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Entropy Estimation of Generalized Half-Logistic Distribution (GHL) Based on Type-II Censored Samples

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Abstract: This paper derives the entropy of a generalized half-logistic distribution based on Type-II censored samples, obtains some entropy estimators by using Bayes estimators of an unknown parameter in the generalized half-logistic distribution based on Type-II censored samples and compares these estimators in terms of the mean squared error and the bias through Monte Carlo simulations.

Keywords: entropy; expected Bayesian estimation; generalized half-logistic distribution; Type-II censored sample

1. Introduction

Shannon [1], who proposed information theory to quantify information loss, introduced statistical entropy. Several researchers have discussed inferences about entropy. Baratpour *et al.* [2] developed the entropy of upper record values and provided several upper and lower bounds for this entropy by using the hazard rate function. Abo-Eleneen [3] suggested an efficient method for the simple computation of entropy in progressively Type-II censored samples. Kang *et al.* [4] derived estimators for the entropy function of a double exponential distribution based on multiply Type-II censored samples by using maximum likelihood estimators (MLEs) and approximate MLEs (AMLEs).

Arora *et al.* [5] obtained the MLE of the shape parameter in a generalized half-logistic distribution (GHL) based on Type-I progressive censoring with varying failure rates. Kim *et al.* [6] proposed Bayes estimators of the shape parameter and the reliability function for the GHL based on progressively

Type-II censored data under various loss functions. Seo *et al.* [7] developed an entropy estimation method for upper record values from the GHLD. Azimi [8] derived the Bayes estimators of the shape parameter and the reliability function for the GHLD based on Type-II doubly censored samples.

The cumulative distribution function (cdf) and probability density function (pdf) of the random variable, X , with the GHLD are, respectively:

$$F(x) = 1 - \left(\frac{2e^{-x}}{1 + e^{-x}} \right)^\lambda \quad (1)$$

and:

$$f(x) = \lambda \left(\frac{2e^{-x}}{1 + e^{-x}} \right)^\lambda \frac{1}{1 + e^{-x}} \quad x > 0, \lambda > 0 \quad (2)$$

where λ is the shape parameter. As a special case, if $\lambda = 1$, then this is the half-logistic distribution.

The rest of this paper is organized as follows: Section 2 develops an entropy estimation method with Bayes estimators of the shape parameter in the GHLD based on Type-II censored samples. Section 3 assesses the validity of the proposed method by using Monte Carlo simulations, and Section 4 concludes.

2. Entropy Estimation

Let X be a random variable with the cdf, $F(x)$, and the pdf, $f(x)$. Then, the differential entropy of X is defined by Cover and Thomas [9] as:

$$H(f) = - \int_{-\infty}^{\infty} f(x) \log f(x) dx \quad (3)$$

Let $X_{1:n}, X_{2:n}, \dots, X_{n:n}$ be the order statistics of random samples $X_1, X_2, \dots, X_n$ from the continuous pdf, $f(x)$. In the conventional Type-II censoring scheme, r is assumed to be known in advance, and the experiment is terminated as soon as the r -th item fails ($r \leq n$). Park [10] provided the entropy of a continuous probability distribution based on Type-II censored samples in terms of the hazard function, $h(x) = f(x)/(1 - F(x))$, as:

$$H(f) = \sum_{i=1}^r \left[1 - \log(n - i + 1) - \int_{-\infty}^{\infty} f_{i:n}(x) \log h(x) dx \right] \quad (4)$$

where $f_{i:n}(x)$ is the pdf of the i -th order statistic, $X_{i:n}$.

He also derived a nonparametric entropy estimator of Equation (4) as:

$$H_m = \sum_{i=1}^r \left[\log \left\{ \frac{n}{2m} (x_{i+m:n} - x_{i-m:n}) \right\} - \log(n - i + 1) \right] - n \left(1 - \frac{r}{n} \right) \log \left(1 - \frac{r}{n} \right) \quad (5)$$

where m is a positive integer smaller than $r/2$, which is referred to as window size; $X_{i-m:n} = X_{1:n}$ for $i - m < 1$ and $X_{i+m:n} = X_{r:n}$ for $i + m > n$.

Theorem 1. The entropy of the GHLD based on Type-II censored samples is:

$$H(f) = \sum_{i=1}^r [1 - \log(n - i + 1) - I] \quad (6)$$

where:

$$I = \log \lambda - \frac{\Gamma(n+1)}{\Gamma(n-i+1)} \sum_{j=1}^{\infty} \frac{2^{-j} \Gamma(n-i+j/\lambda+1)}{j \Gamma(n+j/\lambda+1)} \quad (7)$$

Proof. Let $u = 1 - F(x)$. Then:

$$\begin{aligned} I &= \int_0^{\infty} f_{i:n}(x) \log h(x) dx \\ &= \frac{\Gamma(n+1)}{\Gamma(i)\Gamma(n-i+1)} \int_0^1 u^{n-i}(1-u)^{i-1} \log \left[\lambda \left(1 - \frac{1}{2} u^{1/\lambda} \right) \right] du \\ &= \log \lambda + \frac{\Gamma(n+1)}{\Gamma(i)\Gamma(n-i+1)} \int_0^1 u^{n-i}(1-u)^{i-1} \log \left(1 - \frac{1}{2} u^{1/\lambda} \right) du \\ &= \log \lambda - \frac{\Gamma(n+1)}{\Gamma(i)\Gamma(n-i+1)} \sum_{j=1}^{\infty} \frac{2^{-j}}{j} \int_0^1 u^{n-i+j/\lambda}(1-u)^{i-1} du \\ &= \log \lambda - \frac{\Gamma(n+1)}{\Gamma(n-i+1)} \sum_{j=1}^{\infty} \frac{2^{-j} \Gamma(n-i+j/\lambda+1)}{j \Gamma(n+j/\lambda+1)} \end{aligned} \quad (8)$$

by using:

$$\log(1-z) = - \sum_{j=1}^{\infty} \frac{z^j}{j} \quad \text{for } |z| < 1 \quad (9)$$

□

Note that entropy Equation (6) depends on the shape parameter, λ . The following subsections estimate λ by using two Bayesian inference methods: one method makes use of a conjugate prior; the other, distributions of hyperparameters that are parameters of a prior.

2.1. Bayes Estimation

Theorem 2. A natural conjugate prior for the shape parameter, λ , is the gamma prior, given by:

$$\pi(\lambda|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}, \quad \alpha, \beta > 0 \quad (10)$$

Then, the Bayes estimator of λ based on the square error loss function (SELF) is obtained as:

$$\hat{\lambda}_S = \frac{r + \alpha}{\beta + h} \quad (11)$$

where:

$$h = (n-r) \log \left(\frac{1 + e^{-x_{r:n}}}{2e^{-x_{r:n}}} \right) + \sum_{i=1}^r \log \left(\frac{1 + e^{-x_{i:n}}}{2e^{-x_{i:n}}} \right) \quad (12)$$

Proof. The likelihood function based on the Type-II censoring scheme is written as:

$$\begin{aligned} L(\lambda) &= \frac{n!}{(n-r)!} [1 - F(x_{r:n})]^{n-r} \prod_{i=1}^r f(x_{i:n}) \\ &= \frac{n!}{(n-r)!} \lambda^r \left(\frac{2e^{-x_{r:n}}}{1 + e^{-x_{r:n}}} \right)^{\lambda(n-r)} \prod_{i=1}^r \left(\frac{2e^{-x_{i:n}}}{1 + e^{-x_{i:n}}} \right)^{\lambda} \frac{1}{1 + e^{-x_{i:n}}} \end{aligned} \quad (13)$$

Then, it is easy to obtain the MLE of λ to be $\hat{\lambda} = r/h$ from Equation (13).

From gamma prior Equation (10) and likelihood function Equation (13), the posterior distribution of λ is obtained as:

$$\begin{aligned} \pi(\lambda|\underline{x}) &= \frac{L(\lambda)\pi(\lambda|\alpha, \beta)}{\int_{\lambda} L(\lambda)\pi(\lambda|\alpha, \beta)d\lambda} \\ &= \frac{(\beta + h)^{r+\alpha}}{\Gamma(r + \alpha)} \lambda^{r+\alpha-1} e^{-(\beta+h)\lambda} \end{aligned} \quad (14)$$

which is a gamma distribution with the parameters $(r + \alpha, \beta + h)$. Here, the Bayes estimator of λ based on the SELF is the posterior mean, and therefore, this completes the proof. $\square$

2.2. E-Bayesian Estimation

Han [11] introduced a new Bayesian estimation method referred to as the expected Bayesian (E-Bayesian) estimation method to estimate failure probability. The present paper uses this method to obtain another Bayes estimator of the shape parameter, λ .

In Subsection 2.1, the gamma prior is supposed to obtain the Bayes estimator of λ . Hence, according to Han [12], the following prior distributions for α and β are considered:

$$\pi_1(\alpha, \beta) = \frac{2(c - \beta)}{c^2}, \quad 0 < \alpha < 1, 0 < \beta < c \quad (15)$$

$$\pi_2(\alpha, \beta) = \frac{1}{c}, \quad 0 < \alpha < 1, 0 < \beta < c \quad (16)$$

$$\pi_3(\alpha, \beta) = \frac{2\beta}{c^2}, \quad 0 < \alpha < 1, 0 < \beta < c \quad (17)$$

which are decreasing, constant and increasing functions of β , respectively.

Theorem 3. For prior distributions (15), (16) and (17), the corresponding E-Bayes estimators of λ based on the SELF are, respectively:

$$\hat{\lambda}_{ES1} = \frac{2}{c^2} \left(r + \frac{1}{2} \right) \left[(c + h) \log \left(1 + \frac{c}{h} \right) - c \right] \quad (18)$$

$$\hat{\lambda}_{ES2} = \frac{1}{c} \left(r + \frac{1}{2} \right) \log \left(1 + \frac{c}{h} \right) \quad (19)$$

$$\hat{\lambda}_{ES3} = \frac{2}{c^2} \left(r + \frac{1}{2} \right) \left[c - h \log \left(1 + \frac{c}{h} \right) \right] \quad (20)$$

Proof. For the prior distribution (15):

$$\begin{aligned}
 \hat{\lambda}_{ES1} &= \int_{\alpha} \int_{\beta} \hat{\lambda}_S \pi_1(\alpha, \beta) d\beta d\alpha \\
 &= \frac{2}{c^2} \int_0^1 (r + \alpha) \int_0^c \frac{c - \beta}{\beta + h} d\beta d\alpha \\
 &= \frac{2}{c^2} \left[(c + h) \log \left(\frac{c + h}{h} \right) - c \right] \int_0^1 (r + \alpha) d\alpha \\
 &= \frac{2}{c^2} \left(r + \frac{1}{2} \right) \left[(c + h) \log \left(1 + \frac{c}{h} \right) - c \right]
 \end{aligned} \tag{21}$$

Similarly:

$$\begin{aligned}
 \hat{\lambda}_{ES2} &= \int_{\alpha} \int_{\beta} \hat{\lambda}_S \pi_2(\alpha, \beta) d\beta d\alpha \\
 &= \frac{1}{c} \left(r + \frac{1}{2} \right) \log \left(1 + \frac{c}{h} \right)
 \end{aligned} \tag{22}$$

and:

$$\begin{aligned}
 \hat{\lambda}_{ES3} &= \int_{\alpha} \int_{\beta} \hat{\lambda}_S \pi_3(\alpha, \beta) d\beta d\alpha \\
 &= \frac{2}{c^2} \left(r + \frac{1}{2} \right) \left[c - h \log \left(1 + \frac{c}{h} \right) \right]
 \end{aligned} \tag{23}$$

□

The following Theorem presents the relationships among E-Bayes estimators $\hat{\lambda}_{ESi}$ ($i = 1, 2, 3$).

Theorem 4. For $0 < c < h$, E-Bayes Estimators satisfy:

$$\hat{\lambda}_{ES3} < \hat{\lambda}_{ES2} < \hat{\lambda}_{ES1} \tag{24}$$

and:

$$\lim_{h \rightarrow \infty} \hat{\lambda}_{ES1} = \lim_{h \rightarrow \infty} \hat{\lambda}_{ES2} = \lim_{h \rightarrow \infty} \hat{\lambda}_{ES3} \tag{25}$$

Proof. From Equations (18)–(20):

$$\hat{\lambda}_{ES1} - \hat{\lambda}_{ES2} = \hat{\lambda}_{ES2} - \hat{\lambda}_{ES3} = \frac{2}{c^2} \left(r + \frac{1}{2} \right) \left[\left(\frac{c}{2} + h \right) \log \left(1 + \frac{c}{h} \right) - c \right] \tag{26}$$

For $0 < c < h$:

$$\begin{aligned}
 \left(\frac{c}{2} + h \right) \log \left(1 + \frac{c}{h} \right) - c &= \left(\frac{c}{2} + h \right) \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{i} \left(\frac{c}{h} \right)^i - c \\
 &= \frac{c^2 + 2ch}{2h} - \frac{c^3 + 2c^2h}{4h^2} + \frac{c^4 + 2c^3h}{6h^3} - \dots - c \\
 &= c \left[\frac{c^2}{12h^2} - \frac{c^3}{12h^3} + \frac{3c^4}{40h^4} - \frac{c^5}{15h^5} + \dots \right] \\
 &= c \left[\frac{c^2}{12h^2} \left(1 - \frac{c}{h} \right) + \frac{3c^4}{40h^4} \left(1 - \frac{8c}{9h} \right) + \dots \right] \\
 &> 0
 \end{aligned} \tag{27}$$

by using:

$$\log(1+z) = \sum_{j=1}^{\infty} (-1)^{j-1} \frac{z^j}{j} \quad \text{for } |z| < 1 \quad (28)$$

That is:

$$\hat{\lambda}_{ES1} - \hat{\lambda}_{ES2} = \hat{\lambda}_{ES2} - \hat{\lambda}_{ES3} > 0 \quad \text{for } 0 < c < h \quad (29)$$

In addition:

$$\begin{aligned} \lim_{h \rightarrow \infty} (\hat{\lambda}_{ES1} - \hat{\lambda}_{ES2}) &= \lim_{h \rightarrow \infty} (\hat{\lambda}_{ES2} - \hat{\lambda}_{ES3}) \\ &= \frac{2}{c} \left(r + \frac{1}{2} \right) \lim_{h \rightarrow \infty} \left[\frac{c^2}{12h^2} \left(1 - \frac{c}{h} \right) + \frac{3c^4}{40h^4} \left(1 - \frac{8c}{9h} \right) + \dots \right] \\ &= 0 \end{aligned} \quad (30)$$

This completes the proof. $\square$

With λ replaced by the estimators, $\hat{\lambda}$, $\hat{\lambda}_S$ and $\hat{\lambda}_{ESi}$ ($i = 1, 2, 3$), in Equation (7), entropy estimators of the GHLD based on Type-II censored samples are obtained as:

$$\hat{H}(f) = \sum_{i=1}^r \left[1 - \log(n-i+1) - \hat{I} \right] \quad (31)$$

$$\hat{H}_S(f) = \sum_{i=1}^r \left[1 - \log(n-i+1) - \hat{I}_S \right] \quad (32)$$

$$\hat{H}_{ESi}(f) = \sum_{i=1}^r \left[1 - \log(n-i+1) - \hat{I}_{ESi} \right] \quad (33)$$

where:

$$\hat{I} = \log \hat{\lambda} - \frac{\Gamma(n+1)}{\Gamma(n-i+1)} \sum_{j=1}^{\infty} \frac{2^{-j} \Gamma(n-i+j/\hat{\lambda}+1)}{j \Gamma(n+j/\hat{\lambda}+1)} \quad (34)$$

$$\hat{I}_S = \log \hat{\lambda}_S - \frac{\Gamma(n+1)}{\Gamma(n-i+1)} \sum_{j=1}^{\infty} \frac{2^{-j} \Gamma(n-i+j/\hat{\lambda}_S+1)}{j \Gamma(n+j/\hat{\lambda}_S+1)} \quad (35)$$

$$\hat{I}_{ESi} = \log \hat{\lambda}_{ESi} - \frac{\Gamma(n+1)}{\Gamma(n-i+1)} \sum_{j=1}^{\infty} \frac{2^{-j} \Gamma(n-i+j/\hat{\lambda}_{ESi}+1)}{j \Gamma(n+j/\hat{\lambda}_{ESi}+1)} \quad (36)$$

3. Application

To evaluate the performance of the estimators, their MSEs and biases are simulated through Monte Carlo simulations. Type-II censored samples are generated from the GHLD with $\lambda = 0.5$ for various Type-II censoring schemes. For each scheme, the MLE and Bayes estimators of the shape parameter, λ , are calculated. For the Bayesian inference, hyperparameter values are chosen as $\alpha = 1$ and $\beta = 2$ with $Var(\lambda) = 0.25$ from gamma prior Equation (10). Then, the Bayes estimator based on the informative prior is obtained. To investigate the effect of the hyperparameter, the Bayes estimator is calculated

based on a vague prior with very small parameters (here, $\alpha = 0.01$ and $\beta = 0.01$). In addition, the E-Bayes estimators, $\hat{\lambda}_{ESi}$ ($i = 1, 2, 3$), are calculated for $c = 2(1)4$, and then, corresponding entropy estimators are obtained. With this procedure repeated 10,000 times, MSEs and biases of these estimators are obtained. Finally, MSEs and biases of nonparametric entropy estimator Equation (5) are simulated for window sizes $m = 2(2)6$. These values are tabulated in Tables 1–5.

Table 1. MSEs(biases) of the maximum likelihood estimator (MLE) and the Bayes estimators.

n	r	$\hat{\lambda}$	$\hat{\lambda}_S$	
			$\alpha = 0.01, \beta = 0.01$	$\alpha = 1, \beta = 2$
20	18	0.00223 (0.01110)	0.00223 (0.01124)	0.00198 (0.01029)
	16	0.00268 (0.01415)	0.00268 (0.01410)	0.00234 (0.01303)
	14	0.00344 (0.01788)	0.00344 (0.01790)	0.00294 (0.01626)
	12	0.00450 (0.02232)	0.00450 (0.02231)	0.00373 (0.01998)
30	28	0.00104(0.00615)	0.00104(0.00624)	0.00096(0.00587)
	26	0.00112(0.00791)	0.00112(0.00801)	0.00104(0.00754)
	24	0.00126(0.00911)	0.00126(0.00921)	0.00115(0.00865)
	22	0.00146(0.01060)	0.00147(0.01072)	0.00133(0.01002)
	20	0.00176(0.01252)	0.00177(0.01264)	0.00159(0.01176)
40	38	0.00060(0.00409)	0.00060(0.00415)	0.00057(0.00395)
	36	0.00063(0.00541)	0.00064(0.00548)	0.00060(0.00523)
	34	0.00069(0.00626)	0.00069(0.00633)	0.00065(0.00604)
	32	0.00077(0.00719)	0.00077(0.00727)	0.00072(0.00693)
	30	0.00086(0.00814)	0.00086(0.00823)	0.00080(0.00783)
	28	0.00097(0.00878)	0.00097(0.00887)	0.00090(0.00842)

Table 2. MSEs (biases) of E-Bayes estimators.

n	r	i	$\hat{\lambda}_{ESi}$		
			$c = 2$	$c = 3$	$c = 4$
20	18	1	0.00231 (0.01555)	0.00211 (0.01088)	0.00196 (0.00633)
		2	0.00211 (0.01081)	0.00190 (0.00396)	0.00180 (−0.00265)
		3	0.00195 (0.00608)	0.00179 (−0.00295)	0.00180 (−0.01162)
	16	1	0.00279 (0.01912)	0.00252 (0.01382)	0.00231 (0.00868)
		2	0.00251 (0.01374)	0.00222 (0.00600)	0.00207 (−0.00144)
		3	0.00230 (0.00836)	0.00203 (−0.00183)	0.00203 (−0.01157)
	14	1	0.00359 (0.02348)	0.00320 (0.01737)	0.00290 (0.01147)
		2	0.00319 (0.01726)	0.00277 (0.00836)	0.00253 (−0.00013)
		3	0.00288 (0.01104)	0.00243 (−0.00064)	0.00243 (−0.01173)
	12	1	0.00469 (0.02875)	0.00413 (0.02155)	0.00369 (0.01464)
		2	0.00411 (0.02140)	0.00349 (0.01098)	0.00312 (0.00109)
		3	0.00364 (0.01406)	0.00294 (0.00040)	0.00294 (−0.01246)

Table 2. Cont.

n	r	$\hat{\lambda}_{ESi}$				
		i	$c = 2$	$c = 3$	$c = 4$	
30	28	1	0.00107 (0.00907)	0.00100 (0.00609)	0.00095 (0.00316)	
		2	0.00100 (0.00606)	0.00093 (0.00166)	0.00091 (−0.00265)	
		3	0.00095 (0.00306)	0.00091 (−0.00277)	0.00093 (−0.00845)	
	26	1	0.00117 (0.01104)	0.00108 (0.00782)	0.00101 (0.00465)	
		2	0.00108 (0.00778)	0.00099 (0.00302)	0.00095 (−0.00162)	
		3	0.00101 (0.00453)	0.00095 (−0.00177)	0.00096 (−0.00789)	
	24	1	0.00131 (0.01248)	0.00121 (0.00898)	0.00113 (0.00555)	
		2	0.00121 (0.00894)	0.00110 (0.00378)	0.00104 (−0.00125)	
		3	0.00112 (0.00540)	0.00104 (−0.00142)	0.00105 (−0.00805)	
	22	1	0.00153 (0.01427)	0.00140 (0.01044)	0.00130 (0.00669)	
		2	0.00140 (0.01039)	0.00126 (0.00475)	0.00118 (−0.00073)	
		3	0.00129 (0.00652)	0.00118 (−0.00094)	0.00118 (−0.00815)	
	20	1	0.00185 (0.01653)	0.00168 (0.01229)	0.00154 (0.00815)	
		2	0.00168 (0.01224)	0.00149 (0.00601)	0.00139 (−0.00002)	
		3	0.00154 (0.00795)	0.00138 (−0.00027)	0.00136 (−0.00819)	
	40	38	1	0.00062 (0.00625)	0.00059 (0.00406)	0.00056 (0.00190)
			2	0.00059 (0.00405)	0.00056 (0.00080)	0.00055 (−0.00240)
			3	0.00056 (0.00184)	0.00055 (−0.00247)	0.00057 (−0.00670)
36		1	0.00066 (0.00769)	0.00062 (0.00537)	0.00059 (0.00308)	
		2	0.00062 (0.00535)	0.00058 (0.00191)	0.00056 (−0.00147)	
		3	0.00059 (0.00301)	0.00056 (−0.00155)	0.00057 (−0.00602)	
34		1	0.00072 (0.00867)	0.00067 (0.00620)	0.00064 (0.00377)	
		2	0.00067 (0.00619)	0.00062 (0.00253)	0.00060 (−0.00105)	
		3	0.00064 (0.00370)	0.00060 (−0.00113)	0.00061 (−0.00587)	
32		1	0.00080 (0.00975)	0.00075 (0.00712)	0.00070 (0.00454)	
		2	0.00075 (0.00710)	0.00068 (0.00322)	0.00065 (−0.00059)	
		3	0.00070 (0.00446)	0.00065 (−0.00069)	0.00066 (−0.00572)	
30		1	0.00090 (0.01086)	0.00083 (0.00805)	0.00078 (0.00529)	
		2	0.00083 (0.00803)	0.00076 (0.00388)	0.00072 (−0.00018)	
		3	0.00078 (0.00520)	0.00072 (−0.00030)	0.00072 (−0.00566)	
28		1	0.00102 (0.01169)	0.00094 (0.00868)	0.00087 (0.00572)	
		2	0.00094 (0.00865)	0.00085 (0.00420)	0.00080 (−0.00014)	
		3	0.00087 (0.00562)	0.00080 (−0.00027)	0.00080 (−0.00600)	

Table 3. MSEs (biases) of the nonparametric entropy estimators.

n	r	H_m		
		$m = 2$	$m = 4$	$m = 6$
20	18	0.05727 (0.20756)	0.06025 (0.22059)	0.08642 (0.27390)
	16	0.06583 (0.22228)	0.07309 (0.24550)	0.10819 (0.31053)
	14	0.07890 (0.24193)	0.09138 (0.27594)	0.13773 (0.35178)
	12	0.09593 (0.26667)	0.11657 (0.31389)	0.17984 (0.40306)
30	28	0.03308 (0.16156)	0.02725 (0.14729)	0.03620 (0.17493)
	26	0.03587 (0.16849)	0.03088 (0.15896)	0.04268 (0.19315)
	24	0.03880 (0.17525)	0.03484 (0.16998)	0.04947 (0.20955)
	22	0.04315 (0.18375)	0.04048 (0.18340)	0.05828 (0.22819)
	20	0.04891 (0.19439)	0.04792 (0.19957)	0.06942 (0.24957)
	18	0.05727 (0.20756)	0.06025 (0.22059)	0.08642 (0.27390)
40	38	0.02460 (0.14259)	0.01640 (0.11477)	0.02001 (0.12952)
	36	0.02624 (0.14731)	0.01816 (0.12217)	0.02306 (0.14138)
	34	0.02786 (0.15161)	0.02000 (0.12880)	0.02599 (0.15110)
	32	0.02972 (0.15641)	0.02225 (0.13612)	0.02943 (0.16125)
	30	0.03206 (0.16178)	0.02487 (0.14381)	0.03325 (0.17169)
	28	0.03429 (0.16653)	0.02762 (0.15142)	0.03753 (0.18264)

Table 4. MSEs (biases) of entropy estimators with the MLE and the Bayes estimators.

n	r	$\hat{H}(f)$	$\hat{H}_S(f)$	
			$\alpha = 0.01, \beta = 0.01$	$\alpha = 1, \beta = 2$
20	18	0.00599 (0.01498)	0.00599 (0.01522)	0.00536 (0.01403)
	16	0.00684 (0.01922)	0.00684 (0.01947)	0.00603 (0.01787)
	14	0.00834 (0.02410)	0.00833 (0.02412)	0.00722 (0.02218)
	12	0.01042 (0.02988)	0.01042 (0.03020)	0.00878 (0.02711)
30	28	0.00291(0.00865)	0.00291(0.00880)	0.00271(0.00830)
	26	0.00303(0.01140)	0.00303(0.01156)	0.00281(0.01092)
	24	0.00331(0.01307)	0.00331(0.01324)	0.00305(0.01248)
	22	0.00376(0.01513)	0.00376(0.01532)	0.00343(0.01438)
	20	0.00441(0.01775)	0.00441(0.01795)	0.00399(0.01679)
	18	0.00599(0.02198)	0.00599(0.02222)	0.00536(0.02003)
40	38	0.00172(0.00590)	0.00172(0.00601)	0.00163(0.00573)
	36	0.00176(0.00805)	0.00176(0.00817)	0.00167(0.00781)
	34	0.00188(0.00932)	0.00189(0.00944)	0.00178(0.00902)
	32	0.00205(0.01069)	0.00205(0.01082)	0.00192(0.01033)
	30	0.00225(0.01205)	0.00225(0.01219)	0.00210(0.01163)
	28	0.00249(0.01288)	0.00250(0.01303)	0.00232(0.01239)

Table 5. MSEs (biases) of entropy with E-Bayes estimators.

n	r	i	$\hat{H}_{ESi}(f)$		
			$c = 2$	$c = 3$	$c = 4$
20	18	1	0.00605 (−0.02246)	0.00568 (−0.01481)	0.00542 (−0.00729)
		2	0.00567 (−0.01471)	0.00534 (−0.00335)	0.00526 (0.00776)
		3	0.00541 (−0.00688)	0.00526 (0.00827)	0.00556 (0.02311)
	16	1	0.00695 (−0.02740)	0.00644 (−0.01892)	0.00609 (−0.01060)
		2	0.00643 (−0.01879)	0.00595 (−0.00622)	0.00579 (0.00605)
		3	0.00606 (−0.01008)	0.00579 (0.00669)	0.00607 (0.02307)
	14	1	0.00850 (−0.03320)	0.00779 (−0.02362)	0.00729 (−0.01425)
		2	0.00778 (−0.02345)	0.00709 (−0.00928)	0.00681 (0.00450)
		3	0.00725 (−0.01358)	0.00679 (0.00532)	0.00704 (0.02371)
	12	1	0.01062 (−0.04023)	0.00962 (−0.02914)	0.00888 (−0.01834)
		2	0.00959 (−0.02891)	0.00858 (−0.01256)	0.00813 (0.00327)
		3	0.00881 (−0.01743)	0.00810 (0.00437)	0.00833 (0.02548)
30	28	1	0.00295 (−0.01359)	0.00281 (−0.00861)	0.00272 (−0.00368)
		2	0.00281 (−0.00856)	0.00269 (−0.00114)	0.00269 (0.00617)
		3	0.00272 (−0.00351)	0.00269 (0.00639)	0.00285 (0.01615)
	26	1	0.00310 (−0.01662)	0.00292 (−0.01131)	0.00280 (−0.00606)
		2	0.00292 (−0.01126)	0.00276 (−0.00335)	0.00272 (0.00444)
		3	0.00279 (−0.00586)	0.00272 (0.00469)	0.00286 (0.01509)
	24	1	0.00340 (−0.01864)	0.00318 (−0.01295)	0.00303 (−0.00733)
		2	0.00318 (−0.01289)	0.00298 (−0.00441)	0.00292 (0.00393)
		3	0.00302 (−0.00709)	0.00292 (0.00421)	0.00306 (0.01534)
	22	1	0.00387 (−0.02112)	0.00360 (−0.01496)	0.00341 (−0.00889)
		2	0.00359 (−0.01489)	0.00333 (−0.00574)	0.00325 (0.00326)
		3	0.00340 (−0.00861)	0.00324 (0.00360)	0.00339 (0.01560)
40	38	1	0.00455 (−0.02424)	0.00420 (−0.01751)	0.00396 (−0.01089)
		2	0.00420 (−0.01743)	0.00386 (−0.00743)	0.00373 (0.00237)
		3	0.00394 (−0.01056)	0.00372 (0.00277)	0.00385 (0.01585)
	36	1	0.00175 (−0.00958)	0.00167 (−0.00589)	0.00163 (−0.00222)
		2	0.00167 (−0.00586)	0.00162 (−0.00035)	0.00163 (0.00511)
		3	0.00163 (−0.00213)	0.00163 (0.00523)	0.00173 (0.01251)
	34	1	0.00181 (−0.01188)	0.00171 (−0.00801)	0.00165 (−0.00417)
		2	0.00171 (−0.00798)	0.00163 (−0.00220)	0.00162 (0.00351)
		3	0.00165 (−0.00406)	0.00162 (0.00364)	0.00171 (0.01127)
	32	1	0.00194 (−0.01333)	0.00183 (−0.00926)	0.00176 (−0.00523)
		2	0.00183 (−0.00923)	0.00173 (−0.00316)	0.00171 (0.00284)
		3	0.00176 (−0.00511)	0.00171 (0.00298)	0.00179 (0.01098)
40	30	1	0.00211 (−0.01490)	0.00199 (−0.01061)	0.00190 (−0.00636)
		2	0.00199 (−0.01058)	0.00187 (−0.00418)	0.00183 (0.00214)
		3	0.00189 (−0.00623)	0.00183 (0.00231)	0.00190 (0.01074)
	28	1	0.00233 (−0.01650)	0.00218 (−0.01195)	0.00207 (−0.00745)
		2	0.00218 (−0.01191)	0.00203 (−0.00514)	0.00198 (0.00155)
		3	0.00207 (−0.00730)	0.00197 (0.00174)	0.00205 (0.01066)
	26	1	0.00258 (−0.01760)	0.00241 (−0.01276)	0.00229 (−0.00797)
		2	0.00241 (−0.01272)	0.00224 (−0.00551)	0.00218 (0.00160)
		3	0.00228 (−0.00780)	0.00217 (0.00181)	0.00225 (0.01130)

From the Tables, the MLE and the Bayes estimator based on the vague prior are very close to each other, and the Bayes estimator based on the informative prior is more efficient than the MLE and the Bayes estimator based on the vague prior. In addition, except for $\hat{\lambda}_{ES1}$ with $c = 2$, E-Bayes estimators outperform the MLE and the Bayes estimator based on the vague prior. The same results are obtained for entropy estimators, and they are more efficient than the nonparametric entropy estimators.

For $\lambda = 1.5$ and $\lambda = 2.5$, the same simulation method is carried out, and the results are consistent with those reported in Tables 1–5. Therefore, the results for $\lambda = 1.5$ and $\lambda = 2.5$ are not reported.

4. Conclusions

This paper discusses entropy estimators for the GHLD with the shape parameter based on Type-II censored samples. The paper derived entropy estimators by using the MLE and Bayes estimators of the shape parameter in the GHLD based on Type-II censored samples and compared them in terms of their MSE and bias. The results show that E-Bayes estimators based on prior Equations (16) and (17) are more efficient than the MLE and the Bayes estimator based on the vague prior for the considered cases. This suggests that the E-Bayesian estimation method based on constant function Equation (16) or increasing function Equation (17) should be used in the case of no prior belief.

Conflicts of Interest

The authors declare no conflict of interest.

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