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Membership Bundling in Platform Competition: To Bundle Add-Ons Together or Separately?

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Abstract

Platforms are increasingly adopting membership bundling strategies to strengthen competitiveness. This paper explores how duopoly platforms bundle their membership services (the base products) with those provided by other platforms (add-ons) through a game-theoretic lens. We focus on the competing platforms' strategic decisions to bundle different add-ons together or separately by examining three key determinants: the quality gap between the base products, the quality of versus consumer preference for the add-ons, and the profit-sharing ratio to partners who offer the add-ons. First, with comparable base-good qualities, symmetric bundling emerges in equilibrium. Specifically, simultaneously bundling add-ons together (or separately) dominates when the add-on quality (or the consumers' preference) mainly drives purchase. Second, significant quality disparity in the base goods leads to asymmetric equilibria: the high-quality platform strategically selects the bundling mode, together or separately, that minimizes the profit-sharing payouts, forcing the low-quality rival to adopt a different strategy. Finally, when the base goods have similar quality, the platform competition can largely yield optimal welfare outcomes. With a significant quality disparity, however, the equilibrium strategies may deviate from social efficiency. Our study advances understanding of platform competition with membership bundling and offers regulatory insights for social planners to strategically intervene in platforms' membership bundling decisions.

Keywords: membership; bundling; platform competition; digital ecosystem

1. Introduction

Digital platforms are continuously pursuing new growth opportunities within their ecosystems [1,2], experimenting with diverse sales strategies to explore consumption potentials. With the widespread acceptance of membership-based services [3], membership bundling—the practice of marketing two or more membership-based products (or services) as a combined package selling at a special price—has become particularly prominent, especially in a competitive context.

For example, as shown in Figure 1, Taobao and JD.COM are two main e-commerce platforms in the Chinese market. While Taobao bundles its membership service with those of NetEase Cloud Music and Quark together for sales, JD.COM bundles its membership service, JD PLUS, with the membership services of NetEase Cloud Music and Baidu Library separately. In the audio streaming industry, Himalaya's service is bundled with the services of Dingdong (an e-commerce platform) and Baidu Netdisk separately, while its competitor Dragonfly FM bundles its own service with those from LETV and Kuaikan Comic together. In the video streaming market, Disney+ and YouTube are competitors in North America.

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While Disney+ is bundled with the memberships of Hulu and ESPN+ together, YouTube is also bundled with YouTube Music and YouTube Kids together. On the other hand, in the Chinese market, Mango TV bundles its membership service with Kuaikan Comic and NetEase Cloud Music separately, and its competitor Tencent Video similarly bundles its membership service with Tencent Comic and Kugou separately.

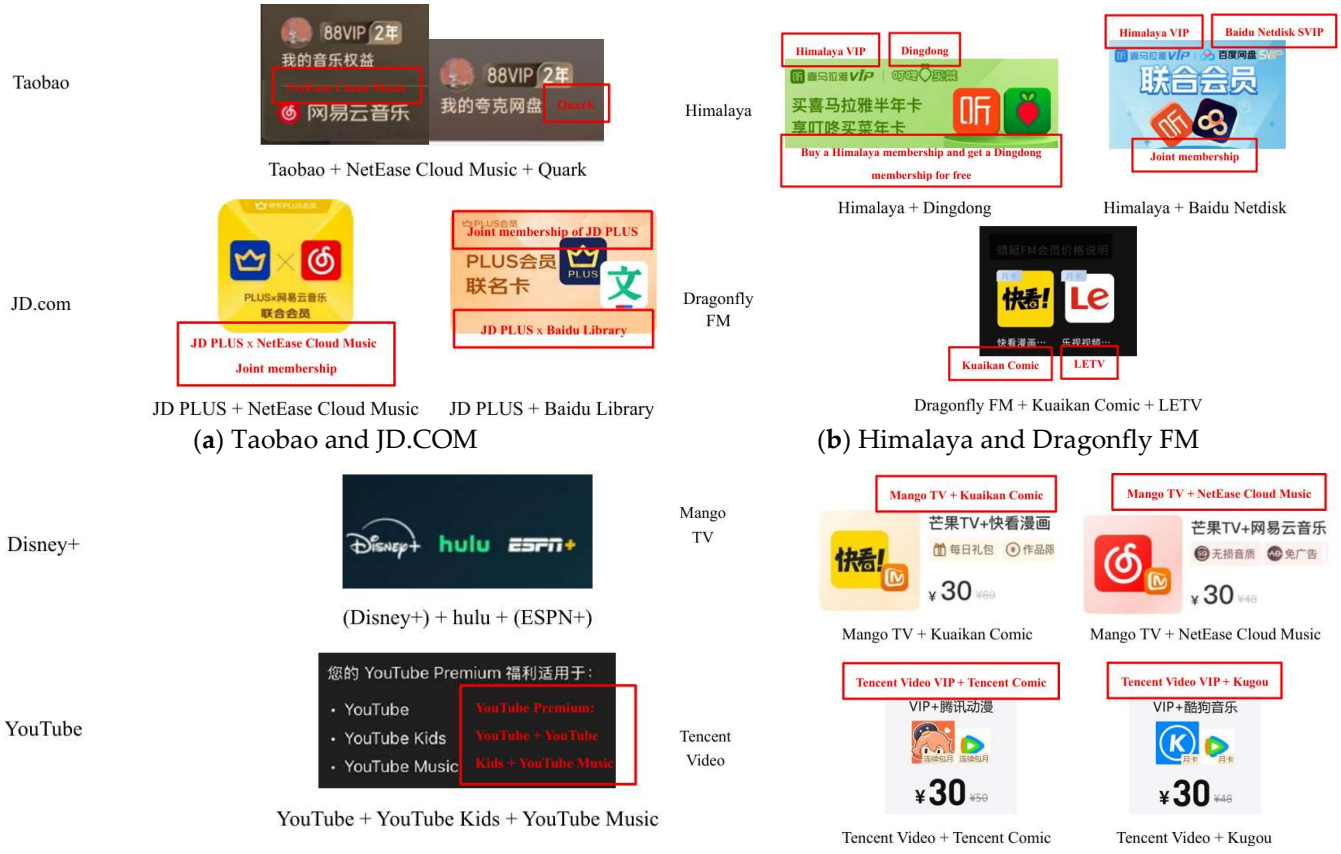


Figure 1. Examples of bundled membership services.

As observed, the ways in which the competing platforms bundle product (or service) portfolios may diverge. This paper aims to answer the following questions: What are the key influencing factors that determine the bundling strategies of competing platforms, i.e., to bundle add-on services together or to bundle them separately? When would competing platforms adopt the same or different bundling strategies? How do the bundling strategies of competing platforms affect consumer surplus and social welfare?

We focus on how the qualities of products (including the platforms' base products and the add-ons), consumer preference, and the profit-sharing arrangements with add-on partners jointly shape the platforms' bundling strategies. First, consumers generally exhibit a strong preference for high-quality products, and bundling various add-ons can enhance the overall quality of the product package. However, bundling different products together may also amplify the degree of horizontal product differentiation [4]. While it can improve quality, it may simultaneously raise misfit costs for consumers if the resulting bundle deviates from their ideal preferences. Platforms must therefore balance these two aspects. We conjecture that if consumers value product quality more than product fit, bundling all add-ons with the base product would likely be the optimal strategy. Second, whether competing platforms adopt the same or different bundling strategies may depend on their inherent competitive advantages. As indicated by Zhang et al. [5], the qualities of core products could significantly influence firms' strategic choices—when core product qualities

were similar, adopting identical strategies often proved advantageous. We thus examine whether a narrow quality gap between base goods leads competing platforms to choose the same bundling approach, that is, to simultaneously bundle all add-ons together or to simultaneously bundle them separately. Third, since platforms share profits from bundled sales with add-on providers, the profit-sharing arrangement can also critically affect their bundling decisions. Generally, if one platform holds a competitive edge (e.g., with a higher-quality base product), it may opt for the bundling strategy with the lower profit-sharing ratio to further increase profit, thereby compelling the rival platform to pursue a different bundling strategy for differentiation. Finally, in terms of consumer surplus and social welfare, if consumers prioritize product quality over horizontal preferences, bundling add-ons together may better satisfy their needs as compared to separate add-on sales, or vice versa if preference fit dominates. Moreover, once platforms incorporate profit-sharing into their strategic decision-making considerations, their competitive behavior may deviate from socially efficient outcomes, as the distribution of profits within the ecosystem may not necessarily be aligned with overall social welfare.

To systematically address the previously mentioned questions and to examine the above conjectures, we use a game-theoretic model to conduct in-depth discussions on how the qualities of products, consumer preference, and profit-sharing patterns shape the bundling strategies employed by two competing platforms. The platforms respectively bundle their base products (or services) with the products (or services) offered by other platforms (i.e., the add-on products). We find that the strategy equilibrium between the two platforms depends largely on the quality gap between their base goods, the quality of the add-ons, the consumers' preference for the add-ons, and the profit-sharing ratio to the add-on partners. Specifically, with a low gap in the quality of the base products, the two platforms indeed will adopt identical bundling strategies. In particular, if the quality of the bundled products plays a dominant role in consumers' purchase decisions, both platforms prefer to bundle their base goods with the two add-ons together. For instance, Disney+ bundles its services with those of Hulu and ESPN+ together, while YouTube bundles YouTube Music and YouTube Kids together as well. When consumers have relatively homogeneous preferences for add-ons, product quality becomes a key factor affecting the purchase decision, and thus the strategy that bundles all add-ons together becomes an equilibrium choice for both parties. On the contrary, if consumers exhibit strong and differentiated preferences for the add-ons, the platforms are more inclined to choose to bundle the add-ons separately. This outcome helps explain the example of Mango TV vs. Tencent Video: as consumers have a strong preference for different add-ons, both platforms adopt separate-bundling strategies. On the other hand, in the presence of a large quality gap between their base goods, the high-quality platform preferentially selects the superior strategy with the lower profit-sharing ratio to partners, and the low-quality platform, accordingly, will choose a different strategy to ease the competition. For instance, the advantageous platform JD.COM cooperates with platforms like NetEase Cloud Music and Baidu Library, which may agree with lower profit-sharing ratios when bundled separately. Meanwhile, its competitor Taobao has launched the 88VIP membership package with different add-ons bundled together to enhance consumer benefits. In this circumstance, equilibrium is manifested as advantageous platforms choosing to bundle separately, while disadvantaged platforms choose to bundle together. Likewise, the equilibrium where disadvantaged platforms choose to bundle separately while advantageous platforms choose to bundle together can be seen in the example of Himalaya vs. Dragonfly FM. Furthermore, when the base goods have similar qualities, the platforms' competition can largely yield optimal welfare outcomes. With significant quality disparity, however, the profit-driven competition strategies may deviate from social efficiency.

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The rest of the paper is organized as follows. Section 2 provides the related literature. Section 3 describes the model setup. Equilibrium results for different bundling cases are presented in Section 4. In Section 5, we derive the corresponding conditions under which different bundling strategies of the platforms can appear as an equilibrium. We also compare the consumer surplus and social welfare under different strategies of the platforms. Section 6 concludes the paper. All proofs are presented in Appendix B.

2. Literature Review

This paper mainly relates to four streams of research: (1) platform competition, (2) product (or service) bundling, (3) product differentiation, and (4) platform governance.

The first related research stream is concerned with platform competition. For example, Zhang et al. [5] examined duopoly platforms that selected between freemium and premium service strategies. Zennyo [6] studied ad-sponsored platforms that introduced freemium models and indicated that the fixed costs for introducing a premium service were the key factor influencing their choices in equilibrium. Adner et al. [7] studied the effects of software compatibility on the competition between two hardware platforms. Li et al. [8] examined platform competition in live-streaming sales with network effects. Bernstein et al. [9] discovered that, in the sharing economy, platform competition would lower the price that the customer paid for using the platform, reduce congestion, and benefit consumers. Xu et al. [10] presented an analysis of blockchain adoption and price competition between two platforms. Yue et al. [11] investigated the reusable packaging adoption strategies for two competing e-commerce platforms in the presence of green consumers.

The second related research stream involves studies on bundling strategies [12,13]. Moon and Shugan [14] found that a monopolist had an incentive to bundle add-on products with core products only if the add-on products were popular, while niche sellers avoided bundling to prevent revenue loss. Zhang et al. [15] studied how a monopolist's bundling strategy for core and ancillary services—bundling one or multiple ancillary services—varied with market heterogeneity and the correlation between ancillary service valuations. Jang and Chung [16] provided application developers with substantive guidelines on optimizing the pricing of base products and the add-ons under different levels of product popularity. Jin et al. [17] investigated how piracy disrupted the effectiveness of bundling for information goods. Bhargava [18] designed an optimal bundled product line for digital products consumed on multiple devices. Chung et al. [4] explored the competition between a multi-product firm adopting a bundling strategy and a single-product firm. Lin et al. [19], Du et al. [20], and Guo et al. [21] all examined product bundling strategies for duopoly platforms. Jena and Ghadge [22] found that in Retailer Stackelberg and Vertical Nash, the manufacturer's bundling strategy was significantly superior to the retailer's bundling strategy. Zhang et al. [23] indicated that the bundling strategy influenced the manufacturer's decision to expand into multichannel operations. Zhang [24] explored how correlated preferences for add-ons influence optimal bundling formats and revealed that the optimal bundling strategy depended on market structure.

Our work contributes to exploring the bundling decisions of duopolistic platforms with the consideration of the influence of the quality gap between the base products of the platform, the quality level of the add-ons, the consumer preferences for different add-ons, and the profit-sharing ratio to the add-on partners. Specifically, a game-theoretic model is constructed to identify the key motivations behind the platforms' decisions to bundle add-ons together or separately. To the best of our knowledge, rare studies examine bundling base product (service) with multiple add-ons together or separately, except Zhang et al. [15] and Zhang [24]. However, while the studies of Zhang et al. [15] and Zhang [24] were closely related to our study, both of them focused on a monopolistic firm, but our study explores

the bundling decisions of duopolistic platforms, in the absence and presence of network effects, respectively. Moreover, considering platforms' bundling decisions, while the studies by Lin et al. [19], Du et al. [20] and Guo et al. [21] were also closely related to our study, Lin et al. [19] examined competing platforms bundling their own platform access and their own content together, and Guo et al. [21] focused on competing platforms jointly launching a membership bundle. Our study differs from these studies in that we consider the duopoly platforms to bundle their membership services with the services offered by other platforms, as exemplified by the examples in the introduction. Specifically, we examine the influences of the quality gap between the base products of the platforms, the quality level of and the consumer preferences for different add-ons, and the profit-sharing ratio to the add-on partners, which is different from the work in Du et al. [20], who considered the effect of profit-sharing ratio but ignored the quality difference and the horizontal differentiation of the products.

The third related research stream is on product differentiation [25–27]. Tookanlou and Wong [28] considered the optimal product line design when consumers are heterogeneous in both horizontal and vertical dimensions. Manda and Jain [29] discussed the relationship among product quality, product differentiation, and demand outsourcing. Bai et al. [30] discussed the noteworthy gaps in the existing livestreaming literature, including the strategy of differentiation of products among livestreaming platforms. Liu et al. [31] examined the quality-differentiated strategies across livestreaming and online channels and pointed out that manufacturers should focus on product types and the capabilities of influencers when choosing and allocating differentiation strategies. Du et al. [32] examined how value-added services enhanced e-commerce platforms' competitiveness.

While the study in Du et al. [32] was closely related to our study, Du et al. [32] focused on the quality differentiation between bundled products. In contrast, in addition to quality differentiation, this study also takes into account the horizontal differentiation between the platforms' base products as well as the differentiation between the add-ons that are bundled with the base products. Specifically, the differentiation level between different add-ons is reflected by consumers' preference (weak or strong) for the add-ons. We explore the competing platforms' decisions to bundle the add-ons together or separately, and offer insights for social planners to guide the platforms' decisions.

The last stream of relevant literature examines platform governance. Several studies have focused on the governance of fraudulent reviews [33,34]. Papanastasiou et al. [35] indicated that some degree of decentralization may be beneficial for resolving a broader range of disputes in online platforms. Huang et al. [36] conducted research on quality regulation strategies of platforms and found that a subsidization strategy could be effective and always improve social welfare. He et al. [37] showed that the introduction of algorithm-based content moderation into platform governance could help sustain content management of digital communities. Kim et al. [38] proposed a detection model integrating agenda-setting theory, specifically designed to understand hate speech content within Korean news platforms, urging media editors to follow ethical agendas to reduce emotional incitement. While these studies predominantly examine platform governance strategies from the perspective of the platforms themselves, this paper considers platforms to expand their ecosystems via bundling add-on services from other platforms and adopts a governmental standpoint to explore how governments can guide platforms' membership bundling strategies to enhance overall social welfare.

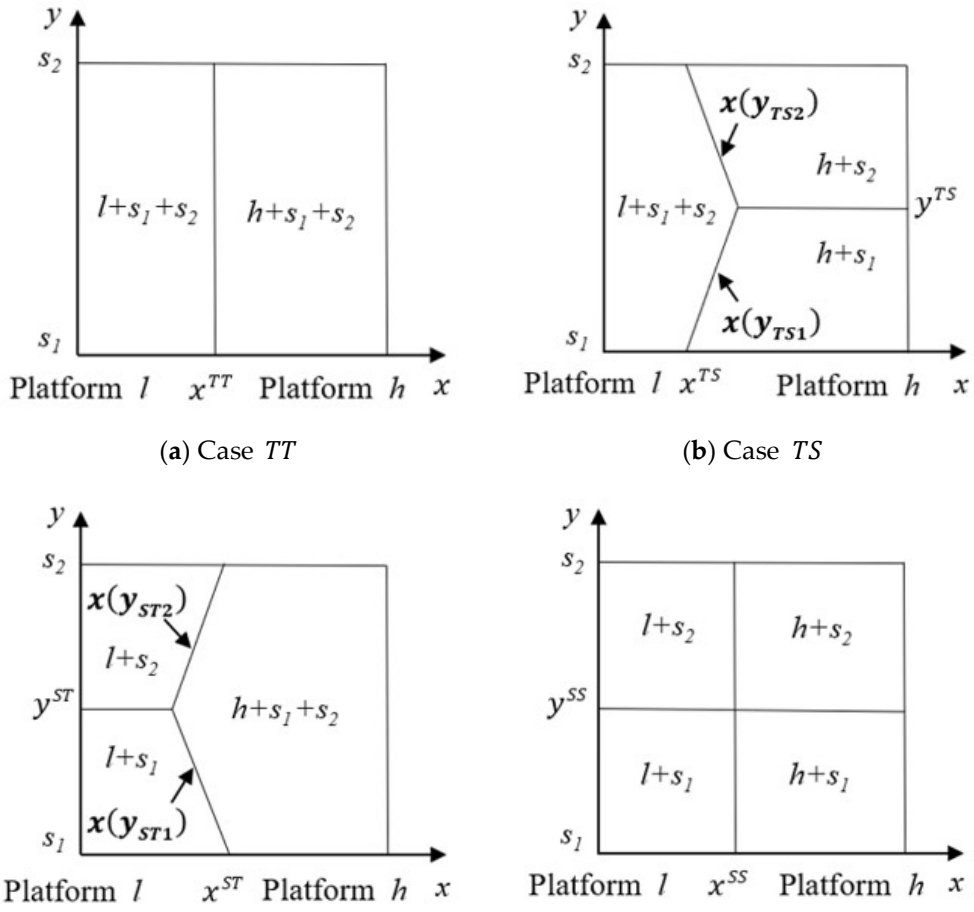
3. Model

Let i ($i = l, h$) denote the two platforms (e.g., Himalaya and Dragonfly in the audio streaming industry) and their corresponding base goods that offer different qualities, with

$v_h > v_l > 0$. Let $v_d = (v_h - v_l)$ denote the quality difference. Both platforms would like to introduce different add-ons from other platforms to stimulate purchase, such as Himalaya's service, which is bundled with the services of Dingdong and Baidu Netdisk separately, while its competitor Dragonfly FM bundles its own service with those from LETV and Kuaikan Comic together.

We focus on the platforms faced with two bundling options, namely, (i) bundling the base good with different add-ons together (denoted by T) and (ii) bundling the base good with different add-ons separately (denoted by S). For ease of exposition, we consider two add-on products, named as s_k ($k = 1, 2$). Without loss of generality, we assume the quality of the two add-ons is the same and is denoted as v_0. Four market configurations are possible, that is, the Case TT in which both platforms bundle the base good and the two add-ons together; the Case TS in which Platform l offers the bundle with the base good and the two add-ons together while Platform h offers two bundled products that bundle its base good with the two add-ons respectively (i.e., separately); and Case ST and Case SS can be interpreted similarly.

Note that the add-ons are provided by other platforms, which would extract a fraction of the revenue from the bundled products. We use γ_T or γ_S to denote the corresponding fraction in the case in which the platforms use the T or the S strategy, respectively. Because the two competing platforms can differ in interface or design, and the add-ons can differ in content or service, consumers would have different preferences for the two base goods as well as for the two add-ons. We thus consider a two-dimensional horizontal differentiation model, which helps to measure the consumers' preference [39], as shown in Figure 2.



(**a**) Case TT (**b**) Case TS

(**c**) Case ST (**d**) Case SS

Figure 2. Market segmentations under the four different bundling scenarios.

Platform l is located at (0, 0) while Platform h is located at (1, 0). The location of the add-on s_1 is at (0, 0) while s_2 is located at (0, 1). Each consumer is characterized by a two-dimensional location (x, y), where x$\in$ [0, 1] and y$\in$ [0, 1]. The x dimension represents the consumer's preference for the base goods of the platforms while the y dimension represents his/her preference for the add-on products. The degree of misfit in the base good (or the add-on) is measured by the distance between the consumer and the base good (or the add-on product) along the x (or y) dimension. Denote the unit misfit cost for the base good (the add-on) as t_x (t_y). The greater the value of t_x (t_y), the stronger is the consumer's preference for their ideal products. The misfit cost for the consumer at (x, y) to use the base good l (or h) can be expressed as $t_x\ x$ (or $t_x\ (1-x)$), and the misfit cost to use the add-on s_1 (or s_2) can be expressed as $t_y\ y$ (or $t_y\ (1-y)$). Specifically, we consider that the preference for the base goods plays a more important role than the preference of the add-ons in determining consumers' purchase decisions, that is, $t_x \geq t_y$ [7].

Although bundling both add-ons with the base good together can significantly enhance the overall product quality, it may also lead to a higher misfit cost. Therefore, the platform must strike a balance between the quality of the add-ons and the misfit cost. Additionally, since the add-ons are provided by other platforms, the bundling profits are also affected by the profit-sharing mechanism with the partners. Thus, this study aims to explore the possible bundled equilibrium outcomes that can be achieved under the joint effects of the three key factors: the quality of the add-ons, the misfit cost of the add-ons, and the profit-sharing ratio to partners who offer the add-ons.

Denote U_{Ti} as the utility for the consumer located at (x, y) to buy from Platform i when Platform i chooses to bundle the two add-ons together (i.e., adopts T strategy). Denote U_{Sik} as the utility for the consumer located at (x, y) to buy from Platform i when Platform i chooses to bundle the two add-ons separately (i.e., adopts S strategy) ($i = l, h; k = 1, 2$). Thus, we have the following consumer utility functions (We assume each consumer purchases only one bundle, excluding the option of buying the base product alone. This approach allows us to examine platform competition in markets where bundled offerings are predominant. In practice, mandatory bundling is widely adopted. For example, Taobao's "88VIP"—with over 53 million members as of October 2025—requires consumers to purchase the bundled package including all add-ons, and consumers cannot opt to buy the core Taobao membership service alone. Furthermore, platforms have now tended to guide consumers in their ecosystems to choose the bundles as their first choice, making a standalone subscription economically unattractive. For example, the annual fee of JD PLUS alone (¥198) is the same as the price of the bundle of JD PLUS and Baidu Library, which positions the Baidu Library membership as a zero-cost add-on, making the bundled option more attractive to consumers. Likewise, the bundled membership package of Tencent Video and Kugou is priced at ¥30 per month, while the standalone Tencent Video subscription costs ¥30 per month as well. This effectively encourages consumers to choose the package as their most rational and preferred option (See Table A2 in Appendix A.4 for details). Accordingly, our study focuses on the competition between platforms through bundled products. This also simplifies the model by avoiding the complexity of accounting for competition between the bundled and the standalone offerings, emphasizing the competing platforms' bundling strategies).

$$U_{Tl} = \theta(v_l + 2v_0) - t_x x - t_y - p_{Tl} \tag{1}$$

$$U_{Sl1} = \theta(v_l + v_0) - t_x x - t_y y - p_{Sl1} \tag{2}$$

$$U_{Sl2} = \theta(v_l + v_0) - t_x x - t_y(1-y) - p_{Sl2} \tag{3}$$

$$U_{Th} = \theta(v_h + 2v_0) - t_x(1-x) - t_y - p_{Th} \tag{4}$$

$$U_{Sh1} = \theta(v_h + v_0) - t_x(1-x) - t_y y - p_{Sh1} \tag{5}$$

$$U_{Sh2} = \theta(v_h + v_0) - t_x(1-x) - t_y(1-y) - p_{Sh2} \tag{6}$$

where Equation (1) (or Equation (4)) is the consumer utility from buying from Platform l (or Platform h) when it chooses to bundle the base good with the two add-ons together; Equations (2) and (3) (or Equations (5) and (6)) are the consumer utility from buying from Platform l (or Platform h) when it chooses to bundle the base good with the two add-ons separately. Specifically, θ represents consumers' willingness to pay for unit quality, and the final terms in the equations indicate the prices of the product bundles.

Table A1 in Appendix A.1 summarizes all the notations.

3.1. Case TT

In this scenario, both platforms adopt T strategy. The utility for the consumer located at (x, y) to buy from Platform l (denoted by U_l^{TT}) or from Platform h (denoted by U_h^{TT}) are given by (1) and (4), respectively.

By setting $U_l^{TT} = U_h^{TT}$, we can derive the indifferent consumer's location as follows:

$$x^{TT} = \frac{p_h^{TT} - p_l^{TT} + \theta(v_l - v_h)}{2t_x} + \frac{1}{2}. \tag{7}$$

As shown in Figure 2a, the market is divided into two regions. Consumers located on the left-hand side of x^{TT} purchase from Platform l, whereas those on the right-hand side purchase from Platform h.

Without loss of generality, the marginal cost of the base good is normalized to zero for both platforms [7,40]. Then, the profit functions for the platforms can be specified as follows:

$$\begin{cases} \pi_l^{TT} &= (1-2\gamma_T) D_l^{TT} p_l^{TT} \\ \pi_h^{TT} &= (1-2\gamma_T) D_h^{TT} p_h^{TT}, \end{cases} \tag{8}$$

where $D_l^{TT} = x^{TT}$ and $D_h^{TT} = 1 - D_l^{TT}$.

3.2. Case TS

In this scenario, consumers can either purchase a bundled product that consists of the base good l and both the add-ons from Platform l, or select between the two bundled products on Platform h that only contain one add-on. The utility for the consumer located at (x, y) to purchase from Platform l (denoted by U_l^{TS}) or from Platform h (denoted by U_h^{TS1} and U_h^{TS2}, respectively) are given by (1), (5), and (6), respectively.

Letting $U_l^{TS} = U_h^{TS1}$, $U_l^{TS} = U_h^{TS2}$ and $U_h^{TS1} = U_h^{TS2}$, we obtain the indifferent curves as follows:

$$\begin{cases} x(y_{TS1}) = \frac{\theta(v_l + v_0 - v_h) + p_h^{TS1} - p_l^{TS} + t_x - t_y}{2t_x} + \frac{t_y}{2t_x} \cdot y \\ x(y_{TS2}) = \frac{\theta(v_l + v_0 - v_h) + p_h^{TS2} - p_l^{TS} + t_x}{2t_x} - \frac{t_y}{2t_x} \cdot y, \\ y^{TS} = \frac{1}{2} + \frac{p_h^{TS2} - p_h^{TS1}}{2t_y} \end{cases} \tag{9}$$

where $x(y_{TS1})$ is the curve in which the consumers are indifferent between purchasing $(l + s_1 + s_2)$ and purchasing $(h + s_1)$, $x(y_{TS2})$ is the curve in which the consumers are indifferent between purchasing $(l + s_1 + s_2)$ and purchasing $(h + s_2)$, and y^{TS} is the curve in which the consumers are indifferent between purchasing $(h + s_1)$ and purchasing $(h + s_2)$,

as shown in Figure 2b. Therefore, we can formulate the demand functions for the bundled products as follows:

$$\begin{cases} D_l^{TS} = \int_0^{y^{TS}} x(y_{TS1})dy + \int_{y^{TS}}^1 x(y_{TS2})dy \\ D_h^{TS1} = \int_0^{y^{TS}} (1 - x(y_{TS1}))dy \\ D_h^{TS2} = \int_{y^{TS}}^1 (1 - x(y_{TS2}))dy \end{cases}. \tag{10}$$

The profit functions of the two platforms can thus be specified as:

$$\begin{cases} \pi_l^{TS} = (1 - 2\gamma_T)D_l^{TS}p_l^{TS} \\ \pi_h^{TS} = \pi_h^{TS1} + \pi_h^{TS2} = (1 - \gamma_S)D_h^{TS1}p_h^{TS1} + (1 - \gamma_S)D_h^{TS2}p_h^{TS2} \end{cases}. \tag{11}$$

To simplify the following analysis, p_h^{TS1} and p_h^{TS2} are assumed to be equal given the equal qualities of the two add-ons [7]. This is also in line with the reality that a real-world platform would like to offer different bundled products with the same price. For example, in the video streaming market, Mango TV offers "Mango TV + Kuaikan Comic" and "Mango TV + NetEase Cloud Music" at the same price, while Tencent Video also provides "Tencent Video + Tencent Comic" and "Tencent Video + Kugou" with an equivalent price. In the presence with $p_h^{TS1} = p_h^{TS2}$, we have $y^{TS} = \frac{1}{2}$.

3.3. Case ST

In case ST, Platform l adopts the S strategy, whereas Platform h opts for the T strategy. The utility for the consumer located at (x, y) to purchase from Platform l (denoted by U_l^{ST1} and U_l^{ST2}, respectively) or Platform h (denoted by U_h^{ST}) are given by (2), (3), and (4), respectively.

As formulated in Figure 2c, consumers are divided into three segments according to their preferences. Solving the indifference equations $U_h^{ST} = U_l^{ST1}$, $U_h^{ST} = U_l^{ST2}$ and $U_l^{ST1} = U_l^{ST2}$, respectively, we can have three indifferent curves as follows:

$$\begin{cases} x(y_{ST1}) = \frac{\theta(v_l - v_0 - v_h) + p_h^{ST} - p_l^{ST1} + t_x + t_y}{2t_x} - \frac{t_y}{2t_x} \cdot y \\ x(y_{ST2}) = \frac{\theta(v_l - v_0 - v_h) + p_h^{ST} - p_l^{ST2} + t_x}{2t_x} + \frac{t_y}{2t_x} \cdot y \\ y^{ST} = \frac{1}{2} + \frac{p_l^{ST1} - p_l^{ST2}}{2t_y} \end{cases}, \tag{12}$$

Thus, we can formulate the demand functions for the two platforms as follows:

$$\begin{cases} D_l^{ST1} = \int_0^{y^{ST}} x(y_{ST1})dy \\ D_l^{ST2} = \int_{y^{ST}}^1 x(y_{ST2})dy \\ D_h^{ST} = \int_0^{y^{ST}} (1 - x(y_{ST1}))dy + \int_{y^{ST}}^1 (1 - x(y_{ST2}))dy \end{cases}. \tag{13}$$

The profit functions can be specified as:

$$\begin{cases} \pi_l^{ST} = \pi_l^{ST1} + \pi_l^{ST2} = (1 - \gamma_S)D_l^{ST1}p_l^{ST1} + (1 - \gamma_S)D_l^{ST2}p_l^{ST2} \\ \pi_h^{ST} = (1 - 2\gamma_T)D_h^{ST}p_h^{ST} \end{cases}. \tag{14}$$

Similar to Case TS, we let $p_l^{ST1} = p_l^{ST2}$ and thus $y^{ST} = \frac{1}{2}$.

3.4. Case SS

When both platforms adopt the S strategy, the utility for the consumer located at (x, y) to buy from Platform l (denoted by U_l^{SS1} and U_l^{SS2}, respectively) or from Platform h (denoted by U_h^{SS1} and U_h^{SS2}, respectively) are given by (2), (3), (5), and (6), respectively.

Letting $U_l^{SS1} = U_l^{SS2}$, $U_h^{SS1} = U_h^{SS2}$, $U_l^{SS1} = U_h^{SS1}$ and $U_l^{SS2} = U_h^{SS2}$, as shown in Figure 2d, we can derive two indifferent curves as follows:

$$\begin{cases} x^{SS} = \frac{1}{2} + \frac{\theta(v_l - v_h) + p_h^{SS1} - p_l^{SS1}}{2t_x} \\ y^{SS} = \frac{1}{2} + \frac{p_l^{SS2} - p_l^{SS1}}{2t_y} \end{cases}. \tag{15}$$

Thus, we can formulate the demand functions for the two platforms as follows:

$$\begin{cases} D_l^{SS} = D_l^{SS1} + D_l^{SS2} = x^{SS}y^{SS} + x^{SS}(1 - y^{SS}) \\ D_h^{SS} = D_h^{SS1} + D_h^{SS2} = (1 - x^{SS})y^{SS} + (1 - x^{SS})(1 - y^{SS}) \end{cases}. \tag{16}$$

Then, the profit functions of the two platforms can be specified as:

$$\begin{cases} \pi_l^{SS} = \pi_l^{SS1} + \pi_l^{SS2} = (1 - \gamma_S)D_l^{SS1}p_l^{SS1} + (1 - \gamma_S)D_l^{SS2}p_l^{SS2} \\ \pi_h^{SS} = \pi_h^{SS1} + \pi_h^{SS2} = (1 - \gamma_S)D_h^{SS1}p_h^{SS1} + (1 - \gamma_S)D_h^{SS2}p_h^{SS2} \end{cases}, \tag{17}$$

Considering $p_l^{SS1} = p_l^{SS2}$ and $p_h^{SS1} = p_h^{SS2}$, we have $y^{SS} = \frac{1}{2}$.

3.5. Timeline of the Game

The timeline of the two-platform game is as follows. In stage 1, the platforms simultaneously decide their bundling strategies, which are the primary decisions that the platforms have to make. In stage 2, the platforms simultaneously price their bundled products. In stage 3, consumers make their purchase decisions.

4. Equilibrium Analysis

We solve the two-platform game using backward induction. Based on the decisions in stage 1, there are four possible subgames in stage 2: the Cases TT, TS, ST, and SS. We first derive the equilibrium for each subgame and then analyze the equilibrium strategies for the platforms in the next section.

The subgame equilibria are provided in Appendix A.2. Comparing the equilibrium prices in the four subgames yields the following results.

Proposition 1. *(Effects on Equilibrium Prices)*

The platforms can charge a higher price by bundling the base good with the two add-ons together than by bundling the base good with the two add-ons separately (i.e., $p_l^{TT} > p_l^{STk}$, $p_l^{TS} > p_l^{SSk}$, $p_h^{TT} > p_h^{TSk}$ and $p_h^{ST} > p_h^{SSk}$ ($k = 1, 2$), if and only if the consumers' preference for the add-ons is weak, i.e., $t_y < \frac{4}{3}\theta v_0$.

Proposition 1 indicates that whether the platforms charge higher or lower prices by using the T strategy than by using the S strategy depends on the consumers' preference for and the quality of the add-ons, as illustrated in Figure 3. Specifically, if t_y is high enough such that $t_y > \frac{4}{3}\theta v_0$, i.e., consumers have a strong preference for the add-on products, the advantage of the bundled products in terms of quality only has a limited influence on the consumers' purchase decision, which prompts the platforms to set a lower price when choosing the T strategy. On the other hand, when t_y is low such that $t_y < \frac{4}{3}\theta v_0$, the quality of the products dominates the consumers' preference in their purchase decision, and the platforms tend to set a higher price when choosing the T strategy rather than choosing the S strategy.

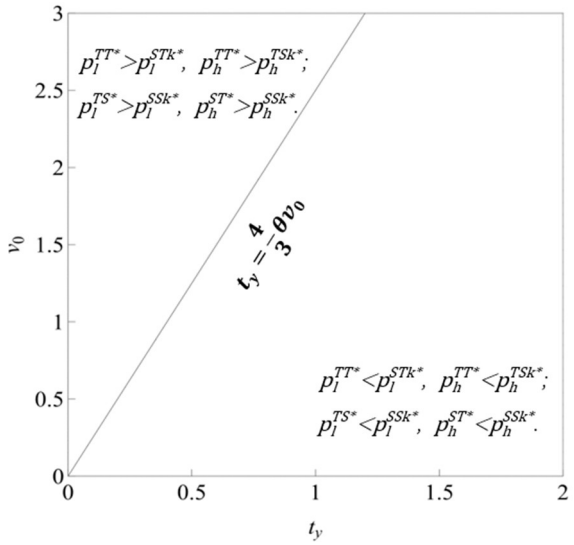


Figure 3. Impact of the misfit cost (t_y) and quality (v_0) of the add-ons on prices.

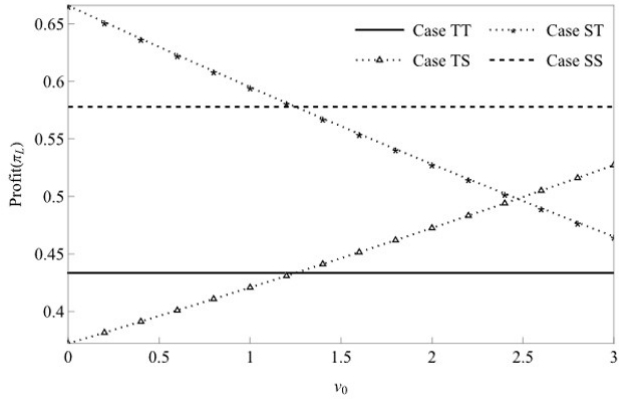
Proposition 2. *(Effects on Equilibrium Profits)*

(a) In Case TS or Case ST , the platform that chooses to bundle with the two add-ons together (or separately) always benefits (or encounters a loss) from an increase in v_0, but encounters loss (or benefits) from an increase in t_y.

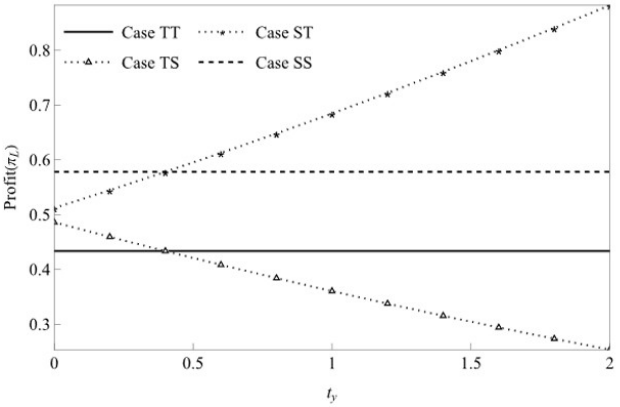
(b) In Case TT and Case SS, neither the quality (i.e., v_0) nor the consumers' preference for the add-ons (i.e., t_y) affects the profits of both platforms.

Proposition 2 indicates the impact of the quality of and the consumers' preference for the add-ons on the platforms' profits. In Case *TS*, as the quality of the add-ons (or the consumers' preference for the add-ons) is strengthened, Platform *l* achieves greater (or lower) profit, as shown in Figure 4. Similar observations can also be obtained for Platform *h* in Case *ST*. In contrast, in Case *ST*, Platform *l*'s profit always increases (or decreases) as the consumers' preference for the add-ons (or the quality of the add-ons) increases. Similar effects are observed for Platform *h* in Case *TS*.

The intuition behind the results is as follows. In Case *TS*, as v_0 increases, compared to Platform *h* choosing to bundle its base good with the two add-ons separately, Platform *l* can significantly increase its service quality by bundling its base good with the two add-ons together. As a result, Platform *l*'s competitive advantage increases, which can ensure it charges a higher price and increases its market demand at the same time, ultimately leading it to a higher profit. In contrast, Platform *h* can only charge lower prices and earn less market demand, which reduces its profit. On the other hand, as the value of t_y becomes higher, the consumers' preference for their ideal add-ons becomes stronger. In other words, the higher t_y leads to greater disutility that the consumers would suffer if they buy the add-on that is different from their preferred one. As a result, when Platform *l* chooses the *T* strategy, the consumers purchasing the bundled product from Platform *l* would incur a higher disutility, and thus the price and demand of Platform *l* would both decrease in t_y, followed by a decrease in its profit. Although the consumer's stronger preference for the add-on also adversely affects the utility of buying from Platform *h*, Platform *h* can slightly increase its prices and demands in the competition, leading to an increase in its profit. Similar explanations can also apply to Case *ST*.



(**a**) Effect of v_0 on Platform l's profit



(**b**) Effect of t_y on Platform l's profit

Figure 4. Impacts of the quality (v_0) and the misfit cost (t_y) of the add-ons on Platform l's profit.

Interestingly, we find that in both Case TT and Case SS, the profits of the two platforms do not vary with the quality of or with the consumers' preference for the add-ons. The reason lies in the fact that when the two platforms offer symmetric bundled products to consumers—that is, simultaneously bundling their own base good with the two add-ons together or separately—competition becomes independent of the add-ons.

5. Equilibrium Bundling Decisions

5.1. Equilibrium Strategy

Proposition 3. *(Equilibrium Bundling Strategy)*

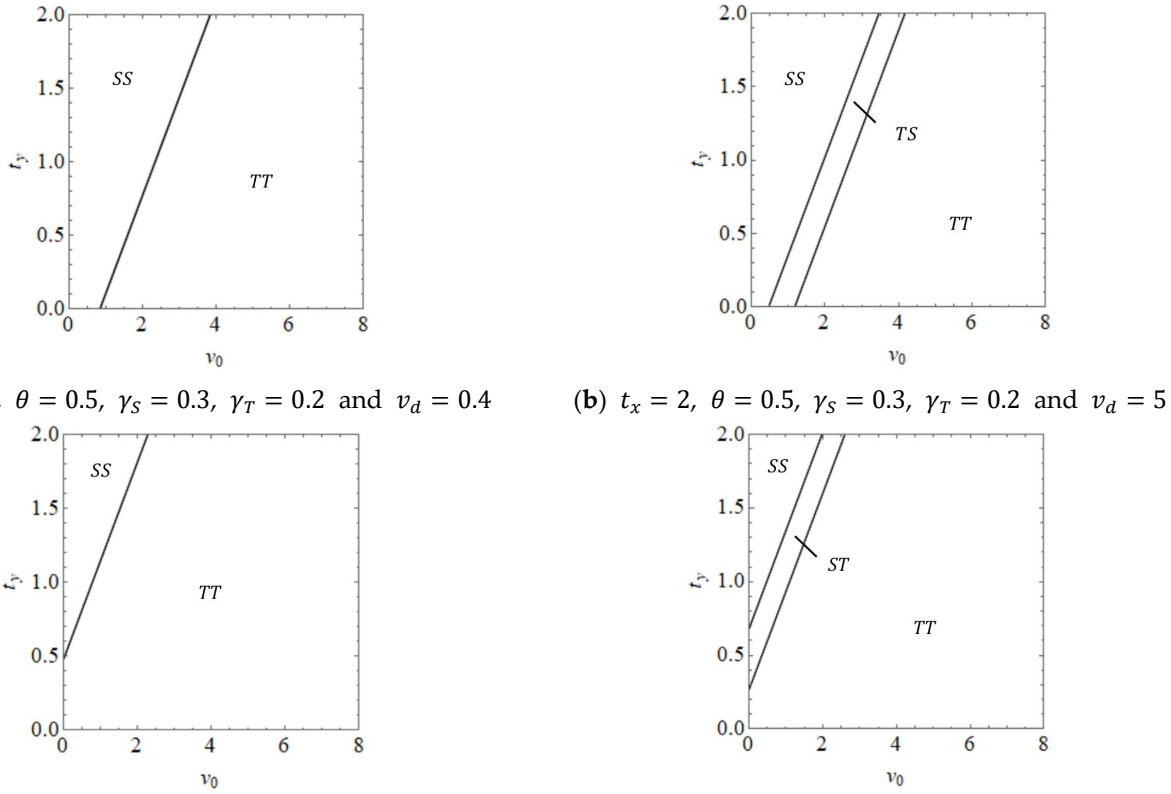
(a) *Case TT can be sustained as the equilibrium when* $v_0 > max\{(\frac{3}{4\theta}t_y - \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) + (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d), (\frac{3}{4\theta}t_y - \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) - (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)\}$.

(b) *Case TS can be sustained as the equilibrium when* $(\frac{3}{4\theta}t_y + \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) - (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d) < v_0 < (\frac{3}{4\theta}t_y - \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) - (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)$ *(i.e., if* $\frac{\gamma_S}{2} < \gamma_T$ *and* $v_d > max\{(\frac{3t_y-4\theta v_0}{4\theta(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})} + \frac{3t_x}{\theta}), (\frac{3t_y-4\theta v_0}{4\theta(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})} - \frac{3t_x}{\theta})\}$*)*.

(c) *Case ST can be sustained as the equilibrium when* $(\frac{3}{4\theta}t_y + \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) + (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d) < v_0 < \left(\frac{3}{4\theta}t_y - \frac{3}{\theta}t_x\left(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}\right) + \left(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}\right)v_d\right)$*(i.e., if* $\gamma_T \leq \frac{\gamma_S}{2}$ *and* $v_d > max\left\{\left(\frac{-3t_y+4\theta v_0}{4\theta\left(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}\right)} + \frac{3t_x}{\theta}\right), \left(\frac{-3t_y+4\theta v_0}{4\theta\left(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}\right)} - \frac{3t_x}{\theta}\right)\right\}$*)*.

(d) *Case SS can be sustained as the equilibrium when* $v_0 < min\{(\frac{3}{4\theta}t_y + \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) - (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d), (\frac{3}{4\theta}t_y + \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) + (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d)\}$ *(i.e.,* $t_y > max\{(\frac{4\theta}{3}v_0 - 4(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})t_x + \frac{4\theta}{3}(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d), (\frac{4\theta}{3}v_0 - 4(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})t_x - \frac{4\theta}{3}(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d)\}$*).*
where $\gamma_1 \equiv 1 - \gamma_S$, $\gamma_2 \equiv 1 - 2\gamma_T$.

Proposition 3 implies that platforms should take into account the quality of the add-on products and the consumers' preferences when deciding whether to bundle the two add-ons separately or together. In addition, we find that the quality difference between the two base goods as well as the profit-sharing ratio also have important impacts on the platforms' equilibrium strategy. Figure 5 shows the conditions for the different equilibria.



(**a**) $t_x = 2$, $\theta = 0.5$, $\gamma_S = 0.3$, $\gamma_T = 0.2$ and $v_d = 0.4$ (**b**) $t_x = 2$, $\theta = 0.5$, $\gamma_S = 0.3$, $\gamma_T = 0.2$ and $v_d = 5$

(**c**) $t_x = 2$, $\theta = 0.5$, $\gamma_S = 0.3$, $\gamma_T = 0.1$ and $v_d = 0.4$ (**d**) $t_x = 2$, $\theta = 0.5$, $\gamma_S = 0.3$, $\gamma_T = 0.1$ and $v_d = 5$

Figure 5. The illustration for different equilibria.

As illustrated in Figure 5a,c, platforms must weigh the relative importance of the quality of the add-ons versus the consumer's preference for the add-ons in determining whether to bundle the add-ons together (Case TT) or to bundle them separately (Case SS). When the quality of the add-ons is high enough such that it plays a more important role in consumers' purchase decisions, the dominant strategy for both platforms is to bundle the base good with the two add-ons together (as indicated in Proposition 3(a)). Specifically, Platform h has a quality advantage when competing with Platform l, and it tends to adopt the T strategy in order to strengthen its competitiveness in the market. For Platform l, to avoid excessive loss of consumers due to a possible increase in the quality gap against Platform h, which may lead to significant profit reduction in the competition, it will also adopt the T strategy.

On the other hand, when consumer preference for the add-ons is strong enough such that consumers' preference for the add-ons plays a more important role in their purchase decisions than the quality of the add-ons does, consumers attach greater importance to

the degree of misfit between the provided products and their desired ones. When both platforms choose to bundle their own base goods with the add-ons separately, consumers have more choices for the bundled products, which can be conducive to reducing the degree of misfit and promoting the sales of the bundled products. Thus, the dominant strategy for both platforms is to bundle the base good with two add-ons separately (as indicated in Proposition 3(d)).

In Figure 5b,d, the cases with both platforms simultaneously choosing the T strategy or simultaneously choosing the S strategy follow the same intuitions. On the other hand, a new case, Case TS, arises in Figure 5b while another new case, Case ST, arises in Figure 5d. In these two cases, neither the quality of nor the consumers' preference for the add-ons plays a dominant role in the consumers' purchase decisions. As a result, the two competing platforms strategically consider different bundling strategies, especially when the quality difference in their base goods is significant (as illustrated by $v_d = 5$ in Figure 5b,d).

Specifically, when the profit-sharing ratio by adopting the T strategy is high, but that by adopting the S strategy is low, such that $(1 - 2\gamma_T) < (1 - \gamma_s)$, Figure 5b shows that TS is likely to arise as an equilibrium. This is because, with an advantage in the quality of the base good, Platform h would preferentially select the S strategy as the preferred choice. In this case, for Platform l, bundling the two add-ons together to improve the quality of its offering can benefit it with a higher price in the competition. As a result, TS arises as the equilibrium. On the contrary, if $(1 - 2\gamma_T) > (1 - \gamma_s)$, as shown in Figure 5d, ST is likely to arise as an equilibrium. On one hand, Platform h has an incentive to adopt the T strategy because of the lower profit-sharing ratio. Platform l, on the other hand, adopts the S strategy to ease the direct competition with its competitor's bundled product.

To sum up, the equilibrium strategies of the platforms depend not only on the add-ons and the quality difference in the platforms' base goods, but also on the profit-sharing ratios.

Proposition 4. *(Possible Equilibrium Combinations)*

(a) *If* $\gamma_T \leq \frac{\gamma_S}{2}$,

(a.1) *Case TT or Case SS can become the equilibrium if* $v_d < \frac{3t_x}{\theta}\left(\frac{\left(\sqrt{\gamma_1}-\sqrt{\gamma_2}\right)^2}{\gamma_1-\gamma_2}\right)$.

(a.2) *Case TT , Case ST, or Case SS can become the equilibrium if* $\frac{3t_x}{\theta}\left(\frac{\left(\sqrt{\gamma_1}-\sqrt{\gamma_2}\right)^2}{\gamma_1-\gamma_2}\right) < v_d$.

(b) *If* $\frac{\gamma_S}{2} < \gamma_T$,

(b.1) *Case TT or Case SS can become the equilibrium if* $v_d < \frac{3t_x}{\theta}\left(\frac{\left(\sqrt{\gamma_1}-\sqrt{\gamma_2}\right)^2}{\gamma_2-\gamma_1}\right)$.

(b.2) *Case TT, Case TS, or Case SS can become the equilibrium if* $\frac{3t_x}{\theta}\left(\frac{\left(\sqrt{\gamma_1}-\sqrt{\gamma_2}\right)^2}{\gamma_2-\gamma_1}\right) < v_d$.

where $\gamma_1 \equiv 1 - \gamma_S$,$\gamma_2 \equiv 1 - 2\gamma_T$.

As illustrated in Figure 5, different combinations of market equilibria appear. According to Proposition 4(a,b), if v_d is small (i.e., $v_d < \frac{3t_x}{\theta}\left(\frac{\left(\sqrt{\gamma_1}-\sqrt{\gamma_2}\right)^2}{\gamma_1-\gamma_2}\right)$ or $v_d < \frac{3t_x}{\theta}\left(\frac{\left(\sqrt{\gamma_1}-\sqrt{\gamma_2}\right)^2}{\gamma_2-\gamma_1}\right)$), only Case TT or Case SS can arise as an equilibrium. On the other hand, if v_d is high enough such that $\frac{3t_x}{\theta}\left(\frac{\left(\sqrt{\gamma_1}-\sqrt{\gamma_2}\right)^2}{\gamma_1-\gamma_2}\right) < v_d$, Case ST can also arise as an equilibrium when γ_T is smaller than $\frac{\gamma_S}{2}$. However, when γ_T is larger than $\frac{\gamma_S}{2}$, Case TS instead can arise as an equilibrium in addition to Case TT and Case SS, if v_d is high enough such that $\frac{3t_x}{\theta}\left(\frac{\left(\sqrt{\gamma_1}-\sqrt{\gamma_2}\right)^2}{\gamma_2-\gamma_1}\right) < v_d$.

The reason for Case ST not emerging as an equilibrium, e.g., in Figure 5b, is that, due to the relatively high profit-sharing ratio to the add-on partners with the T strategy, i.e., $\frac{\gamma_S}{2} < \gamma_T$, $\pi_h^{ST} > \pi_h^{SS}$ cannot be satisfied for Platform h, who has a competitive edge in implementing bundling because of its superior base good. Similarly, the absence of the equilibrium TS in Figure 5d follows the same logic. That is, due to the relatively high

profit-sharing ratio in the S strategy, i.e., $\gamma_T \leq \frac{\gamma_S}{2}$, Platform h is unlikely to adopt the S strategy in that the T strategy yields higher profit for Platform h given Platform l adopting the T strategy.

5.2. Comparison of Consumer Surplus and Social Welfare

Next, we examine which bundling case can generate the highest level of consumer surplus and social welfare. Consumer surplus is the total consumer net utility from purchasing the bundled products, and social welfare is defined as the sum of the consumer surplus and the platforms' profits (more details can be found in the Appendices B.9 and B.10). By comparing the values of the consumer surpluses and social welfares in the four scenarios, we derive the following two propositions.

Proposition 5. *(Comparison in Consumer Surplus)*

(a) The consumer surplus is higher in Case TT than in Case SS if and only if $t_y < \frac{4}{3}\theta v_0$.
(b) The consumer surplus is higher in Case ST than in Case TS if and only if $t_y < \frac{4}{3}\theta v_0$.
(c) When $t_y < \frac{4}{3}\theta v_0$, *there exists a threshold* v_d', *such that Case TT will generate a higher consumer surplus than Case ST if and only if* $v_d < v_d'$
(d) When $t_y > \frac{4}{3}\theta v_0$, *there exists a threshold* v_d'', *such that Case SS will generate a higher consumer surplus than Case TS if and only if* $v_d < v_d''$.

From Proposition 5(a), when the consumers' preference for the add-ons is relatively weak (i.e., $t_y < \frac{4}{3}\theta v_0$), product quality plays an important role in consumers' purchase decisions, and, in such a case, the consumer surplus in Case TT is higher than that in Case SS. The reason is that the total value of the base good bundled with the two add-ons together is always higher than that bundled with only one add-on. However, when the consumers' preference is strong (i.e., $t_y > \frac{4}{3}\theta v_0$), consumers are more willing to purchase products that are more consistent with their own preferences. Although the bundled products under the T strategy have higher quality, the bundled products under the S strategy have significantly lower misfit cost for the consumers. Therefore, Case SS can generate a higher consumer surplus than Case TT.

Similarly, from Proposition 5(b), consumer surplus in Case ST is also higher than that in Case TS when $t_y < \frac{4}{3}\theta v_0$. This is mainly because the quality of Platform h's bundled products in Case ST is superior to that of Case TS, which leads to a higher surplus for consumers purchasing from Platform h. Conversely, when $t_y > \frac{4}{3}\theta v_0$, Case TS instead generates higher consumer surplus than Case ST. Due to the consumers' strong preference for the add-ons, Platform h, who owns the superior base good, can effectively improve the consumers' benefit by adopting the S strategy rather than by adopting the T strategy, in that a lower mismatching cost can significantly increase the consumer surplus as long as the consumers' preference for the add-ons is strong.

From Proposition 5(c), when $t_y < \frac{4}{3}\theta v_0$, Case TT rather than Case ST brings higher consumer surplus if and only if $v_d < v_d'$. That is, the small gap in the quality of the base goods ensures a higher consumer benefit when the two platforms adopt the symmetric bundling strategies TT rather than when they adopt the asymmetric bundling ones ST. This observation also holds in the case of consumers' strong preference for the add-ons, i.e., $t_y > \frac{4}{3}\theta v_0$, in which Case SS will generate a higher consumer surplus than Case TS if and only if $v_d < v_d''$ (Proposition 5(d)). On the other hand, in the presence of a large gap in the quality of the base goods, i.e., $v_d > v_d'$ (or $v_d > v_d''$), asymmetric bundling strategies ST (or TS) bring in a higher consumer surplus in that the consumers can benefit from predominantly purchasing from Platform h.

Proposition 6. *(Comparison in Social Welfare)*

(a) *The social welfare in Case TT is higher than that in Case SS if and only if* $t_y < \frac{4}{3}\theta v_0$.
(b) *The social welfare in Case ST is higher than that in Case TS if and only if* $t_y < \frac{4}{3}\theta v_0$.
(c) *When* $t_y < \frac{4}{3}\theta v_0$, *there exists a threshold,* v_d''', *such that Case TT will bring a higher social welfare than Case ST if and only if* $v_d < v_d'''$.
(d) *When* $t_y > \frac{4}{3}\theta v_0$, *there exists a threshold,* v_d'''', *such that Case SS will bring a higher social welfare than Case TS if and only if* $v_d < v_d''''$.

It is clear in Proposition 6 that the comparison of the social welfare in the four scenarios leads to similar results to those in the comparison of the consumer surplus. When the consumers' preference for the add-ons is relatively weak, Case TT can lead to both higher consumer surplus and higher profits. Therefore, the social welfare is higher in Case TT than in Case SS. On the other hand, when the consumers' preference for the add-ons is strong, because consumers can have the options to choose the bundle products that better fit their preferences in Case SS, which significantly decreases the misfit cost, Case SS generates relatively high social welfare. The comparison between Case ST and Case TS follows the same logic.

In the existence of the consumers' weak (or strong) preference for the add-ons, from Proposition 6(c) (or 6(d)), the social welfare under Case TT (or Case SS) is further greater than that under Case ST (or Case TS) if and only if $v_d < v_d'''$ (or $v_d < v_d''''$). The reason for the higher social welfare under the platforms' symmetric bundling strategies than under the asymmetric bundling ones lies in the fact that the symmetrically bundled products can significantly decrease the misallocation cost when the platforms' base products have comparable qualities.

5.3. Extension with Consideration of Network Effects

In this subsection, we further examine the influences of network effects on the platforms' bundling decisions. Specifically, following the literature [5,21], consumers would derive an additional utility αN_i^m from Platform i $(i = h, l)$, where α is the intensity of network effects and N_i^m represents the expected size of demand of Platform i in a rational expectation equilibrium. Therefore, the higher the demand of Platform i, the higher the additional value delivered to the consumers. This is in line with the reality that a platform's value to each user increases with the size of its user base, say, through user communication within the platform.

We examine how the inclusion of this consumer-side network effects affects the results, ceteris paribus. We observe that a stronger network effect (an increased α) would intensify the price competition and lower the equilibrium prices of the platforms. Aside from this, the extended model shows qualitatively similar results to those obtained in the main model (see Propositions (A1)–(A3) in Appendix A.3), which supports the robustness and relevance of the findings regarding the platforms' bundling strategies.

6. Conclusions and Implications

6.1. Key Findings

This study explores the membership bundling decisions of two competing platforms. We first find that, in equilibrium, in the presence of a low quality gap between their base goods, the two platforms tend to adopt symmetric bundling strategies. Specifically, they will bundle their base goods with both add-ons together if consumers have a weak preference for the add-ons, but the quality of the add-on is sufficiently high to become the decisive factor in consumers' purchase decision. Otherwise, the platforms are willing to bundle their base goods with the two add-ons separately. In the presence of a high quality

gap between their base goods, the high-quality platform strategically chooses the bundling strategy to minimize profit-sharing payouts, forcing the low-quality platform to adopt a divergent strategy. Similar to the platforms' equilibrium strategies, the comparisons in the consumer surplus as well as in the social welfare across different bundling cases also depend on the quality of the add-on, the consumers' preference for the add-ons, and the quality gap between the base goods. We can also verify that the results are robust with the consideration of network effects (see Appendix A.3).

6.2. Implications

The findings of this study offer guidance on whether competing platforms should bundle their base goods (or services) with different add-ons together or separately. In particular, we provide specific insights that competing platforms should implement the same bundling strategies if and only if the quality gap between their base goods is not too large. With comparable qualities in base goods, on one hand, both platforms should implement the *T* strategy when consumers exhibit weak preference for the add-ons such that the quality of the add-ons plays a dominant role in the consumers' purchase decision. On the other hand, when consumers exhibit a strong preference for the add-ons, the *S* strategy should be adopted by both platforms.

For example, the fact that both Disney+ and YouTube choose the *T* strategy (as shown in Table 1) may be because consumers comparably value the video services provided on Disney+ and YouTube, and, meanwhile, consumers have similar preferences for the different entertainment programs provided by Hulu and ESPN+ as well as by YouTube Music and YouTube Kids. On the other hand, the underlying reason for the *S* strategy adopted by Mango TV and Tencent Video may be attributed to the consumers' distinct preferences for the services bundled with Mango TV or Tencent Video. For example, for the two services bundled with Tencent Video, Tencent Comic provides a wide range of domestic and international animation content, while Kugou offers digital music streaming services. These two add-ons offer diversified services for which consumers may have distinct preferences. To attract consumers and meet their diversified needs, bundling the add-ons separately is thus likely to facilitate a win–win scenario.

Table 1. Mapping the bundling strategies to real-world examples.

Examples	Bundled Products	Strategy Cases
Disney+ YouTube	Disney+ + Hulu + ESPN YouTube + YouTube Music + YouTube Kids	Case *TT*
Taobao JD.COM	Taobao + NetEase Cloud Music + Quark JD PLUS + NetEase Cloud Music; JD PLUS + Baidu Library	Case *TS*
Himalaya Dragonfly FM	Himalaya + Dingdong; Himalaya + Baidu Netdisk Dragonfly FM + LETV + Kuaikan Comic	Case *ST*
Mango TV Tencent Video	Mango TV + Kuaikan Comic; Mango TV + NetEase Cloud Music Tencent Video + Tencent Comic; Tencent Video + Kugou	Case *SS*

Nevertheless, when the qualities of the base goods vary significantly between the two competing platforms, it is better for the two platforms to adopt different bundling strategies. As shown in Table 1, JD.COM, as a high-quality platform, enjoys superior resource endowments, which ensure its market competitiveness. It is easier for other platforms, such

as NetEase Cloud Music and Baidu Library, to expand when their membership services are bundled with JD PLUS. Hence, these platforms, which offer the add-ons, may be willing to negotiate with JD.COM at a relatively low profit-sharing ratio in the bundling cooperation, leading JD.COM to be inclined to bundle with these platforms separately. In response, Taobao offers a single package that bundles all the third-party memberships together to bring greater benefits to its consumers.

On the other hand, Dragonfly FM is renowned for its high-quality content, which attracts users who have requirements for in-depth content. Furthermore, both LETV and Kuaikan Comic are actively working to develop their own content ecosystems and expand their market influence. As a result, both of them are willing to be bundled with Dragonfly FM at a lower profit-sharing ratio, incentivizing Dragonfly FM to bundle these add-on services together into a single package to form a vast ecosystem. Himalaya, however, attracting users who are seeking diverse entertainment categories, chooses to bundle different partners' services separately to distinguish itself from Dragonfly FM (see Table 1).

Overall, our research demonstrates that the quality difference of base products, consumer preference, and profit-sharing ratios are critical factors influencing the bundling strategies of competing platforms. Platform managers should undertake comprehensive market research to better understand competitors' product positioning, delineate the distinctions between their own offerings and those of rivals, systematically assess consumer preference, and explore feasible profit-sharing models, which will aid in formulating more targeted market strategies—particularly in light of the current advantageous environment encouraging data sharing. For example, Article 6(11) of the EU Data Markets Act (DMA) [41] requires a platform to provide other parties with access to ranking, query, click, and view data. In addition to the paradigm of data sharing advocated by DMA, the EU Digital Services Act (DSA) also emphasizes enhancing data sharing through enhanced "transparency" [42].

Furthermore, as regulatory restrictions imposed by the DMA require digital platforms to unbundle their own products or services (Article 5(8) [41], "prohibits a gatekeeper from requiring business or end-users to subscribe to one of its CPS as a condition to access another of its CPS"), bundling with services from other platforms represents a promising alternative. According to U.S. antitrust provisions on bundling, which require bundling to follow the "reasonableness principle", the bundling behavior should not harm consumer welfare. Therefore, it is essential to develop bundling strategies that essentially incorporate the aforementioned factors, with particular emphasis on consumer preference. Since regulatory frameworks require "explicit consent" of consumers for bundled offerings, proactively integrating consumer preferences not only supports compliance but also serves as a self-verification mechanism to enhance strategic alignment and market acceptance.

Our study also has implications for social planners, such as policymakers in the government economic agencies. Notably, it can be considered to enhance the assessment of the core/base service qualities of digital platforms. This can more effectively identify whether there exists "significant difference" in the quality of the base services provided by competing platforms in the market, thereby allowing for a clearer distinction regarding the intensity of intervention in the platforms' ecosystem development strategies—such as through bundling add-ons from other platforms. Specifically, when competing platforms offer base services of comparable qualities, their strategic bundling choices can largely coincide with the welfare-enhancing outcomes (considering both consumer surplus and social welfare). In such cases, the economic agencies should primarily adopt an observant approach. However, when significant quality disparity exists between the competing platforms' base services, their strategic decisions would be predominantly influenced by the profit-sharing arrangements with the add-on partners, which may substantially deviate

from the welfare optimization. Policymakers should prioritize regulatory attention to the latter scenario to ensure societal interests. More specifically, when one platform's base service is clearly superior to its competitor's, policymakers should encourage the leading platform to bundle add-ons—either jointly or separately—depending on whether add-on quality or consumer preference is the main driver of purchase. Simultaneously, the competitor should be guided to adopt a differentiated strategy.

6.3. Limitations

Our model presents several limitations. First, our analysis focuses solely on the competition between the platforms that offer the base goods or services. In fact, the platforms that provide the add-ons can also be competitors against each other. Second, our research considers a predetermined profit-sharing mechanism between the platforms that offer the base goods and the platforms that offer the add-ons. However, the cooperation between these platforms does not always involve pre-agreed profit-sharing arrangements; in practice, these mechanisms are often subject to, say, negotiation. Third, our paper focuses primarily on platforms' one-period (or static) bundling decisions but does not consider lock-in effects, which would necessitate a shift to a multi-period framework where consumers may purchase bundled products from different platforms over time.

6.4. Future Directions

This paper can be extended for future research in the following aspects: first, the analysis could be extended to account for multi-tier competition, particularly that among platforms offering base goods and that among platforms offering add-ons. Second, future work could examine the negotiation process between base-good and add-on platforms and their influence on equilibrium bundling strategies. Finally, integrating lock-in effects into platforms' bundling decisions is also a direction worthy of future research. Investigating these aspects would contribute to a more comprehensive understanding of bundling decisions in complex platform ecosystems.

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Appendix A. Supplementary Details to the Main Paper

Appendix A.1. Summary of Key Notations

Table A1. Summary of key notations.

Scenario	Notation	Description
	i	Platform i ($i = l,\ h$)
	j	Strategy j ($j = T,\ S$)
	m	Case m (m =$TT,\ TS,\ ST,$ and SS)
	v_i	The quality for Platform i's base good ($i = l,\ h$)
	v_d	The quality difference in the two base goods
	v_0	The quality of the add-on
	s_k	The kth add-on ($k = 1, 2$)
	$t_x,\ t_y$	Unit misfit cost for using the base good and the add-on, respectively
	γ_j	The ratio of revenue that a third-party platform can gain from the bundled products ($j = T,\ S$)
	π_o^m	The profits of the partner platforms under Case m (m =$TT,\ TS,\ ST,$ and SS)
TT	$p_i^{TT},\ D_i^{TT},\ \pi_i^{TT}$	Price, demand, and profit of Platform i ($i = l,\ h$)
	U_i^{TT}	The utility that the consumer obtains from purchasing the bundled product of Platform i ($i = l,\ h$)
TS	$p_l^{TS},\ D_l^{TS},\ \pi_l^{TS}$	Price, demand, and profit of Platform l
	U_l^{TS}	The utility that the consumer obtains from purchasing the bundled product of Platform l
	$p_h^{TSk},\ D_h^{TSk},\ \pi_h^{TSk}$	Price, demand, and profit of the product that bundles h with s_k ($k = 1, 2$)
	U_h^{TSk}	The utility that the consumer obtains from purchasing the product that bundles h with s_k ($k = 1, 2$)
ST	$p_l^{STk},\ D_l^{STk},\ \pi_l^{STk}$	Price, demand, and profit of the product that bundles l with s_k ($k = 1, 2$)
	U_l^{STk}	The utility that the consumer obtains from purchasing the product that bundles l with s_k ($k = 1, 2$)
	$p_h^{ST},\ D_h^{ST},\ \pi_h^{ST}$	Price, demand, and profit of Platform h
	U_h^{ST}	The utility that the consumer obtains from purchasing the bundled product of Platform h
SS	$p_i^{SSk},\ D_i^{SSk},\ \pi_i^{SSk}$	Price, demand, and profit of the product that bundles i with s_k ($i = l,\ h;\ k = 1, 2$)
	U_i^{SSk}	The utility that the consumer obtains from purchasing the product that bundles i with s_k ($i = l,\ h;\ k = 1, 2$)

Appendix A.2. Equilibrium Prices and Profits

Case *TT*. When both platforms choose to bundle their base good with the two add-ons together, solving the first-order conditions for the two profit-maximizing platforms yields the equilibrium prices, demands, and profits as summarized by the following Lemma A1.

Lemma A1. *In Case TT, the equilibrium prices for the two bundled products on the two platforms are*

$$\begin{cases} p_l^{TT*} = t_x - \frac{1}{3}\theta(v_h - v_l) \\ p_h^{TT*} = t_x + \frac{1}{3}\theta(v_h - v_l) \end{cases}, \tag{A1}$$

the equilibrium demands for the bundled products are

$$\begin{cases} D_l^{TT*} = \frac{1}{2} - \frac{1}{6t_x}\theta(v_h - v_l) \\ D_h^{TT*} = \frac{1}{2} + \frac{1}{6t_x}\theta(v_h - v_l) \end{cases}, \tag{A2}$$

and the equilibrium profits for the two platforms are

$$\begin{cases} \pi_l^{TT*} = (1 - 2\gamma_T)\frac{(3t_x - \theta(v_h - v_l))^2}{18t_x} \\ \pi_h^{TT*} = (1 - 2\gamma_T)\frac{(3t_x + \theta(v_h - v_l))^2}{18t_x} \end{cases}, \tag{A3}$$

It can be seen from Lemma A1 that, in equilibrium, $p_h^{TT*} > p_l^{TT*}$, $D_h^{TT*} > D_l^{TT*}$ and $\pi_h^{TT*} > \pi_l^{TT*}$. This result is intuitive because $v_h - v_l = v_d > 0$. That is, the base good in Platform h has a valuation advantage over its competitor. Thus, Platform h can charge a higher price as well as gain a larger market share, and consequently earn more profit than

Platform l (To ensure that both platforms have positive demands (i.e., $0 < D_i^{TT*} < 1$ for $i = l, h$), it is required that $0 < v_d < \frac{3t_x}{\theta}$).

Case *TS*. When Platform l chooses to bundle the base good with the add-ons together while Platform h chooses to bundle separately, solving the first-order conditions for the two profit-maximizing platforms yields the equilibrium prices, demands, and profits as summarized by the following Lemma A2.

Lemma A2. *In Case TS, the equilibrium prices for the bundled products on the two platforms are*

$$\begin{cases} p_l^{TS*} = t_x - \left(\frac{1}{3}\theta(v_h - v_l - v_0) + \frac{1}{4}t_y\right) \\ p_h^{TS1*} = p_h^{TS2*} = t_x + \left(\frac{1}{3}\theta(v_h - v_l - v_0) + \frac{1}{4}t_y\right) \end{cases}, \tag{A4}$$

the equilibrium demands for the bundled products on the two platforms are

$$\begin{cases} D_l^{TS*} = \frac{1}{2} - \frac{t_y}{8t_x} - \frac{\theta(v_h - v_0 - v_l)}{6t_x} \\ D_h^{TS1*} = D_h^{TS2*} = \frac{1}{4} + \frac{t_y}{16t_x} + \frac{\theta(v_h - v_0 - v_l)}{12t_x} \end{cases}, \tag{A5}$$

and the equilibrium profits are

$$\begin{cases} \pi_l^{TS*} = (1 - 2\gamma_T)\frac{\left(12t_x - 3t_y + 4\theta(v_l - v_h + v_0)\right)^2}{288t_x} \\ \pi_h^{TS*} = \pi_h^{TS1*} + \pi_h^{TS2*} = (1 - \gamma_S)\frac{\left(12t_x + 3t_y - 4\theta(v_l - v_h + v_0)\right)^2}{288t_x} \end{cases}. \tag{A6}$$

Case *ST*. When Platform l chooses to bundle separately while Platform h chooses to bundle together, solving the first-order conditions yields the equilibrium prices, demands, and profits as summarized by the following Lemma A3.

Lemma A3. *In Case ST, the equilibrium prices for the bundled products on the two platforms are*

$$\begin{cases} p_l^{ST1*} = p_l^{ST2*} = t_x - \left(\frac{1}{3}\theta(v_h - v_l + v_0) - \frac{1}{4}t_y\right) \\ p_h^{ST*} = t_x + \left(\frac{1}{3}\theta(v_h - v_l + v_0) - \frac{1}{4}t_y\right) \end{cases}, \tag{A7}$$

the equilibrium demands for the bundled products on the two platforms are

$$\begin{cases} D_l^{ST1*} = D_l^{ST2*} = \frac{1}{4} + \frac{t_y}{16t_x} - \frac{\theta(v_h - v_l + v_0)}{12t_x} \\ D_h^{ST*} = \frac{1}{2} - \frac{t_y}{8t_x} + \frac{\theta(v_h - v_l + v_0)}{6t_x} \end{cases}, \tag{A8}$$

and the equilibrium profits are

$$\begin{cases} \pi_l^{ST*} = \pi_l^{ST1*} + \pi_l^{ST2*} = (1 - \gamma_S)\frac{\left(12t_x + 3t_y - 4\theta(v_h - v_l + v_0)\right)^2}{288t_x} \\ \pi_h^{ST*} = (1 - 2\gamma_T)\frac{\left(12t_x - 3t_y + 4\theta(v_h - v_l + v_0)\right)^2}{288t_x} \end{cases}. \tag{A9}$$

Case *SS*. When both platforms choose to bundle separately, solving the first-order conditions leads to the equilibrium prices, demands, and profits as summarized by the following Lemma A4.

Lemma A4. *In Case SS, the equilibrium prices for the bundled products on the two platforms are*

$$\begin{cases} p_l^{SS1*} = p_l^{SS2*} = t_x - \frac{1}{3}\theta(v_h - v_l) \\ p_h^{SS1*} = p_h^{SS2*} = t_x + \frac{1}{3}\theta(v_h - v_l) \end{cases}, \tag{A10}$$

the equilibrium demands for bundled products on the two platforms are

$$\begin{cases} D_l^{SS1*} = D_l^{SS2*} = \frac{1}{4} - \frac{1}{12t_x}\theta(v_h - v_l) \\ D_h^{SS1*} = D_h^{SS2*} = \frac{1}{4} + \frac{1}{12t_x}\theta(v_h - v_l) \end{cases}, \tag{A11}$$

and the equilibrium profits are

$$\begin{cases} \pi_l^{SS*} = \pi_l^{SS1*} + \pi_l^{SS2*} = (1-\gamma_S)\frac{(3t_x - \theta(v_h - v_l))^2}{18t_x} \\ \pi_h^{SS*} = \pi_h^{SS1*} + \pi_h^{SS2*} = (1-\gamma_S)\frac{(3t_x + \theta(v_h - v_l))^2}{18t_x} \end{cases}. \tag{A12}$$

Note that in this case, in equilibrium, because consumers' purchase decisions are no longer related to the add-ons, the equilibrium prices are independent of v_0 and t_y.

Appendix A.3. Extension: Influences of Network Effects

This extension considers the influences of network effects on the platforms' bundling decisions. We assume a consumer would derive an additional utility αN_i^m from Platform i $(i = h,\ l)$, where α is the intensity of network effects and N_i^m represents the expected size of demand of Platform i in a rational expectation equilibrium.

(1) ***Case TT***

In this scenario, both platforms adopt T strategy. The utility for the consumer located at (x, y) to buy from Platform l (denoted by U_l^{TT}) or from Platform h (denoted by U_h^{TT}) can be respectively derived as:

$$\begin{cases} U_l^{TT} = \theta(v_l + 2v_0) + \alpha N_l^{TT} - t_x x - t_y - p_l^{TT} \\ U_h^{TT} = \theta(v_h + 2v_0) + \alpha N_h^{TT} - t_x(1-x) - t_y - p_h^{TT} \end{cases}. \tag{A13}$$

in which p_l^{TT} and p_h^{TT} are the bundled product prices of Platform l and Platform h with network effects, respectively. By setting $U_l^{TT} = U_h^{TT}$, we can derive the indifferent consumer's location as follows:

$$x^{TT} = \frac{-\alpha N_h^{TT} + \alpha N_l^{TT} + p_h^{TT} - p_l^{TT} + t_x - \theta v_h + \theta v_l}{2t_x}. \tag{A14}$$

In the fulfilled expectation equilibrium, the realized demand of platforms equals the expected market size of consumers: $N_i^{TT} = D_i^{TT}$ $(i = l, h)$. Then, letting $N_l^{TT} = D_l^{TT} = x^{TT}$ and $N_h^{TT} = D_h^{TT} = (1 - x^{TT})$.

We have:

$$\begin{cases} D_l^{TT} = -\frac{-\alpha + p_h^{TT} - p_l^{TT} + t_x - \theta v_h + \theta v_l}{2(\alpha - t_x)} \\ D_h^{TT} = -\frac{-\alpha - p_h^{TT} + p_l^{TT} + t_x + \theta v_h - \theta v_l}{2(\alpha - t_x)} \end{cases}. \tag{A15}$$

The profit functions for the platforms can be specified as follows:

$$\begin{cases} \pi_l^{TT} = (1 - 2\gamma_T) D_l^{TT} p_l^{TT} \\ \pi_h^{TT} = (1 - 2\gamma_T) D_h^{TT} p_h^{TT} \end{cases}. \tag{A16}$$

Solving the first-order conditions for the two profit-maximizing platforms yields the equilibrium prices, demands, and profits as summarized as follows:

$$\begin{cases} p_l^{TT*} = \frac{1}{3}(-3\alpha + 3t_x - \theta v_h + \theta v_l) \\ p_h^{TT*} = \frac{1}{3}(-3a + 3t_x + \theta v_h - \theta v_l) \end{cases}, \tag{A17}$$

$$\begin{cases} D_l^{TT*} = \frac{3\alpha - 3t_x + \theta v_h - \theta v_l}{6\alpha - 6t_x} \\ D_h^{TT*} = \frac{3\alpha - 3t_x - \theta v_h + \theta v_l}{6\alpha - 6t_x} \end{cases}, \tag{A18}$$

$$\begin{cases} \pi_l^{TT*} = \frac{(1-2r_T)(3\alpha - 3t_x + \theta v_h - \theta v_l)^2}{18(t_x - \alpha)} \\ \pi_h^{TT*} = \frac{(1-2r_T)(3\alpha - 3t_x - \theta v_h + \theta v_l)^2}{18(t_x - \alpha)} \end{cases}. \tag{A19}$$

(2) ***Case TS***

In this scenario, consumers can either purchase a bundled product that consists of the base good l and both the add-ons from Platform l, or select between the two bundled products on Platform h that only contain one add-on. The utility for the consumer located at (x, y) to purchase from Platform l (denoted by U_l^{TS}) or from Platform h (denoted by U_h^{TS1} and U_h^{TS2}, respectively) can be written as:

$$\begin{cases} U_l^{TS} = \theta(v_l + 2v_0) + \alpha N_l^{TS} - t_x x - t_y - p_l^{TS} \\ U_h^{TS1} = \theta(v_h + v_0) + \alpha N_h^{TS} - t_x(1-x) - t_y y - p_h^{TS1} \\ U_h^{TS2} = \theta(v_h + v_0) + \alpha N_h^{TS} - t_x(1-x) - t_y(1-y) - p_h^{TS2} \end{cases}, \tag{A20}$$

in which p_l^{TS} is the bundled product price of Platform l considering network effects, and p_h^{TS1} (or p_h^{TS2}) represents the price of the product that bundles the base good h with the add-on s_1 (or s_2) considering network effects.

Letting $U_l^{TS} = U_h^{TS1}$, $U_l^{TS} = U_h^{TS2}$ and $U_h^{TS1} = U_h^{TS2}$, we obtain the indifferent curves as follows:

$$\begin{cases} x(y_{TS1}) = \frac{-\alpha N_h^{TS} + \alpha N_l^{TS} + p_h^{TS1} - p_l^{TS} + t_x - t_y + y t_y + \theta v_0 - \theta v_h + \theta v_l}{2t_x} \\ x(y_{TS2}) = \frac{-\alpha N_h^{TS} + \alpha N_l^{TS} + p_h^{TS2} - p_l^{TS} + t_x - y t_y + \theta v_0 - \theta v_h + \theta v_l}{2t_x} \\ y^{TS} = \frac{1}{2} + \frac{p_h^{TS2} - p_h^{TS1}}{2t_y} \end{cases}. \tag{A21}$$

To simplify the following analysis, p_h^{TS1} and p_h^{TS2} are assumed to be equal given the equal valuations of the two add-ons [7]. In the fulfilled expectation equilibrium, the realized demand of platforms equals the expected market size of consumers: $N_i^{TS} = D_i^{TS}$ $(i = l, h)$. Then, letting $N_l^{TS} = D_l^{TS} = \int_0^{y^{TS}} x(y_{TS1})dy + \int_{y^{TS}}^1 x(y_{TS2})dy$ and $N_h^{TS} = D_h^{TS} = D_h^{TS1} + D_h^{TS2} = \int_0^{y^{TS}} (1 - x(y_{TS1}))dy + \int_{y^{TS}}^1 (1 - x(y_{TS2}))dy$.

We have:

$$\begin{cases} D_l^{TS} = -\frac{-4\alpha + 4p_h^{TS1} - 4p_l^{TS} + 4t_x - 3t_y + 4\theta v_0 - 4\theta v_h + 4\theta v_l}{8(\alpha - t_x)} \\ D_h^{TS} = D_h^{TS1} + D_h^{TS2} = -\frac{-4\alpha - 4p_h^{TS1} + 4p_l^{TS} + 4t_x + 3t_y - 4\theta v_0 + 4\theta v_h - 4\theta v_l}{8(\alpha - t_x)} \end{cases}. \tag{A22}$$

Then, the profit functions for the platforms can be specified as follows:

$$\begin{cases} \pi_l^{TS} = (1 - 2\gamma_T) D_l^{TS} p_l^{TS} \\ \pi_h^{TS} = \pi_h^{TS1} + \pi_h^{TS2} = (1 - \gamma_S) D_h^{TS1} p_h^{TS1} + (1 - \gamma_S) D_h^{TS2} p_h^{TS2} \end{cases}. \tag{A23}$$

Solving the first-order conditions for the two profit-maximizing platforms yields the equilibrium prices, demands, and profits as summarized as follows:

$$\begin{cases} p_l^{TS*} = \frac{1}{12}(-12\alpha + 12t_x - 3t_y + 4\theta v_0 - 4\theta v_h + 4\theta v_l) \\ p_h^{TS1*} = p_h^{TS2*} = \frac{1}{12}(-12\alpha + 12t_x + 3t_y - 4\theta v_0 + 4\theta v_h - 4\theta v_l) \end{cases}, \tag{A24}$$

$$\begin{cases} D_l^{TS*} = \frac{12\alpha - 12t_x + 3t_y - 4\theta v_0 + 4\theta v_h - 4\theta v_l}{24\alpha - 24t_x} \\ D_h^{TS*} = D_h^{TS1*} + D_h^{TS2*} = \frac{12\alpha - 12t_x - 3t_y + 4\theta v_0 - 4\theta v_h + 4\theta v_l}{24\alpha - 24t_x} \end{cases}, \tag{A25}$$

$$\begin{cases} \pi_l^{TS*} = \frac{(1-2r_T)(12\alpha-12t_x+3t_y-4\theta v_0+4\theta v_h-4\theta v_l)^2}{288(t_x-\alpha)} \\ \pi_h^{TS*} = \frac{(1-r_S)(12\alpha-12t_x-3t_y+4\theta v_0-4\theta v_h+4\theta v_l)^2}{288(t_x-\alpha)} \end{cases}. \tag{A26}$$

(3) ***Case ST***

In case ST, Platform l adopts the S strategy, whereas Platform h opts for the T strategy. The utility for the consumer located at (x, y) to purchase from Platform l (denoted by U_l^{ST1} and U_l^{ST2}, respectively) or Platform h (denoted by U_h^{ST}) can be written as:

$$\begin{cases} U_l^{ST1} = \theta(v_l + v_0) + \alpha N_l^{ST} - t_x x - t_y y - p_l^{ST1} \\ U_l^{ST2} = \theta(v_l + v_0) + \alpha N_l^{ST} - t_x x - t_y(1-y) - p_l^{ST2} \\ U_h^{ST} = \theta(v_h + 2v_0) + \alpha N_h^{ST} - t_x(1-x) - t_y - p_h^{ST} \end{cases}, \tag{A27}$$

in which p_h^{ST} is the bundled product price of Platform h considering network effects, and p_l^{ST1} (or p_l^{ST2}) represents the price of the bundled product that bundles the base good l with the add-on s_1 (or s_2) considering network effects.

Solving the indifference equations $U_h^{ST} = U_l^{ST1}$, $U_h^{ST} = U_l^{ST2}$ and $U_l^{ST1} = U_l^{ST2}$, respectively, we can have three indifferent curves as follows:

$$\begin{cases} x(y_{ST1}) = \frac{-\alpha N_h^{ST}+\alpha N_l^{ST}+p_h^{ST}-p_l^{ST1}+t_x+t_y-yt_y-\theta v_0-\theta v_h+\theta v_l}{2t_x} \\ x(y_{ST2}) = \frac{-\alpha N_h^{ST}+\alpha N_l^{ST}+p_h^{ST}-p_l^{ST2}+t_x+yt_y-\theta v_0-\theta v_h+\theta v_l}{2t_x} \\ y^{ST} = \frac{1}{2} + \frac{p_l^{ST1}-p_l^{ST2}}{2t_y} \end{cases}, \tag{A28}$$

Similar to Case TS, we let $p_l^{ST1} = p_l^{ST2}$ and thus $y^{ST} = \frac{1}{2}$. In the fulfilled expectation equilibrium, the realized demands of platforms equal the expected market sizes: $N_i^{ST} = D_i^{ST}$ $(i = l, h)$. Then, letting $N_l^{ST} = D_l^{ST} = D_l^{ST1} + D_l^{ST2} = \int_0^{y^{ST}} x(y_{ST1})dy + \int_{y^{ST}}^1 x(y_{ST2})dy$ and $N_h^{ST} = D_h^{ST} = \int_0^{y^{ST}} (1 - x(y_{ST1}))dy + \int_{y^{ST}}^1 (1 - x(y_{ST2}))dy$.

We have:

$$\begin{cases} D_l^{ST} = D_l^{ST1} + D_l^{ST2} = -\frac{-4\alpha+4p_h^{ST}-4p_l^{ST1}+4t_x+3t_y-4\theta v_0-4\theta v_h+4\theta v_l}{8(\alpha-t_x)} \\ D_h^{ST} = -\frac{-4\alpha-4p_h^{ST}+4p_l^{ST1}+4t_x-3t_y+4\theta v_0+4\theta v_h-4\theta v_l}{8(\alpha-t_x)} \end{cases}. \tag{A29}$$

Then, the profit functions for the platforms can be specified as follows:

$$\begin{cases} \pi_l^{ST} = \pi_l^{ST1} + \pi_l^{ST2} = (1-\gamma_s)D_l^{ST1}p_l^{ST1} + (1-\gamma_s)D_l^{ST2}p_l^{ST2} \\ \pi_h^{ST} = (1-2\gamma_T)D_h^{ST}p_h^{ST} \end{cases}. \tag{A30}$$

Solving the first-order conditions for the two profit-maximizing platforms yields the equilibrium prices, demands, and profits as summarized as follows:

$$\begin{cases} p_l^{ST1*} = p_l^{ST2*} = \frac{1}{12}(-12\alpha + 12t_x + 3t_y - 4\theta v_0 - 4\theta v_h + 4\theta v_l) \\ p_h^{ST*} = \frac{1}{12}(-12\alpha + 12t_x - 3t_y + 4\theta v_0 + 4\theta v_h - 4\theta v_l) \end{cases}, \tag{A31}$$

$$\begin{cases} D_l^{ST*} = D_l^{ST1*} + D_l^{ST2*} = \frac{12\alpha-12t_x-3t_y+4\theta v_0+4\theta v_h-4\theta v_l}{24\alpha-24t_x} \\ D_h^{ST*} = \frac{12\alpha-12t_x+3t_y-4\theta v_0-4\theta v_h+4\theta v_l}{24\alpha-24t_x} \end{cases}, \tag{A32}$$

$$\begin{cases} \pi_l^{ST*} = \frac{(1-r_S)(12\alpha-12t_x-3t_y+4\theta v_0+4\theta v_h-4\theta v_l)^2}{288(t_x-\alpha)} \\ \pi_h^{ST*} = \frac{(1-2r_T)(12\alpha-12t_x+3t_y-4\theta v_0-4\theta v_h+4\theta v_l)^2}{288(t_x-\alpha)} \end{cases}. \tag{A33}$$

(4) ***Case SS***

When both platforms adopt the S strategy, the utility for the consumer located at (x, y) to buy from Platform l (denoted by U_l^{SS1} and U_l^{SS2}, respectively) or from Platform h (denoted by U_h^{SS1} and U_h^{SS2}, respectively) can be written as:

$$\begin{cases} U_l^{SS1} = \theta(v_l + v_0) + \alpha N_l^{SS} - t_x x - t_y y - p_l^{SS1} \\ U_l^{SS2} = \theta(v_l + v_0) + \alpha N_l^{SS} - t_x x - t_y(1-y) - p_l^{SS2} \\ U_h^{SS1} = \theta(v_h + v_0) + \alpha N_h^{SS} - t_x(1-x) - t_y y - p_h^{SS1} \\ U_h^{SS2} = \theta(v_h + v_0) + \alpha N_h^{SS} - t_x(1-x) - t_y(1-y) - p_h^{SS2} \end{cases}, \tag{A34}$$

in which p_l^{SS1} or p_l^{SS2} represents the price of the bundled product containing the add-on s_1 or s_2 on Platform l considering network effects, respectively. Similarly, p_h^{SS1} or p_h^{SS2} represents the prices of the bundled products on Platform h considering network effects.

Letting $U_l^{SS1} = U_l^{SS2}$, $U_h^{SS1} = U_h^{SS2}$, $U_l^{SS1} = U_h^{SS1}$ and $U_l^{SS2} = U_h^{SS2}$, we can derive two indifferent curves as follows:

$$\begin{cases} x^{SS} = \frac{-\alpha N_h^{SS} + \alpha N_l^{SS} + p_h^{SS1} - p_l^{SS1} + t_x - \theta v_h + \theta v_l}{2t_x} \\ y^{SS} = \frac{1}{2} + \frac{p_l^{SS2} - p_l^{SS1}}{2t_y} \end{cases}. \tag{A35}$$

Considering $p_l^{SS1} = p_l^{SS2}$ and $p_h^{SS1} = p_h^{SS2}$, we have $y^{SS} = \frac{1}{2}$. In the fulfilled expectation equilibrium, the realized demands of platforms equal the expected demands: $N_i^{SS} = D_i^{SS}$ $(i = l, h)$. Then, letting $N_l^{SS} = D_l^{SS} = D_l^{SS1} + D_l^{SS2} = x^{SS} y^{SS} + x^{SS}(1 - y^{SS})$ and $N_h^{SS} = D_h^{SS} = D_h^{SS1} + D_h^{SS2} = (1 - x^{SS}) y^{SS} + (1 - x^{SS})(1 - y^{SS})$.

We have:

$$\begin{cases} D_l^{SS} = D_l^{SS1} + D_l^{SS2} = -\frac{-\alpha + p_h^{SS1} - p_l^{SS1} + t_x - \theta v_h + \theta v_l}{2(\alpha - t_x)} \\ D_h^{SS} = D_h^{SS1} + D_h^{SS2} = -\frac{-\alpha - p_h^{SS1} + p_l^{SS1} + t_x + \theta v_h - \theta v_l}{2(\alpha - t_x)} \end{cases}. \tag{A36}$$

Then, the profit functions for the platforms can be specified as follows:

$$\begin{cases} \pi_l^{SS} = \pi_l^{SS1} + \pi_l^{SS2} = (1 - \gamma_s) D_l^{SS1} p_l^{SS1} + (1 - \gamma_s) D_l^{SS2} p_l^{SS2} \\ \pi_h^{SS} = \pi_h^{SS1} + \pi_h^{SS2} = (1 - \gamma_s) D_h^{SS1} p_h^{SS1} + (1 - \gamma_s) D_h^{SS2} p_h^{SS2} \end{cases}. \tag{A37}$$

Solving the first-order conditions for the two profit-maximizing platforms yields the equilibrium prices, demands, and profits as summarized as follows:

$$\begin{cases} p_l^{SS1*} = p_l^{SS2*} = \frac{1}{3}(-3\alpha + 3t_x - \theta v_h + \theta v_l) \\ p_h^{SS1*} = p_h^{SS2*} = \frac{1}{3}(-3\alpha + 3t_x + \theta v_h - \theta v_l) \end{cases}, \tag{A38}$$

$$\begin{cases} D_l^{SS*} = D_l^{SS1*} + D_l^{SS2*} = \frac{3\alpha - 3t_x + \theta v_h - \theta v_l}{6\alpha - 6t_x} \\ D_h^{SS*} = D_h^{SS1*} + D_h^{SS2*} = \frac{3\alpha - 3t_x - \theta v_h + \theta v_l}{6\alpha - 6t_x} \end{cases}, \tag{A39}$$

$$\begin{cases} \pi_l^{SS*} = \frac{(1 - r_S)(3\alpha - 3t_x + \theta v_h - \theta v_l)^2}{18(t_x - \alpha)} \\ \pi_h^{SS*} = \frac{(1 - r_S)(3\alpha - 3t_x - \theta v_h + \theta v_l)^2}{18(t_x - \alpha)} \end{cases}. \tag{A40}$$

Based on the above results, we can have the following propositions on equilibrium bundling strategies, consumer surplus, and social welfare.

Proposition A1. *(Equilibrium Bundling Strategy Considering Network effects)*

(a) *Case TT can be sustained as the equilibrium when* $v_0 > max\{(\frac{3}{4\theta}t_y - \frac{3}{\theta}(t_x-\alpha)(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})+(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d),\ (\frac{3}{4\theta}t_y - \frac{3}{\theta}(t_x-\alpha)(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) - (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)\}$.

(b) *Case TS can be sustained as the equilibrium when* $(\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x-\alpha)(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) -(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d) < v_0 < (\frac{3}{4\theta}t_y - \frac{3}{\theta}(t_x-\alpha)(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) - (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)$

(c) *Case ST can be sustained as the equilibrium when* $(\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x-\alpha)(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) +(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d) < v_0 < (\frac{3}{4\theta}t_y - \frac{3}{\theta}(t_x-\alpha)(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) + (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)$

(d) *Case SS can be sustained as the equilibrium when* $v_0 < min\{(\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x-\alpha)(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) -(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d), (\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x-\alpha)(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) + (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d)\}$
where $\gamma_1 \equiv 1-\gamma_S$, $\gamma_2 \equiv 1-2\gamma_T$.

The equilibrium strategies in Proposition A1 show that, regardless of whether the network effects are considered or not, our main conclusion in Proposition 3 remains qualitatively the same. In both cases, platforms should take into account the quality of the add-on products, the consumers' preferences, and the profit-sharing ratios when deciding whether to bundle the two add-ons separately or together.

Next, we examine which bundling case can generate the highest level of consumer surplus and social welfare. The consumer surplus generated in the Case TT can be formulated as follows:

$$\begin{aligned} CS^{TT} &= \int_0^1 \int_0^{x^{TT}} U_l^{TT} dxdy + \int_0^1 \int_{x^{TT}}^1 U_h^{TT} dxdy \\ &= \int_0^1 \int_0^{\frac{3\alpha-3t_x+\theta v_h-\theta v_l}{6\alpha-6t_x}} (\alpha - t_x - xt_x - t_y + 2\theta v_0 + \frac{\theta v_h}{3} + \frac{2\theta v_l}{3} + \frac{\alpha(3\alpha-3t_x+\theta v_h-\theta v_l)}{6(\alpha-t_x)})dxdy \\ &+\int_0^1 \int_{\frac{3\alpha-3t_x+\theta v_h-\theta v_l}{6\alpha-6t_x}}^1 (\alpha - 2t_x + xt_x - t_y + 2\theta v_0 + \frac{2\theta v_h}{3} + \frac{\theta v_l}{3} + \frac{\alpha(3\alpha-3t_x-\theta v_h+\theta v_l)}{6(\alpha-t_x)})dxdy \\ &= \frac{1}{36(\alpha-t_x)^2}(-45t_x^3 + 18\alpha^2(3\alpha - 2t_y + 4\theta v_0 + \theta v_h + \theta v_l) + 18t_x^2(8\alpha - 2t_y + 4\theta v_0 + \theta v_h + \theta v_l) \\ &+t_x(-153\alpha^2 + 72\alpha t_y - 144\alpha\theta v_0 - 36\alpha\theta v_h + \theta^2 v_h^2 - 36\alpha\theta v_l - 2\theta^2 v_h v_l + \theta^2 v_l^2)), \end{aligned} \tag{A41}$$

Similarly, we can derive the consumer surplus generated for the remaining three cases as follows.

$$\begin{aligned} CS^{TS} &= \int_0^{\frac{1}{2}} \int_0^{x_{yTS1}} U_l{}^{TS} dxdy + \int_{\frac{1}{2}}^1 \int_0^{x_{yTS2}} U_l{}^{TS} dxdy + \int_0^{\frac{1}{2}} \int_{x_{yTS1}}^1 U_h{}^{TS1} dxdy + \int_{\frac{1}{2}}^1 \int_{x_{yTS2}}^1 U_h{}^{TS2} dxdy = \\ &\frac{1}{576(\alpha-t_x)^2 t_x}(-720t_x^4 + 3\alpha^2 t_y^2 - 4t_x^2(612\alpha^2 - 3t_y^2 - 4\theta^2 v_0^2 + 144\alpha\theta v_h - 4\theta^2 v_h^2 + 144\alpha\theta v_l + 8\theta^2 v_h v_l - 4\theta^2 v_l^2 + 8\theta v_0(54\alpha + \theta v_h - \theta v_l) - \\ &6t_y(30\alpha - \theta v_0 + \theta v_h - \theta v_l)) + 6\alpha t_x(-60\alpha t_y - t_y^2 + 48\alpha(3\alpha + 3\theta v_0 + \theta v_h + \theta v_l)) + 72t_x^3(-5t_y + 4(8\alpha + 3\theta v_0 + \theta v_h + \theta v_l))), \end{aligned} \tag{A42}$$

$$\begin{aligned} CS^{ST} &= \int_0^{\frac{1}{2}} \int_0^{x_{yST1}} U_l{}^{ST1} dxdy + \int_{\frac{1}{2}}^1 \int_0^{x_{yST2}} U_l{}^{ST2} dxdy + \int_0^{\frac{1}{2}} \int_{x_{yST1}}^1 U_h{}^{ST} dxdy + \int_{\frac{1}{2}}^1 \int_{x_{yST2}}^1 U_h{}^{ST} dxdy = \frac{1}{576(\alpha-t_x)^2 t_x} \\ &(-720t_x^4 + 3\alpha^2 t_y^2 - 4t_x^2(612\alpha^2 - 3t_y^2 - 4\theta^2 v_0^2 + 144\alpha\theta v_h - 4\theta^2 v_h^2 + 144\alpha\theta v_l + 8\theta^2 v_h v_l - 4\theta^2 v_l^2 + 8\theta v_0(54\alpha - \theta v_h + \theta v_l) - \\ &6t_y(30\alpha - \theta v_0 - \theta v_h + \theta v_l)) + 6\alpha t_x(-60\alpha t_y - t_y^2 + 48\alpha(3\alpha + 3\theta v_0 + \theta v_h + \theta v_l)) \\ &+72t_x^3(-5t_y + 4(8\alpha + 3\theta v_0 + \theta v_h + \theta v_l))), \end{aligned} \tag{A43}$$

$$\begin{aligned} CS^{SS} &= \int_0^{\frac{1}{2}} \int_0^{x^{SS}} U_l{}^{SS1} dxdy + \int_{\frac{1}{2}}^1 \int_0^{X^{ss}} U_l{}^{SS2} dxdy + \int_0^{\frac{1}{2}} \int_{x^{SS}}^1 U_h{}^{SS1} dxdy \\ &+\int_{\frac{1}{2}}^1 \int_{x^{SS}}^1 U_h{}^{SS2} dxdy = \frac{1}{36(\alpha-t_x)^2}(-45t_x^3 + 9\alpha^2(6\alpha - t_y + 4\theta v_0 + 2\theta v_h + 2\theta v_l) \\ &+t_x(-153\alpha^2 + 18\alpha t_y - 72\alpha\theta v_0 - 36\alpha\theta v_h + \theta^2 v_h^2 - 36\alpha\theta v_l - 2\theta^2 v_h v_l + \theta^2 v_l^2) + 9t_x^2(-t_y + 2(8\alpha + 2\theta v_0 + \theta v_h + \theta v_l))). \end{aligned} \tag{A44}$$

By comparing the values of the consumer surpluses in the four scenarios, we can derive the following proposition.

Proposition A2. *(Comparison in Consumer Surplus Considering Network Effects)*

(a) *The consumer surplus is higher in Case TT than in Case SS if and only if* $t_y < \frac{4}{3}\theta v_0$.

(b) *The consumer surplus is higher in Case SS than in Case TS if and only if* $t_y < \frac{4}{3}\theta v_0$.

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(c) *When* $t_y < \frac{4}{3}\theta v_0$, *there exists a threshold* v_d^1, *such that Case TT will generate a higher consumer surplus than Case ST if and only if* $v_d < v_d^1$

(d) *When* $t_y > \frac{4}{3}\theta v_0$, *there exists a threshold* v_d^2, *such that Case SS will generate a higher consumer surplus than Case TS if and only if* $v_d < v_d^2$.

From Proposition A2, the results of the comparison of consumer surplus among the four cases with network effects are also similar to those without, as shown in Proposition 5.

Note that social welfare is given by $SW^m = \pi_l^{m*} + \pi_h^{m*} + \pi_o^{m*} + CS^m$. Thus, we can obtain the social welfare as follows:

$$\begin{gathered} SW^{TT} = \frac{1}{36(\alpha - t_x)^2}(-9t_x^3 + 18t_x^2(2\alpha - 2t_y + 4\theta v_0 + \theta v_h + \theta v_l) - \\ t_x(45\alpha^2 - 72\alpha t_y + 144\alpha\theta v_0 + 36\alpha\theta v_h - 5\theta^2 v_h^2 + 36\alpha\theta v_l + 10\theta^2 v_h v_l - 5\theta^2 v_l^2) + \\ 2\alpha(9\alpha^2 - 18\alpha t_y + 36\alpha\theta v_0 + 9\alpha\theta v_h - 2\theta^2 v_h^2 + 9\alpha\theta v_l \end{gathered} \tag{A45}$$

$$\begin{gathered} SW^{TS} = \frac{1}{576(\alpha - t_x)^2 t_x}(-144t_x^4 + 3\alpha^2 t_y^2 + 72t_x^3(-5t_y + 4(2\alpha + 3\theta v_0 + \theta v_h + \theta v_l)) + \\ 2\alpha t_x(-21t_y^2 - 12t_y(15\alpha - 4\theta v_0 + 4\theta v_h - 4\theta v_l) + \\ 16(9\alpha^2 - 2\theta^2 v_0^2 - 2\theta^2 v_h^2 + 9\alpha\theta v_l - 2\theta^2 v_l^2 + \theta v_0(27\alpha + 4\theta v_h - 4\theta v_l) + \\ \theta v_h(9\alpha + 4\theta v_l))) - 8t_x^2(-6t_y^2 - 15t_y(6\alpha - \theta v_0 + \theta v_h - \theta v_l) + \\ 2(45\alpha^2 - 5\theta^2 v_0^2 - 5\theta^2 v_h^2 + 36\alpha\theta v_l - 5\theta^2 v_l^2 + 2\theta v_0(54\alpha + 5\theta v_h - 5\theta v_l) + 2\theta v_h(18\alpha + 5\theta v_l)))) \end{gathered} \tag{A46}$$

$$\begin{gathered} SW^{ST} = \frac{1}{576(\alpha - t_x)^2 t_x}(-144t_x^4 + 3\alpha^2 t_y^2 + 72t_x^3(-5t_y + 4(2\alpha + 3\theta v_0 + \theta v_h + \theta v_l)) + \\ 2\alpha t_x(-21t_y^2 - 12t_y(15\alpha - 4\theta v_0 - 4\theta v_h + 4\theta v_l) + \\ 16(9\alpha^2 - 2\theta^2 v_0^2 - 2\theta^2 v_h^2 + 9\alpha\theta v_l - 2\theta^2 v_l^2 + \theta v_h(9\alpha + 4\theta v_l) + \theta v_0(27\alpha - 4\theta v_h + 4\theta v_l))) \\ -8t_x^2(-6t_y^2 - 15t_y(6\alpha - \theta v_0 - \theta v_h + \theta v_l) + 2(45\alpha^2 - 5\theta^2 v_0^2 - 5\theta^2 v_h^2 + 36\alpha\theta v_l - 5\theta^2 v_l^2 \\ +2\theta v_h(18\alpha + 5\theta v_l) + 2\theta v_0(54\alpha - 5\theta v_h + 5\theta v_l)))) \end{gathered} \tag{A47}$$

$$\begin{gathered} SW^{SS} = \frac{1}{36(\alpha - t_x)^2}(-9t_x^3 + 9t_x^2(4\alpha - t_y + 4\theta v_0 + 2\theta v_h + 2\theta v_l) - \\ t_x(45\alpha^2 - 18\alpha t_y + 72\alpha\theta v_0 + 36\alpha\theta v_h - 5\theta^2 v_h^2 + 36\alpha\theta v_l + 10\theta^2 v_h v_l - 5\theta^2 v_l^2) + \\ \alpha(-9\alpha t_y + 2(9\alpha^2 + 18\alpha\theta v_0 + 9\alpha\theta v_h - 2\theta^2 v_h^2 + 9\alpha\theta v_l + 4\theta^2 v_h v_l - 2\theta^2 v_l^2))) \end{gathered} \tag{A48}$$

Proposition A3. *(Comparison in Social Welfare Considering Network Effects)*

(a) *The social welfare in Case TT is higher than that in Case SS if and only if* $t_y < \frac{4}{3}\theta v_0$.

(b) *The social welfare in Case ST is higher than that in Case TS if and only if* $t_y < \frac{4}{3}\theta v_0$.

(c) *When* $t_y < \frac{4}{3}\theta v_0$, *there exists a threshold,* v_d^3, *such that Case TT will bring a higher social welfare than Case ST if and only if* $v_d < v_d^3$.

(d) *When* $t_y > \frac{4}{3}\theta v_0$, *there exists a threshold,* v_d^4, *such that Case SS will bring a higher social welfare than Case TS if and only if* $v_d < v_d^4$.

Similarly, we can observe that the comparison of social welfare in the four cases leads to similar results in the absence of and in the presence of network effects (as shown in Propositions 6 and A3). Therefore, the main insights derived from our main model are robust with network effects.

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Appendix A.4. Details of Membership Subscription Prices

Table A2. Illustration of the same prices for standalone subscriptions and bundled options.

Membership	Original Price	Webpage Screenshot
JD.PLUS	¥198 per year (https://plus.jd.com/index (accessed on 15 December 2025))	
Baidu Library	¥138 per year (https://wenku.baidu.com/ndwelfare/browse/vipcenter?_wkts_=1765659061108&bdQuery=%E7%99%BE%E5%BA%A6%E6%96%87%E5%BA%93 (accessed on 15 December 2025))	
JD.PLUS + Baidu Library	¥198 per year	
Tencent Video	¥30 per month (https://m.film.qq.com/magic-act/dfwcp7agtt4wa009utf2yrpz65/index_index.html?ovscroll=0&page=index&__no_magic_qrcode=1 (accessed on 15 December 2025))	
Kugou	¥15 per month (https://vip.kugou.com/recharge.html?from=music&version=7713&action=newSuperVipType&entrance=2001 (accessed on 15 December 2025))	
Tencent Video + Kugou	¥30 per month (https://m.film.qq.com/magic-act/dfwcp7agtt4wa009utf2yrpz65/index_index.html?ovscroll=0&page=index&__no_magic_qrcode=1 (accessed on 15 December 2025))	

Appendix B. Proofs of Lemmas and Propositions

In this appendix, we present the mathematical proofs of all results in the paper. Specifically, we assume that the values of the base products of the platforms are high enough, so the consumer market can be fully covered.

Appendix B.1. Proof of Lemma A1

In Case TT, by computing and substituting D_l^{TT} and D_h^{TT} into π_l^{TT} and π_h^{TT} in Equation (8) and using the first-order conditions:

$$\begin{cases} \frac{\partial \pi_l^{TT}}{\partial p_l^{TT}} = (1-2\gamma_T)\frac{p_h^{TT}-2p_l^{TT}+t_x-\theta(v_h-v_l)}{2t_x} = 0 \\ \frac{\partial \pi_h^{TT}}{\partial p_h^{TT}} = (1-2\gamma_T)\frac{p_l^{TT}-2p_h^{TT}+t_x+\theta(v_h-v_l)}{2t_x} = 0 \end{cases}. \tag{A49}$$

We can derive p_i^{TT*} $(i = l, h)$ as:

$$\begin{cases} p_l^{TT*} = t_x - \frac{1}{3}\theta(v_h - v_l) \\ p_h^{TT*} = t_x + \frac{1}{3}\theta(v_h - v_l) \end{cases}. \tag{A50}$$

We can verify that the second-order derivatives of the profit functions are negative. Based on the equilibrium prices, we can derive the equilibrium demands and equilibrium profits in Lemma A1.

In equilibrium, when both Platform l and Platform h bundle base goods with two add-ons together, the condition that consumers have positive demand for each bundled product should be satisfied to ensure the effectiveness of the strategy. That is, $0 < D_i^{TT*} < 1$ for $i = l, h$, which is equivalent to $0 < x^{TT} < 1$. Therefore, $0 < v_d < \frac{3t_x}{\theta}$ is required.

The proof of Lemma A1 is completed. □

Appendix B.2. Proof of Lemma A2

In Case TS, to simplify the following analysis, p_h^{TS1} and p_h^{TS2} are assumed to be equal, given the equal values of the two add-on products. Substituting $x(y_{TS1})$, $x(y_{TS2})$ and y^{TS} in Equation (9) into D_l^{TS}, D_h^{TS1} and D_h^{TS2} in Equation (10), and then substitute the obtained results into π_l^{TS} and π_h^{TS} in Equation (11). Using the first-order conditions:

$$\begin{cases} \frac{\partial \pi_l^{TS}}{\partial p_l^{TS}} = (1-2\gamma_T)\frac{\left(4p_h^{TS1}-8p_l^{TS}+4t_x-3t_y+4\theta v_0-\theta(v_h-v_l)\right)}{8t_x} = 0 \\ \frac{\partial \pi_h^{TS}}{\partial p_h^{TS1}} = \frac{\partial \pi_h^{TS}}{\partial p_h^{TS2}} = (1-\gamma_S)\frac{\left(-8p_h^{TS1}+4p_l^{TS}+4t_x+3t_y-4\theta v_0+\theta(v_h-v_l)\right)}{8t_x} = 0 \end{cases}. \tag{A51}$$

We can derive p_l^{TS*} and p_h^{TSk*} $(k = 1, 2)$ as:

$$\begin{cases} p_l^{TS*} = t_x - \left(\frac{1}{3}\theta(v_h - v_l - v_0) + \frac{1}{4}t_y\right) \\ p_h^{TS1*} = p_h^{TS2*} = t_x + \left(\frac{1}{3}\theta(v_h - v_l - v_0) + \frac{1}{4}t_y\right) \end{cases}. \tag{A52}$$

We can verify that the second-order derivatives of the profit functions are negative.

Based on the equilibrium prices, we can derive the equilibrium demands and equilibrium profits in Lemma A2.

In equilibrium, the condition that consumers have positive demand for each bundled product should be satisfied to ensure the effectiveness of the strategy. That is, $0 < D_l^{TS*} < 1$, $0 < D_h^{TS1*} < 1$, and $0 < D_h^{TS2*} < 1$. Therefore, $3t_y - 12t_x < 4\theta(v_0 - v_d) < 3t_y + 12t_x$ is required.

The proof of Lemma A2 is completed. □

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Appendix B.3. Proof of Lemma A3

Similar to Case TS, in Case ST we let $p_l^{ST1} = p_l^{ST2}$. Substituting $x(y_{ST1})$, $x(y_{ST2})$ and y^{TS} in Equation (12) into D_l^{ST1}, D_l^{ST2} and D_h^{ST} in Equation (13), and then substitute the obtained results into π_l^{ST} and π_h^{ST} in Equation (14). Using the first-order conditions,

$$\begin{cases} \frac{\partial \pi_l^{ST}}{\partial p_l^{ST1}} = \frac{\partial \pi_l^{ST}}{\partial p_l^{ST2}} = (1-\gamma_S)\frac{\left(4p_h^{ST} - 8p_l^{ST1} + 4t_x + 3t_y - 4\theta v_0 - \theta(v_h - v_l)\right)}{8t_x} = 0 \\ \frac{\partial \pi_h^{ST}}{\partial p_h^{ST}} = (1-2\gamma_T)\frac{\left(-8p_h^{ST} + 4p_l^{ST1} + 4t_x - 3t_y + 4\theta v_0 - \theta(v_h - v_l)\right)}{8t_x} = 0 \end{cases}. \tag{A53}$$

We can derive p_l^{STk*} and p_h^{ST*} $(k = 1, 2)$ as

$$\begin{cases} p_l^{ST1*} = p_l^{ST2*} = t_x - \left(\frac{1}{3}\theta(v_h - v_l + v_0) - \frac{1}{4}t_y\right) \\ p_h^{ST*} = t_x + \left(\frac{1}{3}\theta(v_h - v_l + v_0) - \frac{1}{4}t_y\right) \end{cases}. \tag{A54}$$

We can verify that the second-order derivatives of the profit functions are negative. Based on the equilibrium prices, we can derive the equilibrium demands and equilibrium profits in Lemma A3.

In equilibrium, the condition that consumers have positive demand for each bundled product should be satisfied to ensure the effectiveness of the strategy. That is, $0 < D_l^{ST1*} < 1$, $0 < D_l^{ST2*} < 1$, and $0 < D_h^{ST*} < 1$. Therefore, $3t_y - 12t_x < 4\theta(v_0 + v_d) < 3t_y + 12t_x$ is required.

The proof of Lemma A3 is completed. □

Appendix B.4. Proof of Lemma A4

Considering $p_l^{SS1} = p_l^{SS2}$ and $p_h^{SS1} = p_h^{SS2}$ in Case SS. Substituting x^{SS} and y^{SS} into D_l^{SSk} and D_h^{SSk} $(k = 1, 2)$ in Equation (16), and then substituting the outcomes of D_l^{SSk} and D_h^{SSk} $(k = 1, 2)$ into Equation (17). Using the first-order conditions,

$$\begin{cases} \frac{\partial \pi_l^{SS}}{\partial p_l^{SS1}} = \frac{\partial \pi_l^{SS}}{\partial p_l^{SS2}} = (1-\gamma_S)\frac{\left(p_h^{SS1} - 2p_l^{SS1} + t_x - \theta(v_h - v_l)\right)}{2t_x} = 0 \\ \frac{\partial \pi_h^{SS}}{\partial p_h^{SS1}} = \frac{\partial \pi_h^{SS}}{\partial p_h^{SS2}} = (1-\gamma_S)\frac{\left(-2p_h^{SS1} + p_l^{SS1} + t_x + \theta(v_h - v_l)\right)}{2t_x} = 0 \end{cases}. \tag{A55}$$

We can derive p_i^{SSk*} $(i = l, h)$ $(k = 1, 2)$ as

$$\begin{cases} p_l^{SS1*} = p_l^{SS2*} = t_x - \frac{1}{3}\theta(v_h - v_l) \\ p_h^{SS1*} = p_h^{SS2*} = t_x + \frac{1}{3}\theta(v_h - v_l) \end{cases}. \tag{A56}$$

We can verify that the second-order derivatives of the profit functions are negative. Based on the equilibrium prices, we can derive the equilibrium demands and equilibrium profits in Lemma A4.

In equilibrium, when both Platform l and Platform h bundle base goods with two add-ons separately, the condition that consumers have positive demand for each bundled product should be satisfied to ensure the effectiveness of the strategy. That is, $0 < D_i^{SSk*} < 1$ for $i = l, h$ and $k = 1, 2$, which is equivalent to $0 < x^{SS} < 1$. Therefore, $0 < v_d < \frac{3t_x}{\theta}$ is required.

The proof of Lemma A4 is completed. □

Appendix B.5. Proof of Proposition 1

According to Lemmas A1 and A3, we have $p_l^{TT} - p_l^{STk} = \frac{1}{3}\theta v_0 - \frac{1}{4}t_y$, where $k = 1, 2$. Therefore, $p_l^{TT} > p_l^{STk}$ if and only if $t_y < \frac{4}{3}\theta v_0$.

Similarly, according to Lemmas A1–4, we have:

$$\begin{cases} p_l^{TS} - p_l^{SSk} = \frac{1}{3}\theta v_0 - \frac{1}{4}t_y \\ p_h^{TT} - p_h^{TSk} = \frac{1}{3}\theta v_0 - \frac{1}{4}t_y \\ p_h^{ST} - p_h^{SSk} = \frac{1}{3}\theta v_0 - \frac{1}{4}t_y \end{cases}. \tag{A57}$$

Therefore, $p_l^{TS} > p_l^{SSk}$, $p_h^{TT} > p_h^{TSk}$ and $p_h^{ST} > p_h^{SSk}$ if and only if $t_y < \frac{4}{3}\theta v_0$. The proof of Proposition 1 is completed. □

Appendix B.6. Proof of Proposition 2

(a) According to Equations (A6) and (A9), we can derive the partial derivatives as follows (As mentioned before, to ensure that each bundled product has positive demand in Case TS, it is required that $3t_y - 12t_x < 4\theta(v_0 - v_d) < 3t_y + 12t_x$. To ensure that each bundled product has positive demand in Case ST, it is required that $3t_y - 12t_x < 4\theta(v_0 + v_d) < 3t_y + 12t_x$. Accordingly, the signs of the derivatives can be determined as in Equation (A58)).

$$\begin{cases} \frac{\partial \pi_l^{TS}}{\partial v_0} = \frac{(1-2\gamma_T)\theta(12t_x - 3t_y + 4(v_0 - v_d)\theta)}{36t_x} > 0 \\ \frac{\partial \pi_l^{TS}}{\partial t_y} = \frac{(-1+2\gamma_T)\theta(12t_x - 3t_y + 4(v_0 - v_d)\theta)}{48t_x} < 0 \\ \frac{\partial \pi_h^{TS}}{\partial v_0} = \frac{(-1+\gamma_S)\theta(12t_x + 3t_y - 4(v_0 - v_d)\theta)}{36t_x} < 0 \\ \frac{\partial \pi_h^{TS}}{\partial t_y} = \frac{(1-\gamma_S)\theta(12t_x + 3t_y - 4(v_0 - v_d)\theta)}{48t_x} > 0 \\ \frac{\partial \pi_l^{ST}}{\partial v_0} = \frac{(-1+\gamma_S)\theta(12t_x + 3t_y - 4(v_0 + v_d)\theta)}{36t_x} < 0 \\ \frac{\partial \pi_l^{ST}}{\partial t_y} = \frac{(1-\gamma_S)\theta(12t_x + 3t_y - 4(v_0 + v_d)\theta)}{48t_x} > 0 \\ \frac{\partial \pi_h^{ST}}{\partial v_0} = \frac{(1-2\gamma_T)\theta(12t_x - 3t_y + 4(v_0 + v_d)\theta)}{36t_x} > 0 \\ \frac{\partial \pi_h^{ST}}{\partial t_y} = \frac{(-1+2\gamma_T)\theta(12t_x - 3t_y + 4(v_0 + v_d)\theta)}{48t_x} < 0 \end{cases}. \tag{A58}$$

Therefore, in Case TS and Case ST, the larger the value of v_0 (t_y), the more favorable (unfavorable) it is for the profit growth of platforms adopting the T strategy.

(b) According to Equations (A3) and (A12), we find that the equilibrium profits in Case TT and Case SS are independent of v_0 and t_y.

The proof of Proposition 2 is completed. □

Appendix B.7. Proof of Proposition 3

(a) In Case TT, for Platform l, it has no incentive to deviate when $\pi_l^{TT} > \pi_l^{ST}$, while for Platform h, it has no incentive to deviate when $\pi_h^{TT} > \pi_h^{TS}$. According to Lemmas A1 and A3, we have $\pi_l^{TT} - \pi_l^{ST} = (1-2\gamma_T)\frac{(3t_x - \theta(v_h - v_l))^2}{18t_x} - (1-\gamma_S)\frac{\left(12t_x + 3t_y - 4\theta(v_h - v_l + v_0)\right)^2}{288t_x} = (1-2\gamma_T)\frac{(3t_x - \theta v_d)^2}{18t_x} - (1-\gamma_S)\frac{\left(12t_x + 3t_y - 4\theta(v_d + v_0)\right)^2}{288t_x}$.

Therefore, $\pi_l^{TT} > \pi_l^{ST}$ if and only if $v_0 > \left(\frac{3}{4\theta}t_y - \frac{3}{\theta}(\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}} - 1)t_x + (\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}} - 1)v_d\right)$.

According to Lemmas A1 and A2, we have $\pi_h^{TT} - \pi_h^{TS} = (1-2\gamma_T)\frac{(3t_x + \theta(v_h - v_l))^2}{18t_x} - (1-\gamma_S)\frac{\left(12t_x + 3t_y - 4\theta(v_l - v_h + v_0)\right)^2}{288t_x} = (1-2\gamma_T)\frac{(3t_x + \theta v_d)^2}{18t_x} - (1-\gamma_S)\frac{\left(12t_x + 3t_y - 4\theta(v_0 - v_d)\right)^2}{288t_x}$.

Therefore, $\pi_h^{TT} > \pi_h^{TS}$ if and only if $v_0 > \left(\frac{3}{4\theta}t_y - \frac{3}{\theta}(\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}} - 1)t_x - (\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}} - 1)v_d\right)$.

Thus, Case TT can be sustained as the equilibrium when $v_0 > max\{(\frac{3}{4\theta}t_y - \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_2} - \sqrt{\gamma_1}}{\sqrt{\gamma_1}}) + (\frac{\sqrt{\gamma_2} - \sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d), (\frac{3}{4\theta}t_y - \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_2} - \sqrt{\gamma_1}}{\sqrt{\gamma_1}}) - (\frac{\sqrt{\gamma_2} - \sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)\}$, where $\gamma_1 \equiv 1 - \gamma_S$, $\gamma_2 \equiv 1 - 2\gamma_T$.

(b) In Case TS, for Platform l, it has no incentive to deviate when $\pi_l^{TS} > \pi_l^{SS}$, while for Platform h, it has no incentive to deviate when $\pi_h^{TS} > \pi_h^{TT}$. According to Lemmas A2 and A4, we have $\pi_l^{TS} - \pi_l^{SS} = (1-2\gamma_T)\frac{\left(12t_x-3t_y+4\theta(v_l-v_h+v_0)\right)^2}{288t_x} - (1-\gamma_S)\frac{(3t_x-\theta(v_h-v_l))^2}{18t_x} = (1-2\gamma_T)\frac{\left(12t_x-3t_y+4\theta(v_0-v_d)\right)^2}{288t_x} - (1-\gamma_S)\frac{(3t_x-\theta v_d)^2}{18t_x}$.

Therefore, $\pi_l^{TS} > \pi_l^{SS}$ if and only if $v_0 > \left(\frac{3}{4\theta}t_y + \frac{3}{\theta}(\sqrt{\frac{(1-\gamma_S)}{(1-2\gamma_T)}}-1)t_x - (\sqrt{\frac{(1-\gamma_S)}{(1-2\gamma_T)}}-1)v_d\right)$.

According to Lemmas A1 and A2, we have $\pi_h^{TS} - \pi_h^{TT} = (1-\gamma_S)\frac{(12t_x+3t_y-4\theta(v_l-v_h+v_0))^2}{288t_x} - (1-2\gamma_T)\frac{(3t_x+\theta(v_h-v_l))^2}{18t_x} = (1-\gamma_S)\frac{(12t_x+3t_y-4\theta(v_0-v_d))^2}{288t_x} - (1-2\gamma_T)\frac{(3t_x+\theta v_d)^2}{18t_x}$.

Therefore, $\pi_h^{TS} > \pi_h^{TT}$ if and only if $v_0 < \left(\frac{3}{4\theta}t_y - \frac{3}{\theta}(\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}}-1)t_x - (\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}}-1)v_d\right)$.

Thus, Case TS can be sustained as the equilibrium when $(\frac{3}{4\theta}t_y + \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) - (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d) < v_0 < (\frac{3}{4\theta}t_y - \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) - (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)$. (i.e., if $\frac{\gamma_S}{2} < \gamma_T$ and $v_d > max\{(\frac{3t_y-4\theta v_0}{4\theta(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})} + \frac{3t_x}{\theta}), (\frac{3t_y-4\theta v_0}{4\theta(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})} - \frac{3t_x}{\theta})\}$), where $\gamma_1 \equiv 1-\gamma_S$, $\gamma_2 \equiv 1-2\gamma_T$.

(c) In Case ST, for Platform l, it has no incentive to deviate when $\pi_l^{ST} > \pi_l^{TT}$, while for Platform h, it has no incentive to deviate when $\pi_h^{ST} > \pi_h^{SS}$. According to Lemmas A1 and A3, we have $\pi_l^{ST} - \pi_l^{TT} = (1-\gamma_S)\frac{\left(12t_x+3t_y-4\theta(v_h-v_l+v_0)\right)^2}{288t_x} - (1-2\gamma_T)\frac{(3t_x-\theta(v_h-v_l))^2}{18t_x} = (1-\gamma_S)\frac{\left(12t_x+3t_y-4\theta(v_d+v_0)\right)^2}{288t_x} - (1-2\gamma_T)\frac{(3t_x-\theta v_d)^2}{18t_x}$.

Therefore, $\pi_l^{ST} > \pi_l^{TT}$ if and only if $v_0 > \left(\frac{3}{4\theta}t_y + \frac{3}{\theta}(\sqrt{\frac{(1-\gamma_S)}{(1-2\gamma_T)}}-1)t_x + (\sqrt{\frac{(1-\gamma_S)}{(1-2\gamma_T)}}-1)v_d\right)$.

According to Lemmas A3 and A4, we have $\pi_h^{ST} - \pi_h^{SS} = (1-2\gamma_T)\frac{\left(12t_x-3t_y+4\theta(v_h-v_l+v_0)\right)^2}{288t_x} - (1-\gamma_S)\frac{(3t_x+\theta(v_h-v_l))^2}{18t_x} = (1-2\gamma_T)\frac{\left(12t_x-3t_y+4\theta(v_d+v_0)\right)^2}{288t_x} - (1-\gamma_S)\frac{(3t_x+\theta v_d)^2}{18t_x}$.

Therefore, $\pi_h^{ST} > \pi_h^{SS}$ if and only if $v_0 < \left(\frac{3}{4\theta}t_y - \frac{3}{\theta}(\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}}-1)t_x + (\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}}-1)v_d\right)$.

Thus, Case ST can be sustained as the equilibrium when $(\frac{3}{4\theta}t_y + \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) + (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d) < v_0 < (\frac{3}{4\theta}t_y - \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) + (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)$. (i.e., if $\gamma_T \le \frac{\gamma_S}{2}$ and $v_d > max\{(\frac{-3t_y+4\theta v_0}{4\theta(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})} + \frac{3t_x}{\theta}), (\frac{-3t_y+4\theta v_0}{4\theta(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})} - \frac{3t_x}{\theta})\}$), where $\gamma_1 \equiv 1-\gamma_S$, $\gamma_2 \equiv 1-2\gamma_T$.

(d) In Case SS, for Platform l, it has no incentive to deviate when $\pi_l^{SS} > \pi_l^{TS}$, while for Platform h, it has no incentive to deviate when $\pi_h^{SS} > \pi_h^{ST}$. According to Lemmas A2 and A4, we have $\pi_l^{SS} - \pi_l^{TS} = (1-\gamma_S)\frac{(3t_x-\theta(v_h-v_l))^2}{18t_x} - (1-2\gamma_T)\frac{\left(12t_x-3t_y+4\theta(v_l-v_h+v_0)\right)^2}{288t_x} = (1-\gamma_S)\frac{(3t_x-\theta v_d)^2}{18t_x} - (1-2\gamma_T)\frac{\left(12t_x-3t_y+4\theta(v_0-v_d)\right)^2}{288t_x}$.

Therefore, $\pi_l^{SS} > \pi_l^{TS}$ if and only if $v_0 < \left(\frac{3}{4\theta}t_y + \frac{3}{\theta}(\sqrt{\frac{(1-\gamma_S)}{(1-2\gamma_T)}}-1)t_x - (\sqrt{\frac{(1-\gamma_S)}{(1-2\gamma_T)}}-1)v_d\right)$.

According to Lemmas A3 and A4, we have $\pi_h^{SS} - \pi_h^{ST} = (1-\gamma_S)\frac{(3t_x+\theta(v_h-v_l))^2}{18t_x} - (1-2\gamma_T)\frac{\left(12t_x-3t_y+4\theta(v_h-v_l+v_0)\right)^2}{288t_x} = (1-\gamma_S)\frac{(3t_x+\theta v_d)^2}{18t_x} - (1-2\gamma_T)\frac{\left(12t_x-3t_y+4\theta(v_d+v_0)\right)^2}{288t_x}$.

Therefore, $\pi_h^{SS} > \pi_h^{ST}$ if and only if $v_0 < \left(\frac{3}{4\theta}t_y + \frac{3}{\theta}(\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}}-1)t_x + (\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}}-1)v_d\right)$.

Thus, Case SS can be sustained as the equilibrium when $v_0 < min\{(\frac{3}{4\theta}t_y + \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) - (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d), (\frac{3}{4\theta}t_y + \frac{3}{\theta}t_x(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) + (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d)\}$. (i.e., $t_y > max\{(\frac{4\theta}{3}v_0 - 4(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})t_x + \frac{4\theta}{3}(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d), (\frac{4\theta}{3}v_0 - 4(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})t_x - \frac{4\theta}{3}(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d)\}$), where $\gamma_1 \equiv 1-\gamma_S$, $\gamma_2 \equiv 1-2\gamma_T$.

The parameter settings of all the figures in Figure 5 are consistent with the model analysis. Notably, in Figure 5a, that is, $\frac{\gamma_S}{2} < \gamma_T$, when all the parameter conditions are satisfied, there exists a scenario where two equilibria can coexist—both adopting the T strategy (i.e., Case TT) or both

adopting the S strategy (i.e., Case SS). According to Lemmas A1 and A4, we can derive that $\pi_l^{SS} - \pi_l^{TT} = (1-\gamma_S)\frac{(3t_x-\theta(v_h-v_l))^2}{18t_x} - (1-2\gamma_T)\frac{(3t_x-\theta(v_h-v_l))^2}{18t_x} = (2\gamma_T-\gamma_S)\frac{(3t_x-\theta(v_h-v_l))^2}{18t_x} > 0$ and $\pi_h^{TT} - \pi_h^{SS} = (1-\gamma_S)\frac{(3t_x+\theta(v_h-v_l))^2}{18t_x} - (1-2\gamma_T)\frac{(3t_x+\theta(v_h-v_l))^2}{18t_x} = (2\gamma_T-\gamma_S)\frac{(3t_x+\theta(v_h-v_l))^2}{18t_x} > 0$. Thus, the equilibrium where both platforms adopt the S strategy makes them more profitable than when both adopt the T strategy.

The proof of Proposition 3 is completed. □

Appendix B.8. Proof of Proposition 4

Based on the analysis and results of Proposition 3, and on the premise of satisfying the basic assumptions of the model, we can obtain the conditions for the market equilibrium of different combinations in Proposition 4. □

Appendix B.9. Proof of Proposition 5

Consumer surplus is the total consumer net utility from purchasing the bundled products; thus, we can derive the consumer surplus in the four cases as follows:

$$\begin{aligned} CS^{TT} &= \int_0^1 \int_0^{x^{TT}} U_l^{TT} dxdy + \int_0^1 \int_{x^{TT}}^1 U_h^{TT} dxdy \\ &= \int_0^1 \int_0^{\frac{p_h^{TT*}-p_l^{TT*}+\theta(v_l-v_h)}{2t_x}+\frac{1}{2}} \theta(v_l+2v_0) - t_x x - t_y - p_l^{TT*} dxdy \\ &+ \int_0^1 \int_{\frac{p_h^{TT*}-p_l^{TT*}+\theta(v_l-v_h)}{2t_x}+\frac{1}{2}}^1 \theta(v_h+2v_0) - t_x(1-x) - t_y - p_h^{TT*} dxdy \\ &= \frac{1}{36}\left(-45t_x - 36t_y + 18(4v_0+v_h+v_l)\theta + \frac{(v_h-v_l)^2\theta^2}{t_x}\right), \end{aligned} \tag{A59}$$

$$\begin{aligned} CS^{TS} &= \int_0^{\frac{1}{2}} \int_0^{x_{yTS1}} U_l^{TS} dxdy + \int_{\frac{1}{2}}^1 \int_0^{x_{yTS2}} U_l^{TS} dxdy \\ &+ \int_0^{\frac{1}{2}} \int_{x_{yTS1}}^1 U_h^{TS1} dxdy + \int_{\frac{1}{2}}^1 \int_{x_{yTS2}}^1 U_h^{TS2} dxdy \\ &= \frac{1}{144t_x}\left(-180t_x^2 + 3t_y^2 - 6\theta t_y(v_0 - v_h + v_l) + 4\theta^2(v_0 - v_h + v_l)^2 + t_x(-90t_y + 72\theta(3v_0 + v_h + v_l))\right), \end{aligned} \tag{A60}$$

$$\begin{aligned} CS^{ST} &= \int_0^{\frac{1}{2}} \int_0^{x_{yST1}} U_l^{ST1} dxdy + \int_{\frac{1}{2}}^1 \int_0^{x_{yST2}} U_l^{ST2} dxdy \\ &+ \int_0^{\frac{1}{2}} \int_{x_{yST1}}^1 U_h^{ST} dxdy + \int_{\frac{1}{2}}^1 \int_{x_{yST2}}^1 U_h^{ST} dxdy \\ &= \frac{1}{144t_x}\left(-180t_x^2 + 3t_y^2 - 6\theta t_2(v_0 + v_h - v_l) + 4\theta^2(v_0 + v_h - v_l)^2 + t_x(-90t_y + 72\theta(3v_0 + v_h + v_l))\right), \end{aligned} \tag{A61}$$

$$\begin{aligned} CS^{SS} &= \int_0^{\frac{1}{2}} \int_0^{x^{SS}} U_l^{SS1} \mathrm{d}x\mathrm{d}y + \int_{\frac{1}{2}}^1 \int_0^{X^{ss}} U_l^{SS2} \mathrm{d}x\mathrm{d}y \\ &+ \int_0^{\frac{1}{2}} \int_{x^{SS}}^1 U_h^{SS1} \mathrm{d}x\mathrm{d}y + \int_{\frac{1}{2}}^1 \int_{x^{SS}}^1 U_h^{SS2} \mathrm{d}x\mathrm{d}y \\ &= \frac{1}{36}\left(-45t_x - 9t_y + \frac{\theta^2(v_h-v_l)^2}{t_x} + 18\theta(2v_0 + v_h + v_l)\right). \end{aligned} \tag{A62}$$

Therefore, we have $CS^{TT} - CS^{SS} = \frac{1}{36}(-45t_x - 36t_y + 18(4v_0 + v_h + v_l)\theta + \frac{(v_h-v_l)^2\theta^2}{t_x}) - \frac{1}{36}(-45t_x - 9t_y + \frac{\theta^2(v_h-v_l)^2}{t_x} + 18\theta(2v_0 + v_h + v_l)) = -\frac{3t_y}{4} + \theta v_0$ and $CS^{TS} - CS^{ST} = \frac{1}{144t_x}(-180t_x^2 + 3t_y^2 - 6\theta t_y(v_0 - v_h + v_l) + 4\theta^2(v_0 - v_h + v_l)^2 + t_x(-90t_y + 72\theta(3v_0 + v_h + v_l))) - \frac{1}{144t_x}(-180t_x^2 + 3t_y^2 - 6\theta t_2(v_0 + v_h - v_l) + 4\theta^2(v_0 + v_h - v_l)^2 + t_x(-90t_y + 72\theta(3v_0 + v_h + v_l))) = \frac{\theta(3t_y - 4\theta v_0)(v_h - v_l)}{36t_x}$.

(a) $CS^{TT} > CS^{SS}$ if and only if $t_y < \frac{4}{3}\theta v_0$.

(b) $CS^{ST} > CS^{TS}$ if and only if $t_y < \frac{4}{3}\theta v_0$.

(c) When $t_y < \frac{4}{3}\theta v_0$, compare CS^{TT} and CS^{ST}, we have $CS^{TT} - CS^{ST}$ $= \frac{1}{144t_x}(-3t_y^2 + t_x(-54t_y + 72\theta v_0) - 4\theta^2 v_0(v_0 + 2v_h - 2v_l) + 6\theta t_y(v_0 + v_h - v_l)) = \frac{1}{144t_x}(-3t_y^2 + t_x(-54t_y + 72\theta v_0) - 4\theta^2 v_0(v_0 + 2v_d) + 6\theta t_y(v_0 + v_d))$

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Consequently, if $v_d < \frac{-54t_xt_y-3t_y^2+72\theta t_xv_0+6\theta t_yv_0-4\theta^2v_0^2}{2\theta(-3t_y+4\theta v_0)}$ (i.e., $v_d < v_d'$), then $CS^{TT} > CS^{ST}$, otherwise $CS^{TT} \le CS^{ST}$.

(d) When $t_y > \frac{4}{3}\theta v_0$, compare CS^{SS} and CS^{TS}, we have $CS^{SS} - CS^{TS} = \frac{1}{144t_x}(-3t_y^2 + 18t_x(3t_y - 4\theta v_0) + 6\theta t_y(v_0 - v_h + v_l) - 4\theta^2 v_0(v_0 - 2v_h + 2v_l)) = \frac{1}{144t_x}(-3t_y^2 + 18t_x(3t_y - 4\theta v_0) + 6\theta t_y(v_0 - v_d) - 4\theta^2 v_0(v_0 - 2v_d))$.

Consequently, if $v_d < \frac{-54t_xt_y+3t_y^2+72\theta t_xv_0-6\theta t_yv_0+4\theta^2v_0^2}{2\theta(-3t_y+4\theta v_0)}$ (i.e., $v_d < v_d''$), then $CS^{SS} > CS^{TS}$, otherwise $CS^{SS} \le CS^{TS}$.

The proof of Proposition 5 is completed. □

Appendix B.10. Proof of Proposition 6

Social welfare is defined as the sum of the consumer surplus and the platforms' total profits; thus, we can derive the social welfare in four cases as follows:

$$\begin{aligned} SW^{TT} &= \pi_l^{TT*} + \pi_h^{TT*} + \pi_o^{TT*} + CS^{TT} = \tfrac{(3t_x-\theta(v_h-v_l))^2}{18t_x} + \tfrac{(3t_x+\theta(v_h-v_l))^2}{18t_x} \\ &\quad - \tfrac{1}{36}\left(45t_x + 36t_y - 18(4v_0 + v_h + v_l)\theta - \tfrac{(v_h-v_l)^2\theta^2}{t_x}\right) \\ &= \tfrac{1}{36}\left(-9t_x - 36t_y + 18(4v_0 + v_h + v_l)\theta + \tfrac{5(v_h-v_l)^2\theta^2}{t_x}\right), \end{aligned} \tag{A63}$$

$$\begin{aligned} SW^{TS} &= \pi_l^{TS*} + \pi_h^{TS*} + \pi_o^{TS*} + CS^{TS} \\ &= \tfrac{\left(12t_x-3t_y+4\theta(v_l-v_h+v_0)\right)^2}{288t_x} + \tfrac{\left(12t_x+3t_y-4\theta(v_l-v_h+v_0)\right)^2}{288t_x} \\ &+ \tfrac{1}{144t_x}\left(-180t_x^2 + 3t_y^2 - 6\theta t_y(v_0 - v_h + v_l) + 4\theta^2(v_0 - v_h + v_l)^2 + t_x\left(-90t_y + 72\theta(3v_0 + v_h + v_l)\right)\right) \\ &= \tfrac{-18{t_x}^2-45t_xt_y+6{t_y}^2-15t_y(v_0-v_h+v_l)\theta+36t_x(3v_0+v_h+v_l)\theta+10(v_0-v_h+v_l)^2\theta^2}{72t_x}, \end{aligned} \tag{A64}$$

$$\begin{aligned} SW^{ST} &= \pi_l^{ST*} + \pi_h^{ST*} + \pi_o^{ST*} + CS^{ST} \\ &= \tfrac{\left(12t_x+3t_y-4\theta(v_h-v_l+v_0)\right)^2}{288t_x} + \tfrac{\left(12t_x-3t_y+4\theta(v_h-v_l+v_0)\right)^2}{288t_x} + \\ &\tfrac{1}{144t_x}\left(-180t_x^2 + 3t_y^2 - 6\theta t_2(v_0 + v_h - v_l) + 4\theta^2(v_0 + v_h - v_l)^2 + t_x\left(-90t_y + 72\theta(3v_0 + v_h + v_l)\right)\right) \\ &= \tfrac{-18{t_x}^2-45t_xt_y+6{t_y}^2-15t_y(v_0+v_h-v_l)\theta+36t_x(3v_0+v_h+v_l)\theta+10(v_0+v_h-v_l)^2\theta^2}{72t_x}, \end{aligned} \tag{A65}$$

$$\begin{aligned} SW^{SS} &= \pi_l^{SS*} + \pi_h^{SS*} + \pi_o^{SS*} + CS^{SS} \\ &= \tfrac{(3t_x-\theta(v_h-v_l))^2}{18t_x} + \tfrac{(3t_x+\theta(v_h-v_l))^2}{18t_x} \\ &\quad + \tfrac{1}{36}\left(-45t_x - 9t_y + \tfrac{\theta^2(v_h-v_l)^2}{t_x} + 18\theta(2v_0 + v_h + v_l)\right) \\ &= \tfrac{1}{36}\left(-9t_x - 9t_y + 18(2v_0 + v_h + v_l)\theta + \tfrac{5(v_h-v_l)^2\theta^2}{t_x}\right). \end{aligned} \tag{A66}$$

Therefore, we have $SW^{TT} - SW^{SS} = \frac{1}{36}(-9t_x - 36t_y + 18(4v_0 + v_h + v_l)\theta + \frac{5(v_h-v_l)^2\theta^2}{t_x}) - \frac{1}{36}(-9t_x - 9t_y + 18(2v_0 + v_h + v_l)\theta + \frac{5(v_h-v_l)^2\theta^2}{t_x}) = -\frac{3t_y}{4} + \theta v_0$ and $SW^{TS} - SW^{ST} = (\frac{-18{t_x}^2-45t_xt_y+6{t_y}^2-15t_y(v_0-v_h+v_l)\theta+36t_x(3v_0+v_h+v_l)\theta+10(v_0-v_h+v_l)^2\theta^2}{72t_x}) - (\frac{-18{t_x}^2-45t_xt_y+6{t_y}^2-15t_y(v_0+v_h-v_l)\theta+36t_x(3v_0+v_h+v_l)\theta+10(v_0+v_h-v_l)^2\theta^2}{72t_x}) = \frac{5(v_h-v_l)\theta(3t_y-4\theta v_0)}{36t_x}$.

(a) $SW^{TT} > SW^{SS}$ if and only if $t_y < \frac{4}{3}\theta v_0$.

(b) $SW^{ST} > SW^{TS}$ if and only if $t_y < \frac{4}{3}\theta v_0$.

(c) When $t_y < \frac{4}{3}\theta v_0$, compare SW^{TT} and SW^{ST}, we have $SW^{TT} - SW^{ST} = -\frac{6{t_y}^2-15t_y(v_0+v_h-v_l)\theta+10v_0(v_0+2v_h-2v_l)\theta^2+9t_x\left(3t_y-4\theta v_0\right)}{72t_x} = -\frac{6{t_y}^2-15t_y(v_0+v_d)\theta+10v_0(v_0+2v_d)\theta^2+9t_x(3t_y-4\theta v_0)}{72t_x}$.

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Consequently, if $v_d < \frac{-27t_xt_y-6t_y^2+36\theta t_xv_0+15\theta t_yv_0-10\theta^2v_0^2}{5\theta(-3t_y+4\theta v_0)}$ (i.e., $v_d < v_d'''$), then $SW^{TT} > SW^{ST}$, otherwise $SW^{TT} \leq SW^{ST}$.

(d) When $t_y > \frac{4}{3}\theta v_0$, compare SW^{SS} and SW^{TS}, we have $SW^{SS} - SW^{TS}$= $\frac{-6{t_y}^2+15t_y(v_0-v_h+v_l)\theta-10v_0(v_0-2v_h+2v_l)\theta^2+9t_x(3t_y-4\theta v_0)}{72t_x} =$ $\frac{-6{t_y}^2+15t_y(v_0-v_d)\theta-10v_0(v_0-2v_d)\theta^2+9t_x(3t_y-4\theta v_0)}{72t_x}$.

We can derive that the condition $SW^{SS} > SW^{TS}$ is equivalent to $v_d < \frac{-27t_xt_y+6t_y^2+36\theta t_xv_0-15\theta t_yv_0+10\theta^2v_0^2}{5\theta(-3t_y+4\theta v_0)}$ (i.e., $v_d < v_d''''$).

The proof of Proposition 6 is completed. □

Appendix B.11. Proof of Extension

Appendix B.11.1. Proof of Case TT

In Case TT, by computing and substituting D_l^{TT} and D_h^{TT} into π_l^{TT} and π_h^{TT} in Equation (A16) and using the first-order conditions:

$$\begin{cases} \frac{\partial \pi_l^{TT}}{\partial p_l^{TT}} = (1-2\gamma_T)\frac{(\alpha-p_h^{TT}+2p_l^{TT}-t_x+\theta v_h-\theta v_l)}{2(a-t_x)} = 0 \\ \frac{\partial \pi_h^{TT}}{\partial p_h^{TT}} = (1-2\gamma_T)\frac{(\alpha+2p_h^{TT}-p_l^{TT}-t_x-\theta v_h+\theta v_l)}{2t_x} = 0 \end{cases}. \tag{A67}$$

We can derive p_i^{TT*} $(i = l, h)$ as:

$$\begin{cases} p_l^{TT*} = \frac{1}{3}(-3\alpha+3t_x-\theta v_h+\theta v_l) \\ p_h^{TT*} = \frac{1}{3}(-3a+3t_x+\theta v_h-\theta v_l) \end{cases}. \tag{A68}$$

We can verify that the second-order derivatives of the profit functions are negative. Based on the equilibrium prices, we can derive the equilibrium demands and equilibrium profits in Case TT.

The proof of Case TT is completed. □

Appendix B.11.2. Proof of Case TS

In Case TS, to simplify the following analysis, p_h^{TS1} and p_h^{TS2} are assumed to be equal, given the equal values of the two add-on products. Substituting D_l^{TS} and D_h^{TS} in Equation (A22) into π_l^{TS} and π_h^{TS} in Equation (A23). Using the first-order conditions:

$$\begin{cases} \frac{\partial \pi_l^{TS}}{\partial p_l^{TS}} = (1-2r_T)\frac{(4\alpha-4p_h^{TS1}+8p_l^{TS}-4t_x+3t_y-4\theta v_0+4\theta v_h-4\theta v_l)}{8(\alpha-t_x)} = 0 \\ \frac{\partial \pi_h^{TS}}{\partial p_h^{TS1}} = \frac{\partial \pi_h^{TS}}{\partial p_h^{TS2}} = (1-\gamma_S)\frac{(4\alpha+8p_h^{TS1}-4p_l^{TS}-4t_x-3t_y+4\theta v_0-4\theta v_h+4\theta v_l)}{8(\alpha-t_x)} = 0 \end{cases}. \tag{A69}$$

We can derive p_l^{TS*} and p_h^{TSk*} $(k = 1, 2)$ as:

$$\begin{cases} p_l^{TS*} = \frac{1}{12}(-12\alpha+12t_x-3t_y+4\theta v_0-4\theta v_h+4\theta v_l) \\ p_h^{TS1*} = p_h^{TS2*} = \frac{1}{12}(-12\alpha+12t_x+3t_y-4\theta v_0+4\theta v_h-4\theta v_l) \end{cases}. \tag{A70}$$

We can verify that the second-order derivatives of the profit functions are negative.

Based on the equilibrium prices, we can derive the equilibrium demands and equilibrium profits.

The proof of Case TS is completed. □

Appendix B.11.3. Proof of Case *ST*

Similar to Case *TS*, in Case *ST* we let $p_l^{ST1} = p_l^{ST2}$. Substituting D_l^{ST} and D_h^{ST} in Equation (A29) into π_l^{ST} and π_h^{ST} in Equation (A30). Using the first-order conditions:

$$\begin{cases} \frac{\partial \pi_l^{ST}}{\partial p_l^{ST1}} = \frac{\partial \pi_l^{ST}}{\partial p_l^{ST2}} = (1-\gamma_S)\frac{(4\alpha - 4p_h^{ST} + 8p_l^{ST1} - 4t_x - 3t_y + 4\theta v_0 + 4\theta v_h - 4\theta v_l)}{8(\alpha - t_x)} = 0 \\ \frac{\partial \pi_h^{ST}}{\partial p_h^{ST}} = (1-2\gamma_T)\frac{(4\alpha + 8p_h^{ST} - 4p_l^{ST1} - 4t_x + 3t_y - 4\theta v_0 - 4\theta v_h + 4\theta v_l)}{8(\alpha - t_x)} = 0 \end{cases}. \tag{A71}$$

We can derive p_l^{STk*} and p_h^{ST*} $(k = 1, 2)$ as:

$$\begin{cases} p_l^{ST1*} = p_l^{ST2*} = \frac{1}{12}(-12\alpha + 12t_x + 3t_y - 4\theta v_0 - 4\theta v_h + 4\theta v_l) \\ p_h^{ST*} = \frac{1}{12}(-12\alpha + 12t_x - 3t_y + 4\theta v_0 + 4\theta v_h - 4\theta v_l) \end{cases}. \tag{A72}$$

We can verify that the second-order derivatives of the profit functions are negative. Based on the equilibrium prices, we can derive the equilibrium demands and equilibrium profits.

The proof of Case *TS* is completed. □

Appendix B.11.4. Proof of Case *SS*

Considering $p_l^{SS1} = p_l^{SS2}$ and $p_h^{SS1} = p_h^{SS2}$ in Case *SS*. Substituting D_l^{SS} and D_h^{SS} in Equation (A36) into Equation (A37). Using the first-order conditions:

$$\begin{cases} \frac{\partial \pi_l^{SS}}{\partial p_l^{SS1}} = \frac{\partial \pi_l^{SS}}{\partial p_l^{SS2}} = (1-\gamma_S)\frac{(\alpha - p_h^{SS1} + 2p_l^{SS1} - t_x + \theta v_h - \theta v_l)}{2(\alpha - t_x)} = 0 \\ \frac{\partial \pi_h^{SS}}{\partial p_h^{SS1}} = \frac{\partial \pi_h^{SS}}{\partial p_h^{SS2}} = (1-\gamma_S)\frac{(\alpha + 2p_h^{SS1} - p_l^{SS1} - t_x - \theta v_h + \theta v_l)}{2(\alpha - t_x)} = 0 \end{cases}. \tag{A73}$$

We can derive p_i^{SSk*} $(i = l, h)$ $(k = 1, 2)$ as:

$$\begin{cases} p_l^{SS1*} = p_l^{SS2*} = \frac{1}{3}(-3\alpha + 3t_x - \theta v_h + \theta v_l) \\ p_h^{SS1*} = p_h^{SS2*} = \frac{1}{3}(-3\alpha + 3t_x + \theta v_h - \theta v_l) \end{cases}. \tag{A74}$$

We can verify that the second-order derivatives of the profit functions are negative. Based on the equilibrium prices, we can derive the equilibrium demands and equilibrium profits.

The proof of Case *SS* is completed. □

Appendix B.11.5. Proof of Proposition A1

(a) In Case *TT*, for Platform *l*, it has no incentive to deviate when $\pi_l^{TT} > \pi_l^{ST}$, while for Platform *h*, it has no incentive to deviate when $\pi_h^{TT} > \pi_h^{TS}$. According to the equilibrium outcomes in Case *TT* and Case *ST*, we have $\pi_l^{TT} - \pi_l^{ST} = \frac{(1-2r_T)(3\alpha - 3t_x + \theta v_h - \theta v_l)^2}{18(t_x - \alpha)} - \frac{(1-r_S)(12\alpha - 12t_x - 3t_y + 4\theta v_0 + 4\theta v_h - 4\theta v_l)^2}{288(t_x - \alpha)} = \frac{(1-2r_T)(3\alpha - 3t_x + \theta v_d)^2}{18(t_x - \alpha)} - \frac{(1-r_S)(12\alpha - 12t_x - 3t_y + 4\theta v_0 + 4\theta v_d)^2}{288(t_x - \alpha)}$.

Therefore, $\pi_l^{TT} > \pi_l^{ST}$ if and only if $v_0 > (\frac{3}{4\theta}t_y - \frac{3}{\theta}(\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}} - 1)(t_x - \alpha) + (\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}} - 1)v_d)$.

According to the equilibrium outcomes in Case *TT* and Case *TS*, we have $\pi_h^{TT} - \pi_h^{TS} = \frac{(1-2r_T)(3\alpha - 3t_x - \theta v_h + \theta v_l)^2}{18(t_x - \alpha)} - \frac{(1-r_S)(12\alpha - 12t_x - 3t_y + 4\theta v_0 - 4\theta v_h + 4\theta v_l)^2}{288(t_x - \alpha)} = \frac{(1-2r_T)(3\alpha - 3t_x - \theta v_d)^2}{18(t_x - \alpha)} - \frac{(1-r_S)(12\alpha - 12t_x - 3t_y + 4\theta v_0 - 4\theta v_d)^2}{288(t_x - \alpha)}$.

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Therefore, $\pi_h^{TT} > \pi_h^{TS}$ if and only if $v_0 > (\frac{3}{4\theta}t_y - \frac{3}{\theta}\left(\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}} - 1\right)(t_x - \alpha) - (\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}} - 1)v_d)$.

Thus, Case TT can be sustained as the equilibrium when $v_0 > max\{(\frac{3}{4\theta}t_y - \frac{3}{\theta}(t_x - \alpha)(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) + (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d), (\frac{3}{4\theta}t_y - \frac{3}{\theta}(t_x - \alpha)(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) - (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)\}$, where $\gamma_1 \equiv 1 - \gamma_S$, $\gamma_2 \equiv 1 - 2\gamma_T$.

(b) In Case TS, for Platform l, it has no incentive to deviate when $\pi_l^{TS} > \pi_l^{SS}$, while for Platform h, it has no incentive to deviate when $\pi_h^{TS} > \pi_h^{TT}$. According to the equilibrium outcomes in Case TS and Case SS, we have $\pi_l^{TS} - \pi_l^{SS} = \frac{(1-2r_T)(12\alpha-12t_x+3t_y-4\theta v_0+4\theta v_h-4\theta v_l)^2}{288(t_x-\alpha)} - \frac{(1-r_S)(3\alpha-3t_x+\theta v_h-\theta v_l)^2}{18(t_x-\alpha)} = \frac{(1-2r_T)(12\alpha-12t_x+3t_y-4\theta v_0+4\theta v_d)^2}{288(t_x-\alpha)} - \frac{(1-r_S)(3\alpha-3t_x+\theta v_d)^2}{18(t_x-\alpha)}$.

Therefore, $\pi_l^{TS} > \pi_l^{SS}$ if and only if $v_0 > (\frac{3}{4\theta}t_y + \frac{3}{\theta}(\sqrt{\frac{(1-\gamma_S)}{(1-2\gamma_T)}} - 1)(t_x - \alpha) - (\sqrt{\frac{(1-\gamma_S)}{(1-2\gamma_T)}} - 1)v_d)$.

According to the equilibrium outcomes in Case TT and Case TS, we have $\pi_h^{TS} - \pi_h^{TT} = \frac{(1-r_S)(12\alpha-12t_x-3t_y+4\theta v_0-4\theta v_h+4\theta v_l)^2}{288(t_x-\alpha)} - \frac{(1-2r_T)(3\alpha-3t_x-\theta v_h+\theta v_l)^2}{18(t_x-\alpha)} = \frac{(1-r_S)(12\alpha-12t_x-3t_y+4\theta v_0-4\theta v_d)^2}{288(t_x-\alpha)} - \frac{(1-2r_T)(3\alpha-3t_x-\theta v_d)^2}{18(t_x-\alpha)}$.

Therefore, $\pi_h^{TS} > \pi_h^{TT}$ if and only if $v_0 < (\frac{3}{4\theta}t_y - \frac{3}{\theta}\left(\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}} - 1\right)(t_x - \alpha) - (\sqrt{\frac{(1-2\gamma_T)}{(1-\gamma_S)}} - 1)v_d)$.

Thus, Case (T_l, S_h) can be sustained as the equilibrium when $(\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x - \alpha)(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) - (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d) < v_0 < (\frac{3}{4\theta}t_y - \frac{3}{\theta}(t_x - \alpha)(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) - (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)$, where $\gamma_1 \equiv 1 - \gamma_S$, $\gamma_2 \equiv 1 - 2\gamma_T$.

(c) In Case ST, for Platform l, it has no incentive to deviate when $\pi_l^{ST} > \pi_l^{TT}$, while for Platform h, it has no incentive to deviate when $\pi_h^{ST} > \pi_h^{SS}$. According to the equilibrium outcomes in Case TT and Case ST, we have $\pi_l^{ST} - \pi_l^{TT} = \frac{(1-r_S)(12\alpha-12t_x-3t_y+4\theta v_0+4\theta v_h-4\theta v_l)^2}{288(t_x-\alpha)} - \frac{(1-2r_T)(3\alpha-3t_x+\theta v_h-\theta v_l)^2}{18(t_x-\alpha)} = \frac{(1-r_S)(12\alpha-12t_x-3t_y+4\theta v_0+4\theta v_d)^2}{288(t_x-\alpha)} - \frac{(1-2r_T)(3\alpha-3t_x+\theta v_d)^2}{18(t_x-\alpha)}$.

Therefore, $\pi_l^{ST} > \pi_l^{TT}$ if and only if $\left(\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x - \alpha)\left(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}\right) + \left(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}\right)v_d\right) < v_0$.

According to the equilibrium outcomes in Case ST and Case SS, we have $\pi_h^{ST} - \pi_h^{SS} = \frac{(1-2r_T)(12\alpha-12t_x+3t_y-4\theta v_0-4\theta v_h+4\theta v_l)^2}{288(t_x-\alpha)} - \frac{(1-r_S)(3\alpha-3t_x-\theta v_h+\theta v_l)^2}{18(t_x-\alpha)} = \frac{(1-2r_T)(12\alpha-12t_x+3t_y-4\theta v_0-4\theta v_d)^2}{288(t_x-\alpha)} - \frac{(1-r_S)(3\alpha-3t_x-\theta v_d)^2}{18(t_x-\alpha)}$.

Therefore, $\pi_h^{ST} > \pi_h^{SS}$ if and only if $v_0 < \left(\frac{3}{4\theta}t_y - \frac{3}{\theta}(t_x - \alpha)\left(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}\right) + \left(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}\right)v_d\right)$.

Thus, Case ST can be sustained as the equilibrium when $(\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x - \alpha)(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) + (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d) < v_0 < (\frac{3}{4\theta}t_y - \frac{3}{\theta}(t_x - \alpha)(\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}}) + (\frac{\sqrt{\gamma_2}-\sqrt{\gamma_1}}{\sqrt{\gamma_1}})v_d)$, where $\gamma_1 \equiv 1 - \gamma_S$, $\gamma_2 \equiv 1 - 2\gamma_T$.

(d) In Case SS, for Platform l, it has no incentive to deviate when $\pi_l^{SS} > \pi_l^{TS}$, while for Platform h, it has no incentive to deviate when $\pi_h^{SS} > \pi_h^{ST}$. According to the equilibrium outcomes in Case TS and Case SS, we have $\pi_l^{SS} - \pi_l^{TS} = (1 - \gamma_S)\frac{(3t_x-\theta(v_h-v_l))^2}{18t_x} - (1 - 2\gamma_T)\frac{(12t_x-3t_y+4\theta(v_l-v_h+v_0))^2}{288t_x} = (1 - \gamma_S)\frac{(3t_x-\theta v_d)^2}{18t_x} - (1 - 2\gamma_T)\frac{(12t_x-3t_y+4\theta(v_0-v_d))^2}{288t_x}$.

Therefore, $\pi_l^{SS} > \pi_l^{TS}$ if and only if $v_0 < \left(\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x - \alpha)\left(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}\right) - \left(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}\right)v_d\right)$.

According to the equilibrium outcomes in Case ST and Case SS, we have $\pi_h^{SS} - \pi_h^{ST} = (1 - \gamma_S)\frac{(3t_x+\theta(v_h-v_l))^2}{18t_x} - (1 - 2\gamma_T)\frac{\left(12t_x-3t_y+4\theta(v_h-v_l+v_0)\right)^2}{288t_x} = (1 - \gamma_S)\frac{(3t_x+\theta v_d)^2}{18t_x} - (1 - 2\gamma_T)\frac{\left(12t_x-3t_y+4\theta(v_d+v_0)\right)^2}{288t_x}$.

Therefore, $\pi_h^{SS} > \pi_h^{ST}$ if and only if $v_0 < \left(\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x-\alpha)\left(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}\right) + \left(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}\right)v_d\right)$.

Thus, Case *SS* can be sustained as the equilibrium when $v_0 < min\{(\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x-\alpha)(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) - (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d), (\frac{3}{4\theta}t_y + \frac{3}{\theta}(t_x-\alpha)(\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}}) + (\frac{\sqrt{\gamma_1}-\sqrt{\gamma_2}}{\sqrt{\gamma_2}})v_d)\}$, where $\gamma_1 \equiv 1-\gamma_S$, $\gamma_2 \equiv 1-2\gamma_T$.

The proof of Proposition A1 is completed. □

Appendix B.11.6. Proof of Proposition A2

According to the values of the consumer surpluses in the four scenarios, we have $CS^{TT} - CS^{SS} = (\frac{1}{36(\alpha-t_x)^2}(-45t_x^3 + 18\alpha^2(3\alpha - 2t_y + 4\theta v_0 + \theta v_h + \theta v_l) + 18t_x^2(8\alpha - 2t_y + 4\theta v_0 + \theta v_h + \theta v_l) + t_x(-153\alpha^2 + 72\alpha t_y - 144\alpha\theta v_0 - 36\alpha\theta v_h + \theta^2 v_h^2 - 36\alpha\theta v_l - 2\theta^2 v_h v_l + \theta^2 v_l^2))) - (\frac{1}{36(\alpha-t_x)^2}(-45t_x^3 + 9\alpha^2(6\alpha - t_y + 4\theta v_0 + 2\theta v_h + 2\theta v_l) + t_x(-153\alpha^2 + 18\alpha t_y - 72\alpha\theta v_0 - 36\alpha\theta v_h + \theta^2 v_h^2 - 36\alpha\theta v_l - 2\theta^2 v_h v_l + \theta^2 v_l^2) + 9t_x^2(-t_y + 2(8\alpha + 2\theta v_0 + \theta v_h + \theta v_l)))) = -\frac{3t_y}{4} + \theta v_0$ and $CS^{TS} - CS^{ST} = (\frac{1}{576(\alpha-t_x)^2 t_x}(-720t_x^4 + 3\alpha^2 t_y^2 - 4t_x^2(612\alpha^2 - 3t_y^2 - 4\theta^2 v_0^2 + 144\alpha\theta v_h - 4\theta^2 v_h^2 + 144\alpha\theta v_l + 8\theta^2 v_h v_l - 4\theta^2 v_l^2 + 8\theta v_0(54\alpha + \theta v_h - \theta v_l) - 6t_y(30\alpha - \theta v_0 + \theta v_h - \theta v_l)) + 6\alpha t_x(-60\alpha t_y - t_y^2 + 48\alpha(3\alpha + 3\theta v_0 + \theta v_h + \theta v_l)) + 72t_x^3(-5t_y + 4(8\alpha + 3\theta v_0 + \theta v_h + \theta v_l)))) - (\frac{1}{576(\alpha-t_x)^2 t_x}(-720t_x^4 + 3\alpha^2 t_y^2 - 4t_x^2(612\alpha^2 - 3t_y^2 - 4\theta^2 v_0^2 + 144\alpha\theta v_h - 4\theta^2 v_h^2 + 144\alpha\theta v_l + 8\theta^2 v_h v_l - 4\theta^2 v_l^2 + 8\theta v_0(54\alpha - \theta v_h + \theta v_l) - 6t_y(30\alpha - \theta v_0 - \theta v_h + \theta v_l)) + 6\alpha t_x(-60\alpha t_y - t_y^2 + 48\alpha(3\alpha + 3\theta v_0 + \theta v_h + \theta v_l)) + 72t_x^3(-5t_y + 4(8\alpha + 3\theta v_0 + \theta v_h + \theta v_l)))) = -\frac{\theta t_x(-3t_y+4\theta v_0)(v_h-v_l)}{36(\alpha-t_x)^2}$.

(a) $CS^{TT} > CS^{SS}$ if and only if $t_y < \frac{4}{3}\theta v_0$.

(b) $CS^{ST} > CS^{TS}$ if and only if $t_y < \frac{4}{3}\theta v_0$.

(c) When $t_y < \frac{4}{3}\theta v_0$, compare CS^{TT} and CS^{ST}, we have $CS^{TT} - CS^{ST} = \frac{1}{576(\alpha-t_x)^2 t_x}(-3\alpha^2 t_y^2 - 72t_x^3(3t_y - 4\theta v_0) + 6\alpha t_x(-36\alpha t_y + t_y^2 + 48\alpha\theta v_0) + 4t_x^2(-3t_y^2 - 4\theta v_0(36\alpha + \theta v_0 + 2\theta v_d) + 6t_y(18\alpha + \theta v_0 + \theta v_d)))$. This difference is a linearly decreasing function of v_d. Therefore, if $v_d < \frac{(-3\alpha^2 t_y^2 - 72t_x^3(3t_y-4\theta v_0) + 6\alpha t_x(-36\alpha t_y + t_y^2 + 48\alpha\theta v_0) + 4t_x^2(-3t_y^2 + 6t_y(18\alpha+\theta v_0) - 4\theta v_0(36\alpha+\theta v_0)))}{(8\theta t_x^2(-3t_y+4\theta v_0))}$ (i.e., $v_d < v_d^1$), then $CS^{TT} > CS^{ST}$, otherwise $CS^{TT} \le CS^{ST}$.

(d) When $t_y > \frac{4}{3}\theta v_0$, compare CS^{SS} and CS^{TS}, we have $CS^{SS} - CS^{TS} = \frac{1}{576(\alpha-t_x)^2 t_x}(-3\alpha^2 t_y^2 + 72t_x^3(3t_y - 4\theta v_0) + 6\alpha t_x(36\alpha t_y + t_y^2 - 48\alpha\theta v_0) - 4t_x^2(3t_y^2 + 6t_y(18\alpha - \theta v_0 + \theta v_d) + 4\theta v_0(-36\alpha + \theta v_0 - 2\theta v_d)))$. This difference is a linearly decreasing function of v_d. Therefore, if $v_d < \frac{(3\alpha^2 t_y^2 - 72t_x^3(3t_y-4\theta v_0) - 6\alpha t_x(36\alpha t_y + t_y^2 - 48\alpha\theta v_0) + 4t_x^2(3t_y^2 + 6t_y(18\alpha-\theta v_0) + 4\theta v_0(-36\alpha+\theta v_0)))}{(8\theta t_x^2(-3t_y+4\theta v_0))}$ (i.e., $v_d < v_d^2$), then $CS^{SS} > CS^{TS}$, otherwise $CS^{SS} \le CS^{TS}$.

The proof of Proposition A2 is completed. □

Appendix B.11.7. Proof of Proposition A3

According to the values of the social welfare in the four scenarios, we have $SW^{TT} - SW^{SS} = -\frac{3t_y}{4} + \theta v_0$ and $SW^{TS} - SW^{ST} = -\frac{\theta t_x(-3t_y+4\theta v_0)(v_h-v_l)}{36(\alpha-t_x)^2}$.

(a) $SW^{TT} > SW^{SS}$ if and only if $t_y < \frac{4}{3}\theta v_0$.

(b) $SW^{ST} > SW^{TS}$ if and only if $t_y < \frac{4}{3}\theta v_0$.

(c) When $t_y < \frac{4}{3}\theta v_0$, compare SW^{TT} and SW^{ST}, we have $SW^{TT} - SW^{ST} = \frac{1}{576(\alpha-t_x)^2 t_x}(-3\alpha^2 t_y^2 - 72t_x^3(3t_y - 4\theta v_0) + 8t_x^2(-6t_y^2 - 2\theta v_0(36\alpha + 5\theta v_0 + 10\theta v_d) + 3t_y(18\alpha + 5\theta v_0 + 5\theta v_d)) + 2\alpha t_x(21t_y^2 + 16\theta v_0(9\alpha + 2\theta v_0 + 4\theta v_d) - 12t_y(9\alpha + 4\theta v_0 + 4\theta v_d)))$. This difference is a linearly decreasing function of v_d. Therefore, if $v_d < \frac{-3\alpha^2 t_y^2 - 72t_x^3(3t_y-4\theta v_0) + 2\alpha t_x(21t_y^2 + 16\theta v_0(9\alpha+2\theta v_0) - 12t_y(9\alpha+4\theta v_0)) + 8t_x^2(-6t_y^2 + 3t_y(18\alpha+5\theta v_0) - 2\theta v_0(36\alpha+5\theta v_0))}{8\theta t_x(-4\alpha+5t_x)(-3t_y+4\theta v_0)}$ (i.e., $v_d < v_d^3$), then $SW^{TT} > SW^{ST}$, otherwise $SW^{TT} \le SW^{ST}$.

(d) When $t_y > \frac{4}{3}\theta v_0$, compare SW^{SS} and SW^{TS}, we have $SW^{SS} - SW^{TS} = \frac{1}{576(\alpha-t_x)^2 t_x}(-3\alpha^2 t_y^2 + 72t_x^3(3t_y - 4\theta v_0) + 2\alpha t_x(21t_y^2 + 12t_y(9\alpha - 4\theta v_0 + 4\theta v_h - 4\theta v_l) + 16\theta v_0(-9\alpha + 2\theta v_0 - 4\theta v_h +$

$4\theta v_l)) - 8t_x^2(6t_y^2 + 3t_y(18\alpha - 5\theta v_0 + 5\theta v_h - 5\theta v_l) + 2\theta v_0(-36\alpha + 5\theta v_0 - 10\theta v_h + 10\theta v_l)))$. This difference is a linearly decreasing function of v_d. Therefore, $SW^{SS} > SW^{TS}$ is equivalent to $v_d < \frac{3\alpha^2 t_y^2 - 72t_x^3(3t_y - 4\theta v_0) - 2\alpha t_x(21t_y^2 + 12t_y(9\alpha - 4\theta v_0) + 16\theta v_0(-9\alpha + 2\theta v_0)) + 8t_x^2(6t_y^2 + 3t_y(18\alpha - 5\theta v_0) + 2\theta v_0(-36\alpha + 5\theta v_0))}{8\theta t_x(-4\alpha + 5t_x)(-3t_y + 4\theta v_0)}$ (i.e., $v_d < v_d^4$).

The proof of Proposition A3 is completed. □

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