materials proceedings

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Proceeding Paper

Cattaneo-Christov Heat Flux Model Effect on Magnetized Maxwell Nanofluid Flow over a Stretching Surface †

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† Presented at the 6th Conference on Emerging Materials and Processes (CEMP 2023), Islamabad, Pakistan, 22–23 November 2023.

Abstract: This study investigates the flow of a magnetohydrodynamic (MHD) Maxwell fluid over a stretching sheet using a Darcy-Forchheimer (DF) model. We employ numerical analysis with a copper (Cu) nanofluid suspended in water, considering Cattaneo–Christov heat flow, viscous dissipation, and joule heating. Nonlinear ordinary differential equations (ODEs) are solved using the bvp4c method in Matlab and we examine the normalized shear stress, temperature profile, and heat flux rate. Our findings reveal insights for practical applications, showing how parameters such as the relaxation Prandtl number, magnetic parameter, Eckert number parameter, and radiation parameter impact system behaviour.

Keywords: Darcy-Forchheimer model; Maxwell fluid; MHD; nanofluid; Cattaneo–Christov heat flux model; bvp4c

Citation: Jameel, M.; Shah, Z.; Khan, M.S.; Ullah, N. Cattaneo-Christov Heat Flux Model Effect on Magnetized Maxwell Nanofluid Flow over a Stretching Surface. *Mater. Proc.* **2024**, *17*, 12. https://doi.org/10.3390/materproc2024017012

Academic Editors: Sofia Javed, Waheed Miran, Erum Pervaiz and Iftikhar Ahmad

Published: 11 April 2024

1. Introduction

The study of magnetohydrodynamics Maxwell flow over a stretching sheet has been significantly influenced by the Darcy-Forchheimer model. The Darcy-Forchheimer model has been investigated by a large number of researchers as a potential tool for analysing the flow patterns shown by a variety of fluids in a variety of configurations [1]. Studying magnetohydrodynamics Maxwell flow over a stretching sheet is a major application of the Darcy-Forchheimer model. This is one of the most important applications of the model. Researchers Muskat, Brinkman, and Forchheimer [2] were among the pioneers in the field who first reported applying the Darcy-Forchheimer model to the investigation of nonlinear systems. The magneto-Darcy-Forchheimer flow of Maxwell fluid over a convectively heated surface was one of the topics that Seddeek researched. Seddeek looked into how the flow characteristics of the Maxwell fluid were affected when magnetic fields and convective heat transfer were taken into consideration. Both the Cattaneo–Christov model and the Darcy-Forchheimer model were taken into consideration as Pal and Mondal [3] investigated the flow of Oldroyd-B fluid with heat flux using the Cattaneo–Christov model. Jha et al. came up with a nonlinear Brinkman–Forchheimer extended Darcy flow model, which further improved the comprehension of how fluids behave when flowing through porous media [4]. In addition to this, Sadiq and Hayat looked at the Darcy-Forchheimer flow of a magneto Maxwell liquid that was surrounded by a sheet that was heated convectively. Because it offers a more accurate picture of how flow behaves in porous media, the Darcy-Forchheimer model has seen extensive usage in the research of magnetohydrodynamics Maxwell flow over a stretching sheet [5]. Because it takes into account both viscous and inertial forces in porous media, the Darcy-Forchheimer model is an excellent choice for analysing the behaviour of fluids in circumstances in which both forces play a substantial role, such as MHD Maxwell flow over a stretching sheet [6,7]. Studies have demonstrated that the Darcy-Forchheimer model provides a better understanding of the flow behaviour in porous media compared to the standard Darcian model. This is because the Darcy-Forchheimer model takes into account the physics of flow in porous media. The Darcy-Forchheimer model contains additional elements to account for the inertial effects in the flow, which become increasingly relevant as the flow velocity increases. These inertial effects in the flow become increasingly important as the flow velocity increases. These additional words allow for a more realistic depiction of the flow behaviour, particularly in scenarios in which the flow is influenced by external forces such as heat transfer or magnetic fields. This is especially useful when describing flows that are affected by such factors [8]. The implementation of the Darcy-Forchheimer model in the investigation of MHD Maxwell flow over a stretching sheet has made a substantial contribution to our improved comprehension of the fluid dynamics involved in these systems. Researchers have been able to obtain insights into how the behaviour of the flow is affected by elements such as magnetic fields, convective heat transfer, and varying viscosity by using this model.

2. Mathematical Formulation

In this study, magnetohydrodynamics (MHD) is used to explore the flow of a non-Newtonian Maxwell nanofluid across a medium that has a Darcy-Forchheimer porous structure. It is presumed that the flow is laminar and two-dimensional (2D), which indicates that it takes place in two dimensions and that the different layers of fluid do not mix with one another (Figure 1). It is assumed that the thermal conductivity of the nanofluid changes as the temperature does, because the nanofluid is composed of nanoparticles of titanium dioxide (TiO_2) that are suspended in water. It is presumed that the flow is radiative as well, which indicates that heat is being transmitted not just by conduction and convection but also through radiation. It is assumed that the surface that is being stretched has a Darcy porosity, which indicates that the flow through the surface is controlled by Darcy's law. The Cattaneo-Christov model is used to represent the heat flux; this model accounts for the fact that the speed at which heat travels has a finite value. It is also taken into account that the flow loses energy owing to friction, which is what is meant by the term "viscous dissipation". Because of these assumptions and limitations, it is possible to create a simplified mathematical model that can be applied to the investigation of the flow of non-Newtonian Maxwell nanofluids inside Darcy-Forchheimer porous structures using MHD.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right)\right) = \mu_{nf}\frac{\partial^2 u}{\partial y^2} - \sigma_{nf}B_0^2 Sin^2\gamma \tag{2}$$

$$(\rho C_p)_{nf}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \lambda_2\Omega_E\right) = \left(k_{nf} + \frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} + \sigma_{nf}B_0^2 u^2, \tag{3}$$

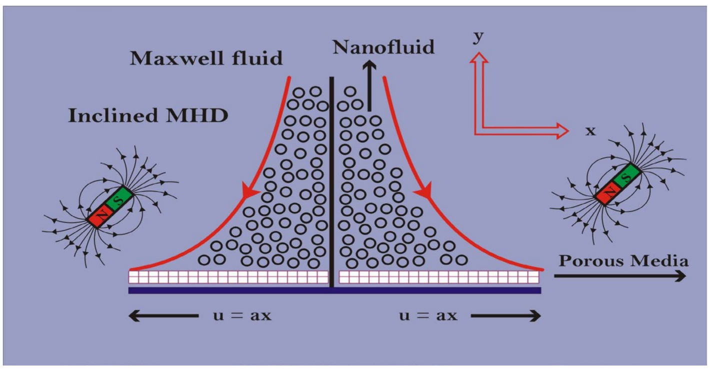

Figure 1. Under the above assumptions, the continuity, momentum, and energy of nanoparticles equations based on a Darcy flow model are as follows: for numerous values of δ_m.

The suitable boundary conditions are

$$u = u_w = ax,\ \ v = 0,\ \ T = T_w\ \ at\ \ y = 0,\ \ u \to 0,\ \ T \to T_\infty,\ \ as\ \ y \to \infty \tag{4}$$

where μ_{nf}, ρ_{nf}, $(\rho C_p)_{nf}$, k_{nf}, and σ_{nf} are demonstrated below and the thermo physical features of the considered nanofluid flow are given in Table 1.

$$\left.\begin{array}{c} D_1 = \frac{\mu_{nf}}{\mu_f} = (1-\phi)^{-2.5} \\ D_2 = \frac{\rho_{nf}}{\rho_f} = (1-\phi) + \phi\frac{\rho_s}{\rho_f} \\ D_3 = \frac{(\rho c_p)_{nf}}{(\rho c_p)_f} = (1-\phi) + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f} \\ D_4 = \frac{k_{nf}}{k_f} = \left[\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+2\phi(k_f-k_s)}\right] \\ D_5 = \frac{\sigma_{nf}}{\sigma_f} = \left[\frac{\sigma_s+2\sigma_f-2\phi(\sigma_f-\sigma_s)}{\sigma_s+2\sigma_f+2\phi(\sigma_f-\sigma_s)}\right] \end{array}\right\} \tag{5}$$

Table 1. Features of nanofluid and base fluid.

Nanoliquid Physical Properties	C_p(J/kgK)	ρ(kg/m^3)	k(W/mK)	σ(Sm^{-1})
Base Liquid (Water)	4179	997.1	0.613	0.05
Copper	385	8933	400	5.97×10^7
Alumina	765	3970	40	------

The appropriate similarity transformation:

$$u = axf'(\eta),\ v = -\sqrt{a\nu_f}f(\eta),\ \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty},\ \eta = \sqrt{\frac{a}{\nu_f}}y \tag{6}$$

With the help of Equation (6), the momentum and heat equation, along with boundary conditions, transform to a non-dimensional form, as follows:

$$D_1 f''' + D_2\left(ff'' - f'^2 + \delta_m\left(2ff'f'' - f^2 f'''\right)\right) - \left(D_5 M Sin^2\gamma\right)f' = 0 \tag{7}$$

$$\begin{array}{c}(D_4 + Rd)\theta'' + \\ EcPr\left(D_1 f''^2 + \delta_m\left(2ff'^2 f'' - f^2 f''^2\right)\right) + D_5 MEcPrf'^2 = 0\end{array} \tag{8}$$

$$f'(0) = 1,\ \ f(0) = 0,\ \ \theta(0) = 1\ f \to 0,\ \ \theta \to 0,\ \ as\ \ \eta \to \infty \tag{9}$$

The Maxwell fluid parameter, magnetic parameter, porosity parameter, local inertia coefficient, Prandtl number, Eckert number, and thermal relaxation parameter are all important factors in the equation.

3. Results and Discussions

The primary objective of this study is to examine the phenomena of flow and heat transfer in the context of stretching sheets, specifically employing a nanofluid composed of copper nanoparticles. The bvp4c technique is employed to transform nonlinear partial differential equations into nonlinear ordinary differential equations, enabling the computation of numerical solutions and facilitating a thorough analysis of the system's dynamics. Figures 2–5 contain graphical depictions and analytical observations pertaining to the behaviour of the system, facilitating the derivation of well-informed conclusions and enabling subsequent investigation. The findings presented in Figure 2 demonstrate that an increase in the Maxwell fluid parameter leads to a corresponding increase in fluid elasticity. This heightened elasticity, in turn, gives rise to higher attractive forces among nanoparticles. Consequently, the formation of particle chains occurs, which hinders the flow of the fluid and subsequently reduces its radial velocity. The influence of the magnetic parameter, represented as M, on fluid velocity is seen in Figure 3. This effect can lead to the emergence of magnetic chains or clusters, particularly in fluids containing magnetic nanoparticles. The study demonstrates a link between fluid temperature and the magnetic parameter M, as depicted in Figure 4. The rise in fluid temperature is a consequence of the Joule heating effect, resulting from an increase in the magnetic parameter. This increase leads to oscillations in magnetic nanoparticles. This phenomenon results in the production of heat and the dispersion of thermal energy. The establishment of this relationship holds significant importance in the examination and manipulation of heat transfer characteristics within magnetic fluid applications. Figure 5 illustrates the correlation between the fluid temperature and the Eckert number parameter, Ec. The outcomes demonstrate that with an upsurge in the Eckert number parameter, there is a corresponding upsurge in the temperature of the fluid. The Eckert number parameter denotes the ratio of the fluid's kinetic energy flux to its thermal energy. When the Eckert number parameter is higher, it indicates a greater conversion of kinetic energy into thermal energy within the fluid. This conversion process is known as viscous dissipation, where the friction between the fluid and its surroundings dissipates the fluid's kinetic energy into heat. A higher value of the Eckert number parameter results in a greater conversion of kinetic energy into thermal energy, consequently leading to an elevated temperature of the fluid.

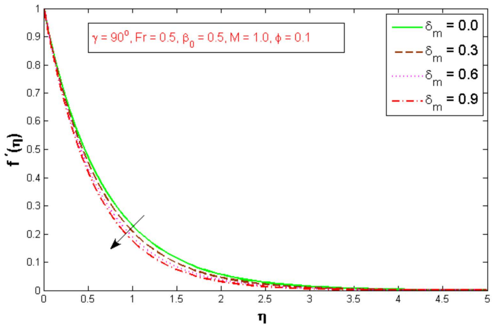

Figure 2. Moderation of $f'(\eta)$ for numerous values of δ_m.

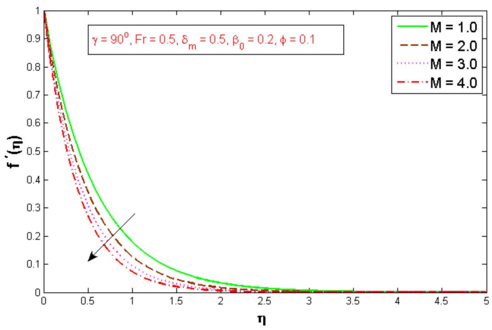

Figure 3. Moderation of $f'(\eta)$ for numerous values of M.

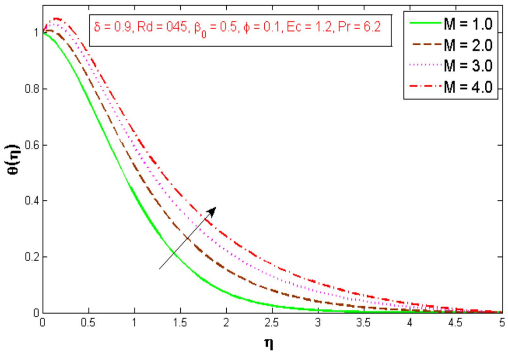

Figure 4. Moderation of $\theta(\eta)$ for numerous values of M.

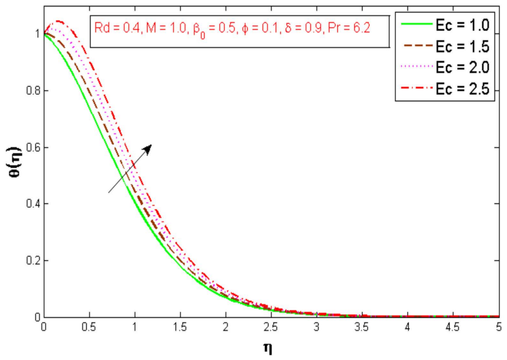

Figure 5. Moderation of $\theta(\eta)$ for numerous values of Ec.

4. Conclusions

This study examines the flow and heat transfer characteristics of a copper nanofluid over a stretching sheet using the bvp4c technique. The Maxwell fluid parameter, magnetic parameter, and Eckert number parameter significantly impact the system's dynamics. An increase in the Maxwell fluid parameter leads to fluid elasticity, particle chains, and reduced radial velocity. The magnetic parameter creates magnetic chains and clusters, affecting fluid temperature and thermal energy production. The Eckert number parameter indicates greater viscous dissipation, leading to elevated fluid temperature. These findings can be used to design and optimize systems for various applications.

Author Contributions: Conceptualization, M.J. and M.S.K.; methodology, Z.S.; software, M.J.; validation, Z.S., N.U. and M.J.; formal analysis, M.J.; investigation, N.U.; resources, Z.S.; data curation, M.S.K.; writing—original draft preparation, M.J.; writing—review and editing, Z.S.; visualization, N.U.; supervision, Z.S.; project administration, Z.S.; funding acquisition, M.J. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data that support the study's findings are available upon reasonable request from the corresponding author.

Conflicts of Interest: The authors declare no conflict of interest.

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