduction, the periodic table, molecular structure and quantum mechanics. In this setting, the concepts of entropy and the second law have been reformulated under the constructive approach of statistical mechanics. It is worth noticing that, despite the phenomenological stance of thermodynamics, rather abstract approaches became common in late 19th century [11], a trend that was deepened in the field of statistical mechanics that followed, as we demonstrate in the next paragraphs.

The literature, as well as the knowledge on thermal physics based on statistical mechanics is rather vast and multifaceted. In this manuscript we shall focus on the aspects relevant to the astrophysical analysis we present in the next section.

The dawn of statistical mechanics is usually attributed to the works of James Clerk Maxwell and Ludwig Boltzmann on the kinetic theory of gases and heat, beginning in 1860. At that time, very few experimental evidences supported an atomistic model for matter (Brownian motion and crystal cleavage to name two) and atoms were considered a working hypothesis. Analogously to the case of Carnot and thermodynamics, many of the concepts associated with statistical mechanics, such as phase space and ensemble (although by another term) can be found already in the highly influential works of Boltzmann.

According to Boltzmann, a gas is consisted of "material points" featuring a distribution of positions and kinetic energy (which he called "*vis viva*" despite the works of Joule, evidencing again Kuhnian aspects in the evolution of science). Each degree of freedom (dimensional components of momentum and position) represents one dimension of the phase space, rendering a gas of N particles 6N dimensional. Based on Liouville's theorem and probabilistic analysis, entropy and the second law is modeled in a way very similar to what we observe nowadays in textbooks. In this approach, entropy is proportional to the number of possible states presented by a system and is expressed by the famous Boltzmann equation:

$$S = k ln \Omega \tag{2}$$

After Boltzmann and Maxwell, other scientists further established the foundations of statistical mechanics, mainly Josiah Willard Gibbs and Albert Einstein [12]. In this context, great effort was invested searching for, what they called, a "rational mechanical basis of thermodynamics", *i.e.*, a mathematical model based on the laws of Newtonian mechanics to explain thermal phenomena.

In contrast with the works of Boltzmann, which were overabundant and contaminated with rather obsolete concepts, Gibbs produced a synthetic, elegant and mathematically rigorous treatment on statistical mechanics [11,12]. Gibbs was not concerned with the atomistic nature of matter because he aimed to generalize the laws of statistical mechanics to a broader realm of "systems with many degrees of freedom". Gibbs justified his results as a useful mathematical tool to treat, among others, thermodynamic systems, not claiming any meaningful connection with real systems. The term "ensemble" was introduced in statistical mechanics by Gibbs and he focused his analysis on the canonical ensemble as opposed to the microcanonical which was Boltzmann's main studied ensemble. The difference between them is that the canonical ensemble considers a system in thermal equilibrium with a much larger system often referred as a heat reservoir, while the microcanonical ensemble is more suitable for isolated systems.

Einstein's contribution to statistical mechanics took an independent route regarding the works of Gibbs. It has been argued that the corpuscular and mechanical approach was the unifying context behind the apparently unrelated Einstein's early papers dealing with aspects such as statistical mechanics, Brownian motion, relativity (by reading the works of an electron-theory supporter Hendrik Antoon Lorentz), and photoelectric effect [1]. He worked on this subject in the early years of the 20th century while working on the patent office in Bern and was unaware of Gibbs' publication. Einstein aimed to confirm an atomistic nature of gases by deriving a consistent statistical mechanical theory. However, it can also be said that, in the same way of Gibbs, Einstein's works were highly abstract and theoretical. For instance, in the present manuscript we are interested in how models account for the confinement of gases. In his 1902 article entitled "*Kinetic theory of thermal equilibrium and of the second law of thermodynamics*" Einstein defines [13] (p. 417) "*an arbitrary system that can be represented by a mechanical system whose state is uniquely determined by a very large number of coordinates p_1…p_n and the corresponding velocities…*". If the system is a gas or a solid, it is not stated. The energy of the system is given by $E = L + V_a + V_i$, where L is the kinetic energy, V_i is the internal potential and V_a is a potential representing external influences on the system. As Einstein poses [13] (p. 418):

> *"Two kinds of external forces shall act upon the masses of the system. One kind of force shall be derivable from potential V_a and shall represent external conditions (gravity, effect of rigid walls without thermal effects, etc.); their potential may contain time explicitly, but its derivative with respect to time should be very small. The other forces shall not be derivable from a potential and shall vary rapidly. They have to be conceived as the forces that produce the influx of heat. If such forces do not act, but V_a depends explicitly on time, then we are dealing with an adiabatic process."*

We would like to emphasize how speculatively the problem is stated, using a very general system whose energy contains a potential that might represent "rigid walls without thermal effects". Regarding V_a, at the end of the article, Einstein adds "*No assumptions had to be made about of the forces that correspond to potential V_a, not even that such forces occur in nature*" [13] (p. 433).

The similar posture can be observed in contemporary statistical mechanical treatments. To illustrate this point we calculate the volume occupied by a gas in the phase space (Ω which accounts for the number of stated of the system) following the procedure found in a reference textbook on the subject [14] (p. 63). The ensuing deduction is reanalyzed in a subsequent section of this manuscript.

This treatment is performed considering the microcanonical ensemble and the Cartesian coordinate system. Consider a gas of N particles of mass m inside a container of volume V. The energy of the system is given by:

$$E = K + U \tag{3}$$

where K is the kinetic energy and U is the potential representing the interactions between the particles of the gas. The phase space is composed by the momentum (p) and position (q) of each particle in the system. One can say that the volume occupied by the system in the phase space (Ω) is the product of the volume occupied by the possible positions of the particles (φ) multiplied by the volume occupied by the possible momenta of the particles (χ). Thus:

$$\Omega(K, U) = \varphi(q)\chi(p) \tag{4}$$

Evaluating K by decomposing the momentum p in the Cartesian reference we have:

$$\sum_{j=1}^{N}\sum_{\alpha=1}^{3}\frac{p_{\alpha j}^2}{2m} = K \tag{5}$$

where summation in j represents the totality of the particles of the system and the summation in α represents the coordinates x, y and z. Equation (5) is the expression for a hyper-sphere of radius:

$$p = \sqrt{2mK} \tag{6}$$

Thus χ is proportional to the volume of a 3N-dimentional spherical shell of radius p, hence:

$$\chi(p) = (2mK)^{\frac{(3N-1)}{2}} \approx (2mK)^{\frac{3N}{2}} \tag{7}$$

where 1 is neglected when compared to N, which is naturally very large.

To assess φ we have to evaluate the possible positions presented by the particles of the system. To accomplish this, we have to take into consideration the influence of U. However, if one considers a gas with rather small density and rather high kinetic energy (as an ideal gas at room temperature) one can consider U as negligible [14]. Thus, each particle contributes to φ with the volume of the container V, and:

$$\varphi(q) = V^N \tag{8}$$

and Ω is given by:

$$\Omega(K, U) = BV^N(2mK)^{\frac{3N}{2}} \tag{9}$$

where B is a constant to guarantee proper proportionality. The entropy of this system can easily be evaluated by using Equation (2) together with Equation (9). To bring this development closer to a more familiar setting, one can apply the known thermodynamic expression:

$$P = k_B T\,\frac{\partial \ln(\Omega)}{\partial V} \tag{10}$$

to Equation (9) and one finds the ideal gas equation:

$$PV = Nk_B T \tag{11}$$

where k_B is the Boltzmann constant and T is the temperature of the gas. Note how, in the line of reasoning about φ, the potential U is disregarded and the parameter V is introduced with the plain argument that the allowed room for each particle of the gas is the volume of the container. At this point, the author of the book inserts a footnote stating that this could be done by making a potential energy of a particle go to infinity once it tries to penetrate the wall of the container [14] (p. 64). Thus, containers are often poorly defined in statistical mechanics and this is crucial for the scientific realism involved in this subject and in this manuscript. We have to consider that the concept of wall is critical to the definition of pressure, a fundamental thermodynamic (and highly phenomenological) parameter [15] (Vol. I, p.1-3).

The physical parameter pressure is defined as the ratio of force to the area over which that force is distributed. In the corpuscular constructive model, the force is attributed to the collisions of the gas particles on the walls. That definition implies the existence of a surface. Historically this surface is often considered to be the wall of the gas container [9] (p. 12).

The poor definition of walls takes its toll on the constructive explanation of the temperature rise of a compressed gas. If a gas is compressed by an embolus, the temperature rise is usually justified by the average kinetic energy increase of the gas particles while they recoil from the incoming walls of the embolus [15] (Vol. I, p. 1-4).

Similar considerations may be raised regarding other issues. For instance, according to the thermodynamic point of view, the "Universe" consists of the system and its surroundings ([8] (p. 52); [9] (p. 62)). In statistical mechanics, the canonical ensemble considers the system in contact with a much larger system that functions as a heat bath or heat reservoir. This larger system is of the same nature of the smaller one, *i.e.*, usually made of material points featuring position and momentum [16]. How the "Universe" in its contemporary meaning, is reduced to this picture is unsettled. Moreover, textbooks often argue that if one observes an entropy decrease in a given system, then there must be a compensating increase elsewhere [9] (p. 63). This kind of argument we call the "elsewhere argument" where the term "elsewhere" is poorly defined as well. We have to take into account that the strength of a constructive theory is tied to the degree in which such theory is thoroughly constructive.

Thermodynamics is a fertile field for controversy. Since the works of Boltzmann and his H-theorem, many assumptions and postulates are constantly being debated [5]. In his articles, Einstein stated that he aimed to fill a conceptual gap in Boltzmann's theory related to the connection between heat phenomena and Newtonian mechanics through the laws of probability, although prominent scientists of the time, like Paul and Tatiana Ehrenfest never recognized the very existence of this gap [12]. Moreover, it is often argued that temperature is related to kinetic energy of the particles of the system. By this point of view, the difference between a moving body (kinetic energy, related to work) and a hot body (thermal energy) is explained in terms of difference between a coherent and a disordered (or incoherent) motion of the particles that constitutes the body [9] (p. 33). In essence, a moving body has a coherent motion of its parts and can be described as a whole by Newtonian mechanics, while a hot body, due to the disordered movement of its parts has to be described statistically. However, this line of reasoning can be problematic once one tries to consider much smaller systems. In the next section we present another arguable example. Furthermore, it is worth noticing that the thermodynamic definition of entropy (Equation (1)) is related to a process undergone by a system, while the statistical mechanical definition of entropy (Equation (2)) is related to a state of the system. Yet, taking into account the multifaceted aspects of the concept of irreversibility [7], much of the foundations of thermodynamics regarding systems in and out of equilibrium, quasi-static processes, irreversibility, and even time-reversal non-variance are still open for debate [5,7,17].

At this point, we would like to list a few key aspects, or concepts, regarding traditional thermodynamics and statistical mechanics that are reflected on the next section dealing with astrophysical thermodynamics: A1; the container of volume V. A2; the heat reservoir. A3; the Universe and the surroundings. A4; the "elsewhere argument". A5; the thermal equilibrium and the concept of "room temperature". This concept is very intuitive and often taken for granted. This is discussed in the next section. A6; the entropy of the universe is always increasing. This concept preponderates in the current prevailing paradigm and is further discussed in the next section.

3. Entropy and the Second Law in Astrophysics and Cosmology

The treatment of entropy in astrophysical systems has generated surprising aspects when compared to the thermodynamic understanding of traditional systems. This fact is evident in a rather recent manuscript entitled "*Is our Universe Natural?*" published by cosmologist Sean M. Carroll [4]. In this section, we consider several technical manuscripts on the subject, but we give special attention on Carroll's review article due to its tone, suitable to a broader audience, and due to the scope of the journal; as a specialist's appreciation on the subject and oriented to a wider audience, our manuscript often refers to Carroll's state of affairs in the field, which is useful for the discussion we address here. In the abstract Carroll declares that cosmologists and physicists are trying to explain "*how surprising aspects of our Universe can arise from simple dynamical principles*" giving a hint of a statistical mechanical perspective, echoing Einstein's stated goal in his 1902 article. He also states that "*When considering both the state in which we find our current Universe, and the laws of physics it obeys, we discover features that seem remarkably unnatural to us*". Thus, Carroll remarks that astrophysical systems frequently seem unnatural.

Basically, the argument following his article is that, despite being an ever increasing parameter (according to the second law of thermodynamics), the entropy of the Universe is not nearly as high as it could be. This emphasizes a disparity between model and observation that should not be overlooked and has puzzled physicists since Boltzmann [12,17]. Furthermore, this indicates that astrophysical systems deserve special consideration. As we show later on, we argue that this is in fact the case. Of course, Carroll bases his assertion on established literature. In the next paragraphs we intend to demonstrate the influence of historical background in the treatment of entropy in astrophysical systems. More specifically, we show how elements of the theory have been utilized paradigmatically, leading to the so called "surprising" results that render our universe "unnatural". We argue that these results should be considered as symptoms of the scope limitation of the prevailing thermodynamic set of practices and patterns of thought.

An astrophysical system is essentially different than a non-astrophysical system. In the first case, gravity is preponderant. Is this sense, gravitational gas clouds plays central role in understanding the evolution of the Universe and, as Carroll declares: "*Unfortunately, we do not have a rigorous definition of entropy for systems coupled to gravity*" [4]. So let us analyze this issue more closely.

The number of manuscripts dealing with the connection between gravity and entropy is in the order of thousands (On 14 July 2014, a Web of Science search with the terms "gravity" and "entropy" retrieved 3505 hits.) The majority relates to the topic of black hole physics and most of them refer to the so-called "Bekenstein-Hawking entropy", which states that the entropy of a black hole is proportional to its event horizon area.

The concept of black hole entropy started in a set of three manuscripts: "*Black holes and Entropy*", and "*Generalized second law of thermodynamics in black-hole physics*", both published by theoretical physicist Jacob D. Bekenstein in 1973 and 1974, respectively [18,19], and the third one called "*Black holes and thermodynamics*" published by theoretical physicist Stephen W. Hawking in 1976 [20].

According to the central idea behind Bekenstein's 1973 article, there is an analogy, in the sense of the second law of thermodynamics, connecting entropy and a black hole's event horizon. As Hawking would publish his "Hawking radiation" paper only one year later [21], at the time the event horizon was considered as a unidirectional membrane allowing particles to go inwards solely. Thus, the area of the event horizon was regarded as an ever increasing parameter in the same fashion as the entropy of an isolated system. Later on in the manuscript, Bekenstein puts forward a wider analogy, relating a black hole system to the first law of thermodynamics.

To develop a black hole entropy expression, Bekenstein bases his analysis in information entropy, which is in turn based on the works of C. E. Shannon "The Mathematical Theory of Communications". The analysis has a number of shortcomings such as violation of the second law in certain cases, the adaptive method to bring forth an "assumed" mathematical expression for the black-hole's entropy and dimensional inconsistency (remedied by an *ad hoc* inclusion of Planck's constant). Bekenstein himself seems aware of the problems when he states "*At the outset it should be clear that the black-hole entropy we are speaking of is not the thermal entropy inside the black hole*", and afterwards, when he finds an expression for a black-hole's temperature T_{bh}, he states similarly "*But we emphasize that one should not regard* T_{bh} *as the temperature of the black hole; such an identification can easily lead to all sorts of paradoxes, and is thus not useful*." Despite the shortcomings, Bekenstein concludes that "*In fact, one can say that the black-hole state is the maximum entropy state of a given amount of matter*".

The works of Bekenstein gained more attention after the publication of "*Black holes and thermodynamics*" by Hawking. His paper is grounded in the previous publications by Bekenstein (thus biased towards information theory as well) and on his assertion made on a prior article, that black holes can emit radiation (based on quantum fluctuations at the event horizon, Hawking stated that particles could tunnel the event horizon towards infinity [21]). The quantum treatment of black holes accounted for some of the inconsistencies present in Bekenstein's previous work but generated other ones. As Hawking would argue, if black holes radiate, they therefore evaporate, decreasing their mass and the area of their horizon, undermining the very starting point of Bekenstein's 1973 manuscript.

Following Carnot's stance, Hawking studied a system consisted of a black hole and its surroundings, thus preserving the validity of the second law. He did so relying on the "elsewhere" argument. While in Carnot's argument, elsewhere could be identified with the atmosphere (that could be approximated by a big volume of an ideal gas), the same cannot be stated for astrophysical systems. Moreover, Hawking departs from the premise that the second law of thermodynamics should hold, which is clearly paradigmatic. This procedure is valid, as long as the resulting model agrees with observation. We will return to this point later on. But there is another matter in his argument we would like to address. To account for the problem, Hawking constructed a closed system by stating "*Consider a black hole surrounded by black-body radiation in a large container at the same temperature as the black hole*." What is this large container? We recall that thermodynamics was established studying systems such as steam engines, pistons, gases in boxes. In these systems the box of volume V actually "contains" the gas inside the volume V. As we pointed out, the walls of the container (or the sensor) is fundamental in the definition of pressure. In traditional systems the gas spreads out if the box is not present (In solid state physics, the phenomena of electronic and heat transport are often quantum mechanically treated by modeling the solid as box of volume V containing an "electron gas". This treatment gives rise to the Born-von Karman boundary condition to the wave function, essential to its solution. In this case the procedure makes sense once we realize that electrons move (almost) freely in a metal bar while being trapped inside the volume of the bar. Furthermore, the procedure is justified by established experimental evidences [22].). This is clearly not the case in Hawking's system. As we demonstrated in previous section, the actual container of volume V plays key role in determining the phase space volume of the system, defining the statistical mechanical parameter "volume V" in the mathematical expressions. This is an interesting point regarding scientific realism. While containers are ill defined in traditional statistical mechanics, they are virtually assumed in astrophysical thermodynamics. Thus Hawking resorted on an *ad hoc* container to establish his argument. Later on in his manuscript, Hawking tries an even more traditional statistical-mechanical approach utilizing a microcanonical ensemble. In his words: "*Consider, for example, a certain amount of energy E placed in an insulated box of volume V. Assume, for simplicity, that this energy can be distributed only among gravitons and black holes*". The volume V appears as a thermodynamic parameter in the next expression of his manuscript. Therefore we observe that the classical statistical-mechanical system of "N particles inside a box of volume V" turns into "N black-holes (or gravitons) inside a box of volume V". Observe that the box has a specific quality of being insulated. The inclusion an insulating box in the black hole system evidences a paradigmatic construction without an effective tie to an actual physical system. Note that placing a system inside a box of volume V is a standard and well accepted procedure in astrophysics [20,23–27].

The manuscripts of Bekenstein and Hawking serve as a basis for several important and recent works in cosmology and other areas of theoretical physics. As Bekenstein indicated, the physics of a volume of space can be encoded on its boundary (event horizon) paving the way for the so called "holographic principle" put forward by scientists such as Gerard 't Hooft [28] and Leonard Susskind [29]. This represented a particular paradigm shift as statistical mechanics is applied to thermodynamics of horizons [30] leading to the generalized second law of thermodynamics. The term "generalized" is used to address the fact that a new manifestation of entropy, related to horizons, has been added to the thermodynamic landscape [31,32]. It is important to note that early works on black hole thermodynamics were oriented towards information theory. This led to the intriguing view, put forward mainly by Erik Verlinde and held by many researchers as an extended holographic picture, that gravity is not a fundamental force, but an emergent phenomenon [33,34].

Many researchers have been investigating the intertwining relation between general relativity and thermodynamics in Black hole physics. In certain conditions, it is possible to derive the Einstein's equations from the proportionality of entropy to the horizon area together with the fundamental relation $\delta Q = TdS$ [35,36]. Furthermore, studies have shown that gravitational field equations in a wide variety of theories, when evaluated on a horizon, reduce to special form of the first law of thermodynamics [31].

Other interesting developments concern the question of the accelerated expansion of the universe. In specific conditions, black holes can be regarded as the time reversed analog of the Big Bang universe and some researchers have employed a thermodynamical approach to address dark energy [37]. As an alternative form of addressing inflation cosmology, by using Einstein's field equations, the question of the first and second law of thermodynamics has been also addressed in the frame of modified gravity models [38,39].

Let us now focus our attention in another interesting issue, this one related to heat capacity. The measurement of the heat capacity of a body is a straightforward procedure and it is routinely done in many scientific labs around the world. The traditional heat capacity measurement of a body is performed by giving it a controlled amount of heat and measuring the magnitude its temperature rise. It is pertinent to mention that the opposite also works. One can subtract heat from a body and measure its temperature fall. By definition, heat capacity (C_{hc}) involves the relation between heat (Q) and temperature (T) of a body:

$$C_{hc} = \frac{\Delta Q}{\Delta T} \tag{12}$$

The heat capacity of solids, liquids and gases has been modeled successfully, even considering different approaches in thermodynamics and solid state physics. Erwin Schrödinger demonstrated that this quantity is always positive [40] and this is the mainstream view on the case. However, many authors claim negative heat capacity in self-gravitating systems [20,23,24]. So, we should look closely on how negative heat capacities are attributed to astrophysical systems.

Lynden-Bell and Silva *et al.* [23,24] presents a classical evaluation of negative heat capacity for self-gravitating systems. Their evaluation involves the virial theorem. In simple terms, the virial theorem states that for an isolated gravitational system in equilibrium, the relation between the kinetic (K) and potential (U) energy is given by:

$$U = -2K \tag{13}$$

In their study the following expression is used:

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V \tag{14}$$

in combination with:

$$E = U + K = -K \tag{15}$$

to derive:

$$C_V = \left(\frac{\partial(-K)}{\partial T}\right)_V = \frac{\partial\left(-{}^{3}\!/_{2}\, N k_B T\right)}{\partial T} = -\frac{3}{2} N k_B \tag{16}$$

This, according to their analysis, demonstrates the negative heat-capacity of self-gravitating systems. Firstly, Equation (14) is valid for confined systems (derivation at constant volume) [14] so the discussion returns to the container issue. However, there is a more interesting issue in this derivation that is also strongly tied to historical context. In physical mathematical expressions, the symbol "E" is usually connected to the "energy of the system". However, energy can be evaluated in different ways. Scientists treat with several kinds of energy: kinetic, potential, thermal, vibrational *etc.* Each kind reflects a different scope in science. As stated in previous section, the concept of energy conservation replaced the obsolete concepts of *vis viva* and the conservation of *caloric*. However, the actual nature of energy is still unknown [15] (Vol. I, p. 4-1). Therefore different fields of physics and chemistry may attribute different meanings to the symbol "E". In thermodynamics, the total energy "E" is related to heat and work. As statistical-mechanics phase-space deals with position and momentum of particles, "*E*" is strongly tied to kinetic energy (this explains the modern view that the temperature of a gas depends on the average speed of the particles it contains). In Raman scattering phenomena, one has to focus on the vibrational energy related to the internal structure of the gas particles (or the solid). In relativistic problems, mass may be considered as energy. To account for properties of a solid, condensed matter physicists focus on three contributions to energy: structural (crystal lattice), electronic and magnetic (spin) [41]. What is worth noticing is that in each particular case, there is an evaluation on which kind of energy is relevant in the scope of the studied system and one has to be careful while transporting assumptions from one case to another.

In his studies on tacit knowledge, Michael Polanyi questioned modern versions of the Laplacean conception of universal knowledge, based on completely formalized, or mathematical, representation of the universe [42]. According to him, the practical context in which formulas are used is of fundamental relevance. In his words:

> *"… formulas are meaningless unless they bear on non-mathematical experiences. In other words, we can use formulas only after we have made sense of the world to the point of asking questions about it and have established the bearing of the formulas on the experience that they are to explain. Mathematical reasoning about experience must include, beside the antecedent non-mathematical finding and shaping of experience, the equally non-mathematical relating of mathematics to such experience and the eventual, also non-mathematical, understanding of experience elucidated by mathematical theory."*

Lynden-Bell as well as Silva *et al*. took the energy "*E"* from the context of the virial theorem applied to an astrophysical system and used it to perform a calculation in a statistical-mechanical context. There is a subtlety here. If one is interested in calculating heat-capacity, one should be careful while taking into account the potential energy. In Equation (15), "*E*" is the sum of kinetic and gravitational potential energy. Notice that the gravitational potential energy is negative having its zero when the massive objects are infinitely separated. One has to realize that a negative variation in gravitational potential energy results in a positive variation in kinetic energy (part of it may be converted in higher temperature).

Let us illustrate the issue considering a *gedanken* experiment proposed by mathematical physicist John C. Baez [43]. In his discussion regarding thermodynamics of gravity bound systems, John Baez emphasizes, according to his understanding, the paradoxical behavior of such systems and put forward fruitful insights. He proposes the following scenario:

- *SCENARIO 1*: Consider a system consisted of an artificial satellite orbiting the Earth. Equation (13) holds valid for this case. Imagine that a small perturbation slightly deflects the satellite's trajectory, starting a falling process. While the satellite is falling, its potential energy is decreasing and the satellite is gaining kinetic energy.

According to John Baez's interpretation, that is consistent with the model proposed by Lynden-Bell and Silva *et al*. to SCENARIO 1, the augmenting kinetic energy of the satellite means increasing its temperature. Thus, as the potential energy decreases, the temperature increases; ergo negative heat capacity.

So, in one hand we have the traditional approach which is strongly tied to laboratory procedures. On the other hand, we have the rather constructive statistical mechanical approach to the negative heat capacity advocated by Lynden-Bell, Silva and Baez. Note that the laboratory procedures imply an "external agent" giving/taking energy to/from the body. In this context, attributing negative heat capacity to a system means that, if energy is added (in the form of heat or work) to this system, its temperature decreases. In this frame, let us take a closer look to SCENARIO 1 along with the constructive Lynden-Bell and Silva *et al*. approach and contrast it with the phenomenological approach to heat capacity. We can raise several questions: Q1; what is the external agent? The only external action to the system is the "small perturbation". Thus there is no significant external energy added to or subtracted from the system. The kinetic and potential energy involved in the argument of SCENARIO 1 are internal to the system. Q2; how would be the opposite process, in which the temperature decreases? Q3; how the increased kinetic energy of the satellite means higher temperature? What is the constructive statistical-mechanical basis to this claim? We have to remember the argument in previous section regarding a coherent or disordered movement of constituents to distinguish between a moving and a hot body [9] (p. 33). One may assert that the satellite will get hot in atmosphere reentry and collision with the ground (a phenomenological observation). But SCENARIO 1 would be essentially the same in the case of a planet without atmosphere. In this case, the falling process involves a coherent motion (essentially not statistical, *i.e.*, not thermal) until ground collision. How the sudden increase in temperature, when the satellite reaches the ground, relates to the gradual increase/decrease in kinetic/potential energy? Q4; if one assumes that the system Earth/satellite has negative heat capacity, does this mean that it cannot reach thermal equilibrium with another system (as Hawking claims for his system [20])? Q5; what happens if there is an action on the system? Suppose we insert the Sun as an external agent. It is known that, as the Sun gives heat to Earth-satellite system its temperature increases. That is surely inconsistent with a negative heat capacity view. Let us act in another way. Suppose we work on the satellite, pushing it downwards significantly (the analogous of compressing a gas with a piston). Will this process decrease the Earth-satellite temperature (being therefore consistent with a negative heat capacity view)? The accepted view is that the temperature actually increases [44].

As discussed before, experiments (and several models) show that heat capacity is always positive. It is a very unnatural behavior if something gets colder after we give energy to it. On the other hand, we often see systems showing negative entropy variation (we turn on a refrigerator and its interior gets colder, winter arrives, cities sprawl, to name just three). So which concept is more fundamental? Which corresponds to a more natural behavior of a system? Which one is worth sustaining to sponsor further progress in science? That entropy always increases or that heat capacity is always positive? In his quantum mechanical treatment, Hawking concluded that a black hole has negative heat capacity and therefore he claims that it cannot establish thermal equilibrium with its surroundings [20]. These are two statements containing very "unnatural" matters. On the other hand, we see Hawking resorting in placing a black hole inside an *ad hoc* insulated box of volume V in order to assure the second law, while the negative heat capacity is asserted as a fact by him. We can raise similar concerns when one utilizes traditional thermodynamic expressions such as:

$$T^{-1} = \partial S/\partial E \tag{17}$$

(which is an expression derived from Equation (1) from Clausius) in astrophysical systems [18,20,23,24,26]. Hawking used this expression in his paper and this was also the case when Bekenstein derived his expression for T_{bh} in his 1973 manuscript. Clausius proposed Equation (1) to define entropy, in an attempt to understand the workings of steam engines. Later on, Bekenstein and Hawking took the reverse route, departing from a concept of entropy derived from information theory and adapted to the realm of black holes, and then used Equation (17) to assign a temperature to the black hole. It is noteworthy that after Hawking published his manuscript about what is now called "Hawking radiation", the expression for T_{bh} has been considered (even by Bekenstein) the actual temperature of the black hole, despite Bekenstein's disclaimer in his prior to 1975 manuscripts.

Another questionable procedure is the usage of canonical ensemble in astrophysical systems [23,25,27]. It is established that the canonical ensemble presupposes thermal equilibrium between the system of interest and a much larger system (a heat reservoir). A system such as the solar system is in thermal equilibrium with what larger system? What could be its constitution? Could it be consisted of particles with positions and momenta? There is a lack of evaluation of what this larger system could be and what would be the consequences if the system of interest would not be in thermal equilibrium with this reservoir. Hawking arrives at a similar conclusion by a different argument. In his words: "*The fact that the temperature of a black hole decreases as the mass increases means that black holes cannot be in stable thermal equilibrium in the situations in which there is an indefinitely large amount of energy available. (...) this implies that the normal statistical-mechanical canonical ensemble cannot be applied to gravitating systems*" [20].

In Section 2 we argued that thermodynamics is fertile to controversy. In this section we addressed some concerns in foundational works in astrophysical thermodynamics. It is worth noticing that even in recent literature, many concepts regarding horizon entropy are treated as debatable [31].

In the light of the previous paragraphs, we observed that the understanding of entropy in astrophysics and cosmology undergoes several shortcomings: S1; disparity between model and observation. S2; questionable procedures (*ad hoc* containers, unsuitable statistical-mechanical ensembles and thermodynamic laws and expressions placed out of scope). S3; general acceptance of negative heat capacity without critical evidence. S4; misusage of physical parameters such as energy "*E*" and volume "*V*". S5; Manuscripts often contradict one another.

It is straightforward to accept the second law when dealing with phenomenological models such as traditional thermodynamics. When we observe the temperature decrease inside a refrigerator, it is easy to understand how entropy is increased in the environment (elsewhere). We know for a fact that engines have limited efficiency, and the amount of heat generated by the engine is surely larger than the amount extracted from the refrigerator's interior. Once again, this is tied to historical context of thermodynamics which has evolved along with engine technology. However when dealing with a constructive model such as statistical mechanics, the elsewhere argument is often called on without any careful evaluation, as if this procedure is no longer necessary (evidencing again the strength of the second law paradigm). This is what happens in Carroll's manuscript. To account for the rather low entropy of our Universe, Carroll resorts on his own version of the "elsewhere argument": the multiverse model. A statistical mechanical calculation is not presented and, according to Carroll, this is not yet possible because there is no rigorous definition of entropy for systems coupled to gravity. Carroll's view on the entropy of the universe has already been criticized in the frame of time reversibility and the so called "past hypothesis" [17]. How we can better model the entropy of the universe? In the next section we suggest an alternative view.

4. Perspectives on the Second Law of Thermodynamics

In the introduction of his manuscript, Carroll states: "*For configurations, the concept of entropy quantifies how likely a situation is. If we find a collection of gas molecules in a high-entropy state distributed uniformly in a box, we are not surprised, whereas if we find the molecules huddled in a low-entropy configuration in one corner of the box, we imagine there must be an explanation*". We understand that, during a gravitational gas-cloud collapse, gravity drives a high-entropy sparsely distributed gas into a rather organized system, composed in general by spherical bodies describing very determined and predictable trajectories, like our solar system. There is an intuitive justification for the negative entropy variation in the gravitational collapse process. For a matter of simplicity, consider a rather small gravitational gas-cloud with total mass equivalent to a small planet. By doing this, we intend to keep the discussion inside the framework of the canonical coordinates of a statistical mechanical system (position q and momentum p of each particle) avoiding questions regarding internal configuration of each particle (such as energy generated by thermonuclear reactions inside stars). We also intend to avoid deviating discussions regarding non classical solutions, such as event horizon formation and the long-range interactions of galaxy-size systems (and the non-extensiveness of the system's entropy—for more on this see [45]). The high-entropy gas turns into low entropy spherical bodies due to the action of gravity. Think about the Earth-Moon system, on how the gas collapsed into two spherical objects orbiting each other so orderly that we can preview their future positions for millennia (note that our cozy high entropy environment is maintained by the Sun, which is strictly not included in this particular reasoning). Once more we go back to the reasoning regarding the distinction between a hot and a moving body. Note how the gravitational gas cloud collapse involves a coherent inward motion of the particles of the gas, hence not rendering a thermal character to the process (this statement is valid in the rather initial stage of the collapse, when the interactions (except gravitational) between the particles of the gas may still be neglected). Some may argue that, regarding the collapse of a larger cloud, such as the one that resulted in our sun, entropy is increased in the surroundings mainly by electromagnetic radiation. Note that this happens as a result of the gravity driven collapse, when other kinds of interactions come into play, namely electromagnetic and nuclear.

John Baez conjectured on this hypothesis and evaluated the negative entropy variation of a gravitational gas cloud collapse. Despite his venturesome insight, his calculation is based on the virial theorem and incurs on several of the above mentioned shortcomings: an elusive confinement in a volume V (he does not mention a container and its characteristics), the usage of the ideal gas entropy expression for his gas cloud and the acceptance of negative specific heat (the intensive correspondent to negative heat capacity). Nevertheless, John Baez sustained the validity of the second law by resorting on the "elsewhere argument" without any kind of evaluation [43].

We encourage the hypothesis that systems coupled to gravity may present negative entropy variation. Let us enumerate interesting aspects of this view:

i *The low entropy of our Universe*: As peculiar as it may seem, a negative entropy variation for an isolated system by the action of gravity can be considered as a reasonable explanation for our fairly organized Universe.

ii *Energy sources*: In thermodynamics, increasing entropy is commonly linked to energy degradation (the energy can no longer be converted into work). On the other hand, we can think about useful energy being generated in a decreasing entropy process. According to our model, the solar system is the result of a gravitational gas cloud collapse, *i.e.*, a negative entropy variation process. Note that practically all sources of energy we have come essentially from the Sun: solar, fossil fuels, biofuels, biomass, hydroelectric, wind, and wave. Regarding nuclear energy, we have to remember that 99% of the so called baryonic matter of our Universe is hydrogen and helium. All heavier elements were formed inside stars or other gravity dependent cosmic phenomena. Note that tidal and geothermal energy are also connected to gravity. It is worth noticing that this entropic view on energy sources may open new routes for scientific endeavors in progress, such as batteries and hydrogen fusion.

iii *The reverse elsewhere argument*: Consider the following process: a system consisted of two bodies with dissimilar temperatures are put in contact in order to reach thermal equilibrium. It can be proved that, after thermal equilibrium, the entropy of the system is higher than the previous configuration when the two bodies had different temperatures. This specific analysis is correct. It is also accepted that, the higher the temperature of the system, the higher its entropy. This leads to a particularly delicate discussion regarding things getting colder spontaneously in nature (like winter). In fact, these processes are usually despised along with an "elsewhere argument". This meets with the day-by-day notion that, if cold and coherent structures emerge, like a refrigerator's interior or a city, then consequently heat and disorder have been generated in the surroundings (such as power plants for instance). In the example of thermal equilibrium between two objects with distinct temperatures, we should verify how one object became warmer than the other in the first place. If we track down the energy source, we should find a negative entropy variation process (see item ii) from where useful energy has been created. Let us reverse the usual argument: we can say that a local increase in entropy is possible when it is coupled with a decrease in entropy elsewhere. That would be the reverse elsewhere argument. Therefore, practically all of our daily experience with automobiles, cities, flora, fauna, rivers, winds, hot sources, cold sinks, "room temperature" *i.e.*, our familiar high entropy environment is possible because there was once a huge gas cloud that collapsed. We have also to consider, in this argument, a negative entropy variation related to nuclear processes.

iv *Our special location*: Thermodynamically speaking, one of the special features of our planet is something that most people take for granted and is known as "room temperature". In the scope of statistical mechanics, this explains why the canonic ensemble (that requires a heat bath or heat reservoir) has been more commonplace than the microcanonical ensemble. This fact reinforces our argument that models that have been proven successful to our day-by-day phenomena are likely to be fruitless when applied to astrophysical systems. The second law is very useful to understand our quotidian processes, however once we leave our particular environment we should look for a more comprehensive model. Our "room temperature" means we live under an energy balance, in the sense that, in first approximation, the same amount of energy that we receive from the Sun are irradiated back to space. In this sense, the unbalance between the Sun and our planet is entropic, as we receive from the Sun a low entropy form of energy (mainly visible light) irradiating back a high entropy form of energy. For every 1 high energy photon we receive, the Earth radiates about 20 low energy photons. This entropic dissymmetry reinforces item ii.

v *Understanding life*: This issue is connected to item iv. When scientists study life under a cosmological perspective, such as in the field of astrobiology, it is always mentioned that life can occur only under very special conditions. The second law and our rather high entropy environment appear to be connected to the necessary conditions for life. It is interesting to note that Schrödinger introduced the concept of "negative entropy" while developing his model for life [46]. It is connected to the amount of entropy that a living organism has to export to the environment in order to keep its own entropy low. The term has been shortened to "negentropy" and has close relation with free enthalpy applied to molecular biology.

vi *Entropy equilibrium and the arrow of time*: In our view, the rather low entropy of our Universe is due to a balance between processes that decrease entropy (such as gravitational gas cloud collapses) and processes that increase entropy (such as thermal processes). In this scope, the statistical mechanical quandary regarding the connection between process reversibility and the arrow of time loses much of its weight.

It is worth noticing that our hypothesis regarding negative entropy variation for gravity coupled systems considers a statistical mechanical point of view *i.e.*, regards configurations of the phase space. We sustain the thermodynamic view of the second law with respect to heat and engine efficiency (the Clausius and Kelvin statements), which are not directly defined in terms of entropy. So, our proposal takes its toll on the connection between thermodynamics and statistical-mechanics. Of course, this connection has never been unanimous and subjected to criticism such as the discussions regarding the arrow of time problem. In the light of the items listed in this section, we believe the hypothesis might render far more fruits than disservices.

5. Statistical Mechanical Evaluation of a Gravitational Gas Cloud Collapse

In the light of previous sections, it is clear that a new statistical mechanical treatment to gravity-bound systems is worth pursuing. Although the main goal of this manuscript is to highlight possible consequences of paradigmatic aspects in the development of science and to promote an alternative stance on the second law, we intend to put forward a new approach in terms of the phase space for this type of system. Our main goal in this section is to demonstrate that a constructive model based on statistical mechanics to sustain our hypothesis is possible. In this sense, the presented model should be considered a first, rather bold step on the issue, and for this reason, we started from the very fundamental ground within statistical mechanics. There are three main assets in this treatment: we do not resort on any kind of container, we use microcanonical ensemble which is more suitable to isolated systems, and we obtain negative entropy variation.

To evaluate the expression for the number of possible states of a self-gravitating cloud, it is important to discuss the relation between phase space, kinetic energy and potential energy. The phase space is composed by the momentum and position of each particle in the system. We can think about two correlated dualities: position/momentum and potential energy/kinetic energy. The connection between the two dualities can be exemplified in a simple physical system, e.g., a pendulum. When the pendulum's potential energy is at a maximum, so is its position (the height of the oscillating mass). On the other hand, when the pendulum's kinetic energy is maximized, its momentum is as well. Note that in both cases one exists in detriment of the other, *i.e.*, they are complimentary. The same conclusion can be obtained for other types of systems. These assumptions seem to be obvious, but still have severe consequences in the following statistical-mechanics development. As stated in Section 3, the connection between kinetic energy and momentum in the phase space is well established. We propose a connection between potential energy and position in a similar fashion used in the kinetic energy and momentum relation (Equation (6)).

For the sake of simplicity, let us consider a rather small gravitational gas-cloud with total mass equivalent to a planet. Here, we focus on the action of gravitational interaction during the initial phase of the gravitational collapse when the cloud is still sparse and the interaction is mainly attractive. In this regime, the collapse can be considered a quasi-static process. Gravity is a long range interaction. For this reason, we consider that the action of gravity in lowering the entropy of the system occurs mainly in its initial phase (of collapse). Once the cloud becomes denser, other types of interactions come into play, namely electromagnetic and, for larger clouds, nuclear. Hence, these other interactions, which are also related to processes that increase entropy, are relevant after the initial phase. Here, we focus in the action of gravity in the entropic state of the gas cloud.

We follow a procedure similar to the one performed in Section 3 from Equation (4) to Equation (7). However, this time we follow through a different perspective. Again the volume occupied by the system in the phase space (Ω) is the product of the volume occupied by the possible positions of the particles (φ) multiplied by the volume occupied by the possible momenta of the particles (χ). Thus:

$$\Omega(K, U) = \varphi(q)\chi(p) \tag{18}$$

where is a position relative to a potential rather than an absolute reference, as commonly attributed in statistical mechanics, and p is the momentum. Consequently, we have a clear connection between φ and the potential energy similar to the connection between χ and the kinetic energy.

Consider our gas-cloud as being consisted of N particles, each one with mass m and a general interaction potential energy given by:

$$U_{ij}(q) = Cq_{ij}^n \tag{19}$$

where C is a constant, n is an integer and q_{ij} is the distance between the i^{th} and j^{th} particles. Given this general formalization, several different attractive and repulsive potentials can be considered. For simplicity, we will consider a monoatomic gas. Assuming no internal energy, the total energy of the gas can be expressed by:

$$E = K + U \tag{20}$$

where K and U are the total kinetic and potential energy of the system, respectively. To evaluate χ, we proceed as in traditional statistical mechanics and the result is the same of Equation (7). In this context, we rewrite Equation (6) in the form:

$$p_\rho = \sqrt{2mK} \tag{21}$$

Thus, according to the classical evaluation, a possible state of the system is one located on an axis of, χ at a distance p_ρ from the origin. This particular case represents a state in which the total kinetic energy of the system is attributed to a single component of a single particle's momentum. Thus, we can think of the radius p_ρ as a characteristic momentum of the system.

To evaluate φ it is better to simplify the problem by performing a few restrictions. Assuming that each particle feels only the potential of the nearest neighbor in each component xyz of the reference system, the expression for the total potential energy is:

$$U = \sum_{j=1}^{N} \sum_{\alpha=1}^{3} C q_{\alpha j}^{n} \tag{22}$$

which is a reasonable approximation to this problem. In the same way p_ρ was defined as a characteristic moment of the system, a characteristic position q_ρ can also be defined for the system. Assuming that the total potential energy of the system can be attributed to a position q_ρ of a single component of a single particle related to the potential of Equation (22), the expression for q_ρ is:

$$q_\rho = \left(\frac{U}{C}\right)^{\frac{1}{n}} \tag{23}$$

Similarly to the case of χ, we can say that φ is proportional to the volume of the hyper-sphere of radius q_ρ:

$$\varphi(q) = \left(\frac{U}{C}\right)^{\frac{3N}{n}} \tag{24}$$

Next, we conclude that:

$$\Omega(K, U) = B(2mK)^{\frac{3N}{2}} \left(\frac{U}{C}\right)^{\frac{3N}{n}} = B\left(p_\rho^{3N}\right)\left(q_\rho^{3N}\right) = B\left(p_\rho^{3N}\right)\left(V_\rho^{N}\right) \tag{25}$$

where B is a constant that guarantees the proper proportionality. Note that we also expressed Ω as a function of a characteristic volume of the system $V_\rho = q_\rho^3$, which may be applied to study systems that are not confined in an arbitrary volume (such as a self-gravitating cloud).

Considering $S = k ln\Omega$ and Equation (25) we obtain a general expression for the entropy of an interacting gas:

$$S(K, U) = k\ln B + \frac{3N}{2} k\ln(2mK) + \frac{3N}{n} k\ln\left(\frac{U}{C}\right) \tag{26}$$

The second term on right side of the Equation (26) is associated with the kinetic energy and the third term is related to the potential energy. The behavior of the third term will depend on the kind of potential considered. If we consider a self-gravitating gas, then C is a negative constant, represented as $-C_0$, and $n = -1$. Hence:

$$S(K, U) = k\ln B + \frac{3N}{2} k\ln(2mK) - 3Nk\ln\left(-\frac{U}{C_0}\right) \tag{27}$$

Let us simplify the analysis. In the beginning of the collapsing process, our cloud is diffuse, *i.e.*, the particles are far apart from each other. Due to weak interactions, the kinetic energy of the system is very low. Thus, consider $U \approx 0$ and $K \approx 0$, and consequently $E \approx 0$. As the total energy of the system is conserved, we can say that, during the collapsing process, $K = -U$, hence:

$$S(K,U) = k\ln B + \frac{3N}{2}k\ln(2mK) - 3Nk\ln\left(\frac{K}{C_0}\right) \tag{28}$$

As the collapsing proceeds, the kinetic energy increases in detriment of the potential energy. It is clear from Equation (28) that there is a negative variation in the entropy in this particular case. Similar analysis can be deduced for any potential.

6. Conclusions

Selected aspects of the current understanding of astrophysical and cosmological entropy have been discussed in depth. We highlighted issues related to the current paradigm, like *ad hoc* containers, scope of expressions and concepts and unsettled ensemble usage. Such issues, we argue, are inherent to interesting albeit often considered unnatural notions of our universe, such as negative heat capacity, multiverse hypothesis based on entropic arguments and the black hole entropy. Furthermore, we emphasize the disparity between model and observation implicated in the realization that the entropy of our universe is not nearly as high as it could be.

In this context, we demonstrated that a new stance on entropy and the second law is legitimated in the scope of astrophysics. We qualitatively justified and formally modeled the decrease of entropy in processes coupled to gravity. Our model is not related to entropy of horizons therefore representing a departure from established cosmological thermodynamics in a fundamental manner. Furthermore, this new approach opens new research routes in many areas such as cosmology, astrophysics, energy sources and life. For instance, the alternative stance on energy sources presented in Section 4 may impact research on fusion power and photosynthesis.

Finally, along with the astrophysical thermodynamics case study, we emphasized the relevance of the awareness of concepts, such as paradigm, tacit knowledge and scientific realism, in the practice of science.

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Conflicts of Interest

The author declares no conflict of interest.

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