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Mitigating Supply Uncertainty in Agricultural Supply Chains: A Trust-Based Punishment Mechanism to Reduce the Bullwhip Effect

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Abstract

This study addresses supply uncertainty in the transportation and production of agricultural products within e-commerce-driven supply chains by developing a stochastic supply chain framework. By embedding an endogenous trust-based punishment mechanism, we characterize the strategic interplay between relational governance and operational decisions. In the single-period setting, higher retailer trust leads to larger order quantities while reducing the supplier's optimal supply effort. This occurs because increased trust lowers the minimum delivery commitment ratio, allowing the retailer to voluntarily share more supply risk in exchange for lower-cost products. This creates a mutually beneficial risk-sharing arrangement between the retailer and supplier. When extending the framework to a dual-channel setting with a reliable backup option, the retailer consistently leverages trust and risk-sharing to lower sourcing costs. Concurrently, fluctuations in trust prompt the retailer to strategically shift order allocations between channels based on the primary supplier's maximum effort capacity. Multi-period analysis shows a positive relationship between trust and the supplier's expected profit, with optimal maximum supply effort stabilizing under repeated cooperation. Numerical experiments demonstrate that dual-channel decentralized decision-making achieves bullwhip effect mitigation quantitatively comparable to the centralized benchmark, while also alleviating moral hazard issues and benefiting retailers. These results indicate that the proposed mechanism effectively curbs opportunistic behavior and promotes sustainable, mutually advantageous supply chain collaboration.

Keywords: agricultural e-commerce; supply uncertainty; trust–punishment mechanism; dynamic programming

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1. Introduction

Agricultural supply chains inherently suffer from severe supply uncertainty, driven primarily by stochastic harvest yields [1,2] and significant quantity deterioration during logistical distribution [3]. These unpredictable disruptions frequently lead to supply–demand mismatches, elevated transaction costs, and heightened operational risks for downstream buyers [4]. To mitigate these negative impacts, formal contracts are widely utilized; however, they often incur high enforcement costs and struggle to adapt to rapid market changes [5,6]. Consequently, trust has long been recognized as a critical informal governance mechanism that complements formal contracts in stabilizing trading relationships [7–9].

The rapid development of agricultural e-commerce platforms (such as Hema Fresh or Zfresh) provides a new operational paradigm for studying trust-based governance [10]. In traditional offline agricultural supply chains, transactions are highly fragmented and information asymmetry is pervasive, making it difficult to continuously quantify or enforce "trust" [11]. In contrast, e-commerce platforms feature high-frequency transactions and fully traceable digital footprints. This digital visibility enables platforms to continuously capture historical delivery data and implement dynamic, data-driven reputation evaluations, which serve as automated, institutional governance tools to coordinate decentralized participants [12]. Furthermore, because fresh food e-commerce relies heavily on rapid delivery and strict quality commitments to online consumers, any upstream fulfillment failure immediately triggers severe stockout penalties, making reliable supply effort coordination critical [13]. Despite its practical relevance, existing operations management research lacks a formal mathematical framework to evaluate how such an endogenous trust mechanism interacts with short-term operational decisions and long-term capacity investments.

To bridge this gap, our paper constructs a stochastic two-echelon supply chain model incorporating an endogenous trust-based punishment mechanism tailored to an e-commerce platform setting. We examine an e-commerce retailer) and an upstream supplier under random supply conditions. The retailer's trust in the supplier continuously updates based on historical fulfillment ratios and directly determines the baseline delivery commitment threshold for the next cycle. If the realized delivery falls short of this threshold, the supplier incurs a financial penalty, linking reputation with economic incentives. To hedge against penalty risks, the supplier can invest in variable supply efforts (e.g., cold-chain improvements or smart-farming technologies), creating an analytical trade-off between variable input costs and reputation preservation. Building upon this baseline, we extend our framework to a dual-channel structure by introducing a higher-cost but capacity-stable backup supplier, capturing the multi-sourcing dynamics common among major e-commerce platforms. Furthermore, we develop a multi-period stochastic dynamic programming model to analyze the long-term asymptotic behavior and stability of this trust-governed system.

To visually encapsulate the interaction logic and decision sequences described above, the conceptual architecture of the trust-governed agricultural e-commerce supply chain investigated in this paper is structurally illustrated in Figure 1.

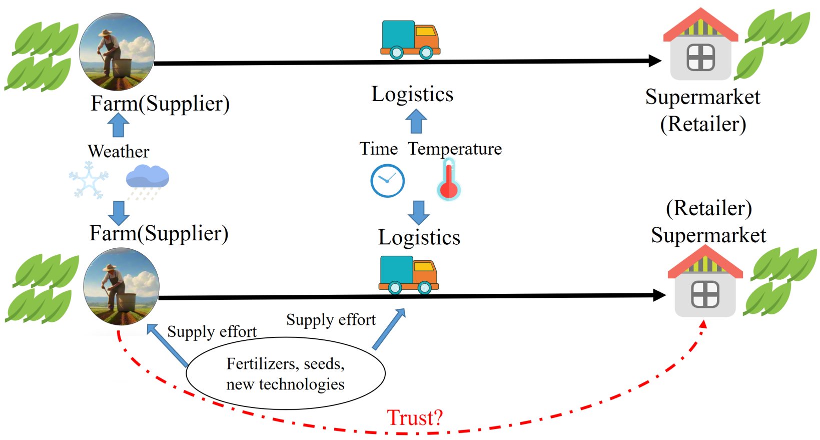

Figure 1. Agricultural Supply Chain Diagram Based on Trust.

To systemically evaluate the efficacy of this governance framework, this study primarily aims to resolve the following core research questions: (1) To mitigate supply uncertainty, what are the supplier's optimal supply effort and the retailer's purchasing plan? (2) What are the strategic choices of suppliers and the retailer under decentralized, centralized, and dual-channel decisions? (3) How does the trust relationship between suppliers and the retailer affect the decision-making and performance of the above supply chain?

By addressing these questions, the main theoretical and practical contributions of this paper are organized as follows. First, we develop a dynamic framework in which trust evolves endogenously based on observable fulfillment performance, thereby transforming trust from a latent relational concept into a state variable embedded in stochastic supply chain optimization. Second, we characterize the joint interaction between ordering decisions, supplier effort, and trust-based contractual constraints, revealing a non-trivial trade-off between coordination efficiency and effort incentives. Third, we extend the model to a dual-sourcing setting and show that trust reshapes sourcing allocation by altering perceived reliability across suppliers, generating a trust-mediated substitution effect. Fourth, we demonstrate that repeated interactions lead to stable supplier effort levels in the long run, while dual sourcing mitigates trust-induced moral hazard and improves supply chain stability.

The rest of this paper is structured as follows. Section 2 reviews the literature on supply uncertainty, supply chain trust, and supply chain coordination. Section 3 gives a detailed model description and hypothesis, and analyzes decisions made by the retailer and supplier under a decentralized supply chain and a centralized supply chain, respectively. Section 4 analyzes the influence of trust value on optimal decisions by obtaining the numerical results of optimal decisions. Section 5 provides a summary of this paper and the future research prospect. See the Appendixes A–E for relevant proof and supplementary analysis of this paper.

2. Literature Review

This paper is positioned at the intersection of three main research streams: supply uncertainty in agricultural supply chains, supply chain coordination mechanisms under uncertainty, and trust-based governance in supply chain management, while each stream offers valuable foundational insights, existing models typically analyze these dimensions in isolation, leaving the conceptual feedback loops between endogenous performance-dependent trust and stochastic operational scaling largely unexamined.

Supply uncertainty remains an inherent vulnerability in agribusiness operations, primarily driven by volatile weather conditions, biological growth variations, and post-harvest quantity deterioration during logistics distribution [14]. Within the quantitative operations management literature, this supply-side friction is systematically formalized using the random supply paradigm, which conceptualizes the realized delivery or harvest quantity as a stochastic multiplicative variable scaling directly with the order or initial input size [15]. A prominent line of inquiry employs proportional random yield distributions to capture unreliable production and procurement processes facing downstream trading entities [16,17]. For instance, models frequently utilize uniform or general stochastic scaling factors to simulate how a supplier's planned capacity translates into highly volatile realized units [18,19]. However, this established paradigm operates under a restrictive transactional assumption: the baseline parameters governing supply reliability are treated as exogenous constants, failing to capture how downstream buyers continuously recalibrate their procurement expectations based on high-frequency operational execution.

To mitigate the adverse consequences of random supply—such as severe supply–demand mismatches and inflated transaction costs—a substantial body of literature focuses on supply chain coordination under uncertainty through contract design. Traditional approaches leverage risk-sharing mechanisms, such as revenue-sharing and buyback contracts, to align decentralized incentives and motivate upstream suppliers to expand their production efforts despite supply disruptions [6,20]. To establish a more robust cushion against catastrophic supply shocks, researchers have expanded single-channel baselines into multi-sourcing and dual-channel sourcing frameworks. This stream extensively evaluates the strategic integration of a reliable but premium-priced backup supplier to hedge against the random disruptions of an unreliable primary channel [21,22]. Optimal volume split and sourcing allocations in these dual-channel environments depend heavily on the correlation of random supplies, supplier capacity limits, and effective landed costs [16,23]. Nevertheless, this stream focuses almost exclusively on physical capacity hedging and mathematical volume allocation, leaving the informal relational governance that underpins continuous contracting completely unmodeled.

Parallel to formal operational contracts, behavioral operations research emphasizes trust-based mechanisms as critical informal governance tools that complement or substitute for high-enforcement formal agreements in stabilizing trading relationships [7,9]. Early analytical formalizations utilize behavioral preference coefficients or "cheap talk" paradigms to define trust as a latent parameter scaling relationship synchronization, supplier compliance, or consumer confidence [24,25]. In decentralized buyer–supplier setups, these soft parameters translate into behavioral reliance factors that smooth joint demand forecasting and capacity reservations under cost-sharing schemes [26,27]. Concurrently, researchers acknowledge the dual nature of relational governance, cautioning that excessive, unmonitored trust can breed organizational blind spots, thereby escalating supplier opportunism and structural inefficiencies [28,29]. Given the multi-period nature of procurement, recent work transitions from static coefficients toward mathematical trust-updating models to track time-varying reputation shifts across sequential transactions [30,31]. However, these dynamic trust frameworks generally abstract away from the physical logistics of supply chains. Stochastic supply degradations, multi-channel portfolio switching, and capacity effort investments are rarely embedded as core state variables, rendering the operational consequences of trust updates invisible.

A systematic cross-examination of these three streams reveals a major theoretical dichotomy: while traditional agricultural supply and coordination models provide sophisticated analytical tools for capacity hedging, they treat the relationship between trading partners as a static transaction parameter. Conversely, behavioral trust governance frameworks capture dynamic relational learning but omit the stochastic physical physics—such as random supply degradation, multi-channel effort coordination, and capacity limits—inherent to fresh agricultural delivery. Our paper explicitly bridges this gap by embedding trust not as an exogenous preference coefficient, but as an endogenous operational state variable that updates dynamically based on observable fulfillment performance. Unlike standard dual-sourcing models, our backup channel transitions from a simple volume buffer into a strategic relational discipline tool. Unlike standard behavioral models, we mathematically map the "dark side" of trust as a localized moral hazard that crowds out supplier effort inputs, establishing the long-term asymptotic convergence of the trust-governed system. The precise positioning of this study against these representative streams is summarized in Table 1.

Table 1. Literature Review Summary.

Representative Studies	Supply Uncertainty	Endogenous Trust	Dual Sourcing	Upstream Effort Input	Multi-Period Dynamics
Dong et al. [16]	√	×	√	×	×
Tang and Kouvelis [18]	√	×	×	×	×
Anderson and Monjardino [1]	√	×	×	×	×
Yan et al. [32]	√	×	×	×	×
Peng et al. [6]	√	×	×	×	×
Xie et al. [20]	√	×	×	×	×
Schmitt and Snyder [21]	√	×	√	×	×
Giri and Bardhan [22]	√	×	√	×	×
Capaldo and Giannoccaro [33]	×	√	×	×	×
Fu et al. [30,31,34]	×	√	×	×	√
Han and Dong [26]	×	√	×	×	√
Özer et al. [24]	×	√	×	×	√
This paper	√	√	√	√	√

3. Model Analysis

This section analyzes a two-tier agricultural supply chain framework under production and logistics uncertainty, progressing systematically from a single-period baseline to a multi-period dynamic environment. We begin by examining a single-period basic case to derive the structural equilibria of order quantities and supply efforts under both decentralized (Stackelberg game) and centralized decision-making modes. This analytical foundation is subsequently extended to a dual-channel configuration incorporating a stable backup supplier to evaluate strategic risk diversification. Finally, we extend the framework into a multi-period stochastic dynamic programming model to capture the long-term co-evolution of the supplier's maximum capacity investment and the retailer's evolving trust value. For reference, all mathematical notations utilized throughout the proofs are systematically summarized in Table A1 of Appendix A.

3.1. Basic Case: Single-Channel Sourcing Framework

We consider a baseline two-tier agricultural supply chain comprising an independent e-commerce retailer and an upstream primary supplier, both acting as risk-neutral and independent economic entities aiming to maximize their respective expected profits under supply uncertainty. To characterize the strategic interplay regarding procurement and operational enhancement, the transaction sequence is modeled as a multi-stage stochastic Stackelberg game, wherein the retailer acts as the market leader and the upstream supplier serves as the follower. The sequence of decisions and physical realizations within each transactional cycle unfolds as illustrated in Figure 2.

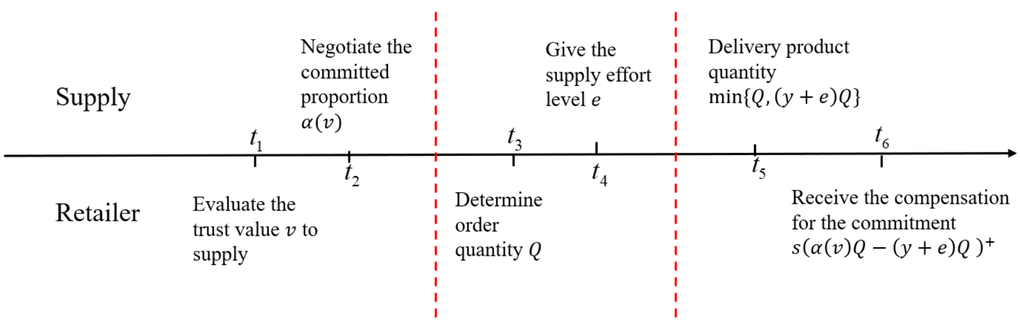

Figure 2. Illustration of the Decision-Making Processes in the Basic Case.

To buffer against disruptions, the supplier commits to sustaining a realized delivery ratio within a baseline interval $[\underline{\alpha}, \overline{\alpha}] \subset [0,1]$. At time t_1, the retailer evaluates the current trust state $v \in [0,1]$. Structurally rationalized by a continuous Beta distribution $B(a_0, b_0)$ derived from historical data [26], this trust value serves as a point estimate to update contract terms. At time t_2, both parties negotiate a trust-contingent delivery commitment threshold formalized as $\alpha(v) = (1-v)\overline{\alpha} + v\underline{\alpha} \in (0,1)$ [24,25,35]. Following this, the strategic game proceeds sequentially. First, the retailer moves at time t_3 by placing an order of quantity Q. Upon observing the order scale, the supplier responds at time t_4 by determining their non-contractable upstream supply effort level e, which incurs a variable investment cost to enhance supply stability. After production and logistics collapse under randomness, the random supply rate $y \sim F(\cdot)$ is realized at time t_5, yielding an actual delivery quantity arriving at the destination expressed as $\min\{Q, (y+e)Q\}$. Finally, if the actual received quantity falls below the trust-dependent commitment threshold, the supplier is contractually obligated to pay a financial penalty to compensate for the supply shortfall. This non-fulfillment compensation is calculated as $s(\alpha(v)Q - (y+e)Q)^+$ and settled at time t_6, where s denotes the unit compensation cost, while the salvage value of unsold items and holding costs are normalized to zero without loss of generality [2,36].

Based on the above decision sequence and a given trust value v, we first analyze the retailer's optimal order quantity and the supplier's optimal supply effort in the decentralized setting.

3.1.1. Decentralized Decision-Making

In decentralized decision-making, both parties independently make their decisions. When confronted with uncertain supply, the retailer determines the order quantity at wholesale price w to meet market demand, while the supplier decides on a supply effort level e [37–39] under the effort coefficient β to mitigate the risk of shortages and produces at unit cost c. Given that the linear assumption effectively captures the fundamental relationship between effort cost and supply effort, this paper models the effort cost as a linear function of supply effort, implying that the additional effort cost incurred by the supplier to ensure greater delivery quantities is represented by βe [38,40–42]. After receiving the agricultural products, the retailer sells them at a price p. We introduce the uncertain supply rate, denoted as y, which follows a random distribution characterized by its probability density function $g(y)$ and cumulative probability density function $G(y)$. Specifically, we assume that the probability density function $g(y)$ of the uncertain supply rate follows a uniform distribution [19,43], that is, $y \sim U(0,1)$. Additionally, to ensure that both the retailer and supplier attain positive profits, we assume $p > w > c > s > 0$. Next, combined with the decision-making process, the supplier's expected profit is given as:

$$\mathbb{E}(\Pi_s) = \int_0^1 \left[w \cdot \min\{(y+e)Q, Q\} - cQ - s(\alpha(v)Q - (y+e)Q)^+ - \beta e\right] g(y)dy, \quad (1)$$

where, in the integral sign, the first item represents the wholesale profit from the actual quantity of products delivered, the second item represents the production cost of the retailer's order, the third item represents the compensation cost incurred due to shortages, and the fourth item represents the cost associated with supply effort aimed at reducing the uncertain supply rate.

Considering the retailer is a market leader, we assume that the market demand D is deterministic for the retailer and satisfies the inequality: $\alpha(v)Q \leq D \leq Q$. Then, the retailer's expected profit is given by:

$$\mathbb{E}(\Pi_r) = \int_0^1 [p \cdot \min\{D, \min\{(y+e)Q, Q\}\} + s(\alpha(v)Q - (y+e)Q)^+ \\ -w \cdot \min\{(y+e)Q, Q\}]g(y)dy, \quad (2)$$

where, in the integral sign, the first item represents the profit generated from the consumer market, the second item represents the compensation received from the supplier due to shortages, and the third item represents the cost associated with the purchase of products from the actual quantity delivered.

According to Equation (2), we explore the optimal decisions of both parties through backward induction and examine how the trust value v impacts these decisions. We summarize these findings in Propositions 1 and 2.

Proposition 1. *In decentralized decision-making under supply uncertainty, $\mathbb{E}(\Pi_s)$ is a concave function with respect to e. The optimal supply effort level e^* with given order quantity Q satisfies: When $0 < \beta \leq (1-\overline{\alpha})wQ$, for each trust value $v \in [0,1]$, $e^* = \frac{wQ-\beta}{wQ}$; When $\beta > (1-\overline{\alpha})wQ$,*

$$e^* = \begin{cases} \dfrac{wQ + s\alpha(v)Q - \beta}{wQ + sQ}, & 0 \leq v < \dfrac{\beta - (1-\overline{\alpha})wQ}{wQ(\overline{\alpha} - \underline{\alpha})} \\ \dfrac{wQ - \beta}{wQ}, & \dfrac{\beta - (1-\overline{\alpha})wQ}{wQ(\overline{\alpha} - \underline{\alpha})} \leq v \leq 1 \end{cases} \quad (3)$$

Proposition 1 suggests that when the effort coefficient β is less than the threshold $(1-\overline{\alpha})wQ$, there is no risk of compensation for the supplier. Additionally, as β exceeds that threshold, the optimal supply effort level e^* increases with the order quantity Q. This indicates that with an increasing Q, the supplier is inclined to elevate the supply effort level to mitigate the risk of shortages and boost wholesale revenue. Furthermore, when the order quantity Q is fixed, it is apparent that the optimal supply effort level e^* diminishes with the trust value v. This implies that as the retailer's trust in the supplier grows, the supplier incurs fewer shortage losses and reduces the supply effort level to minimize cost input.

Proposition 2. *Under supply uncertainty, $\mathbb{E}(\Pi_r)$ is a concave function with respect to Q. The optimal order quantity Q^* in the decentralized decision-making is:*
When $0 < \beta \leq (1-\overline{\alpha})wQ_d$, for each trust value $v \in [0,1]$, $Q^* = Q_d$; When $\beta > (1-\overline{\alpha})wQ_d$,

$$Q^* = \begin{cases} \sqrt{\dfrac{\beta^2(p-s-w) + 2\beta Dp(s+w) + D^2p(s+w)^2}{\alpha(v)^2 s(ps - w(s+w)) + 2\alpha(v)sw(p+s+w) + w(pw + (s+w)(s+2w))}}, & 0 \leq v < \dfrac{\beta - (1-\overline{\alpha})wQ_d}{wQ_d(\overline{\alpha} - \underline{\alpha})} \\ \sqrt{\dfrac{\beta^2(p-w) + 2\beta Dpw + D^2pw^2}{w^2(p+2w)}}, & \dfrac{\beta - (1-\overline{\alpha})wQ_d}{wQ_d(\overline{\alpha} - \underline{\alpha})} \leq v \leq 1. \end{cases} \quad (4)$$

where the order boundary value $Q_d = \sqrt{\frac{\beta^2(p-w)+2\beta Dpw+D^2pw^2}{w^2(p+2w)}}$.

Proposition 2 indicates that when the effort coefficient β is less than the threshold $(1-\overline{\alpha})wQ_d$, the trust–punishment mechanism is weakened as the delivery rate of the supplier improves. However, when β surpasses that threshold, the optimal order quantity Q^* rises with the effort coefficient β. This implies that as β increases, the supplier finds it increasingly challenging to manage supply uncertainty. Consequently, the retailer is more inclined to increase the order quantity to mitigate the risk of market shortages and enhance sales profit. However, it should be noted that this also increases the bullwhip effect and the risk of supplier shortages. Furthermore, it is evident from the proposition that the optimal order quantity Q^* increases with the trust value v. This suggests that when the retailer has greater trust in the supplier, the retailer is more willing to share the supplier's shortage risk. Consequently, the retailer places larger orders to boost actual delivery quantity, even at the cost of potentially reducing sales revenue. This strategic decision aims to create a mutually beneficial outcome for both parties.

Proposition 3. *In decentralized decision-making, the optimal decisions and the optimal profits of both parties have the following characteristics:*
1. The optimal order quantity Q^* is non-decreasing with the trust value v, i.e., $\frac{\partial Q^*}{\partial v} \geq 0$;
2. When $pD > \beta$ and $0 \leq v < \frac{\beta-(1-\overline{\alpha})wQ_d}{wQ_d(\overline{\alpha}-\underline{\alpha})}$, the optimal supply effort level e^* decreases with the trust value v, i.e., $\frac{\partial e^*}{\partial v} < 0$;
3. When $\frac{w-s}{2} - c \geq 0$, increasing the retailer's trust in the supplier can raise the supplier's optimal expected profit but reduce the retailer's optimal expected profit.

Proposition 3 uncovers how informal trust alters the operational equilibrium and risk-sharing dynamics under decentralized decision-making. Structurally, a higher trust value v expands the trading scale ($\frac{\partial Q^*}{\partial v} \geq 0$) because an enhanced supplier reputation diminishes the retailer's perceived transaction risk, encouraging larger procurement commitments.

Conversely, the counter-intuitive negative relationship between trust and supply effort ($\frac{\partial e^*}{\partial v} < 0$) uncovers a localized moral hazard driven by an operational "insurance effect." As trust grows, the negotiated commitment threshold $\alpha(v)$ systematically decreases, lowering the supplier's margin of liability and shielding them from non-fulfillment penalties under random supply shocks. Consequently, the marginal economic benefit of investing in costly effort diminishes, prompting the supplier to rationally crowd out effort inputs to save on variable costs. This demonstrates that while trust expands market scale, excessive trust creates a safety cushion that dampens short-term supply performance incentives.

Furthermore, Proposition 3 clarifies the distributional tension on expected profits. Under the margin condition $\frac{w-s}{2} - c \geq 0$, rising trust drives a redistribution of economic surplus, systematically inflating the supplier's profit while eroding the retailer's returns. By lowering the commitment threshold $\alpha(v)$, the retailer voluntarily absorbs a higher portion of upstream stochastic supply risks in exchange for trade volume, allowing the supplier to capture a relational premium by simultaneously reducing penalty exposure and operational investment costs.

3.1.2. Centralized Model in Basic Case

Centralized decision-making is inherently linked with uniformity, contributing to heightened reliability and compliance, thereby rendering it a more effective approach to decision-making. Within the centralized decision-making framework, the retailer collaboratively shares market demand denoted as D, and both the retailer and supplier jointly determine the production process. Specifically, we use Q^c to represent the retailer's order quantity and e^c to represent the supplier's supply effort level. This sharing of market demand affords the supplier a comprehensive understanding of the discrepancy between market demand and the order quantity, reducing the risk of inventory hoarding. Furthermore, it allows the adjustment of the supply effort level in response to this discrepancy. Consequently, we build an expected profit model for the integrated supply chain,

$$\mathbb{E}(\Pi_c) = \int_0^1 [p\min\{D, \min\{(y+e^c)Q^c, Q^c\}\} - cQ^c - \beta e^c]g(y)dy, \tag{5}$$

where the first item signifies the profit from sales under actual supply conditions, the second item denotes the actual production cost, and the third item accounts for the supplier's expenditure on supply improvement efforts aimed at reducing supply uncertainty.

Next, we will examine how $\mathbb{E}(\Pi_c)$ changes relative to supply effort level e^c and order quantity Q^c. The results are as follows.

Proposition 4. *In centralized decision-making, $\mathbb{E}(\Pi_c)$ is a concave function with respect to e^c and Q^c. We have some claims as follows:*
1. When $Dp < \beta$, both parties cannot cooperate.
2. When $Dp \geq \beta$, the optimal order quantity Q^{c*} and supply effort level e^{c*} are

$$Q^{c*} = \sqrt{\frac{2Dp\beta - \beta^2}{2pc}}, \quad e^{c*} = \frac{pD - \beta}{pQ^{c*}}. \tag{6}$$

Proposition 4 establishes the joint concavity of the centralized system's expected profit, defining the analytical boundaries for supply chain integration. The threshold condition $Dp \geq \beta$ uncovers a fundamental feasibility constraint: system-wide cooperation and capacity alignment are economically sustainable if and only if the market revenue potential is large enough to amortize the upstream operational setup frictions and effort investment costs.

When market benefits are considerable or the marginal cost of effort is small ($Dp \geq \beta$), the unique system-first-best decisions are structurally derived as expressed in (6). By operating as a single, consolidated economic entity, the central planner internalizes the classic double marginalization effect. This integrated configuration coordinates localized decision margins, establishing an aggressive procurement order quantity Q^{c*} to capture downstream demand while simultaneously motivating an intensified supply effort level e^{c*} to compress the variance of the random supply rate. Conversely, when the operating environment degrades—such that $Dp < \beta$ due to an excessive effort cost coefficient or a constrained market size—the central planner's optimization hits a corner solution. Under this distressed landscape, the marginal utility of investing in supply chain reliability is completely cannibalized by the high variable costs, driving the optimal effort level to zero. This analytical divergence demonstrates that macro-level improvements in the market capacity and technical operating environments are non-optional prerequisites for fostering meaningful supply chain alignment and long-term buyer–supplier collaboration.

3.2. Dual-Channel Case

Building upon the theoretical framework of the base case, we extend it to the dual-channel scenario, wherein the retailer operates with two suppliers: a primary supplier a, with whom they maintain a long-term relationship, and a backup supplier b. As the two primary suppliers currently hold a monopoly in the local market, potential new competitors may encounter significant barriers to entry during a business cycle. We assume that supplier a possesses an uncertain supply, whereas supplier b is a reliable partner who provides a certain supply. To analyze their optimal decision-making strategies, we also employ a three-stage stochastic model, encompassing both decentralized and centralized decision-making contexts. In this model, the retailer takes on the role of the leader, and two suppliers act as followers within a Stackelberg game framework. The intricate sequence of this decision-making process is visually depicted in Figure 3, much like in the base case.

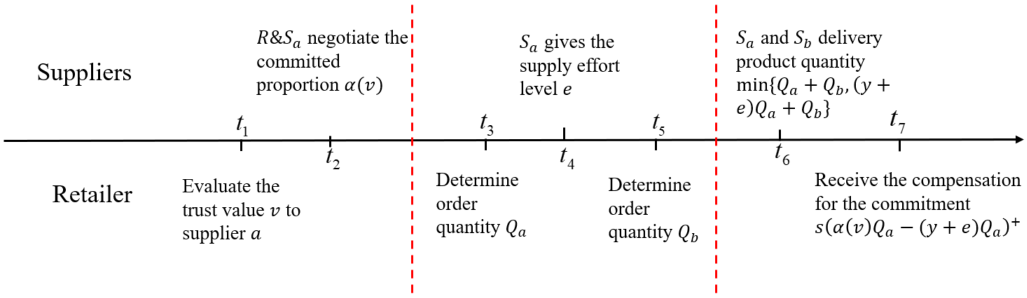

Figure 3. Illustration of the Decision-Making Processes in Dual-Channel Case.

In Figure 3, unlike the decision-making process in the base case, the retailer orders Q_a from supplier a at time t_3 and Q_b from supplier b at time t_5. Upon completing the production of these orders at time t_6, the retailer receives a product quantity, denoted as $\min\{Q_a + Q_b, (y+e)Q_a + Q_b\}$, which is the combined delivery from both suppliers.

Based on the above decision processes, we will first examine the retailer's order quantities and supply effort level of the supplier a within the decentralized model.

3.2.1. Decentralized Model Under a Traditional Arrangement

In the face of uncertain supply, the retailer independently determines order quantities Q_a and Q_b at wholesale prices w_a and w_b, respectively, to fulfill market demand. Meanwhile, supplier a determines a supply effort level to minimize the risk of shortages and produces at a unit cost of c_a. Conversely, supplier b produces at a unit cost of c_b. To ensure that the retailer gives preference to ordering products from supplier a and accounting for the cost of certain supply as a significant factor, we assume that the wholesale price w_a is less than w_b and the production cost c_a is lower than c_b. Therefore, considering this decision-making process, we can derive the expected profit of supplier a as:

$$\mathbb{E}(\Pi_{s_a}) = \int_0^1 (w_a \cdot \min\{(y+e)Q_a, Q_a\} - c_a Q_a - s(\alpha(v)Q_a - (y+e)Q_a)^+ - \beta e) g(y) dy. \quad (7)$$

Clearly , the expected profit equation for supplier a aligns with that of the supplier in the base case. In addition, as supplier b is considered a reliable partner for the retailer, its capacity matches the order quantity placed by the retailer, and its profit solely comprises wholesale revenue. Then, the profit of supplier b:

$$\Pi_{s_b} = (w_b - c_b)Q_b. \quad (8)$$

Furthermore, the relationship of market demand D with order quantities Q_a and order quantities Q_b satisfies inequality $\alpha(v)Q_a + Q_b \le D \le Q_a + Q_b$. Then, the retailer's expected profit is given by:

$$\begin{aligned}\mathbb{E}(\Pi_R) = \int_0^1 & [p \cdot \min\{D, \min\{(y+e)Q_a + Q_b, Q_a + Q_b\}\} + s(\alpha(v)Q_a - (y+e)Q_a)^+ \\ & - w_a \cdot \min\{(y+e)Q_a, Q_a\} - w_b Q_b] g(y) dy,\end{aligned} \quad (9)$$

where, in the integral sign, the first item represents profit from the consumer market, the second item represents compensation received from supplier a for shortages, the third item represents costs associated with purchasing the products delivered by supplier a, and the fourth item represents the purchase cost of products ordered from supplier b.

According to these Equations ((7)–(9)), we explore the optimal decisions of three parties through backward induction. We summarize these findings in Proposition 5.

Proposition 5. *In decentralized decision-making, $\mathbb{E}(\Pi_R)$ is convex and concave with respect to Q_a and Q_b, respectively. Furthermore, we have some claims as follows:*
1. When $0 \leq e_{max} < \frac{w_a Q_L + s\alpha(v) Q_L - \beta}{(w_a+s)Q_L}$, the optimal order quantities (Q_a^*, Q_b^*) is $(0, D)$ and the optimal supply effort level e^* is 0.
2. When $\frac{w_a Q_L + s\alpha(v) Q_L - \beta}{(w_a+s)Q_L} \leq e_{max} < \frac{w_a Q_{U_1} + s\alpha(v) Q_{U_1} - \beta}{(w_a+s)Q_{U_1}}$, the optimal order quantities (Q_a^*, Q_b^*) is

$$
\begin{aligned}
Q_a^* &= \frac{\beta}{w_a(1-e_{max}) + s(\alpha(v) - e_{max})} \\
Q_b^* &= D - \frac{(w_a w_b + s w_b + p w_a + p s \alpha(v))Q_a^* - p\beta}{p(w_a+s)},
\end{aligned}
\tag{10}
$$

and the optimal supply effort level e^* is e_{max}.
3. When $\frac{w_a Q_{U_1} + s\alpha(v) Q_{U_1} - \beta}{(w_a+s)Q_{U_1}} \leq e_{max} < \frac{w_a Q_{U_2} - \beta}{w_a Q_{U_2}}$, the optimal order quantities (Q_a^*, Q_b^*) is

$$
\begin{aligned}
Q_a^* &= \begin{cases} \dfrac{pD(w_a+s) + p\beta}{(w_a+s)w_b + p(w_a + s\alpha(v))}, 0 \leq v < \dfrac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}} \\ \dfrac{\beta}{w(1-e_{max})}, \quad \dfrac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}} \leq vs. \leq 1 \end{cases} \\
Q_b^* &= \begin{cases} 0, \quad 0 \leq v < \dfrac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}} \\ D - \dfrac{(w_a w_b + p w_a)Q_a^* - p\beta}{p w_a}, \quad \dfrac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}} \leq vs. \leq 1 \end{cases}
\end{aligned}
\tag{11}
$$

and the optimal supply effort level is

$$
e^* = \begin{cases} \dfrac{wQ_a^* + s\alpha(v)Q_a^* - \beta}{(w+s)Q_a^*}, \quad 0 \leq v < \dfrac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}} \\ e_{max}, \quad \dfrac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}} \leq vs. \leq 1 \end{cases}
\tag{12}
$$

4. When $\frac{wQ_{U_2} - \beta}{wQ_{U_2}} \leq e_{max} \leq 1$, the optimal order quantities (Q_a^*, Q_b^*) is

$$
Q_a^* = \frac{pDw_a + p\beta}{w_a w_b + p w_a} \quad Q_b^* = 0,
\tag{13}
$$

and the optimal supply effort level e^* is $\frac{wQ_a^* - \beta}{wQ_a^*}$, where $e_{max} \in [0,1]$ is the maximum value of supplier a's supply effort level, Q_L is the equivalent order quantity with 0 placed to supplier a, $Q_{U_1} = \frac{pD(w_a+s)+p\beta}{(w_a+s)w_b + p(w_a+s\alpha(v))}$ and $Q_{U_2} = \frac{pDw_a + p\beta}{w_a w_b + p w_a}$.

Proposition 5 highlights that when the supply effort level is excessively low, the retailer opts solely for supplier b as a supply partner. Moreover, if the lower bound of the commitment proportion $\underline{\alpha}$ is too small, and the retailer places excessive trust in supplier a, the retailer takes on more shortage risk in the base case. However, in the dual-channel scenario, the retailer refrains from cooperating with supplier a and becomes entirely risk-averse. Additionally, in the dual-channel scenario, when the supply effort level e is limited and has a small maximum value, the retailer also avoids cooperating with supplier a. For moderate maximum values of e, the retailer selects both supplier a and b as supply partners to share the supply uncertainty. Nevertheless, for high maximum values of e, the retailer chooses only supplier a as a supply partner, prioritizing lower procurement costs over the risk of stock shortages. These findings offer comprehensive ordering guidance for the retailer.

To explore the impact on the optimal decisions and profits of the three parties, we can derive the following Corollary by combining Propositions 3 and 5.

Corollary 1. *In dual-channel decentralized decision-making, several key corollaries can be drawn.*
1. The optimal order quantity Q_a^* is non-decreasing with the trust value v, i.e., $\frac{\partial Q_a^*}{\partial v} \geq 0$.
2. In Case 3 of Proposition 5, only if $0 \leq v < \frac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}}$, the optimal supply effort level e^* decreases with the trust value v, i.e., $\frac{\partial e^*}{\partial v} < 0$.
3. The optimal order quantity Q_b^* decreases with the trust value v in interval $[0, \frac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}})$ and is equal to a positive constant or 0 in interval $\left[\frac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}}, 1\right]$.
4. When $\frac{w_a - s}{2} - c_a \geq 0$, increasing the retailer's trust in the supplier can raise the supplier a's optimal expected profit but reduce the retailer's optimal expected profit, while the expected profit of supplier b depends on the optimal order quantity Q_b^*.

Corollary 1 systematically delineates how informal trust updates govern sourcing portfolio allocation and risk-sharing dynamics in a dual-channel structure. Driven by the diminished transaction risk perception, the retailer scales up their procurement from the primary supplier ($\frac{\partial Q_a^*}{\partial v} \geq 0$) as trust accumulates. However, this relational expansion triggers a dual behavioral and operational shift constrained by the primary supplier's maximum capacity boundary e_{max}. In the lower trust threshold ($v < \frac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}}$), the operational "insurance effect" persists: the relaxation of the commitment threshold $\alpha(v)$ cushions the primary supplier against non-fulfillment penalties, prompting them to systematically crowd out their supply effort ($\frac{\partial e^*}{\partial v} < 0$). To hedge against the severe stockout risks induced by this localized moral hazard, the retailer strategically leverages the backup channel, aggressively allocating orders to supplier b ($\frac{\partial Q_b^*}{\partial v} < 0$ as v increases). This reveals that within this volatile relational interval, the backup supplier functions not merely as a physical capacity buffer, but as a relational governance tool to discipline upstream opportunism.

Crucially, when trust crosses the critical threshold ($v \geq \frac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}}$), the contractual commitment threshold drops below the primary supplier's maximum capability. Backed by high relational capital, the retailer becomes sufficiently confident in the primary channel's baseline delivery, rendering the expensive backup channel redundant. Consequently, procurement from the backup supplier asymptotically drops to a positive constant or zero, concentrating the entire supply chain footprint onto the primary channel. Under the margin condition $\frac{w_a - s}{2} - c_a \geq 0$, this relational concentration allows supplier a to extract an increasing relational premium—inflating their expected profit while compressing the retailer's margins—whereas supplier b's economic surplus is entirely subordinated to their residual order allocation Q_b^*.

3.2.2. Centralized Model in Dual-Channel Case

In a manner similar to Section 3.1.2, the retailer engages in collaborative demand sharing, with both the retailer and the suppliers jointly shaping the ordering production process. Specifically, we use Q_a^c and Q_b^c to represent the retailer's order quantities from supplier a and supplier b, respectively, and e^c to denote the supply effort level of supplier a. This collaborative demand-sharing provides supplier a with a holistic understanding of the gap between market demand and the ordering quantities, which, in turn, reduces the risk of excess supply effort. Furthermore, it permits adjustments to supplier a's supply effort level and the allocation of orders by the retailer in response to this gap. Consequently, we construct an expected profit model for this integrated supply chain:

$$\mathbb{E}(\Pi_C) = \int_0^1 [p \cdot \min\{D, \min\{(y + e^c)Q_a^c + Q_b^c, Q_a^c + Q_b^c\}\} - c_a Q_a^c - c_b Q_b^c - \beta e^c] g(y) dy, \quad (14)$$

where the first item signifies the profit generated from sales under actual supply conditions, the second item denotes the actual production cost incurred by supplier a, the third item accounts for the production cost of supplier b, and the fourth item represents the supplier's investment in supply improvement efforts, intended to mitigate supply uncertainty.

Next, we will examine how $\mathbb{E}(\Pi_C)$ changes relative to supply effort level e^c and order quantities (Q_a^c, Q_b^c). The results are as follows.

Proposition 6. *In centralized decision-making, $\mathbb{E}(\Pi_C)$ is a concave function with respect to e^c, Q_a^c and Q_b^c, respectively. As the optimal decisions of the three parties cannot reach an agreement, none of them can cooperate, and the centralized case will degenerate into the decentralized case.*

Proposition 6 establishes the joint concavity of the centralized expected profit function $\mathbb{E}(\Pi_C)$, ensuring the existence and uniqueness of the global system-first-best decisions $(e^{c*}, Q_a^{c*}, Q_b^{c*})$. However, in the absence of a formal coordination contract, this integrated optimum serves strictly as an idealized analytical benchmark rather than an achievable market equilibrium. Because the independent supply chain members are inherently self-interested economic entities, their localized decision incentives diverge from the system-wide profit-maximization objective, causing the cooperative centralized structure to organically degenerate into the decentralized Stackelberg game.

Within this centralized baseline, the optimal sourcing portfolio is structurally governed by the marginal efficiency trade-offs between the two channels. When the primary supplier a maintains high supply capacity or investment efficiency, the system-first-best policy concentrates order allocation onto channel a to capture lower marginal costs. Conversely, as supply frictions or capacity constraints tighten on channel a, the central planner dynamically shifts volume to the reliable backup supplier b to hedge against downstream stockout costs. In a decentralized market without risk-sharing alignment, the retailer bears the entire burden of ordering cost inputs while the primary supplier absorbs non-contractable effort risks, preventing them from voluntarily replicating these system-first-best frontiers. This theoretical gap underscores the operational necessity of evaluating decentralized performance-contingent mechanisms to bridge the ensuing efficiency loss.

4. Multi-Period Extension Model

In the preceding section, we examined the optimal supply effort level and order quantities for a single period in both the basic and dual-channel cases. When it comes to decentralized decision-making, we perform a dynamic analysis involving the parameter v to ascertain the optimal decision. However, in the real world, the retailer's trust in the supplier fluctuates based on order delivery outcomes. Therefore, we will investigate the impact of trust values on optimal decision-making in a multi-period context.

In multi-period trading, trust is an endogenous variable, which is obtained from the weighted average of trust in the previous trading and historical trading data. Let $y_t \in [0,1]$ represent the supply rate of an order in period $t \in \mathbb{N}^+$, where $y_t = 1$ indicates full supply at the time of delivery. The supplier invests an effort level e_t to enhance the supply level. Consequently, the actual supply rate of the order is the sum of supply rate y_t and supply effort level e_t, expressed as $Y_t = \min\{y_t + e_t, 1\}$. Additionally, the retailer's trust in the supplier is influenced by this actual supply rate. We use the scalar v_t to denote the retailer's trust value in the supplier at the start of period t. Trust evolves as an exponentially smoothed moving average of the actual supply rate. The trust at the end of period t (and the beginning of $t+1$) is

$$v_{t+1} = \lambda v_t + (1-\lambda) Y_t, \tag{15}$$

and $0 \leq v_t \leq 1$ for all t under the assumptions that the initial trust value satisfies $v_1 \in [0,1]$ and $\lambda \in (0,1)$. In particular, $v_{t+1} = \lambda v_t + (1-\lambda)$ if $Y_t = 1$, and $v_{t+1} = \lambda v_t$ if $Y_t = 0$. The constant parameter λ characterizes the sharpness of the retailer's memory and is henceforth referred to as market memory. A higher value of λ corresponds to retailers with longer-lasting memories. By iterating Equation (15), we have

$$v_{t+1} = (1-\lambda)\sum_{\tau=0}^{t-1} \lambda^{\tau} Y_{t-\tau} + \lambda^{t} v_t. \tag{16}$$

This dynamic equation suggests that the actual supply rate per period has a persistent but diminishing impact. If $Y_t = 0$, then $v_{t+\tau}$ is lower by $(1-\lambda)\lambda^{\tau-1}$ compared to the case when $Y_t = 1$, where $\tau \in \mathbb{N}$.

From Sections 3.1 and 3.2, we have learned that the retailer's trust in the supplier influences the commitment proportion and, consequently, the optimal decisions for all parties within a single period. However, we must also consider the critical role of the maximum supply effort level in optimal decision-making. In multi-period scenarios, after negotiating the commitment proportion, the supplier has to establish a maximum supply effort level e_{max}. As the retailer places orders with the supplier, the retailer incentivizes the supplier to enhance the supply level. The supplier, in response, selects the optimal supply effort level from the supply effort feasible set $(0, e_{max}]$ based on the product order magnitude and the commitment proportion. Let $E_t \in [0,1]$ be the maximum supply effort level in period t. Next, we explore the optimal decisions of both parties under dynamic trust values in the multi-period. This decision-making process is visually illustrated in Figure 4.

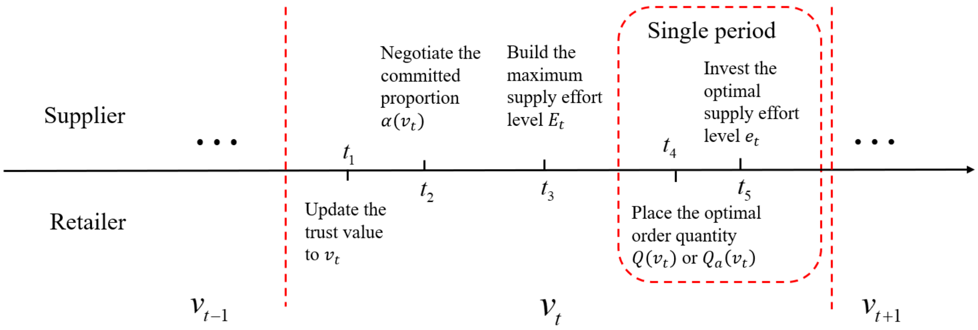

Figure 4. Illustration of the Decision-Making Processes in the Multi-period.

In the dynamic Stackelberg game, building on the theoretical results of a single period, the retailer can place the optimal order quantity $Q(v_t)$ or $Q_a(v_t)$ with a fixed trust value, while the supplier invests the optimal supply effort level e_t. Here, $Q(v_t)$ or $Q_a(v_t)$ is a function of the optimal order quantity Q^* or Q_a^* based on the trust value v_t. Furthermore, the supplier indirectly acquires trust information through the negotiated commitment proportion, establishes the maximum supply effort level E_t, and informs the retailer of this maximum supply effort level. After the transaction at period t, the retailer updates the trust value to v_{t+1} according to Equation (15).

To optimize the maximum supply effort level, we first determine the retailer's optimal order quantity from both centralized and decentralized decision-making perspectives, taking the maximum supply level into account. In centralized decision-making, as Equation (6), the feasible set $\mathbf{Q}_t^c(E_t)$ for order quantity $Q^c(v_t)$ at the maximum supply effort level of the t period is

$$\mathbf{Q}_t^c(E_t) = \left[\max\left\{D, \frac{pD-\beta}{E_t(p-c)+c}\right\}, \frac{pD-\beta}{c}\right]. \tag{17}$$

In decentralized decision-making, from Equation (3), the feasible set $\mathbf{Q}_t(v_t, E_t)$ for order quantity $Q(v_t)$ or $Q_a(v_t)$ at the maximum supply effort level of the t period is

$$\mathbf{Q}_t(v_t, E_t) = \begin{cases} \left[\dfrac{\beta}{w + s\alpha(v_t)}, +\infty\right), & 0 \le v_t < \dfrac{\overline{\alpha} - E_t}{\overline{\alpha} - \underline{\alpha}} \\ \left[\dfrac{\beta}{w}, +\infty\right), & \dfrac{\overline{\alpha} - E_t}{\overline{\alpha} - \underline{\alpha}} \le v_t \le 1 \end{cases}. \tag{18}$$

To determine the retailer's optimal order quantity, we formulate the following optimization objectives in both basic and dual-channel cases. For the basic case, we have

$$Q_b^*(v_t, E_t) = \max_{Q \in \mathbf{Q}_t(v_t, E_t) \backslash [0, D)} \mathbb{E}(\Pi_r(Q, v_t, E_t)) \quad \text{and} \quad Q_b^{c*}(v_t, E_t) = \max_{Q^c \in \mathbf{Q}_t^c(E_t)} \mathbb{E}(\Pi_c(Q^c, E_t)), \tag{19}$$

where $\mathbb{E}(\Pi_r(Q, v_t, E_t))$ represents the retailer's expected profit $\mathbb{E}(\Pi_r)$ based on the trust value v_t and the maximum supply effort level E_t. Similarly, $\mathbb{E}(\Pi_c(Q^c, E_t))$ represents the integrated supply chain's expected profit $\mathbb{E}(\Pi_c(Q^c, e^c(Q^c)))$ based on the maximum supply effort level E_t, where $e^c(\cdot)$ is a continuous function of the order quantity from Equation (6). In the dual-channel case, we have

$$Q_d^*(v_t, E_t) = \max_{Q_a \in \mathbf{Q}_t(v_t, E_t)} \mathbb{E}(\Pi_R(Q_a, v_t, E_t)), \tag{20}$$

where $\mathbb{E}(\Pi_R(Q_a, v_t, E_t))$ represents the retailer's expected profit $\mathbb{E}(\Pi_R(Q_a, Q_b(Q_a)))$ based on the trust value v_t and the maximum supply effort level E_t, where $Q_b(\cdot)$ is a continuous function of Q_a.

By Equations (19) and (20), the retailer determines the optimal order quantity for the t period, taking into account the trust value and maximum supply effort level. Subsequently, the supplier determines the optimal supply effort level, also considering the trust value and maximum supply effort level. Therefore, the supplier's expected profits in the decentralized decision-making are denoted as

$$S_b(v_t, E_t) = \mathbb{E}(\Pi_s[e(Q_b^*(v_t, E_t)), v_t, E_t]) \quad \text{and} \quad S_d(v_t, E_t) = \mathbb{E}(\Pi_s[e(Q_d^*(v_t, E_t)), v_t, E_t]), \tag{21}$$

and the supplier's expected profit in the centralized decision-making is given by

$$S_c(v_t, E_t) = \mathbb{E}(\Pi_s[e^c(Q_b^{c*}(v_t, E_t)), v_t, E_t]), \tag{22}$$

where $e(\cdot)$ is a continuous function of $Q_b^*(v_t, E_t)$, $Q_b^{c*}(v_t, E_t)$ or $Q_d^*(v_t, E_t)$ from Equation (3), as well as of v_t and E_t, and $\mathbb{E}([\Pi_m(e^c(Q), v_t, E_t])$ represents the supplier's expected profit $\mathbb{E}(\Pi_m)$ based on the trust value v_t and the maximum supply effort level E_t.

Next, we investigate the properties of the function $S_i(\cdot, E_t)$ with respect to v_t for each $E_t \in [0, 1]$ in the case $i \in \{d, b, c\}$. We present these results in Proposition 7.

Proposition 7. *When $\frac{w-s}{2} - c \ge 0$ or $\frac{w_a - s_a}{2} - c_a \ge 0$, for each case $i \in \{b, d, c\}$, the following claims holds:*
(1) When $0 \le E_t < e_t^* \le 1$, the optimal order quantity Q_t^* in the case b is non-increasing with v_t.
(2) When $0 \le e_t^* \le E_t \le 1$, the supplier's expected profit $S_i(\cdot, E_t)$ is non-decreasing with v_t in the t-period, where e^* represents the unlimited optimal effort level in period t.

When the maximum effort level is too small ($<e_t^*$), greater retailer trust in the supplier results in a smaller order quantity. This indicates that the supplier's supply capacity is limited, leading the retailer to reduce their order quantity to share the supplier's out-of-stock risk. Conversely, when the maximum supply effort level is sufficiently large ($\ge e_t^*$), the supplier's expected profit $S_i(\cdot, E_t), i \in \{b, d, c\}$ consistently rises with the trust value v_t.

This suggests that through a trust negotiation mechanism, the retailer shares the supply risk with the supplier, thereby reducing the supplier's compensation for stockout. This dynamic fosters a long-term relationship between both parties.

Let H_t represent the history from the start of period 1 to the beginning of period t, where $H_t = (v, E, y, v_1, E_1, y_1, \ldots, v_{t-1}, E_{t-1}, y_{t-1}, v_t)$. A policy for the supplier is an anticipative decision rule that specifies $E_t \in [0, 1]$ for each H_t. We assume that the supplier is risk neutral, it uses geometric discounting with $\delta \in (0, 1)$ as a single-period discount factor, and the expected value of the profit during period t is credited at the beginning of period t. Then, the supplier's expected value is

$$V_i(v, \pi_i) = \mathbb{E}_v^{\pi_i}\left(\sum_{t=1}^{\infty} \delta^t \left[S_i(v_t, E_{i,t}) - \frac{\eta}{2}E_{i,t}^2\right]\right), \quad i \in \{b, d, c\}, \tag{23}$$

where $\frac{\eta}{2}E_{i,t}^2$ represents the maintenance cost of maximum supply effort level in the case i. Let $V_i(\cdot)$ be termed the supplier's value function and π_i^* is said to be an optimal policy for the supplier in the case i if its expected value is maximal for each initial states $v \in [0, 1]$: $V_i(v) = V_i(v, \pi_i^*)$.

Given the initial trust value v, the supplier's dynamic optimization problem is to find a policy that maximizes Equation (23). This problem corresponds to the following dynamic program.

$$V_i(v) = \max_{E_i \in [0,1]} \left\{S_i(v, E_i) - \frac{\eta}{2}E_i^2 + \delta \int_0^1 V_i(\lambda v + (1-\lambda)\min\{y + e(v, E_i), 1\})g(y)dy\right\}, \tag{24}$$

where $e(v, E_i) = e(Q_i^*(v, E_i))$ represents a binary function that depends on both the trust value and the maximum supply effort level. The dynamic Equation (24) reflects a tactical approach to determine the decision variable, which is the maximum supply effort level. An optimal policy with a strategic view is illustrated in the following proposition.

Proposition 8. *For each case $i \in \{b, d, c\}$ and a fixed initial state v, under Proposition 7-(ii), the following claims hold true:*
(i) When $\frac{w-s}{2} - c \geq 0$ (or $\frac{w-s_a}{2} - c_a \geq 0$) and $\lambda + \frac{\partial e(v,E_i)}{\partial v}(1-\lambda) \geq 0$, the supplier's value function $V_i(v)$ is non-decreasing with respect to $v \in [0, 1]$.
(ii) There exists an optimal deterministic policy $\pi_i^* = \{E_i^*, E_i^*, \ldots\}$ and a unique value function $V_i(v)$ for the supplier.

Proposition 8-(i) emphasizes that the supplier's value function features the monotonicity, the same as their expected profit in a single period. It underscores that a retailer with a longer-lasting memory positively influences the value function in multi-period trading. Proposition 8-(ii) posits that the supplier strategically decides their maximum effort supply level, leading to a unique total expected discounted profit with a given initial state v. Additionally, the optimal policy remains stable and can be obtained through value iteration or policy iteration, facilitating the supplier in establishing a fixed supply effort level. Over successive trading periods, the retailer's trust value in the supplier gradually stabilizes.

5. Numerical Analysis

To validate our theoretical findings, the parameter matrix is calibrated by bifurcating empirical market sourcing and established literature paradigms [6,19,44]. Crucially, all core financial price-and-cost parameters are strictly anchored on empirical agribusiness marketplace statistics from the Ministry of Agriculture and Rural Affairs (MOA), the United States Department of Agriculture (USDA), the 21food platform, and the China Edible

Fungi Business Network [45–48]. These benchmarks for fresh produce and stable grains demonstrate a stable price-to-cost scaling structure (c:w:p = 2:5:15 CNY/kg). Accordingly, we directly set the baseline parameters as $p = 15$, $w = w_a = 5$, and $c = c_a = 2$. For the dual-channel extension, the backup channel variables are symmetrically initialized as $w_b = 6$ and $c_b = 3$, reflecting the typical premium landed cost of secure capacity adjustments.

Conversely, the remaining behavioral and contract parameters—which are unobservable from open marketplace transactions—are calibrated following standard settings in upstream operations research literature [6,19,44]. These baseline values are configured as follows: market demand $D = 200$, penalty rate $s = 0.5$, effort cost barrier $\beta = 500$, upper commitment bound $\overline{\alpha} = 0.8$, lower commitment bound $\underline{\alpha} = 0.5$, demand volatility factor $\lambda = 0.5$, trust discount factor $\delta = 0.8$, and capital transition friction $\eta = 400$.

5.1. Comparative Study: Single-Period Single-Channel and Dual-Channel Operations

In Section 3, we derived optimal decisions for both suppliers and retailers under (i) centralized and decentralized structures in single-channel systems, and (ii) decentralized structures in dual-channel systems, within a single-period framework. This section extends the analysis through numerical experiments to examine how single or dual-sourcing strategies influence supply chain participants' decisions, channel selection behaviors, and the bullwhip effect across different decision-making frameworks.

Figure 5 illustrates the impacts of trust v on single-channel outcomes. Within the gray effective boundary of the relational contract, trust serves as an efficient transactional lubricant that stabilizes order volatility. However, when trust transcends the critical threshold ($v = 0.795$), the retailer's unconditional risk absorption deactivates the trust–punishment mechanism. This structural collapse triggers a non-linear jump in order quantity, expanding the bullwhip effect. This yields a vital governance insight: while balanced trust curbs demand distortion, excessive or blind trust removes essential contract frictions, causing severe double marginalization and over-ordering inefficiencies that pull decentralized profits far below the centralized integrated benchmark seen in Figure 5b.

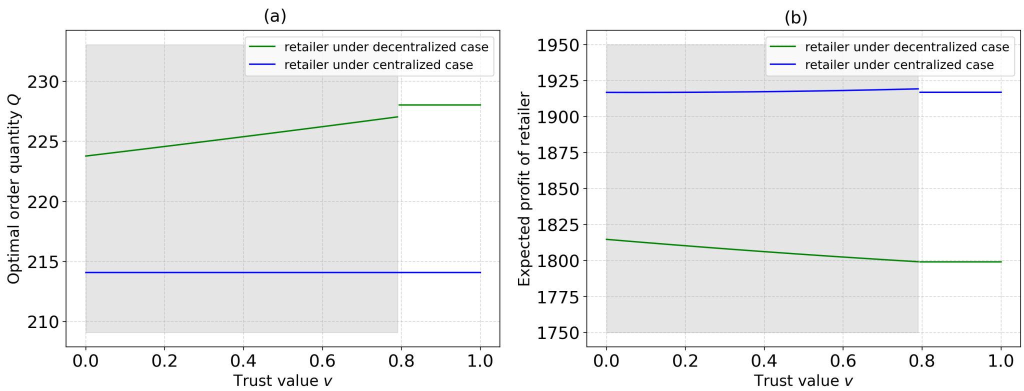

Figure 5. Optimal outcomes under different trust values: (**a**) impact of trust value v on the optimal order quantity Q; (**b**) impact of trust value v on the expected profit of the retailer.

Figure 6 captures parameter interactions under dual-channel procurement. Figure 6a shows that higher supplier effort compresses total order quantities; enhanced upstream reliability reduces downstream stockout variance, enabling the retailer to streamline safety stock outlays. The inner subgraph reveals that higher trust systematically shifts order allocation toward Supplier a. Figure 6b tracks the portfolio risk trade-offs: under constrained upstream effort, severe stockout penalties drive heavy sourcing from the high-cost backup Supplier b, cannibalizing initial profits. As Supplier a's capacity matures, the mitigated supply uncertainty allows the retailer to leverage primary channel cost advantages, driving sustained profit recovery and proving the economic viability of multi-sourcing as a capacity-hedging tool.

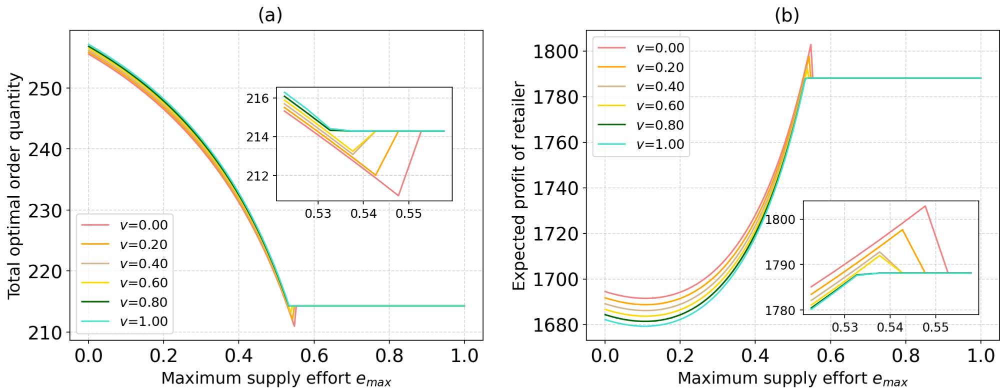

Figure 6. Optimal dual-channel outcomes under different trust values v and maximum supply efforts e_{max}: (**a**) total optimal order quantity (with the inner subgraph magnifying the adjustment behavior around $e_{max} \in [0.52, 0.56]$); (**b**) expected profit of the retailer (with the inner subgraph clarifying localized profit peaks).

A cross-examination of Figures 5 and 6 yields a critical structural insight: the trust–punishment mechanism effectively mitigates the bullwhip effect across both single- and dual-channel networks. Notably, the total decentralized procurement volume under the dual-channel configuration closely approaches the idealized centralized single-channel baseline—both being significantly lower than the inflated decentralized single-channel volume. This convergence provides rigorous numerical evidence that the dual-sourcing structure significantly amplifies the bullwhip-suppression capability of the behavioral contract mechanism. Intriguingly, within a single-period landscape, the retailer maintains a systematic preference for single-channel procurement. This preference is driven by a self-reinforcing operational loop: a concentrated, larger order scale in a single channel maximizes the supplier's marginal return on effort, which in turn motivates upstream capacity optimization, compresses shortage risks, and yields superior expected retail profits.

Figures 7 and 8 track the primary supplier's non-contractable effort and profit. The downward trajectory of effort relative to trust in Figure 7a reflects the contract-theoretic "Insurance Effect". As goodwill increases, behavioral alignment softens immediate penalty threats, providing an empirical "insurance" that allows the supplier to scale down economically inefficient variable effort expenditures to minimize localized operational costs. Crucially, this downward slope does not imply an escalation of supplier opportunism; as long as trust operates within the shaded gray effective regime, the mechanism successfully forces a strictly positive effort floor that bounds moral hazard, optimizing cost efficiency rather than driving draining, infinite effort. Symmetrically, Figure 8a shows that the supplier maintains maximum effort under capacity constraints to avoid channel displacement, but stabilizes at a cost-efficient equilibrium once core decentralized demand is satisfied.

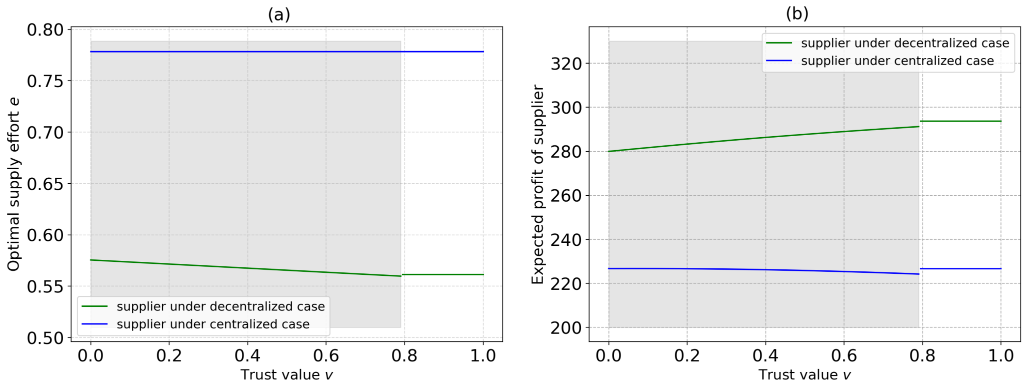

Figure 7. Optimal outcomes under single-channel case: (**a**) optimal supply effort e versus trust value v; (**b**) expected profit of the supplier versus trust value v.

A comparative baseline across Figures 7 and 8 provides vital observations regarding upstream behavior and its managerial implications. First, as rational, independent actors, suppliers under both decentralized structures maintain lower optimal effort profiles relative to the centralized integrated baseline, avoiding the full absorption of variable operational risks when systemic coordination contracts are absent. Second, critically, the dual-channel configuration exhibits an even lower effort level than the decentralized single-channel baseline. This phenomenon is not driven by behavioral shirking, but represents a rational economic response to dual-channel order diversion. Because the dual-sourcing structure splits the total procurement scale, the main supplier's localized marginal utility of effort scales down, driving an optimal, cost-conscious recalibration of capacity. Consequently, single-period suppliers maintain a profound preference for decentralized single-channel structures, where concentrated downstream procurement scales maximize upstream profit margins.

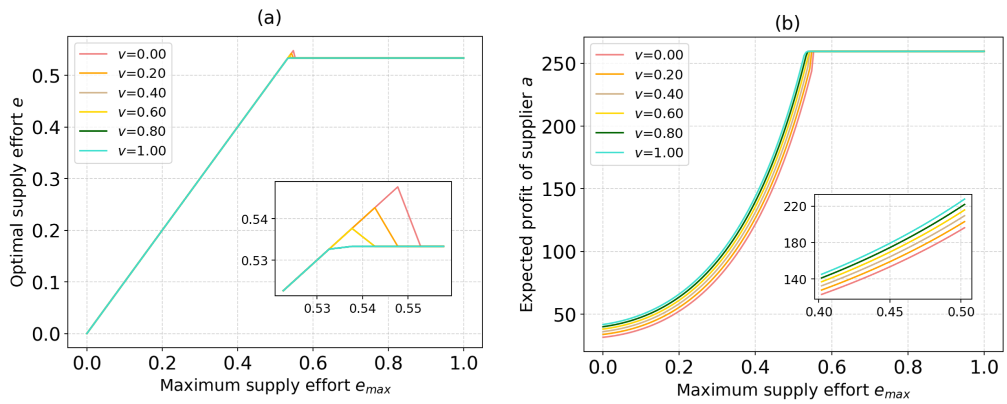

Figure 8. Optimal outcomes for supplier a under dual-channel case: (**a**) optimal supply effort e across different maximum supply efforts e_{max} and trust values v (with the inner subgraph showing localized behavior around $e_{max} \in [0.52, 0.56]$); (**b**) expected profit of supplier a across different maximum supply efforts e_{max} and trust values v (with the inner subgraph magnifying the localized trends around $e_{max} \in [0.40, 0.50]$).

As mapped in Figure 9, we isolate the structural interaction between trust, effort thresholds, and order allocation between Supplier a and b, where Figure 9a,b illustrate the optimal order quantities allocated to Supplier a and Supplier b across different maximum supply efforts e_{max}, respectively. Figure 9a unveils a clear three-phase strategic trajectory that serves as a practical guide for sourcing portfolio design. In the initial Risk Sourcing Phase under low upstream effort regimes, severe supply uncertainty forces the retailer to execute heavy diversification, shifting orders to the higher-cost backup channel (Supplier

b) to secure supply continuity. This shifts into the Channel Substitution Phase as Supplier a's effort capacity crosses into intermediate scales; here, the main supplier's enhanced fulfillment reliability, coupled with its baseline cost advantage, triggers a rapid reallocation, attracting an increasing order share and displacing the backup channel shown in Figure 9b. Finally, in the Strategic Equilibrium Phase beyond the critical optimal capacity threshold, Supplier a maintains a stable production output, and the order portfolio reaches a steady-state Nash equilibrium, providing clear managerial guidance for dual-channel agricultural portfolio designs.

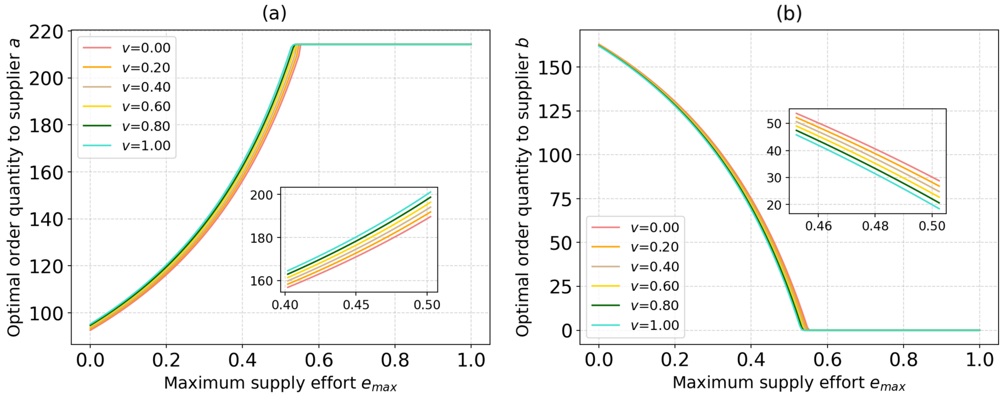

Figure 9. Optimal order allocation under dual-channel case across different trust values v and maximum supply efforts e_{max}: (**a**) optimal order quantity allocated to Supplier a (with the inner subgraph magnifying the steady growth layer around $e_{max} \in [0.40, 0.50]$); (**b**) optimal order quantity allocated to Supplier b (with the inner subgraph showing the rapid displacement zone around $e_{max} \in [0.45, 0.50]$).

5.2. Multi-Period Decision-Making

Within multi-period decision horizons, the systemic operational dynamics evolve from isolated tactical choices into dynamic, long-term strategic interactions. We initially maximize the retailer's multi-period cumulative expected profit to ascertain the optimal intertemporal order trajectory, followed by optimizing the primary supplier's objective function to determine the non-contractable supply effort profile. Finally, the boundaries of the upstream maximum capacity investment are established to calibrate the dynamic trust transition policy governed by Equation (24).

5.2.1. Retailer's Optimal Order Quantity and Expected Profit

This subsection investigates how the long-term intertemporal trust accumulation and upstream capacity constraints jointly dictate the retailer's optimal ordering behavior and financial trajectory.

Figure 10a reveals that a higher maximum supply effort capability expands the supplier's actual delivery distribution, allowing the retailer to compress its planned order placement. However, when the capacity threshold is breached ($E > 0.54$), the relational punishment mechanism undergoes structural relaxation, triggering a sudden, non-linear surge in order placement. From a behavioral economics lens, a profound strategic bifurcation appears across different capacity regimes regarding the role of trust. Under highly constrained effort regimes, an increase in trust correlates with a reduction in order placement. In this vulnerable state, long-term relational trust serves as a stabilizing anchor, preventing panicky over-ordering and smoothing the bullwhip effect. Conversely, once the capacity crosses the critical milestone ($E = 0.54$) and becomes unconstrained according to Proposition 3, higher trust motivates the retailer to actively and strategically absorb the supplier's residual stochastic supply risk. This shift drives an expansion of the optimal order quantity, converting emotional goodwill into a tangible transactional volume.

A cross-horizon comparative analysis encompassing Figures 5, 6 and 10 yields a robust managerial insight: the downstream retailer demonstrates a systematic, unwavering preference for single-channel procurement across both short-term and multi-period horizons. Under the trust–punishment mechanism, the retailer consistently avoids the premium-priced backup channel. Instead, it chooses to invest in multi-period trust building and voluntarily assumes localized supply risks in exchange for the long-term cost benefits of the primary channel, proving that the behavioral governance framework successfully incentivizes enduring, resilient buyer–supplier partnerships over short-term transactional switching.

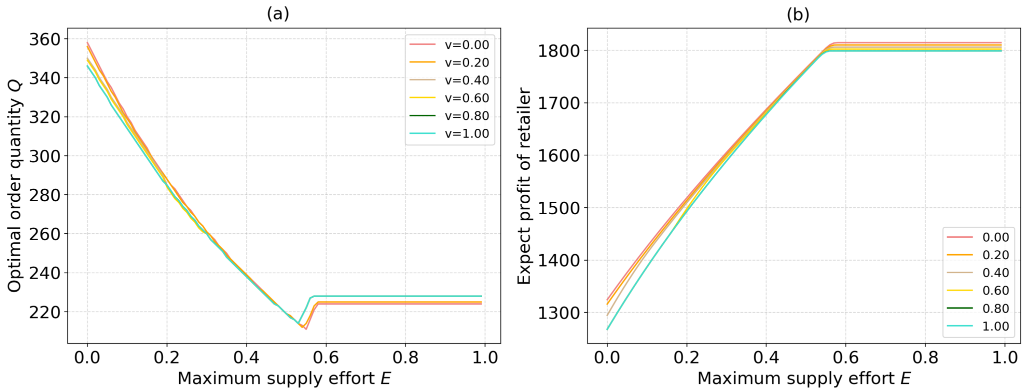

Figure 10. Optimal multi-period outcomes under the decentralized basic scenario across different trust values v and maximum supply efforts E: (**a**) optimal order quantity Q versus maximum supply effort E; (**b**) expected profit of the retailer versus maximum supply effort E.

5.2.2. Supplier's Optimal Supply Effort and Expected Profit

We next evaluate the dynamic impacts of intertemporal trust and maximum capacity capability on the primary supplier's optimal operational effort and long-term financial returns. Since the structural trajectory of the optimal supply effort remains invariant across behavioral regimes, our analysis focuses on the decentralized scenario of the baseline single-channel model.

Figure 11a maps the structural link from maximum capacity capability E to the optimal non-contractable supply effort e under decentralized execution, exhibiting a threshold-matching profile consistent with the dual-channel behavior. Figure 11b tracks the associated financial rewards under decentralization, uncovering a key trade-off between marginal capacity costs and market revenue. In highly constrained effort zones, the supplier's minimal marginal cost expenditure drives a rapid, highly profitable expansion of early returns. As the effort scale increases, the escalating variable operational costs outpace the incremental wholesale revenues, creating a localized profit contraction. However, following the structural relaxation of the contract mechanism, the supplier's optimal effort stabilizes at its cost-efficient equilibrium. Concurrently, the massive bullwhip-effect-driven order surge from the retailer creates a sharp, non-linear spike in supplier revenue, which eventually plateaus at a high-volume steady state.

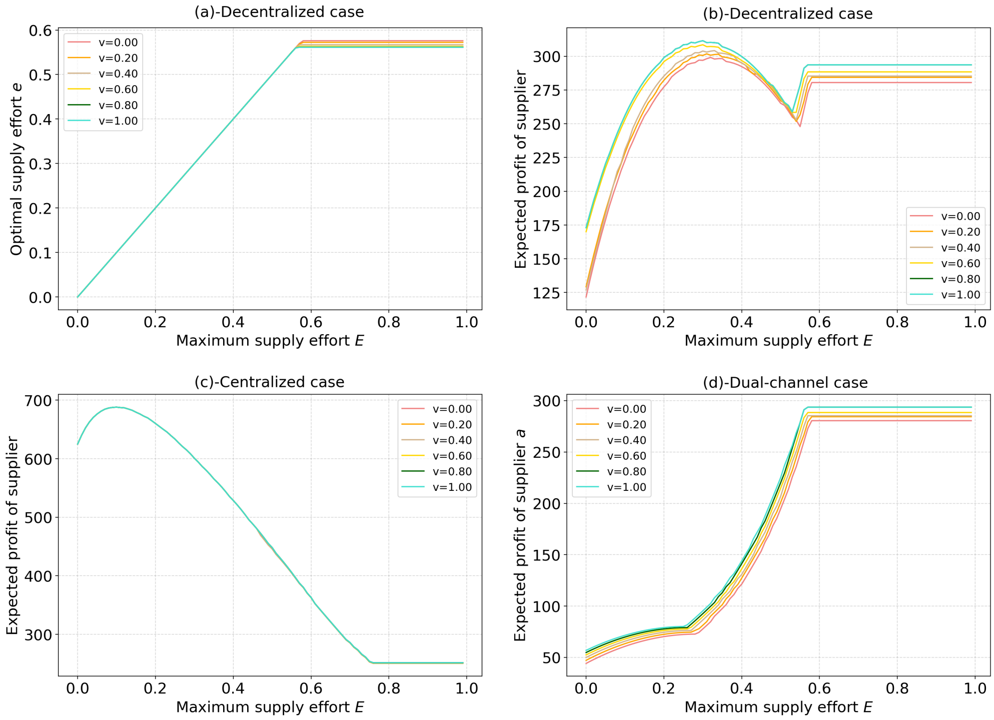

Figure 11. Optimal supply effort e and expected profit under different decision cases.

In sharp contrast, Figure 11c demonstrates the centralized financial landscape, highlighting a key structural risk of centralized integration. Intuitively, high initial order volumes combined with low-capacity investment yield high centralized returns. As the integrated supply effort expands, the escalating system-wide costs drain overall profitability. Crucially, unlike the decentralized framework where the behavioral trust mechanism serves as a flexible, shock-absorbing governance tool, the centralized architecture completely lacks a relational trust-regulation mechanism. Consequently, when structural capacity limits are reached, the system suffers an unmitigated amplification of the bullwhip effect, causing centralized profits to deteriorate much more severely than in decentralized systems.

Finally, Figure 11d analyzes the primary supplier's profit path under dual-channel decentralization. Under low-capacity constraints, the retailer maintains sourcing from Supplier a while absorbing their localized supply risk, but the presence of the higher-cost backup option (Supplier b) limits the primary supplier's early profit growth. As the primary supplier's maximum capacity capability matures, their optimal effort investment scales up efficiently, mitigating the downstream risk burden. This performance improvement triggers a rapid structural shift in the retailer's portfolio, shifting the majority of procurement volume from the high-cost backup Supplier b to the primary Supplier a. The resulting wholesale cost savings accelerate upstream profit growth, demonstrating how a primary supplier can leverage capacity development as a strategic weapon to exclude backup competitors and secure a dominant channel share.

5.3. Sensitivity Analysis

Recognizing the inherent variability in real-world applications, we conducted a sensitivity analysis using the initial setting parameters as the comparison criteria to examine how variations in key input parameters influence the model's output. This analysis not only verifies the model's robustness but also offers practical insights for decision-making under uncertainty. To emphasize the key findings, we present the sensitivity analysis results for $v = 0.2$ as a representative case, while the complete results for other parameter values are provided in the Appendixes A–E. Moreover, since the centralized decision-making scenario does not account for trust considerations, our sensitivity analysis focuses solely on the decentralized decision-making case.

For the basic case, we perform a comprehensive evaluation of the effects of price, demand, and key parameters of the trust punishment mechanism on the decision-making and revenue of all supply chain participants. The detailed results are summarized in Table 2.

As shown in Table 2, the decisions of both supply chain participants and the expected revenues on sales prices are positively correlated with market demand. Notably, the retailer's expected revenues exhibit a more significant increase, consistently exceeding 20%. Additionally, higher market demand incentivizes suppliers to enhance their supply efforts, thereby reducing supply uncertainty and mitigating the bullwhip effect. For downstream retailers, expanding market size is crucial in encouraging suppliers to improve their supply efforts. Regarding the effects of increased out-of-stock penalty costs and a higher commitment lower bound, the changes in decisions and expected income for both sides remain within 4%. This limited impact is attributed to the low out-of-stock volume under the supply effort and trust penalty mechanism, which minimizes suppliers' stockout losses and effectively curbs opportunistic behavior when shortages are severe. In contrast, difficulties in improving supply levels and declining wholesale prices significantly dampen suppliers' motivation to enhance supply conditions. Notably, a reduction in wholesale prices can result in an expected income loss for suppliers of up to 50%. This finding underscores the necessity of safeguarding the interests of upstream agricultural supply chain participants to ensure the long-term sustainability of the entire supply chain.

Table 2. Numerical results of parameter analysis: Basic Case.

p	Q	e	π_s	%	π_r	%	D	Q	e	π_s	%	π_r	%
11	208	0.539	242	−15	1069	−41	100	144	0.344	99	−65	808	−55
13	217	0.558	265	−6.0	1436	−21	150	184	0.483	185	−35	1303	−28
15	225	0.572	283	0	1810	0	200	225	0.572	283	0	1810	0
17	231	0.582	299	6.0	2190	21	250	264	0.633	387	37	2324	28
19	236	0.591	312	10	2573	42	300	304	0.678	495	75	2841	57
s	Q	e	π_s	%	π_r	%	β	Q	e	π_s	%	π_r	%
0	228	0.561	294	3.9	1799	−0.6	300	198	0.701	341	20	1912	5.6
0.25	226	0.567	288	1.8	1805	−0.3	400	212	0.633	310	9.5	1859	2.7
0.5	225	0.572	283	0	1810	0	500	225	0.572	283	0	1810	0
0.75	223	0.576	279	−1.4	1815	0.28	600	237	0.517	261	−7.8	1766	−2.4
1	222	0.581	274	−3.2	1820	0.55	700	250	0.467	241	−15	1724	−4.8
w	Q	e	π_s	%	π_r	%	$\underline{\alpha}$	Q	e	π_s	%	π_r	%
3	292	0.473	−71	−125	2153	19	0.3	225	0.569	285	0.7	1808	−0.1
4	251	0.529	124	−56	1975	9.2	0.4	225	0.57	284	0.4	1809	−0.1
5	225	0.572	283	0	1810	0	0.5	225	0.572	283	0	1810	0
6	205	0.605	422	49	1656	−8.5	0.6	224	0.573	282	−0.4	1812	0.1
7	191	0.633	546	93	1508	−17	0.7	224	0.574	281	−0.7	1813	0.2

A relatively small maximum supply effort e_{max} means limited capacity for additional investment to ensure output. As evidenced in Table 3, when e_{max} increases, the retailer demonstrates significant order transfer behavior, shifting procurement from backup supplier b to primary supplier a. This transition yields mutual benefits for both the retailer and primary supplier a while reducing total order quantity. These findings suggest that a supplier's efforts to enhance supply capacity and reduce supply uncertainty can create a win-win situation for supply chain members and effectively mitigate the bullwhip effect. Furthermore, as the wholesale price w_b of backup supplier b increases, the retailer correspondingly reduces orders from supplier b to control procurement costs. When the maximum supply capacity of primary supplier a exceeds the retailer's demand threshold, the retailer completely ceases procurement from backup supplier b. These conclusions imply that in low supply uncertainty scenarios (where the primary supplier demonstrates strong supply willingness and adequate capacity), even with dual-channel procurement options, the retailer will prioritize the primary supplier due to its competitive wholesale pricing advantage.

Table 3. Numerical results of parameter analysis: Dual-channel.

e_{max}	w_b	Q_a	Q_b	e	π_{sa}	%	π_{sb}	%	π_r	%
0.2	5	117	137	0.200	56	0	275	−29.3	1825	7.9
	5.5	117	133	0.200	56	0	334	−14.1	1757	3.8
	6	117	130	0.200	56	0	389	0	1692	0
	6.5	117	126	0.200	56	0	440	13.1	1628	−3.8
	7	117	122	0.200	56	0	487	25.2	1566	−7.4
0.5	5	192	40	0.500	203	0	80	−1.2	1804	1.9
	5.5	192	33	0.500	203	0	83	2.5	1786	0.8
	6	192	27	0.500	203	0	81	0	1771	0
	6.5	192	20	0.500	203	0	72	−11.1	1759	−0.7
	7	192	14	0.500	203	0	56	−30.9	1750	−1.2
0.8	5	225	0	0.556	286	10.0	0	0	1799	0.6
	5.5	220	0	0.544	272	4.6	0	0	1795	0.4
	6	214	0	0.533	260	0	0	0	1788	0
	6.5	209	0	0.522	247	−5.0	0	0	1778	−0.6
	7	205	0	0.511	236	−9.2	0	0	1765	−1.3

6. Conclusions

Amidst the supply uncertainty inherent in the transportation and production of agricultural products, retailers frequently opt to place orders exceeding current market demand to mitigate potential supply shortages. However, this asymmetric inflation of orders misleads suppliers, prompting unnecessary capacity adjustments that exacerbate the bullwhip effect and trigger supply chain disruptions. To address this challenge, this paper investigates an endogenous, performance-dependent trust–punishment mechanism tailored to modern digital procurement environments.

Our baseline single-period analysis uncovers a critical, counter-intuitive behavioral trade-off: higher retailer trust systematically expands procurement order quantities but marginally crowds out the supplier's optimal supply effort. This occurs because elevated trust lowers the contractually enforced delivery commitment threshold, effectively acting as an operational "insurance policy" where the retailer voluntarily absorbs upstream random supply risks in exchange for lower-cost products. When extending this framework to a dual-channel configuration, we show that the retailer's strategic sourcing allocation is non-monotonically governed by the interaction between trust updates and the primary supplier's maximum capacity boundaries. Specifically, the backup supplier acts not merely as a physical capacity hedge, but as a relational governance tool that structurally disciplines supplier opportunism. Finally, our infinite-horizon multi-period dynamic planning model demonstrates that while localized moral hazard exists in short-term horizons, repeated long-term interactions driven by performance-dependent updates lead to stable, asymptotically convergent supplier effort levels, quantitatively mitigating the bullwhip effect to a degree comparable to the centralized benchmark.

In addition to our existing findings, several intriguing opportunities for further research remain: (i) Modeling market demand as a distribution-free random variable where only the first two moments are known, utilizing distributionally robust optimization; and (ii) extending the framework to a network configuration involving competitive multi-retailer e-commerce structures and a shared upstream supplier to analyze horizontal externalities on trust accumulation.

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Conflicts of Interest: The authors declare that they have no conflicts of interest in this work.

Appendix A. The Notation Table

Table A1. Notation Table.

Parameter	Parameter Meaning
Common parameter	
y	uncertain supply rate, where $y \in [0,1]$
p	Unit retail price
D	Deterministic market demand
v	The trust value of retailers in suppliers, where $v \in [0,1]$
s	Unit compensation cost
β	Effort coefficient
$\overline{\alpha}$	Commitment proportion upper bound, where $\overline{\alpha} \in [0,1]$
$\underline{\alpha}$	Commitment proportion lower bound, where $\underline{\alpha} \in [0,1]$ and $\underline{\alpha} < \overline{\alpha}$
e	Decision variable representing the supplier's supply effort level
Basic case	
w	Unit wholesale price
c	Unit production cost
Q	Decision variable, representing the retailer's order quantity, where $Q \geq D$
Dual-channel case	
c_a	Unit production cost of unstable supplier a
c_b	Unit production cost of backup supplier b, where $c_b \geq c_a$
w_a	Unit wholesale price of unstable supplier a
w_b	Unit wholesale price of backup supplier b, where $w_b \geq w_a$
Q_a	Decision variable representing retailer's order quantity to supplier a
Q_b	Decision variable representing retailer's order quantity to supplier b
Multi-period extension	
λ	The sharpness of the retailer's memory, where $\lambda \in [0,1]$
δ	Discount factor, where $\delta \in [0,1]$
η	Coefficient of maintenance cost

Appendix B. Basic Case

Since the supplier promises to deliver a proportion of the order quantity and has an uncertain supply, we analyze the expected profit of both parties from three scenarios.

When the stochastic capacity of the supplier, denoted as $(y+e)Q$, is less than or equal to the committed order quantity, $\alpha(v)Q$, i.e., $0 < (y+e)Q \leq \alpha(v)Q$, the supplier provides compensation to the retailer. Subsequently, the supplier's expected profit function is represented by

$$\mathbb{E}\left(\Pi_s^1\right) = \int_0^{\max\{\alpha(v)-e,0\}} [w(y+e)Q - cQ - s(\alpha(v)Q - (y+e)Q) - \beta e]g(y)dy, \tag{A1}$$

and the retailer's expected profit function is denoted as

$$\mathbb{E}\left(\Pi_r^1\right) = \int_0^{\max\{\alpha(v)-e,0\}} [p(y+e)Q + s(\alpha(v)Q - (y+e)Q) - w(y+e)Q]g(y)dy. \tag{A2}$$

When the stochastic capacity $(y+e)Q$ is less than the order quantity Q but greater than the committed order quantity $\alpha(v)Q$, i.e., $0 < \alpha(v)Q < (y+e)Q < Q$, the retailer does not receive any compensation from the supplier. In this scenario, the supplier's expected profit function is represented by

$$\mathbb{E}\left(\Pi_s^2\right) = \int_{\max\{\alpha(v)-e,0\}}^{1-e} [w(y+e)Q - cQ - \beta e]g(y)dy, \tag{A3}$$

and the retailer's expected profit is denoted as

$$\mathbb{E}\left(\Pi_r^2\right) = \int_{\max\{\alpha(v)-e,0\}}^{1-e} [p\min\{D, (y+e)Q\} - w(y+e)Q]g(y)dy. \tag{A4}$$

When the stochastic capacity $(y+e)Q$ is greater than or equal to the order quantity Q, i.e., $Q \leq (y+e)Q$, the retailer will obtain the order quantity Q, and the supplier will have to pay for the excessive supply effort. In this case, the supplier's expected profit function is denoted as

$$\mathbb{E}\left(\Pi_s^3\right) = \int_{1-e}^{1} (wQ - cQ - \beta e)g(y)dy, \tag{A5}$$

and the retailer's expected profit is represented by

$$\mathbb{E}\left(\Pi_r^3\right) = \int_{1-e}^{1} (pD - wQ)g(y)dy. \tag{A6}$$

Appendix B.1. Proof of Proposition 1

Proof. According to the Leibniz formula and the expected profit Equation (1) for the supplier, we analyze the influence of the supply effort level e on the expected profit of the supplier. Additionally, we compute the first- and second-order differentiations of Equation (1) with respect to e.

First-order differentiation: When $0 \leq e < \alpha(v)$,

$$\begin{aligned}
\frac{\partial \mathbb{E}(\Pi_s)}{\partial e} &= \frac{\partial \mathbb{E}(\Pi_s^1)}{\partial e} + \frac{\partial \mathbb{E}(\Pi_s^2)}{\partial e} + \frac{\partial \mathbb{E}(\Pi_s^3)}{\partial e} \\
&= \int_0^{\alpha(v)-e} (wQ + sQ - \beta)g(y)dy + \int_{\alpha(v)-e}^{1-e} (wQ - \beta)g(y)dy - \int_{1-e}^{1} \beta g(y)dy \\
&= (1-e)wQ + (\alpha(v) - e)sQ - \beta.
\end{aligned}$$

When $\alpha(v) \le e \le 1$,

$$\frac{\partial \mathbb{E}(\Pi_s)}{\partial e} = \frac{\partial \mathbb{E}(\Pi_s^2)}{\partial e} + \frac{\partial \mathbb{E}(\Pi_s^3)}{\partial e} = \int_0^{1-e} (wQ-\beta)g(y)dy - \int_{1-e}^{1} \beta g(y)dy = (1-e)wQ - \beta.$$

Second-order differentiation: When $0 \le e < \alpha(v)$, $\frac{\partial^2 \mathbb{E}(\Pi_s)}{\partial (e)^2} = -(w+s)Q < 0$. When $\alpha(v) \le e \le 1$, $\frac{\partial^2 \mathbb{E}(\Pi_s)}{\partial (e)^2} = -wQ < 0$.
Because of $\frac{\partial^2 \mathbb{E}(\Pi_s)}{\partial (e)^2} < 0$, we can know that $\mathbb{E}(\Pi_m)$ is a concave function of e. Given the order quantity Q and effort coefficient $\beta > (1-\overline{\alpha})wQ$, when $0 \le e < \alpha(v)$, i.e., $0 \le v < \frac{\beta-(1-\overline{\alpha})wQ}{wQ(\overline{\alpha}-\underline{\alpha})}$, the optimal supply effort level can be solved from the equation $((1-e)wQ + (\alpha(v)-e)sQ - \beta = 0)$, that is $e^* = \frac{wQ+s\alpha(v)Q-\beta}{wQ+sQ}$; when $\alpha(v) \le e \le 1$, i.e., $\frac{\beta-(1-\overline{\alpha})wQ}{wQ(\overline{\alpha}-\underline{\alpha})} \le v \le 1$, the optimal supply effort level can be solved from the equation $((1-e)wQ - \beta = 0)$, that is $e^* = \frac{wQ-\beta}{wQ}$. Specially, if the effort coefficient $0 \le \beta \le (1-\overline{\alpha})wQ$, the optimal supply effort level has the unique value $e^* = \frac{wQ-\beta}{wQ}$. □

Appendix B.2. Proof of Proposition 2

Proof. According to the Leibniz formula and the expected profit Equation (2) for the retailer, we analyze the influence of the order quantity Q on the expected profit of the retailer. Since the optimal supply effort level e^* is related to the order quantity Q, we define the function $e(Q) = \frac{wQ+s\alpha(v)Q-\beta}{wQ+sQ}$. Additionally, we compute the first- and second-order differentiations of Equation (2) with respect to Q.
First-order differentiation: When $0 \le e(Q) < \alpha(v)$,

$$\begin{aligned}\frac{\partial \mathbb{E}(\Pi_r)}{\partial Q} =& \frac{\partial \mathbb{E}(\Pi_r^1)}{\partial Q} + \frac{\partial \mathbb{E}(\Pi_r^2)}{\partial Q} + \frac{\partial \mathbb{E}(\Pi_r^3)}{\partial Q} \\ =& \int_0^{\alpha(v)-e(Q)} \Big[(p-w-s)(y+e(Q)) + (p-w-s)e^{'}(Q)Q + s\alpha(v))\Big]g(y)dy \\ &+ \int_{\alpha(v)-e(Q)}^{\frac{D}{Q}-e(Q)} \Big[(p-w)(y+e(Q)) + (p-w)e^{'}(Q)Q\Big]g(y)dy \\ &- \int_{\frac{D}{Q}-e(Q)}^{1-e(Q)} \Big[w(y+e(Q)) + we^{'}(Q)Q\Big]g(y)dy - \int_{1-e(Q)}^{1} wg(y)dy\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbb{E}(\Pi_r)}{\partial Q} =& \frac{D^2p + \beta^2 p/(s+w)^2 - \beta(\beta-2Dp)/(s+w)}{2Q^2} \\ &+ \frac{-2\alpha(v)sw(p+s+w) + \alpha(v)^2 s[-ps+w(s+w)] - w[pw+(s+w)(s+2w)]}{2(s+w)^2}.\end{aligned}$$

When $\alpha(v) \le e(Q) < \frac{D}{Q}$,

$$\begin{aligned}\frac{\partial \mathbb{E}(\Pi_r)}{\partial Q} =& \frac{\partial \mathbb{E}(\Pi_r^2)}{\partial Q} + \frac{\partial \mathbb{E}(\Pi_r^3)}{\partial Q} \\ =& \int_0^{\frac{D}{Q}-e(Q)} \Big[(p-w)(y+e(Q)) + (p-w)e^{'}(Q)Q\Big]g(y)dy \\ &- \int_{\frac{D}{Q}-e(Q)}^{1-e(Q)} (w(y+e(Q)) + we^{'}(Q)Q)g(y)dy - \int_{1-e(Q)}^{1} wg(y)dy \\ =& \frac{\beta^2(p-w) + 2\beta Dpw + D^2pw^2}{2w^2Q^2} - \frac{p+2w}{2}.\end{aligned}$$

When $\frac{D}{Q} \le e(Q) \le 1$,

$$\frac{\partial \mathbb{E}(\Pi_r)}{\partial Q} = -\int_0^{1-e(Q)} (w(y+e(Q)) + we^{'}(Q)Q)g(y)dy - \int_{1-e(Q)}^{1} wg(y)dy = -\frac{\beta^2}{2Q^2 w} - w < 0.$$

Since $\frac{\partial \mathbb{E}(\Pi_r)}{\partial Q} < 0$ when $\frac{D}{Q} \leq e(Q) \leq 1$, we can analyze the two cases $\alpha(v) \leq e(Q) < \frac{D}{Q}$ and $\frac{D}{Q} \leq e(Q) \leq 1$ together.
Second-order differentiation: When $0 \leq e(Q) < \alpha(v)$,

$$\frac{\partial^2 \mathbb{E}(\Pi_r)}{\partial Q^2} = -\frac{D^2 p + \beta^2 p/(s+w)^2 - \beta(\beta - 2Dp)/(s+w)}{Q^3} < 0.$$

When $\alpha(v) \leq e(Q) \leq 1$,

$$\frac{\partial^2 \mathbb{E}(\Pi_r)}{\partial Q^2} = -\frac{\beta^2(p-w) + 2\beta D p w + D^2 p w^2}{w^2 Q^3} < 0.$$

Because of $\frac{\partial^2 \mathbb{E}(\Pi_r)}{\partial Q^2} < 0$, we can know that $\mathbb{E}(\Pi_r)$ is the concave function of Q. When $0 \leq e(Q) < \alpha(v)$, i.e., $0 \leq v < \frac{\beta-(1-\overline{\alpha})wQ}{wQ(\overline{\alpha}-\underline{\alpha})}$, the optimal order quantity can be solved from the equation ($\frac{\partial \mathbb{E}(\Pi_r)}{\partial Q} = 0$), that is

$$Q^* = \sqrt{\frac{\beta^2(p-s-w) + 2\beta D p(s+w) + D^2 p(s+w)^2}{\alpha(v)^2 s(ps - w(s+w)) + 2\alpha(v) s w(p+s+w) + w(pw + (s+w)(s+2w))}}.$$

When $\alpha(v) \leq e(Q) \leq 1$, i.e., $\frac{\beta-(1-\overline{\alpha})wQ}{wQ(\overline{\alpha}-\underline{\alpha})} \leq v \leq 1$, the optimal order quantity can be solved from the equation ($\frac{\partial \mathbb{E}(\Pi_r)}{\partial Q} = 0$), that is

$$Q^* = \sqrt{\frac{\beta^2(p-w) + 2\beta D p w + D^2 p w^2}{w^2(p+2w)}}.$$

Finally, we define the order boundary value $Q_d = \sqrt{\frac{\beta^2(p-w)+2\beta D p w + D^2 p w^2}{w^2(p+2w)}}$. Based on this order boundary value, we determine the optimal order quantities in the two cases mentioned above. Specially, if the effort coefficient $0 \leq \beta \leq (1-\overline{\alpha})wQ_d$, the optimal ordering quantity has the unique value $Q^* = \sqrt{\frac{\beta^2(p-w)+2\beta D p w + D^2 p w^2}{w^2(p+2w)}}$. □

Appendix B.3. Proof of Proposition 3

Proof. The proof of part 1: When $0 \leq v < \frac{\beta-(1-\overline{\alpha})wQ_d}{wQ_d(\overline{\alpha}-\underline{\alpha})}$, we employ the chain rule for derivatives to calculate the first-order derivative of Equation (4) concerning the variable v, i.e.,

$$\frac{\partial Q^*}{\partial v} = \frac{\partial Q^*}{\partial \alpha(v)} \frac{\partial \alpha(v)}{\partial v}$$
$$= \frac{s(\overline{\alpha}-\underline{\alpha})[\alpha(v)(ps - w(s+w)) + w(p+s+w)]\sqrt{p(\beta + D(s+w))^2 - \beta^2(s+w)}}{[\alpha(v)^2 s(ps - w(s+w)) + 2\alpha(v) s w(p+s+w) + w(pw + (s+w)(s+2w))]^{3/2}} > 0.$$

When $1 \geq vs. \geq \frac{\beta-(1-\overline{\alpha})wQ_d}{wQ_d(\overline{\alpha}-\underline{\alpha})}$, $\frac{\partial Q^*}{\partial v} = 0$. Therefore, the optimal ordering quantity Q^* is non-decreasing with the trust value v, i.e, $\frac{\partial Q^*}{\partial v} \geq 0$.
The proof of part 2: When $0 \leq v < \frac{\beta-(1-\overline{\alpha})wQ_d}{wQ_d(\overline{\alpha}-\underline{\alpha})}$, as the optimal ordering quantity Q^* depends on the trust value v, with the exception of the commitment proportion $\alpha(v)$, we

utilize the chain rule to compute the first-order derivative of Equation (3) with respect to v, that is

$$\begin{aligned}\frac{\partial e^*}{\partial v} &= \frac{\partial e^*}{\partial \alpha(v)}\frac{\partial \alpha(v)}{\partial v} + \frac{\partial e^*}{\partial Q^*}\frac{\partial Q^*}{\partial v} \\ &= \frac{s(\overline{\alpha}-\underline{\alpha})}{w+s}\left(-1+\frac{\beta[\alpha(v)(ps-w(s+w))+w(p+s+w)]\left[p(\beta+D(s+w))^2-\beta^2(s+w)\right]^{-1/2}}{\left[\alpha(v)^2 s(ps-w(s+w))+2\alpha(v)sw(p+s+w)+w(pw+(s+w)(s+2w))\right]^{1/2}}\right).\end{aligned}$$

We can verify that the following inequation holds due to $pD > \beta$:

$$\begin{aligned}&\left[p(\beta+D(s+w))^2-\beta^2(s+w)\right]\cdot\left[\alpha(v)^2 s(ps-w(s+w))+(2\alpha(v)s+w)w(p+s+w)+w(s+w)^2\right] \\ &\qquad -\beta^2[\alpha(v)(ps-w(s+w))+w(p+s+w)]^2 \\ &= \alpha(v)(s+w)\left[2\beta Dps+\beta^2(-s+w)+D^2ps(s+w)\right][\alpha(v)ps-\alpha(v)w(s+w)+2w(p+s+w)] \\ &\quad + w\Big[(-2\beta^2(s+w)+2\beta Dp(s+w)+D^2p(s+w)^2)(pw+(s+w)(s+2w)) \\ &\quad +\beta^2(p-s-w)(s+w)^2\Big] > 0.\end{aligned}$$

Hence, we have $\frac{\partial e^*}{\partial v} < 0$. When $1 \geq v \geq \frac{\beta-(1-\overline{\alpha})wQ_d}{wQ_d(\overline{\alpha}-\underline{\alpha})}$, $\frac{\partial e^*}{\partial v} = 0$. Then, the optimal supply effort level e^* decreases with the trust value $v \in \left[0, \frac{\beta-(1-\overline{\alpha})wQ_d}{wQ_d(\overline{\alpha}-\underline{\alpha})}\right)$, i.e, $\frac{\partial e^*}{\partial v} < 0$. If $0 < \beta < Q_d w(1-\alpha(v))$, $\frac{wQ_d-\beta}{wQ_d} > \frac{wQ_d+s\alpha(v)Q_d-\beta}{(w+s)Q_d}$. Therefore, at the trust segment point, the optimal supply effort level will experience a sudden increase.

The proof of part 3: Following a process akin to that of part 2's proof, when we compute the first-order derivatives of Equations (1) and (2) with respect to the variable v, we observe that the following equations are satisfied.

For the supplier: when $0 \leq e \leq \alpha(v)$,

$$\begin{aligned}\frac{\partial \mathbb{E}(\Pi_s)}{\partial v} &= \frac{\partial \mathbb{E}(\Pi_s)}{\partial \alpha(v)}\frac{\partial \alpha(v)}{\partial v} + \frac{\partial \mathbb{E}(\Pi_s)}{\partial Q^*}\frac{\partial Q^*}{\partial v} + \frac{\partial \mathbb{E}(\Pi_s)}{\partial e^*}\frac{\partial e^*}{\partial v} \\ &= \left[-c+we^*+\frac{1-e^{*2}}{2}w-s\alpha(v)(\alpha(v)-e^*)+\frac{s}{2}(\alpha(v)^2-e^{*2})\right]\frac{\partial Q^*}{\partial v}+sQ^*(\overline{\alpha}-\underline{\alpha})(\alpha(v)-e^*) \\ &\geq \left(\frac{w-s}{2}-c\right)\frac{\partial Q^*}{\partial v}+sQ^*(\overline{\alpha}-\underline{\alpha})(\alpha(v)-e^*) \geq 0,\end{aligned}$$

when $\alpha(v) < e \leq 1$,

$$\begin{aligned}\frac{\partial \mathbb{E}(\Pi_s)}{\partial v} &= \frac{\partial \mathbb{E}(\Pi_s)}{\partial Q^*}\frac{\partial Q^*}{\partial v} + \frac{\partial \mathbb{E}(\Pi_s)}{\partial e^*}\frac{\partial e^*}{\partial v} \\ &= \left[-c+we^*+\frac{1-e^{*2}}{2}w-s\alpha(v)(\alpha(v)-e^*)+\frac{s}{2}(\alpha(v)^2-e^{*2})\right]\frac{\partial Q^*}{\partial v} \geq \left(\frac{w-s}{2}-c\right)\frac{\partial Q^*}{\partial v} \geq 0.\end{aligned}$$

For the retailer: when $0 \leq e \leq \alpha(v)$,

$$\frac{\partial \mathbb{E}(\Pi_r)}{\partial v} = \frac{\partial \mathbb{E}(\Pi_r)}{\partial \alpha(v)}\frac{\partial \alpha(v)}{\partial v} + \frac{\partial \mathbb{E}(\Pi_r)}{\partial Q^*}\frac{\partial Q^*}{\partial v} = -sQ^*(\overline{\alpha}-\underline{\alpha}) \leq 0,$$

when $\alpha(v) < e \leq 1$, $\frac{\partial \mathbb{E}(\Pi_r)}{\partial v} = \frac{\partial \mathbb{E}(\Pi_r)}{\partial Q^*}\frac{\partial Q^*}{\partial v} = 0$. Therefore, the supplier's optimal profit is non-decreasing as the retailer's trust in the supplier grows, while the retailer's optimal profit is non-increasing with an increase in the retailer's trust. □

Appendix B.4. Proof of Proposition 4

Proof. According to the Leibniz formula and the expected profit Equation (5), we analyze the influence of the supply effort level e^c and the order quantity Q^c on the expected profit for the integrated supply chain. Additionally, similar to the proof procedure of Proposition 1 or 2, we compute the first- and second-order differentiations of Equation (5) with respect to Q^c and e^c.
First-order differentiation on Q^c:

$$\frac{\partial \mathbb{E}(\Pi_c)}{\partial Q^c} = \int_0^{\frac{D}{Q^c}-e^c} p(y+e^c)g(y)dy - c = \frac{p}{2}\left[\left(\frac{D}{Q^c}\right)^2 - (e^c)^2\right] - c.$$

First-order differentiation on e^c:

$$\frac{\partial \mathbb{E}(\Pi_c)}{\partial e^c} = \int_0^{\frac{D}{Q^c}-e^c} pQ^c g(y)dy - \beta = pD - pQ^c e^c - \beta.$$

Second-order differentiation on Q^c and e^c, respectively:

$$\frac{\partial^2 \mathbb{E}(\Pi_c)}{\partial (Q^c)^2} = -p\frac{D^2}{Q^{c3}}, \quad \frac{\partial^2 \mathbb{E}(\Pi_c)}{\partial (e^c)^2} = -pQ^c,$$
$$\frac{\partial^2 \mathbb{E}(\Pi_c)}{\partial Q^c \partial e^c} = -pe^c, \quad \frac{\partial^2 \mathbb{E}(\Pi_c)}{\partial e^c \partial Q^c} = -pe^c.$$

Since $\frac{\partial^2 \mathbb{E}(\Pi_c)}{\partial (Q^c)^2} < 0$ and $\frac{\partial^2 \mathbb{E}(\Pi_c)}{\partial (e^c)^2} < 0$, $\mathbb{E}(\Pi_c)(Q^c, e^c)$ is a concave function with respect to e^c and Q^c, respectively.
Furthermore, the determinant of the Hessian Matrix H is

$$|H| = \begin{vmatrix} \frac{\partial^2 \mathbb{E}(\Pi_c)}{\partial (Q^c)^2} & \frac{\partial^2 \mathbb{E}(\Pi_c)}{\partial Q^c \partial e^c} \\ \frac{\partial^2 \mathbb{E}(\Pi_c)}{\partial e^c \partial Q^c} & \frac{\partial^2 \mathbb{E}(\Pi_c)}{\partial (e^c)^2} \end{vmatrix} = 2c.$$

Since the determinant of H is more than 0 and the stagnation point of $\mathbb{E}(\Pi_c)$ is the maximum point. Therefore, the optimal order quantity Q^{c*} and supply effort level e^{c*} are

$$Q^{c*} = \sqrt{\frac{2Dp\beta - \beta^2}{2pc}},$$
$$e^{c*} = \frac{pD - \beta}{pQ^{c*}}.$$

□

Appendix C. Dual Channel Case

As supplier a commits to delivering a proportion of the order quantity with an uncertain supply, we analyze the expected profits of both the retailer and supplier a under three scenarios.

When the stochastic capacity of the supplier a, denoted as $(y+e)Q_a$, is less than or equal to the committed order quantity, $\alpha(v)Q_a$, i.e., $0 < (y+e)Q_a \le \alpha(v)Q_a$, the supplier a provides compensation to the retailer. Subsequently, the expected profit function of supplier a is represented by

$$\mathbb{E}(\Pi^1_{s_a}) = \int_0^{\max\{\alpha(v)-e,0\}} [w_a(y+e)Q_a - c_aQ_a - s(\alpha(v)Q_a - (y+e)Q_a) - \beta e]g(y)dy, \quad \text{(A7)}$$

and the retailer's expected profit function is denoted as

$$\mathbb{E}(\Pi_R^1) = \int_0^{\max\{\alpha(v)-e,0\}} [p((y+e)Q_a + Q_b) + s(\alpha(v)Q_a - (y+e)Q_a) \\ -w_a(y+e)Q_a - w_bQ_b]g(y)dy. \tag{A8}$$

When the stochastic capacity $(y+e)Q_a$ is less than the order quantity Q_a but greater than the committed order quantity $\alpha(v)Q_a$, i.e., $0 < \alpha(v)Q_a < (y+e)Q_a < Q_a$, the retailer does not receive any compensation from the supplier a. In this scenario, the expected profit function of supplier a is represented by

$$\mathbb{E}(\Pi_{s_a}^2) = \int_{\max\{\alpha(v)-e,0\}}^{1-e} [w_a(y+e)Q_a - c_aQ_a - \beta e]g(y)dy, \tag{A9}$$

and the retailer's expected profit is denoted as

$$\mathbb{E}(\Pi_R^2) = \int_{\max\{\alpha(v)-e,0\}}^{1-e} [p \cdot \min\{D, (y+e)Q_a + Q_b\} - w_a(y+e)Q_a - w_bQ_b]g(y)dy. \tag{A10}$$

When the stochastic capacity $(y+e)Q_a$ is greater than or equal to the order quantity Q_a, i.e., $0 < Q_a \leq (y+e)Q_a$, the retailer will obtain the order quantity Q_a, and the supplier a will have to pay for the excessive supply effort. In this case, the expected profit function of supplier a is denoted as

$$\mathbb{E}(\Pi_{s_a}^3) = \int_{1-e}^{1} (w_aQ_a - c_aQ_a - \beta e)g(y)dy, \tag{A11}$$

and the retailer's expected profit function is represented by

$$\mathbb{E}(\Pi_R^3) = \int_{1-e}^{1} (pD - w_aQ_a - w_bQ_b)g(y)dy. \tag{A12}$$

Appendix C.1. Proof of Proposition 5

Proof. According to the Leibniz formula and the expected profit Equation (9) for the retailer, we analyze the influence of the order quantities Q_a and Q_b on the expected profit of the retailer. Additionally, we compute the first- and second-order differentiations of Equation (9) with respect to Q_b and Q_a, respectively.
First-order differentiation with respect to Q_b:

$$\frac{\partial \mathbb{E}(\Pi_R)}{\partial Q_b} = \frac{\partial \mathbb{E}(\Pi_R^1)}{\partial Q_b} + \frac{\partial \mathbb{E}(\Pi_R^2)}{\partial Q_b} + \frac{\partial \mathbb{E}(\Pi_R^3)}{\partial Q_b} \\ = \int_0^{\frac{D-Q_b}{Q_a}-e} (p - w_b)g(y)dy - \int_{\frac{D-Q_b}{Q_a}-e}^{1} w_bg(y)dy = \left(\frac{D-Q_b}{Q_a} - e\right)p - w_b.$$

Second-order differentiation with respect to Q_b:

$$\frac{\partial^2 \mathbb{E}(\Pi_R)}{\partial Q_b^2} = -\frac{p}{Q_a} < 0.$$

Because of $\frac{\partial^2 \mathbb{E}(\Pi_R)}{\partial Q_b^2} < 0$, we can know that $\mathbb{E}(\Pi_R)$ is a concave function of Q_b. Given the order quantity Q_a, the optimal order quantity can be solved from the equation $(\frac{\partial \mathbb{E}(\Pi_R)}{\partial Q_b} = 0)$, that is

$$Q_b^* = D - Q_a\left(\frac{w_b}{p} + e\right).$$

Given that the optimal supply effort level e^* has a relationship with the order quantity Q_a, as indicated in Proposition 1, we can define the function $e(Q_a)$ as follows: when $0 \le e(Q_a) < \alpha(v)$, $e(Q_a) = \frac{wQ_a + s\alpha(v)Q_a - \beta}{wQ_a + sQ_a}$; when $\alpha(v) \le e(Q_a) \le 1$, $e(Q_a) = \frac{wQ_a - \beta}{wQ_a}$. Similarly, since the optimal order quantity Q_b^* is also related to the order quantity Q_a, we can define the function $Q_b(Q_a)$ as follows: $Q_b(Q_a) = D - Q_a\left(\frac{w_b}{p} + e(Q_a)\right)$. In this scenario, it is important to note that the expected profit $\mathbb{E}(\Pi_R)$ is solely dependent on the value of Q_a, i.e., $\mathbb{E}(\Pi_R) = \mathbb{E}(\Pi_R)(Q_a)$.

First-order differentiation with respect to Q_a: when $0 \le e(Q_a) < \alpha(v)$,

$$\begin{aligned}\frac{\partial \mathbb{E}(\Pi_R)}{\partial Q_a} =& \frac{\partial \mathbb{E}(\Pi_R^1)}{\partial Q_a} + \frac{\partial \mathbb{E}(\Pi_R^2)}{\partial Q_a} + \frac{\partial \mathbb{E}(\Pi_R^3)}{\partial Q_a} \\ =& \int_0^{\alpha(v)-e(Q_a)} \Big[(p - w_a - s)(y + e(Q_a)) + (p - w_a - s)e'(Q_a)Q_a + s\alpha(v))\Big] g(y)dy \\ &+ \int_{\alpha(v)-e(Q_a)}^{\frac{D-Q_b(Q_a)}{Q_a} - e(Q_a)} \Big[(p - w_a)(y + e(Q_a)) + (p - w_a)e'(Q_a)Q_a\Big] g(y)dy \\ &- \int_{\frac{D-Q_b(Q_a)}{Q_a} - e(Q_a)}^{1-e(Q_a)} \Big[w_a(y + e(Q_a)) + w_a e'(Q_a)Q_a\Big] g(y)dy - \int_{1-e(Q_a)}^{1} w_a g(y)dy \\ =& -\frac{\beta^2}{2Q_a^2(w_a + s)} + \frac{w_b^2}{2p} + \frac{[-1 - 2\alpha(v) + \alpha^2(v))]sw_a + 2\alpha(v)sw_b + 2w_a(w_b - w_a)}{2(s + w_a)}\end{aligned}$$

when $\alpha(v) \le e(Q_a) \le 1$, similar to the proof of Proposition 2, we can discuss the two cases $\alpha(v) \le e(Q_a) < \frac{D-Q_b}{Q_a}$ and $\frac{D-Q_b}{Q_a} \le e(Q_a) \le 1$ together, i.e.,

$$\begin{aligned}\frac{\partial \mathbb{E}(\Pi_R)}{\partial Q_a} =& \frac{\partial \mathbb{E}(\Pi_R^2)}{\partial Q_a} + \frac{\partial \mathbb{E}(\Pi_R^3)}{\partial Q_a} \\ =& \int_0^{\frac{D-Q_b(Q_a)}{Q_a} - e(Q_a)} \Big[(p - w_a)(y + e(Q_a)) + (p - w_a)e'(Q_a)Q_a\Big] g(y)dy \\ &- \int_{\frac{D-Q_b(Q_a)}{Q_a} - e(Q_a)}^{1-e(Q_a)} \Big[w_a(y + e(Q_a)) + we'(Q_a)Q_a\Big] g(y)dy - \int_{1-e(Q_a)}^{1} w_a g(y)dy \\ =& -\frac{\beta^2}{2Q_a^2 w_a} - w_a + w_b + \frac{w_b^2}{2p}\end{aligned}$$

Second-order differentiation with respect to Q_a: when $0 \le e(Q_a) < \alpha(v)$, $\frac{\partial^2 \mathbb{E}(\Pi_R)}{\partial Q_a^2} = \frac{\beta^2}{Q_a^3(w_a + s)} > 0$; when $\alpha(v) \le e(Q_a) \le 1$, $\frac{\partial^2 \mathbb{E}(\Pi_R)}{\partial Q_a^2} = \frac{\beta^2}{Q_a^3 w_a} > 0$.

Because of $\frac{\partial^2 \mathbb{E}(\Pi_R)}{\partial Q_a^2} > 0$, we can know that $\mathbb{E}(\Pi_R)$ is the convex function of Q_a. When $\alpha_L > \alpha(v) > e(Q_a) \ge 0$, $\frac{\partial \mathbb{E}(\Pi_R)}{\partial Q_a} < 0$ and $\mathbb{E}(\Pi_R)$ decreases with respect to Q_a. The optimal order quantities (Q_a^*, Q_b^*) is $(0, D)$. When $\max\{\alpha_L, e(Q_a)\} \le \alpha(v)$, the optimal order quantity can be solved from the equation ($\frac{\partial \mathbb{E}(\Pi_R)}{\partial Q_a} = 0$), that is

$$Q_a' = \frac{\beta\sqrt{p}}{\sqrt{\alpha(v)^2 psw_a + 2\alpha(v)ps(w_b - w_a) - pw_a(s + 2(w_a - w_b)) + (s + w_a)w_b^2}},$$

where

$$\alpha_L = \max\left\{\frac{ps(w_a - w_b) + \sqrt{p^2s^2(w_b - w_a)^2 - psw_a\left[pw_a(-s + 2(w_b - w_a)) + (s + w_a)w_b^2\right]}}{psw_a}, 0\right\}.$$

When $\alpha(v) \leq e(Q_a) \leq 1$, the optimal order quantity can be solved from the equation $(\frac{\partial \mathbb{E}(\Pi_R)}{\partial Q_a} = 0)$, that is

$$Q_a^{'} = \frac{\beta\sqrt{p}}{\sqrt{2pw_a w_b - 2pw_a^2 + w_a w_b^2}},$$

Clearly, $\mathbb{E}(\Pi_R)$ takes the minimum value at $Q_a^{'}$. We define the equivalent order quantity placed to supplier a as Q_L satisfying $\mathbb{E}(\Pi_R)(Q_L) = \mathbb{E}(\Pi_R)(0) = D(p - w_b)$. Additionally, considering that Q_b is non-negative and the relationship between $\alpha(v)$ and $e(Q_a)$, we denote the maximum value of order quantity Q_a as $Q_{U_1} = \frac{pD(w_a+s)+p\beta}{(w_a+s)w_b+p(w_a+s\alpha(v))}(\alpha(v) \geq e)$ or $Q_{U_2} = \frac{pDw_a+p\beta}{w_a w_b+pw_a}(e > \alpha(v))$. Given that supplier a's supply effort level is constrained, there exists a maximum value, denoted as e_{max}, within the range $[0,1]$. Consequently, there is also a maximum value, denoted as $Q_{max} = \frac{\beta}{w_a(1-e_{max})+s(\alpha(v)-e_{max})}$ or $\frac{\beta}{w_a(1-e_{max})}$, for supplier a's supply quantity. Based on the magnitude relationship between Q_L, Q_{U_1}, Q_{U_2}, and Q_{max}, we have some claims as follows:
1. When $0 \leq Q_{max} < Q_L$, i.e, $0 \leq e_{max} < \frac{w_a Q_L + s\alpha(v)Q_L - \beta}{(w_a+s)Q_L}$, whether $0 \leq v < \frac{\overline{\alpha}-\alpha_L}{\overline{\alpha}-\underline{\alpha}}$ or $\frac{\overline{\alpha}-\alpha_L}{\overline{\alpha}-\underline{\alpha}} \leq vs. \leq 1$, the optimal order quantities (Q_a^*, Q_b^*) is $(0, D)$ and the optimal supply effort level e^* is 0.
2. When $Q_L \leq Q_{max} < Q_{U_1}$, i.e, $\frac{w_a Q_L + s\alpha(v)Q_L - \beta}{(w_a+s)Q_L} \leq e_{max} < \frac{w_a Q_{U_1} + s\alpha(v)Q_{U_1} - \beta}{(w_a+s)Q_{U_1}}$, based on the cut-off point $\frac{\overline{\alpha}-e_{max}}{\overline{\alpha}-\underline{\alpha}}$ of the trust value v, we can determine that the optimal order quantities (Q_a^*, Q_b^*) is $(Q_{max}, Q_b^*(Q_{max}))$ and the optimal supply effort level e^* is e_{max}.
3. When $Q_{U_1} \leq Q_{max} < Q_{U_2}$, i.e, $\frac{w_a Q_{U_1} + s\alpha(v)Q_{U_1} - \beta}{(w_a+s)Q_{U_1}} \leq e_{max} < \frac{w_a Q_{U_2} - \beta}{w_a Q_{U_2}}$, based on the cut-off point $\frac{\overline{\alpha}-e_{max}}{\overline{\alpha}-\underline{\alpha}}$ of the trust value v, we can determine that the optimal order quantities (Q_a^*, Q_b^*) is $(Q_{U_1}, 0)$ or $(Q_{max}, Q_b^*(Q_{max}))$, and the optimal supply effort level e^* is $e(Q_a^*)$ or e_{max}.
4. When $Q_{U_2} \leq Q_{max} \leq 1$, i.e, $\frac{w_a Q_{U_2} - \beta}{w_a Q_{U_2}} \leq e_{max} \leq 1$, the optimal order quantities (Q_a^*, Q_b^*) is $(Q_{U_2}, 0)$ and the optimal supply effort level e^* is $\frac{w_a Q_a^* - \beta}{w_a Q_a^*}$. □

Appendix C.2. Proof of Corollary 1

Proof. The proof of part 1: From Cases 2 and 3 of Proposition 5, we employ the chain rule for derivatives to calculate the first-order derivative of Equations (10) and (11) concerning the variable v.
When $0 \leq e_{max} < \frac{w_a Q_L + s\alpha(v)Q_L - \beta}{(w_a+s)Q_L}$ and $\frac{w_a Q_{U_2} - \beta}{w_a Q_{U_2}} \leq e_{max} \leq 1$, $\frac{\partial Q_a^*}{\partial v} = 0$
When $\frac{w_a Q_L + s\alpha(v)Q_L - \beta}{(w_a+s)Q_L} \leq e_{max} < \frac{w_a Q_{U_1} + s\alpha(v)Q_{U_1} - \beta}{(w_a+s)Q_{U_1}}$,

$$\frac{\partial Q_a^*}{\partial v} = \frac{\partial Q_a^*}{\partial \alpha(v)}\frac{\partial \alpha(v)}{\partial v} = \frac{s(\overline{\alpha}-\underline{\alpha})\beta}{[w_a(1-e_{max}) + s(\alpha(v) - e_{max})]^2} > 0.$$

When $\frac{w_a Q_{U_1} + s\alpha(v)Q_{U_1} - \beta}{(w_a+s)Q_{U_1}} \leq e_{max} < \frac{w_a Q_{U_2} - \beta}{w_a Q_{U_2}}$, if $0 \leq v < \frac{\overline{\alpha}-e_{max}}{\overline{\alpha}-\underline{\alpha}}$,

$$\frac{\partial Q_a^*}{\partial v} = \frac{(pD(w_a+s)+p\beta)[ps(\overline{\alpha}-\underline{\alpha})]}{[(w_a+s)w_b + p(w_a + s\alpha(v))]^2} > 0,$$

if $\frac{\overline{\alpha}-e_{max}}{\overline{\alpha}-\underline{\alpha}} \leq vs. \leq 1$, the optimal order quantity Q_a^* is larger than that in interval $v \in \left[0, \frac{\overline{\alpha}-e_{max}}{\overline{\alpha}-\underline{\alpha}}\right)$. Therefore, the optimal ordering quantity Q_a^* is non-decreasing with the trust value v.

As the optimal ordering quantity Q_a^* depends on the trust value v, with the exception of the commitment proportion $\alpha(v)$, we utilize the chain rule to compute the first-order

derivative of Equation (3) with respect to v.
When $0 \leq e_{max} < \frac{w_a Q_{U_1} + s\alpha(v) Q_{U_1} - \beta}{(w_a + s) Q_{U_1}}$ and $\frac{w_a Q_{U_2} - \beta}{w_a Q_{U_2}} \leq e_{max} \leq 1$, $\frac{\partial e^*}{\partial v} = 0$.
When $\frac{w_a Q_{U_1} + s\alpha(v) Q_{U_1} - \beta}{(w_a + s) Q_{U_1}} \leq e_{max} < \frac{w_a Q_{U_2} - \beta}{w_a Q_{U_2}}$, if $0 \leq v < \frac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}}$,

$$\frac{\partial e^*}{\partial v} = \frac{\partial e^*}{\partial \alpha(v)} \frac{\partial \alpha(v)}{\partial v} + \frac{\partial e^*}{\partial Q_a^*} \frac{\partial Q_a^*}{\partial v} = \frac{s(\overline{\alpha} - \underline{\alpha})}{w + s} \left(-1 + \frac{\beta}{D(w_a + s) + \beta} \right) < 0,$$

if $\frac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}} \leq vs. \leq 1$, $\frac{\partial e^*}{\partial v} = 0$. Therefore, only if $0 \leq v < \frac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}}$, the optimal supply effort level e^* decreases with the trust value v under Case 3 of Proposition 5, i.e, $\frac{\partial e^*}{\partial v} < 0$.
The proof of part 2: From Cases 2 and 3 of Proposition 5, we utilize the chain rule to compute the first-order derivative of Q_b^* in Equation (10) with respect to v.
When $0 \leq e_{max} < \frac{w Q_L + s\alpha(v) Q_L - \beta}{(w + s) Q_L}$ and $\frac{w_a Q_{U_2} - \beta}{w_a Q_{U_2}} \leq e_{max} \leq 1$, $\frac{\partial Q_b^*}{\partial v} = 0$.
When $\frac{w_a Q_L + s\alpha(v) Q_L - \beta}{(w_a + s) Q_L} \leq e_{max} < \frac{w_a Q_{U_1} + s\alpha(v) Q_{U_1} - \beta}{(w_a + s) Q_{U_1}}$,

$$\begin{aligned}
\frac{\partial Q_b^*}{\partial v} &= \frac{\partial Q_b^*}{\partial \alpha(v)} \frac{\partial \alpha(v)}{\partial v} + \frac{\partial Q_b^*}{\partial Q_a^*} \frac{\partial Q_a^*}{\partial v} = \frac{ps(\overline{\alpha} - \underline{\alpha})}{p(w_a + s)} Q_a^* - \frac{(w_a w_b + s w_b + p w_a + ps\alpha(v))}{p(w_a + s)} \frac{\partial Q_a^*}{\partial v} \\
&= \frac{\beta s(\overline{\alpha} - \underline{\alpha})}{p(w_a + s)[w(1 - e_{max}) + s(\alpha(v) - e_{max})]} \left(1 - \frac{(w_a w_b + s w_b + p w_a + ps\alpha(v))}{[w(1 - e_{max}) + s(\alpha(v) - e_{max})]} \right) < 0.
\end{aligned}$$

When $\frac{w_a Q_{U_1} + s\alpha(v) Q_{U_1} - \beta}{(w_a + s) Q_{U_1}} \leq e_{max} < \frac{w_a Q_{U_2} - \beta}{w_a Q_{U_2}}$, $\frac{\partial Q_b^*}{\partial v} = 0$. Hence, the optimal order quantity Q_b^* decreases with the trust value v in interval $[0, \frac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}})$ and is equal to a positive constant or 0 in interval $\left[\frac{\overline{\alpha} - e_{max}}{\overline{\alpha} - \underline{\alpha}}, 1 \right]$.
The proof of part 3: Following a process akin to that of part 2's proof, when we compute the first-order derivatives of Equations (7) and (9) with respect to the variable v, we observe that the following equations are satisfied.
For the supplier a: when $0 \leq v < \frac{\overline{\alpha} - e}{\overline{\alpha} - \underline{\alpha}}$,

$$\begin{aligned}
\frac{\partial \mathbb{E}(\Pi_{s_a})}{\partial v} &= \frac{\partial \mathbb{E}(\Pi_{s_a})}{\partial \alpha(v)} \frac{\partial \alpha(v)}{\partial v} + \frac{\partial \mathbb{E}(\Pi_{s_a})}{\partial Q_a^*} \frac{\partial Q_a^*}{\partial v} + \frac{\partial \mathbb{E}(\Pi_{s_a})}{\partial e^*} \frac{\partial e^*}{\partial v} \\
&= \left[-c_a + w_a e^* + \frac{1 - e^{*2}}{2} w_a - s\alpha(v)(\alpha(v) - e^{*2}) + \frac{s}{2}(\alpha(v)^2 - e^{*2}) \right] \frac{\partial Q_a^*}{\partial v} + sQ_a^*(\overline{\alpha} - \underline{\alpha})(\alpha(v) - e^*) \\
&\geq \left(\frac{w_a - s}{2} - c_a \right) \frac{\partial Q_a^*}{\partial v} + sQ^*(\overline{\alpha} - \underline{\alpha})(\alpha(v) - e^*) \geq 0,
\end{aligned}$$

when $\frac{\overline{\alpha} - e}{\overline{\alpha} - \underline{\alpha}} \leq vs. \leq 1$,

$$\begin{aligned}
\frac{\partial \mathbb{E}(\Pi_{s_a})}{\partial v} &= \frac{\partial \mathbb{E}(\Pi_{s_a})}{\partial \alpha(v)} \frac{\partial \alpha(v)}{\partial v} + \frac{\partial \mathbb{E}(\Pi_{s_a})}{\partial Q_a^*} \frac{\partial Q_a^*}{\partial v} + \frac{\partial \mathbb{E}(\Pi_{s_a})}{\partial e^*} \frac{\partial e^*}{\partial v} \\
&= \left[-c_a + w_a e^* + \frac{1 - e^{*2}}{2} w_a \right] \frac{\partial Q_a^*}{\partial v} \geq \left(\frac{w_a}{2} - c_a \right) \frac{\partial Q_a^*}{\partial v} \geq 0.
\end{aligned}$$

For the retailer: when $0 \leq v < \frac{\overline{\alpha} - e}{\overline{\alpha} - \underline{\alpha}}$,

$$\frac{\partial \mathbb{E}(\Pi_R)}{\partial v} = \frac{\partial \mathbb{E}(\Pi_R)}{\partial \alpha(v)} \frac{\partial \alpha(v)}{\partial v} + \frac{\partial \mathbb{E}(\Pi_R)}{\partial Q_a^*} \frac{\partial Q_a^*}{\partial v} = -sQ_a^*(\overline{\alpha} - \underline{\alpha}) \leq 0,$$

when $0 \leq v < \frac{\overline{\alpha} - e}{\overline{\alpha} - \underline{\alpha}}$, $\frac{\partial \mathbb{E}(\Pi_R)}{\partial v} = 0$. Therefore, the supplier a's optimal expected profit is non-decreasing as the retailer's trust in the supplier a grows, while the retailer's optimal expected profit is non-increasing with an increase in the retailer's trust. Since the relationship between Π_{s_b} and v depends only on Q_b^*, the variation in the optimal expected profit for supplier b with respect to changes in trust value aligns with the alterations observed in Q_b^* as trust value fluctuates. □

Appendix C.3. Proof of Proposition 6

Proof. According to the Leibniz formula and the expected profit Equation (14), we analyze the influence of the supply effort level e^c and the order quantities Q_a^c and Q_b^c on the expected profit for the integrated supply chain. Additionally, similar to the proof procedure of Proposition 4, we compute the first- and second-order differentiations of Equation (14) with respect to e^c, Q_a^c and Q_b^c, respectively.

First-order differentiation on e^c:

$$\frac{\partial \mathbb{E}(\Pi_C)}{\partial e^c} = \int_0^{\frac{D-Q_b^c}{Q_a^c}-e^c} pQ_a^c g(y)dy - \beta = p(D - Q_b^c - e^c Q_a^c) - \beta.$$

First-order differentiation on Q_a^c:

$$\frac{\partial \mathbb{E}(\Pi_C)}{\partial Q_a^c} = \int_0^{\frac{D-Q_b^c}{Q_a^c}-e^c} p(y+e^c)g(y)dy - c_a = \frac{p}{2}\left(\left(\frac{D-Q_b^c}{Q_a^c}\right)^2 - (e^c)^2\right) - c_a.$$

First-order differentiation on Q_b^c:

$$\frac{\partial \mathbb{E}(\Pi_C)}{\partial Q_b^c} = \int_0^{\frac{D-Q_b^c}{Q_b^c}-e^c} (p-c_b)g(y)dy - \int_{\frac{D-Q_b^c}{Q_a^c}-e^c}^1 c_b g(y)dy = p\left(\frac{D-Q_b^c}{Q_a^c} - e^c\right) - c_b.$$

Second-order differentiation on Q_a^c, Q_b^c and e^c, respectively:

$$\frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial (Q_a^c)^2} = -p\frac{(D-Q_b^c)^2}{(Q_a^c)^3}, \quad \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial (Q_b^c)^2} = -\frac{p}{Q_a^c}, \quad \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial (e^c)^2} = -pQ_a^c,$$
$$\frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial Q_a^c \partial Q_b^c} = \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial Q_b^c \partial Q_a^c} = -\frac{p(D-Q_b^c)}{(Q_a^c)^2}, \quad \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial Q_b^c \partial e^c} = \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial e^c \partial Q_b^c} = -p$$
$$\frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial Q_a^c \partial e^c} = \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial e^c \partial Q_a^c} = -pe^c.$$

Since $\frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial (Q_a^c)^2} < 0$, $\frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial (Q_b^c)^2} < 0$ and $\frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial (e^c)^2} < 0$, $\mathbb{E}(\Pi_C)$ is a concave function with respect to e^c, Q_a^c and Q_b^c, respectively.

Furthermore, the determinant of the Hessian Matrix H is

$$|H| = \begin{vmatrix} \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial (e_a^c)^2} & \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial e^c \partial Q_a^c} & \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial e^c \partial Q_b^c} \\ \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial Q_a^c \partial e^c} & \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial (Q_a^c)^2} & \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial Q_a^c \partial Q_b^c} \\ \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial Q_b^c \partial e^c} & \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial Q_b^c \partial Q_a^c} & \frac{\partial^2 \mathbb{E}(\Pi_C)}{\partial (Q_b^c)^2} \end{vmatrix}.$$

However, at the stagnation point of $\mathbb{E}(\Pi_C)$, the determinant of the Hessian Matrix H is

$$|H| = Q_a^c e^{c*} p^3 \left(\frac{D-Q_b}{Q_a} - 1\right)^2 > 0,$$

and $\frac{\partial \mathbb{E}(\Pi_C)}{\partial Q_a^c} > 0$. Hence, there are no optimal order quantities and supply effort levels. So, none of the three parties can cooperate, and the centralized case will degenerate into the decentralized case. □

Appendix D. Multi-Period Extension

Appendix D.1. Proof of Proposition 7

Proof. For the case b, from Proposition 4, when $0 \le e_t^* \le E_t \le 1$, the supplier's expected profit $S_b(\cdot, E_t)$ increases with the trust value v_t. Similar to the proof procedure of Proposition 6, we can deduce that the optimal ordering quantity Q_t^* is non-decreasing with the trust value v_t in the t-period. Furthermore, $S_b(\cdot, E_t)$ is also non-decreasing with the trust value v_t.

For the case d, as indicated by Corollary 1, it is evident that $S_d(\cdot, E_t)$ is non-decreasing with the trust value v_t for each $E_t \in [0, 1]$.

For the case c, from Proposition 3, the optimal decisions (e_t^{c*}, Q_t^{c*}) is not related to the trust value in the t-period. Additionally, $\frac{\partial \Pi_s}{\partial v_t} = sQ_t^{c*}(\overline{\alpha} - \underline{\alpha})(\alpha(v_t) - e^*) \ge 0$. So, we obtain that $S_c(\cdot, E_t)$ is non-decreasing with the trust value v_t for each $E_t \in [0, 1]$.

For the case b, when $0 \le E_t \le e_t^* \le 1$, the optimal supply effort of supplier is E_t. Similar to Equation (2), we can rewrite the expected profit of the retailer in period t as:

$$\begin{aligned}\mathbb{E}(\Pi_{r_t}) = \int_0^1 &[p \min\{D_t, \min\{(y + E_t)Q_t, Q_t\}\} + s(\alpha(v_t)Q_t - (y + E_t)Q_t)^+ \\ &-w \min\{(y + E_t)Q_t, Q_t\}]g(y)dy.\end{aligned} \tag{A13}$$

According to the Leibniz formula and the expected profit Equation (A13) for the retailer, we analyze the influence of the order quantity Q_t on the expected profit of the retailer. Next, we compute the first- and second-order differentiations of Equation (A13) with respect to Q_t.

First-order differentiation: When $0 \le E_t < \alpha(v_t)$,

$$\begin{aligned}\frac{\partial \mathbb{E}(\Pi_{r_t})}{\partial Q_t} = &\int_0^{\alpha(v_t) - E_t} [(p - w - s)(y + E_t) + s\alpha(v_t))]g(y)dy + \int_{\alpha(v_t) - E_t}^{\frac{D_t}{Q_t} - E_t} [(p - w)(y + E_t)]g(y)dy \\ &- \int_{\frac{D_t}{Q_t} - E_t}^{1 - E_t} w(y + E_t)g(y)dy - \int_{1 - E_t}^1 wg(y)dy\end{aligned}$$

$$\frac{\partial \mathbb{E}(\Pi_{r_t})}{\partial Q_t} = \frac{D_t^2 p}{2Q_t^2} + \frac{1}{2}\Big(\alpha(v_t)s - w + E_t^2(s + w - p) - 2E_t(\alpha(v_t)s + w)\Big)$$

When $\alpha(v_t) \le E_t < \frac{D_t}{Q_t}$,

$$\begin{aligned}\frac{\partial \mathbb{E}(\Pi_{r_t})}{\partial Q_t} &= \int_0^{\frac{D_t}{Q_t} - E_t} (p - w)(y + E_t)g(y)dy - \int_{\frac{D_t}{Q_t} - E_t}^{1 - E_t} w(y + E_t)g(y)dy - \int_{1 - E_t}^1 wg(y)dy \\ &= \frac{D_t^2 p}{2Q_t^2} + \frac{1}{2}\Big(-E_t^2 p + (-1 + (-2 + E_t)E_t)w\Big).\end{aligned}$$

When $\frac{D_t}{Q_t} \le E_t \le 1$,

$$\frac{\partial \mathbb{E}(\Pi_{r_t})}{\partial Q_t} = -\int_0^{1 - E_t} w(y + E_t)g(y)dy - \int_{1 - E_t}^1 wg(y)dy = -\frac{1 - E_t^2}{2}w - wE_t < 0.$$

Since $\frac{\partial \mathbb{E}(\Pi_{r_t})}{\partial Q_t} < 0$ when $\frac{D_t}{Q_t} \le E_t \le 1$, we can analyze the two cases $\alpha(v_t) \le E_t < \frac{D_t}{Q_t}$ and $\frac{D_t}{Q_t} \le E_t \le 1$ together.

Second-order differentiation:

$$\frac{\partial^2 \mathbb{E}(\Pi_{r_t})}{\partial Q_t^2} = -\frac{D_t^2 p}{Q_t^3} < 0.$$

Because of $\frac{\partial^2 \mathbb{E}(\Pi_{r_t})}{\partial Q_t^2} < 0$, we can know that $\mathbb{E}(\Pi_{r_t})$ is the concave function of Q_t. When $0 \leq E_t < \alpha(v)$, the optimal order quantity in period t can be solved from the equation $(\frac{\partial \mathbb{E}(\Pi_{r_t})}{\partial Q_t} = 0)$, that is

$$Q_t^* = \frac{D_t\sqrt{p}}{\sqrt{-(\alpha(v_t) - E_t)^2 s + E_t^2(p-w) + w(1+2E_t)}}.$$

When $\alpha(v_t) \leq E_t \leq 1$, the optimal order quantity in period t can be solved from the equation $(\frac{\partial \mathbb{E}(\Pi_{r_t})}{\partial Q_t} = 0)$, that is

$$Q_t^* = \frac{D_t\sqrt{p}}{\sqrt{E_t^2(p-w) + w(1+2E_t)}}.$$

Hence, when $0 \leq E_t < e_t^*$, we can know that the retailer's optimal ordering quantity Q_t^* in period t is

$$Q_t^* = \begin{cases} \dfrac{D_t\sqrt{p}}{\sqrt{-(\alpha(v_t) - E_t)^2 s + E_t^2(p-w) + w(1+2E_t)}}, & 0 \leq v_t < \dfrac{\overline{\alpha} - E_t}{\overline{\alpha} - \underline{\alpha}} \\ \dfrac{D_t\sqrt{p}}{\sqrt{E_t^2(p-w) + w(1+2E_t)}}, & \dfrac{\overline{\alpha} - E_t}{\overline{\alpha} - \underline{\alpha}} \leq v_t \leq 1 \end{cases}.$$

At the same time, we compute the first-order differentiation of Q_t^* with respect to v_t,

$$\frac{\partial Q_t^*}{\partial v_t} = -\frac{D_t\sqrt{p}}{\left[-(\alpha(v_t) - E_t)^2 s + E_t^2(p-w) + w(1+2E_t)\right]^{3/2}} s(\alpha(v_t) - E_t)(\overline{\alpha} - \underline{\alpha}) \leq 0.$$

This implies the optimal ordering quantity Q_t^* is non-increasing with the trust value v_t in the case b when $0 \leq E_t < e_t^*$. □

Appendix D.2. Proof of Proposition 8

Proof. The first part of the proof: Based on the insights from Proposition 7, we ascertain that $(S_i(\cdot, E_i), i \in b, d, c)$ is non-decreasing with the trust value v_t for each $0 \leq E_i \leq 1$. Subsequently, we employ the finite-horizon equivalent of Equation (24) for $t = 1, 2, \ldots$ beginning with $V_{i,0}(\cdot) = 0$:

$$\begin{aligned} V_{i,t}(v) &= \max_{E_i \in [0,1]} \{J_{i,t}(v, E_i)\}, \\ J_{i,t}(v, E_i) &= S_i(v, E_i) - \frac{\eta}{2}E_i^2 + \delta \int_0^1 V_{i,t-1}(\lambda v + (1-\lambda)\min\{y + e(v, E_i), 1\})g(y)dy. \end{aligned} \tag{A14}$$

Additionally, we initialize an inductive proof with $V_{i,0}(\cdot) = 0$. In cases where $\lambda + \frac{\partial e(v,E_i)}{\partial v}(1-\lambda) \geq 0$, we proceed with the induction by leveraging the monotonicity of $S_i(\cdot, E_i)$ and the understanding that the monotonicity of $V_{i,t-1}(\cdot)$ implies the monotonicity of $J_{i,t}(\cdot, E_i)$. Consequently, we establish the monotonicity of $V_{i,t}(\cdot)$.

The second step of the proof: we show that $V_{i,t}(\cdot)$ converges point-wise to $V_i(\cdot)$ as $t \to \infty$, and the limit function $V_i(\cdot)$ satisfies Equation (24). Convergence is implied by $0 < \delta < 1$ and $0 \leq vs. \leq 1$ because Equation (A14) is a contraction mapping on a complete metric space. Banach's contraction-mapping fixed-point theorem implies that there is a unique fixed point $V_i(\cdot)$ which satisfies Equation (24). The optimal decision-making E_i can be obtained by Equation (24). Then, we can obtain an optimal deterministic policy π_i in the infinite horizon.

The last step confirms that $V_i(\cdot)$ inherits the monotonicity of $V_{i,t}(\cdot)$. For all $\epsilon > 0$, there exists $N_\epsilon < \infty$ such that $n \geq N_\epsilon$ implies $|V_i(v) - V_{i,t}(v)| \leq \epsilon/2$ for all $0 \leq vs. \leq 1$. Therefore, for all $0 \leq vs. \leq v^0 \leq 1$, $\epsilon > 0$, and $n \geq N_\epsilon$, the monotonicity of $V_{i,t}(\cdot)$ implies

$$V_i(v^0) - V_i(v) = \left[V_{i,t}(v^0) - V_{i,t}(v)\right] + \left[V_i(v^0) - V_{i,t}(v^0)\right] - \left[V_i(v) - V_{i,t}(v)\right]$$
$$\geq \left[V_i(v^0) - V_{i,t}(v^0)\right] - \left[V_i(v) - V_{i,t}(v)\right] \geq -\epsilon$$

Let $\epsilon \to 0$, we have $V_i(v^0) - V_i(v) \geq 0$ for all $0 \leq vs. \leq v^0 \leq 1$. □

Appendix D.3. Convergence Process of Suppliers' Expected Value

According to Equation (24) in Section 3.2.2, we observe that the supplier's expected value evolves dynamically, gradually converging to a constant value over time.

This convergence is consistently observed across different cases. Consequently, we focus on presenting this convergence process in Figure A1.

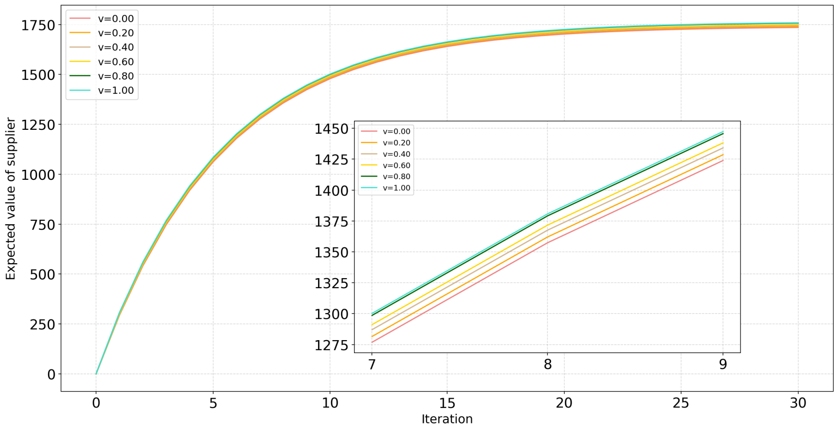

Figure A1. Expected value of supplier under decentralized case.

Figure A1 depicts the iterative process of the supplier's expected value under varying trust values in the decentralized scenario. Notably, after approximately 25 iterations, the expected value stabilizes. Furthermore, a higher trust value correlates with a greater expected value, indicating reduced loss caused by stockouts for the supplier. Remarkably, the convergence process in the centralized and dual-channel cases reflects that of the decentralized process. Specifically, the expected value of the centralized supplier converges to 1722, while that of the dual-channel supplier a converges to 1313.

At the same time, varying initial states v exert different influences on the optimal maximum supply efforts, as illustrated in Figure A2. It is evident that despite fluctuations in trust levels, the supplier's optimal maximum supply effort remains relatively stable. Specifically, in the centralized single-channel case, the optimal maximum supply effort is approximately 0.10. In the decentralized single-channel case, it rises to about 0.27. For the dual-channel case, the effort further increases to around 0.57. Notably, the optimal maximum supply effort in the decentralized case is significantly higher than in the centralized case, indicating that the trust–punishment mechanism effectively motivates suppliers to fulfill downstream demand. Furthermore, due to supplier competition in the dual-channel scenario, the optimal maximum supply effort surpasses that of the other two cases by a considerable margin.

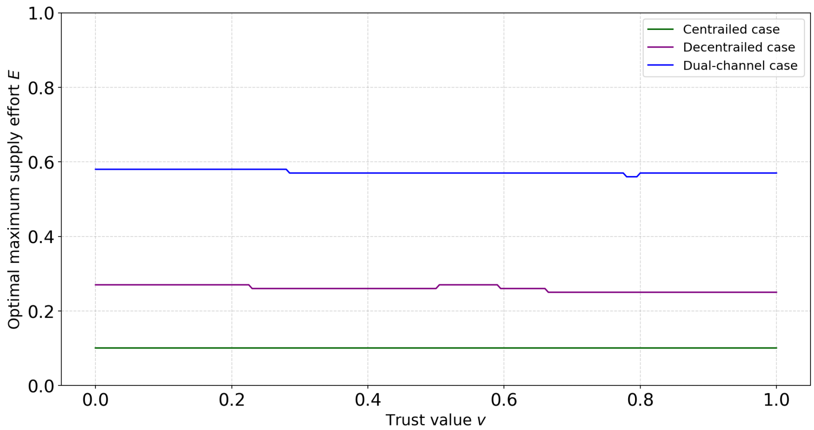

Figure A2. Optimal maximum supply effort E varies with trust value v.

Appendix E. Sensitivity Analysis of Multi-Period Case

Through multi-period sensitivity analysis, we investigate how key intertemporal parameters—the production maintenance cost coefficient η, the transactional linkage coefficient λ, and the discount factor δ—shape supply chain decision-making and channel stability.

Appendix E.1. Sensitivity Analysis on the Maintenance Cost Coefficient η

We first evaluate how the maintenance cost coefficient η influences system decisions when the initial trust state is low ($v = 0.2$). As presented in Table A2, escalating maintenance costs systematically suppress the supplier's investment capacity, leading to diminished supply stability (e decreases from 0.3 to 0.15). This upstream capacity degradation forces a defensive shift in the retailer's procurement strategy, compelling higher order quantities (Q rises from 261 to 303) to buffer against supply uncertainty. From an economics perspective, this defensive over-ordering under constrained capacity directly intensifies the bullwhip effect, leading to systemic profit contraction for both actors.

Table A2. Numerical results of parameter analysis: basic case under multi-period.

η	Expected Value	Q	e	$\mathbb{E}(\Pi_s)$	%	$\mathbb{E}(\Pi_r)$	%
100	1740	261	0.3	301	2.0	1602	2.8
200	1715	269	0.27	299	1.4	1576	1.2
300	1692	269	0.27	299	1.4	1576	1.2
400	1671	274	0.25	295	0.0	1558	0.0
500	1651	274	0.25	295	0.0	1558	0.0
600	1632	274	0.25	295	0.0	1558	0.0
800	1597	279	0.23	290	−1.7	1541	−1.1
1000	1565	279	0.23	290	−1.7	1541	−1.1
1500	1500	288	0.2	282	−4.4	1514	−2.8
2000	1446	296	0.17	269	−8.8	1486	−4.6
3000	1364	303	0.15	260	−11.9	1467	−5.8

Furthermore, while higher initial trust allows the retailer to lower her order quantity and reduce contractual penalties, a critical intertemporal moral hazard arises. In a long-term single-channel setup, the dilution of the punishment intensity softens the contract's enforcement power, permitting the supplier to maintain a lower effort profile (0.3 under single-channel vs. 0.559 under dual-channel). This divergence from single-period benchmarks demonstrates that in the absence of channel competition, long-term relational reliance can induce upstream opportunistic behavior, as the supplier deliberately shirks on variable effort to maximize localized profit margins at the retailer's expense.

Table A3. Numerical results of parameter analysis: dual channel under multi-period.

η	Expected Value	Q_a	Q_b	e	$\mathbb{E}(\Pi_{sa})$	%	$\mathbb{E}(\Pi_{sb})$	%	$\mathbb{E}(\Pi_r)$	%
100	1577	218	0	0.559	267	0.0	0	/	1808	0.0
200	1484	218	0	0.559	267	0.0	0	/	1808	0.0
300	1390	218	0	0.559	267	0.0	0	/	1808	0.0
400	1296	218	0	0.559	267	0.0	0	/	1808	0.0
500	1203	218	0	0.559	267	0.0	0	/	1808	0.0
600	1111	218	0	0.559	267	0.0	0	/	1808	0.0
800	927	218	0	0.559	267	0.0	0	/	1808	0.0
1000	744	218	0	0.559	267	0.0	0	/	1808	0.0
1500	340	130	142	0.08	61	−77.2	355	/	1728	−4.4
2000	331	130	143	0.07	59	−77.9	358	/	1727	−4.5
3000	319	130	146	0.05	56	−79.0	365	/	1726	−4.5

For the dual-channel scenario, Table A3 reveals a critical capacity–cost milestone. When production maintenance costs cross into prohibitive regimes ($\eta \geq 1500$), Supplier a experiences a structural breakdown in effort utility, scaling back their effort from 0.559 to 0.08. This supply risk triggers an immediate reallocation of order volumes from the primary channel (Q_a drops to 130) to the premium backup channel (Q_b steps up to 142), while this strategic diversification stabilizes supply security, the higher backup wholesale price compromises the retailer's financial frontier. This proves that high upstream operational friction naturally forces the supply chain to transition from single- to dual-channel structures, acting as a capacity-hedging tool at the expense of retailer profitability.

The core distinction from the single-channel baseline lies in the contract-theoretic value of an active threat point. The potential introduction of Supplier b exerts continuous competitive pressure on Supplier a, forcing them to commit to a significantly higher effort floor (0.559 vs. 0.3) across stable cost zones. By maintaining an alternative sourcing option, the retailer leverages a credible mechanism threat that bounds upstream moral hazard, effectively mitigating supply uncertainty and maximizing intertemporal profits until primary cost structures become prohibitively high.

Appendix E.2. Sensitivity Analysis on the Linkage Parameter λ

To analyze how the transactional linkage coefficient λ influences intertemporal operations under single- and dual-channel configurations, we conduct numerical evaluations across multi-period horizons, as tabulated in Tables A4 and A5.

Table A4. Numerical results of parameter analysis: basic case under parameter λ.

λ	Expected Value	Q	$\mathbb{E}(\Pi_r)$	%	e	$\mathbb{E}(\Pi_s)$	%
0.1	1674	264	1593	1.08	0.290	301	0.67
0.3	1674	264	1593	1.08	0.290	301	0.67
0.5	1671	269	1576	0.0	0.270	299	0.0
0.7	1664	269	1576	0.0	0.270	299	0.0
0.9	1647	269	1576	0.0	0.270	299	0.0

Table A4 illustrates the parameter impacts under the baseline single-channel structure. As λ increases, a critical structural friction emerges: the escalating linkage factor shrinks the supplier's optimal non-contractable effort profile (e drops from 0.290 to 0.270). This operational contraction indicates that an overly aggressive contractual weight softens the marginal efficacy of immediate supply effort, allowing a rational supplier to prioritize cost containment. Consequently, the diminished upstream fulfillment reliability forces the retailer to scale up defensive ordering (Q expands from 264 to 269) to cushion stockout uncertainty. This friction triggers severe demand distortion, amplifying the bullwhip effect and pulling both retail profits ($\mathbb{E}(\Pi_r)$ drops from 1593 to 1576, representing a 1.08% contraction compared to the $\lambda = 0.5$ baseline) and supplier returns ($\mathbb{E}(\Pi_s)$ drops from 301 to 299, representing a 0.67% contraction) downward.

Table A5. Numerical results of parameter analysis: dual channel under parameter λ.

λ	Expected Value	Q_a	$\mathbb{E}(\Pi_{sa})$	%	Q_b	$\mathbb{E}(\Pi_{sb})$	%	e	$\mathbb{E}(\Pi_r)$	%
0.10	1299	225	284	0.0	0	0	/	0.572	1810	0.0
0.30	1298	225	284	0.0	0	0	/	0.572	1810	0.0
0.50	1296	225	284	0.0	0	0	/	0.572	1810	0.0
0.70	1290	225	284	0.0	0	0	/	0.572	1810	0.0
0.90	1271	225	284	0.0	0	0	/	0.572	1810	0.0

In sharp contrast, Table A5 isolates the parameter dynamics under a dual-channel architecture, revealing a robust operational insulation effect. Across the entire parametric spectrum ($\lambda \in [0.1, 0.9]$), the primary supplier's optimal effort and the retailer's sourcing strategy remain strictly invariant ($e = 0.572$, $Q_a = 225$, $Q_b = 0$). From a contract-theoretic perspective, the presence of an active alternative channel acts as a permanent mechanism threat that effectively locks the supplier into a high-performance equilibrium, rendering them impervious to changes in λ. This competitive enforcement protects downstream margins ($\mathbb{E}(\Pi_r) = 1810$), proving that a dual-sourcing structure functions as a powerful strategic shield that neutralizes internal contract friction and preserves channel efficiency.

Appendix E.3. Sensitivity Analysis on the Discount Factor δ

To evaluate the intertemporal operational impacts of the discount factor δ, we analyze the multi-period simulated systems across a parametric spectrum of $\delta \in [0.70, 0.95]$. Numerical simulations reveal that variations in the discount factor δ exert no structural influence on the optimal ordering decisions, non-contractable supply effort levels, or channel revenues in either the single-channel baseline or the dual-channel configuration. Under the single-channel structure, decisions and profits remain perfectly constant at $Q = 269$, $e = 0.270$, $\mathbb{E}(\Pi_r) = 1576$, and $\mathbb{E}(\Pi_s) = 299$. Symmetrically, within the dual-channel landscape, the operational vector stays locked at $Q_a = 225$, $Q_b = 0$, $e = 0.572$, and $\mathbb{E}(\Pi_r) = 1810$.

This complete invariance indicates that once the behavioral trust–punishment mechanism settles into its multi-period steady-state equilibrium under a specific trust transition policy, the localized marginal returns on capacity investment and safety-stock adjustments become tightly bounded by the physical capacity thresholds and cost parameters. Consequently, while a change in δ naturally scales the absolute systemic discounted values across periods, it does not alter the relative operational trade-offs or marginal decision thresholds, proving the structural robustness of both sourcing architectures against shifts in the players' intertemporal patience.

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