

Article

Aircraft Structural Design and Life-Cycle Assessment through Digital Twins

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Abstract: Numerical modeling tools are essential in aircraft structural design, yet they face challenges in accurately reflecting real-world behavior due to factors like material properties scatter and manufacturing-induced deviations. This article addresses the potential impact of digital twins on overcoming these limitations and enhancing model reliability through advanced updating techniques based on machine learning. Digital twins, which are virtual replicas of physical systems, offer a promising solution by integrating sensor data, operational inputs, and historical records. Machine learning techniques enable the calibration and validation of models, combining experimental inputs with simulations through continuous updating processes that refine digital twins, improving their accuracy in predicting structural behavior and performance throughout an aircraft's life cycle. These refined models enable real-time monitoring and precise damage assessment, supporting decision making in diverse contexts. By integrating sensor data and updating techniques, digital twins contribute to improved design and maintenance operations by providing valuable insights into structural health, safety, and reliability. Ultimately, this approach leads to more efficient and safer aviation operations, demonstrating the potential of digital twins to revolutionize aircraft structural analysis and design. This article explores various advancements and methodologies applicable to structural assessment, leveraging machine learning tools. These include the utilization of physics-informed neural networks, which enable the handling of diverse uncertainties. Such approaches empower a more informed and adaptive strategy, contributing to the assurance of structural integrity and safety in aircraft structures throughout their operational life.

Keywords: digital twins; finite-element models; damage-tolerant design; structural design; model updating; data-driven design



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1. Introduction

Over the past few decades, fatigue and damage-tolerant designs have become widespread in many applications, with recent trends showcasing innovative approaches applied to airframes. Traditionally, fatigue design focuses on ensuring structural integrity under cyclic loading conditions, but strategies now integrate the material's behavior, structural optimization, and sophisticated analysis tools to enhance an airframe's resistance to fatigue [1,2]. The incorporation of life-cycle analysis of structural residual strength enables proactive maintenance strategies and early identification of potential issues [3]. These evolving trends collectively contribute to the development of resilient airframes that not only meet

stringent fatigue and damage-tolerance requirements but also prioritize durability, reduced weight, and enhanced overall performance [4].

The digital evolution in aircraft design and operation has exerted a profound influence on critical aspects of airworthiness, particularly in the realms of damage-tolerant design and adherence to the Limit of Validity (LOV) rule [5]. The integration of advanced digital tools, notably sophisticated simulation and analysis techniques, has bolstered the aviation industry’s capacity to develop and implement damage-tolerant designs. This transformative shift empowers engineers and decision makers to conduct improved assessments of structural responses to diverse stress scenarios, enabling the creation of aircraft that can resiliently manage potential damage while upholding stringent airworthiness standards [6]. In this context of airworthiness, the impact of the LOV rule has been substantial. Digitalization has ushered in a more dynamic and adaptive approach to LOV considerations, aligning them more closely with the real-time health and integrity of essential aircraft components. The continuous stream of data from embedded sensors within the aircraft structure allows for ongoing monitoring, enabling precise and responsive adjustments to LOV parameters. This real-time adaptability ensures that operational limits are not only accurately defined but also continually refined based on the actual structural conditions of the aircraft, thereby optimizing safety margins and contributing to sustained airworthiness throughout the aircraft’s operational life [6].

As aircraft continue to push the boundaries of technological advancements, the need for multi-fidelity reduced-order surrogate modeling becomes critical. The exploitation of reduced-order models (ROMs) can offer efficient approximations for complex structural designs. In damage-tolerant design, these models are instrumental in predicting and assessing structural responses to potential damage scenarios, facilitating design decisions and proactive mitigation strategies. Additionally, ROMs streamline the analysis of critical parameters, aiding in the rapid assessment and optimization of aircraft structures to meet safety standards [7]. Figure 1 illustrates an example of the application of ROMs in structural assessment, utilizing correlation analysis and the design of experiments to evaluate a range of material properties, including those influenced by environmental conditions. This study generates valuable response surfaces that can seamlessly integrate into simulation models. The integration of artificial intelligence (AI) and machine learning (ML) in these approaches has further revolutionized aircraft design, offering capabilities ranging from automated generative design to rapid data analysis for identifying optimization opportunities [8,9]. The use of AI tools is now pervasive in many areas. Just to give some examples in transportation, deep learning—a subset of machine learning—is used in fields such as traffic control [10] and traffic accidents [11].

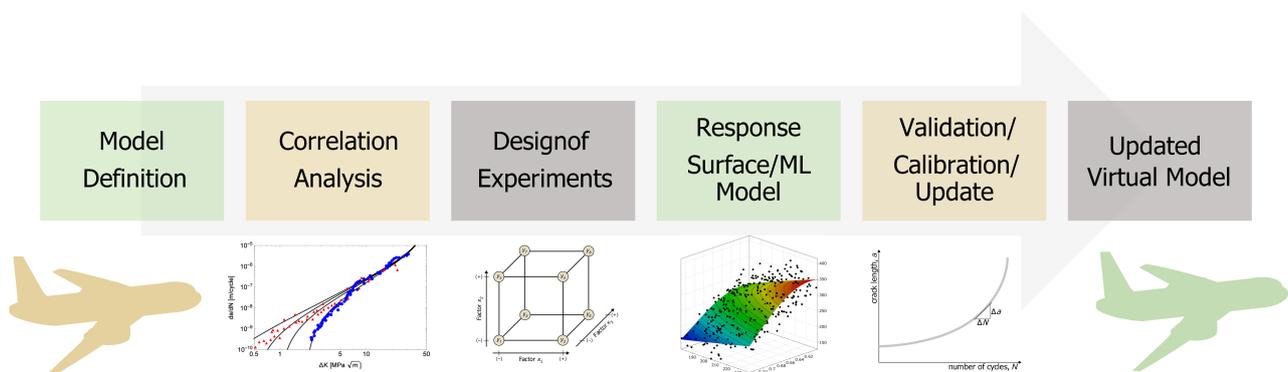


Figure 1. Virtual models from design for operational assessment.

More recently, a transformative approach has emerged in the field of life-cycle simulation for aircraft structures through the integration of digital-twin technology and model updating techniques, as highlighted by Tuegel [12]. The digital twin of a physical object is its digital representation, which (i) accompanies its physical counterpart from conception

to disposal; (ii) is updated in real time via sensors in the physical object; and (iii) informs all decision-making processes concerning the physical object using analysis, simulation, machine learning, and other AI techniques.

The origin of the notion of “digital twin” was briefly discussed by IBM [13]. A quick search on the SCOPUS bibliographic database shows that the number of articles with ‘digital twin’ in their titles shows a strong increase in recent years, as shown in Figure 2. Although the data shown in this figure start in 2015, there are several occurrences with earlier dates, but they do not correspond to the current notion of digital twins.

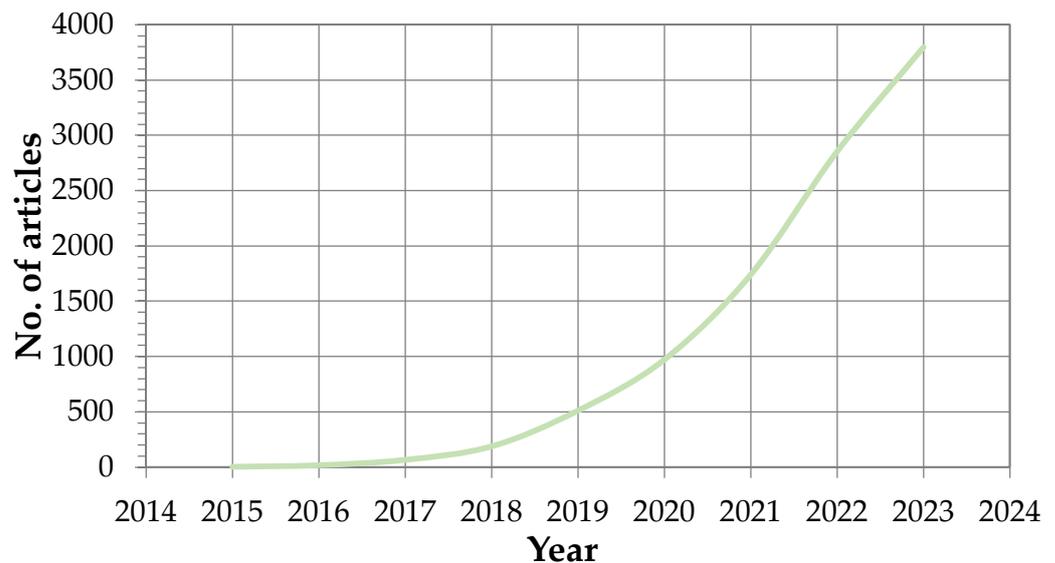


Figure 2. Number of papers with “digital twin” in their titles. The SCOPUS database was assessed on 21 February 2024.

Since these digital twins are a virtual representation of a physical asset, process, or system, they rely on real-time data integration from sensors, simulations, and other sources, as succinctly stated by Jones et al. [14]. This groundbreaking technology, when applied to aircraft structures, empowers engineers and decision makers to comprehensively and continuously monitor and analyze structural behavior throughout the entire life cycle [15]. Diverse approaches have been explored to harness the data available from sensorized structures, capitalizing on the insights they provide. Algorithms and analyses transform these data into valuable information crucial for comprehensive structural assessment. This integration not only facilitates real-time monitoring but also enhances the predictive capabilities of the digital models, allowing for a thorough understanding of structural behavior under varying conditions. The utilization of sensor data, therefore, becomes an integral component in the ongoing evolution of structural assessment practices, enabling a more informed, structural digital-twin approach to ensure the resilience, safety, and optimal performance of critical structures, as proposed in [16]. These digital-twin concepts were extensively discussed for the aeronautical sector in a position paper of the American Institute of Aeronautics and Astronautics (AIAA) and the Aerospace Industries Association (AIA) [17]. The essence of the digital-twin concept is elucidated in Figure 3, showcasing diverse applications such as failure analysis, performance validation, and design optimization. Leveraging the ability of digital twins to replicate the behavior and performance of physical aircraft, these virtual counterparts offer a means to understand structural responses under various operational and environmental conditions, presenting significant advancements for the aerospace sector across multiple dimensions, as noted in [18].

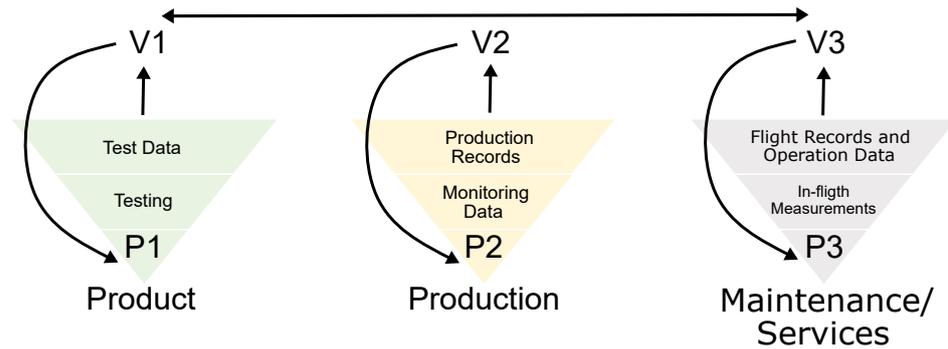


Figure 3. Illustration of the life-cycle interplay between the physical (P) and virtual (V) assets. P1, P2, and P3 represent the physical assets, whereas V1, V2, and V3 represent the corresponding instances of the virtual assets (adapted from the AIAA and AIA position paper [17]).

In tandem with digital advancements, collaborative tools and readily accessible cloud-based solutions have facilitated global teamwork in aircraft design and development [19]. Additionally, the application of advanced simulations, including computational fluid dynamics (CFD) and finite-element analysis (FEA), has evolved to offer precise and comprehensive multi-physical assessments, covering aerodynamics, structural integrity, and thermal performance. Collectively, these advances have inaugurated a new era in aircraft design, marked by heightened efficiency, sustainability, and innovation throughout the aerospace product life cycle [20].

Capitalizing on the diverse artificial intelligence tools that have emerged in the last decade, including machine learning, presents numerous opportunities to enhance the fields of CFD and FEA. In CFD, the integration of ML algorithms improves the accuracy and speed of fluid dynamics simulations, enabling more precise modeling of complex flow phenomena [21]. Similarly, in FEA, AI tools, particularly machine learning algorithms, offer the potential to greatly improve prediction accuracy by capturing complex relationships between input parameters and structural responses. These techniques prove invaluable for surrogate modeling, calibration, and updating of finite-element models [22,23], enhancing precision in predicting complex non-linearities or uncertainties. Furthermore, machine learning's capability to identify and extract patterns from empirical data allows for informed adjustments to model parameters, resulting in refined models that closely align with real-world behaviors. The application of machine learning is particularly promising in addressing challenges related to intricate geometries and varied material properties arising from the complexity and stochasticity of manufacturing and assembly processes.

Taking into account the need to improve aging aircraft management and ensure the safety of older aircraft while optimizing operational costs, this article primarily aims to discuss the potential benefits of integrating digital twins with model updating techniques within the context of aircraft structure life-cycle simulation. Such integration holds the disruptive potential to improve current design philosophies in the civil aircraft sector, contributing to the expanding body of knowledge in aircraft structural simulation. By inspiring further exploration and adoption of digital twin technologies in aerospace engineering, this approach becomes a tool for addressing challenges in simulating and defining the life cycle of aircraft structures and maintenance intervals. The synergy of information from design models and sensor data presents high-impact potential, significantly improving design, analysis, and maintenance processes. This, in turn, enhances overall safety, performance, and cost-effectiveness in aircraft operations, contributing to greener aviation [24].

The main objective of this article is to present new approaches that can bring together digital twins and model updating techniques based on AI tools to take advantage of aircraft structure simulations and sensor data. Model updating refers to refining computer models with actual performance data. This article will highlight how this combination can improve the way aircraft are designed, predict maintenance needs from a structural point of view, and manage an aircraft's lifespan for optimal performance. Finally, the goal is to present

innovative frameworks for digital-twin employment, considering the latest advances in AI tools.

2. Model Updating Techniques

In contemporary engineering design practices, finite-element analysis stands out as a widely employed computational technique for structural design and predicting responses under diverse loading conditions [25]. However, discrepancies between computed outcomes and experimental measurements can limit the model's suitability for precise assessments of structural behavior. Various strategies for model calibration have been developed across different fields, such as those proposed in [26] for composite fuselages and those discussed in [27] for wings.

Model updating techniques involve numerical procedures aimed at incrementally refining a simulation model, often a finite-element model, by adjusting inherent parameters and assumptions. This refinement results in a gradual convergence of the model's behavior, including static and dynamic structural responses, to that of the current physical structure under analysis [28]. In several areas of engineering, model updating has found application, e.g., trains [29], bridges [30], and power transformers [31], among others. In the context of structural damage assessment, methodologies for updating numerical models entail minimizing residuals across relevant structural characteristics by comparing predictions from the numerical model with observed responses from the real structure. This process is treated as an optimization problem, where the objective function quantifies the disparity between the finite-element model and observed measurements. The optimization focus is on refining parameters intrinsic to the numerical model, concurrently serving as designated design variables. Numerous optimization strategies for numerical model updating have been extensively studied for diverse applications [28,32–34].

As outlined in [35], model updating methodologies fall into two main categories: (i) direct methods (typically non-iterative); and (ii) indirect methods (iterative). Iterative methods are more commonly utilized due to their capacity to offer a broader spectrum of parameters for updating. Additionally, they possess the ability to overcome the limitations encountered by direct methods [36].

Among the examples of model updating techniques presented in Figure 4 and proposed by Alkayem et al. [35], computational “intelligence” techniques have been extensively explored in recent years. This group of techniques includes the Nelder–Mead simplex method; the sequential quadratic programming technique; fuzzy sets; simulated annealing; evolutionary computation/algorithms; machine learning techniques; and hybrid optimization methods [32,37,38]. Evolutionary algorithms constitute a class of computational techniques inspired by the principles of biological evolution and natural selection. Rooted in the field of optimization, these algorithms replicate the iterative processes of selection, reproduction, and mutation observed in the natural world to solve or optimize complex problems across diverse domains. By representing potential solutions as individuals within a population and evaluating their fitness based on a defined objective function, evolutionary algorithms iteratively guide the population toward improved solutions over generations. Through the recombination of genetic information and the introduction of random variations, these algorithms explore solution spaces comprehensively, searching for optimal or near-optimal configurations [39].

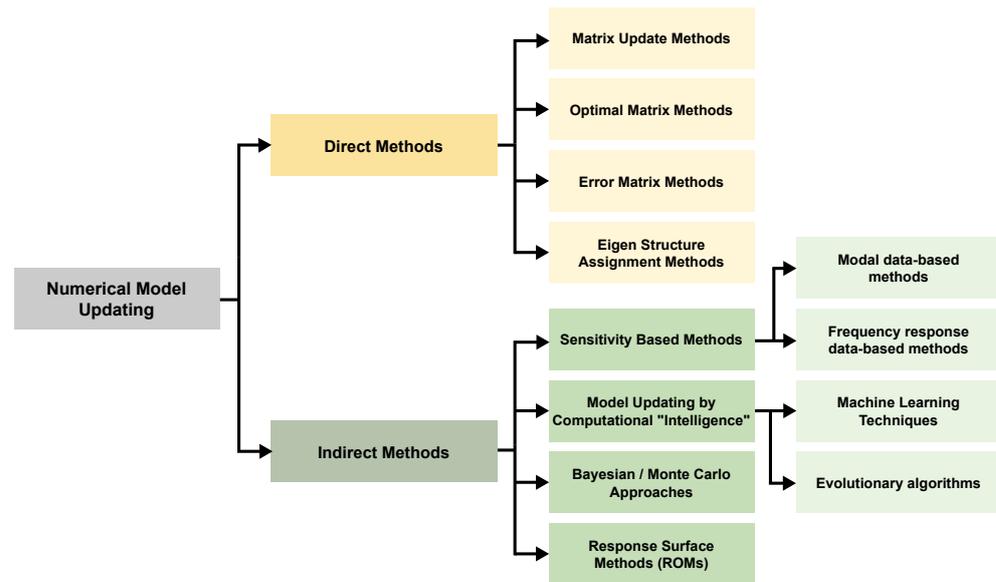


Figure 4. Examples of model updating techniques applicable to finite-element models (adapted from [35]).

Among the computational “intelligence” techniques, machine learning techniques have emerged as very promising tools. These techniques offer novel avenues for enhancing the accuracy and efficiency of this process. Machine learning techniques are typically subdivided into three main groups: (i) supervised learning; (ii) unsupervised learning; and (iii) reinforcement learning. The main difference between supervised and unsupervised learning is whether the model knows what the outputs will be. In reinforced learning the model learns how to respond through rewards or punishments (positive or negative reinforcements) [40].

Harnessing the power of machine learning algorithms, such as convolutional neural networks (CNNs), graph neural networks (GNNs), recurrent neural networks (RNNs), and physics-informed neural networks (PINNs), among others, enables the identification of intricate complex relationships between the numerical model and measured data. This enables the identification of patterns, correlations, and complex dependencies that might be challenging to discern using traditional methods. These machine learning approaches facilitate the development of data-driven surrogate models capable of approximating the mapping between input parameters and structural responses. The surrogates can significantly expedite the iterative optimization process inherent in model updating by replacing computationally expensive finite-element simulations with rapid predictions from the trained machine learning models, as proposed by Ribeiro et al. [41]. Consequently, the fusion of machine learning techniques with finite-element model updating has a high potential impact, not only elevating the accuracy of predictions but also introducing an element of computational efficiency that is indispensable for real-world and complex engineering applications.

Ribeiro et al. [42] introduced an application of GNNs for investigating structural problems. GNNs are a class of neural networks designed to perform inference on data that are structured as graphs. They are particularly well suited to problems where the data are inherently graphical, such as social networks, molecular structures, and communication networks. GNNs operate by applying neural network transformations to the features associated with the nodes and edges of a graph, with the goal of learning a representation that captures the structural information of the input graph. In the seminal work in [43], a flexible framework for building graph networks (GNs) was introduced. This extended the previous work on GNNs by incorporating the global attributes of graphs, denoted as \mathbf{u} , into the input. These global features allow the GN to capture properties that are not limited to local node or edge characteristics. The model processes input data that include these

global features \mathbf{u} , along with node features \mathbf{X} and edge features \mathbf{E} . During the forward pass through the network, the GNN performs a series of computations that update the features of edges and nodes and aggregate information across the graph. These updates and aggregations are governed by update functions (φ) and aggregation functions (ρ), respectively. This refinement aggregates the data across the graph to produce new global attributes \mathbf{u}' , along with updated node and edge attributes, \mathbf{X}' and \mathbf{E}' , respectively. These updated features encapsulate the relational information inherent in the graph's structure. This was illustrated in the authors' GN block diagram, which is shown in Figure 5.

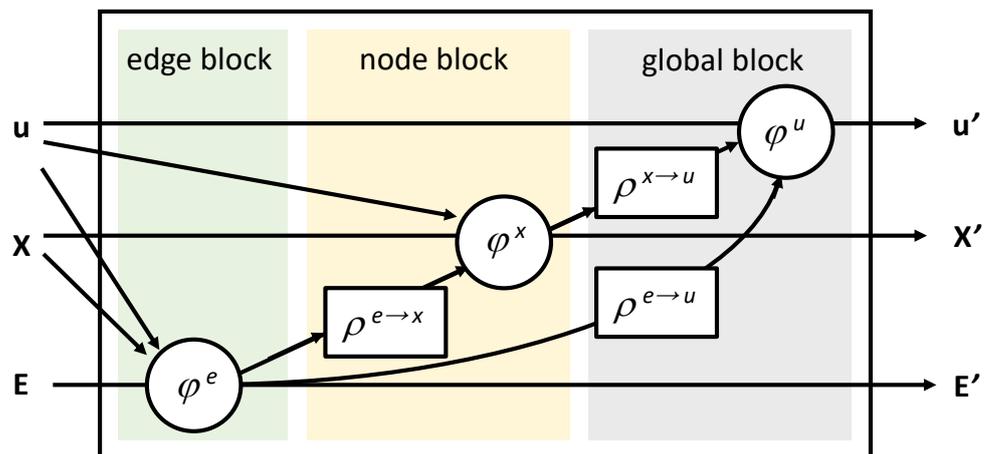


Figure 5. Graph network block diagram. \mathbf{u} —global features; \mathbf{X} —node features; \mathbf{E} —edge features; φ —update functions; ρ —aggregation functions; \mathbf{u}' —update global features; \mathbf{X}' —update node features; and \mathbf{E}' —update edge features.

An example of a structural situation in the aerospace industry is shown in Figure 6, which illustrates a comparison of ground truth (FEM results) and prediction (GNN results) for von Mises stress using the developed dataset. The results are shown through ground truth, prediction, absolute difference fields, and prediction vs. ground truth with coefficient of determination (R^2) analysis, considering the deviations of all nodes of the FEM mesh. The R^2 value provided illustrates the capability and quality of the methodology developed. Details of the development of the methodology can be found in [42,44] and are not repeated here for conciseness. This model demonstrates the method's precision in structural applications of aerospace interest. This innovative approach facilitates the instantaneous estimation of stress–strain fields, reducing the necessity for domain discretization and the solving of differential equations. As a result, it offers a notably low computational burden relative to traditional methods, demonstrating its potential for significantly reducing computational costs and streamlining the analysis and optimization of complex structural systems.

Model updating techniques are increasingly becoming a cornerstone in the aerospace industry, demonstrating their versatility and critical applicability across a wide array of projects. For example, Patelli et al. [45] explored and implemented two of the most recognized stochastic model updating techniques—sensitivity-based updating and Bayesian model updating—specifically focusing on the DLR AIRcraft MODel (AIRMOD) structure. This highlights not only the practical application of these methodologies but also their relevance in refining and enhancing aerospace structures for improved performance and reliability. Similarly, Zhao et al. [46] applied model updating techniques to a flying-wing aircraft, underscoring the broad utility of these methods in addressing the unique challenges presented by different aerospace projects. Furthermore, Goller et al. [47] extended the application of these techniques through two illustrative examples: an antenna reflector and a full-scale satellite model. These examples highlight the adaptability of model updating procedures in tackling the complexities inherent in aerospace engineering.

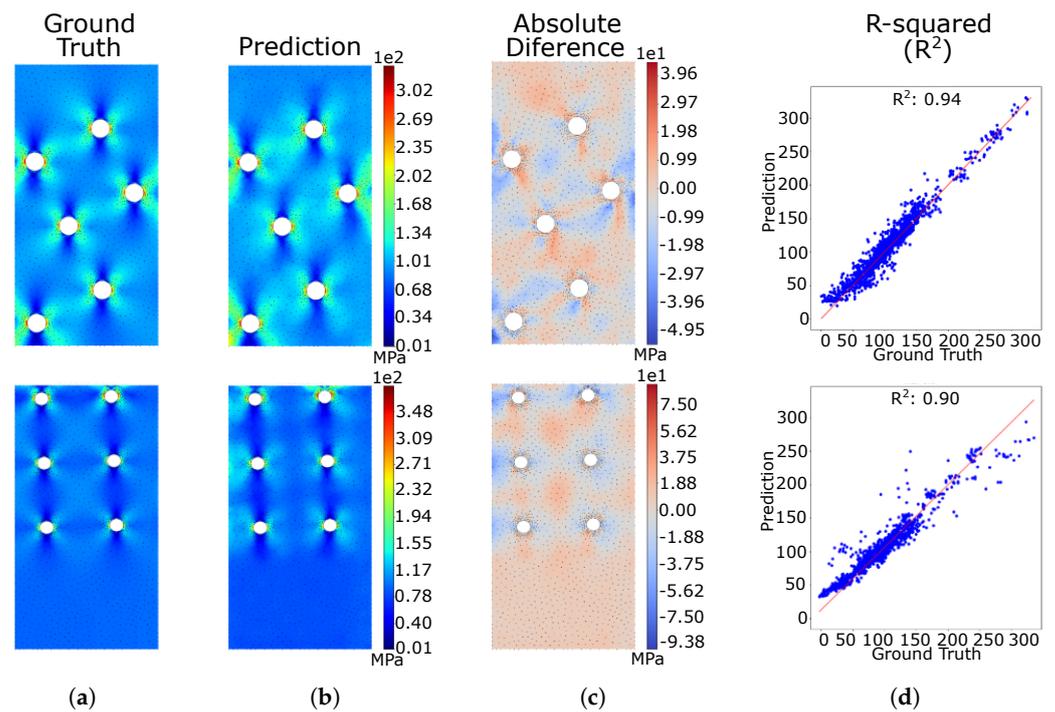


Figure 6. Comparison of ground truth (FEM results) and prediction (GNN results) for von Mises stress. The results are shown through ground truth, prediction, and absolute difference fields, as well as a ground truth vs. prediction plot, with an R^2 value provided. (a) von Mises stress field—ground truth (FEM results); (b) von Mises stress field—prediction (GNN results); (c) absolute difference field; (d) prediction vs. ground truth R^2 analysis.

3. Machine Learning Capabilities in Structural Design

Traditional structural design approaches have long relied on established engineering principles, analytical methods, and empirical data to develop, analyze, and design structures that meet performance and safety requirements. These models and simulations are employed to predict the behavior of materials and structures under various conditions, with a focus on ensuring stability, strength, and durability. In contrast, machine learning approaches represent a paradigm shift by leveraging data-driven techniques to extract patterns, learn from experiences, and make predictions without explicit programming. Looking at structural design, machine learning algorithms can analyze vast amounts of data, including material properties, environmental factors, and historical performance, to identify complex relationships and patterns that may not be apparent through conventional methods. This allows for more nuanced insights, optimization opportunities, and the potential to discover innovative design solutions. While traditional approaches offer well-established methodologies based on physical principles, machine learning introduces a complementary path based on the ability to uncover intricate patterns within large datasets, fostering a more adaptive and data-centric approach to structural design. Figure 7 illustrates the main difference between these two approaches, as proposed by Málaga-Chuquitaype [48]. The traditional design approach involves the careful consideration of all options. It uses transparent code with predictable output but eventually presents a limited scope. In contrast, the AI approach may be opaque and present unexpected outputs but handles complex multi-dimensional problems with adaptability.

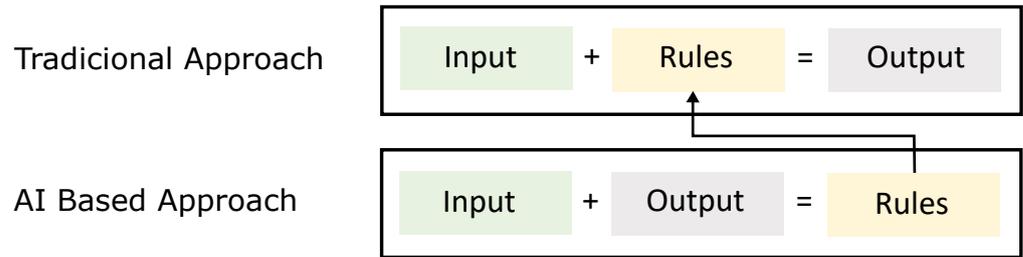


Figure 7. Traditional approach vs. machine learning approach (adapted from Málaga-Chuquitaype [48]).

PINNs represent a powerful fusion of traditional design approaches and machine learning methodologies [49]. These networks embed physical principles and governing equations into their architectures, allowing them to assimilate domain-specific knowledge and constraints. In the context of structural design, PINNs enable the integration of fundamental engineering principles, such as the laws of physics and material behavior, into the learning process. By incorporating these physics-based constraints, PINNs offer a unique advantage over conventional machine learning approaches. They provide a framework for seamlessly combining knowledge derived from traditional design methods with the capacity of neural networks to learn complex patterns and relationships from data. The application of PINNs in structural design allows us to take advantage of both traditional and machine learning approaches. Traditional design methods contribute foundational principles, boundary conditions, and known relationships, ensuring that the learned models align with established engineering standards. Simultaneously, the neural network component of PINNs harnesses the capacity to adapt and uncover intricate patterns within large datasets, offering a data-driven perspective to complement traditional design intuition. In this way, it is possible to reduce the gap between numerical predictions and experimental observations, enabling the estimation of missing physics and uncertainties [50]. This hybrid approach facilitates more accurate predictions, efficient optimization, and the exploration of innovative design solutions. Figure 8 schematically represents the potential between the traditional design and the adaptability and pattern recognition capabilities of data analyses, providing a promising avenue for advancing structural design processes.

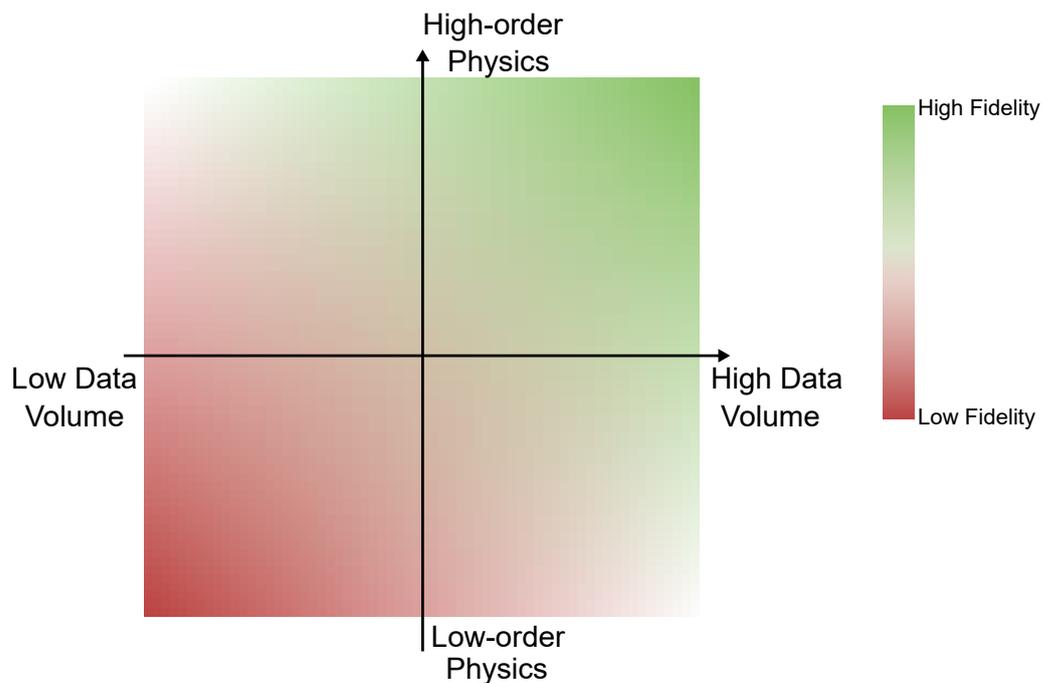


Figure 8. Potential of large data analyses combined with high-order physics.

Nowadays, different tools are available to create physics-informed neural networks, including NVIDIA Modulus (previously SimNet) [51], DeepXDE [52], IDRLnet [53], NeuroDiffEq [54], and finite basis physics-informed neural networks (FBPINNs) [55], among others. The current state of the art in physics-informed machine learning was assessed in [56], particularly its application to digital twins, prognostics, and health management. The authors highlighted that the majority of applications are associated with 1D and 2D partial differential equations, revealing challenges in dealing with complex structures. This underscores the necessity for additional research and development in this field.

4. Damage-Tolerant Design

Damage-tolerant design is a critical design philosophy in aircraft engineering aimed at ensuring the continued structural integrity and safe operation of an aircraft even in the presence of defects, cracks, or unexpected events that instigate structural damage [4]. This approach acknowledges that despite rigorous maintenance and operational practices, aircraft components may still experience wear, tear, or unforeseen incidents. To address this, damage-tolerant design involves the consideration of factors such as material properties, structural configuration, and load distribution to minimize the likelihood of catastrophic failure resulting from localized damage. By employing redundancy, load redistribution mechanisms, and fracture-resistant materials, damage-tolerant design enhances the ability of aircraft structures to withstand and accommodate various forms of damage, thereby extending the service life of aircraft while maintaining safety standards. This approach holds particular significance in the civil aviation industry, where safety and reliability are paramount, ensuring that even under challenging circumstances, aircraft can continue to operate safely until scheduled maintenance interventions can be executed.

The integration of digital-twin technologies and model updating techniques holds the potential to revolutionize the traditional approach to damage-tolerant design in the aerospace industry. Digital twins can provide dynamic and real-time information about an aircraft's structural health, continuously monitoring and analyzing its condition throughout its service. This data feedback enables engineers and decision makers to accurately assess the impact of various forms of damage on the aircraft's structural integrity and performance. By incorporating real-world data into simulations, model updating techniques can refine predictive models to better reflect the actual behavior of the aircraft under different damage scenarios. This iterative process leads to enhanced accuracy in predicting how damage will propagate and affect an aircraft's load distribution, ultimately influencing design decisions and maintenance strategies [57].

Currently, the real-world data of aircraft structures have been enhanced through the integration of advanced structural health monitoring (SHM) technologies [58]. SHM systems employ an array of sensors, including strain gauges, accelerometers, and thermocouples, among others, to continuously monitor the structural integrity of critical components. The real-time data collected from these sensors offer insights into stress distributions, load variations, and potential defects, enabling operators to promptly detect and assess the extent of damage. This proactive approach to damage detection aligns seamlessly with the damage-tolerant design philosophy and digital-twin developments, allowing timely interventions and maintenance to mitigate the progression of flaws, reduce maintenance costs, and extend the operational lifespan of civil aircraft.

Figure 9 schematically presents a typical curve of damage as a function of time, from a structural point of view. The reduction of the probability of failure is linked to maintenance operations, which restore the residual strength of the structure by repairing all damage detected in the structure. For normal conditions, maintenance operations are pre-programmed with time intervals Δt based on the design assumptions and fatigue characteristics of the materials. Modeling the fatigue process may be carried out at different levels of fidelity and for random multiaxial loadings, which has been discussed in detail, e.g., by Zhou and Tao [59], where the interplay between the dynamic response of the structure and high cycle fatigue behavior was considered in detail. Specifically, for fatigue

crack growth (FCG), approaches based on an equivalent stress-intensity factor range, using the Paris law or more advanced FCG laws, can be used together with crack path sensing in physical structures so as to predict crack growth and take corrective measures as required. An example of this approach was given in [60] concerning the propagation of fatigue cracks in aluminum alloy fuselages, where cycles correspond to pressurization in flight and depressurization when taxiing. Khalid et al. [61] gave a thorough review of sensing and structural health monitoring of aeronautical structures. Using digital twins, proactive maintenance planning can be used by detecting and predicting potential structural issues before they escalate, aligning well with the principles of damage-tolerant design by allowing timely interventions to mitigate further damage. These time intervals can be adjusted for each aircraft and considering the specific service conditions ($\Delta t_1, \Delta t_2, \Delta t_3, \dots, \Delta t_n$), improving the operational time without compromising the structural integrity of the aircraft.

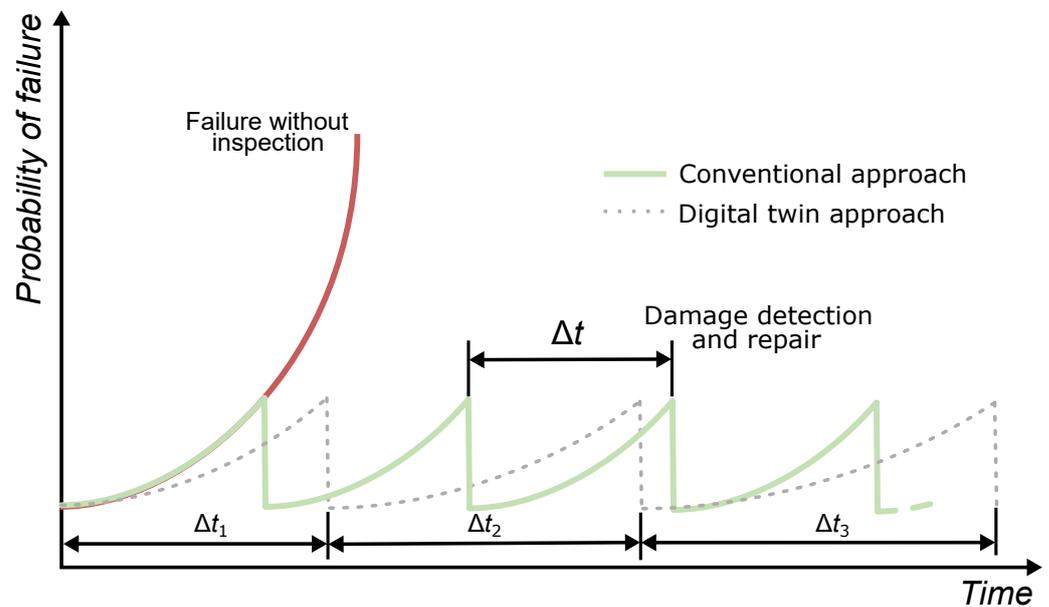


Figure 9. Example of possible impact of digital twins and model updating techniques on damage assessment (continuous line—assumed behaviour, conventional approach; dashed line—digital twin approach).

With structural health monitoring systems, the integration of data from various sensors and sources requires the development of robust data fusion techniques that can handle different types of data streams, such as strain measurements, temperature readings, and vibration patterns. Once the data are pre-processed and integrated, advanced analytic techniques, including machine learning and artificial intelligence algorithms, can be applied to extract meaningful insights, identify patterns, and detect anomalies based on the numerical models and the respective updates. These insights can be used to update and refine digital-twin models, making them more representative of real-world conditions and enabling more accurate predictive simulations. Ultimately, by effectively managing the abundance of data from SHM systems, the accuracy and reliability of structural numerical models and digital twins can be substantially improved, leading to safer and more efficient aircraft operations [62]. The combination of digital twins and model updating techniques can enable more efficient utilization of resources in the damage-tolerant design process [63]. Traditional methods often involve conservative assumptions to account for uncertainties in structural behavior under damage conditions, leading to over-engineered components and increased weight. By leveraging real-time data and accurate simulations, digital twins can provide a deeper understanding of actual loading conditions and stress distributions within damaged structures. This information can guide the refinement of finite-element models and allow for more accurate predictions of residual strength and fatigue life. Consequently,

aircraft manufacturers can optimize component designs and maintenance operations to restore the aircraft’s residual strength, achieving a balance between safety and weight savings. This optimized approach aligns with the philosophy of damage-tolerant design, aiming to maximize the structural lifespan while minimizing unnecessary material, fuel consumption, and maintenance costs.

By combining the data analysis of SHM systems with GNNs, ANNs, or PINNs, it is possible to develop new disruptive approaches. GNNs and ANNs can estimate stress/strain fields using numerical models, as illustrated in Figure 6. These models can then be calibrated and updated with data from structural sensing technologies, enabling fast evaluations and multi-fidelity structural assessments. Examples of approaches using PINNs related to damage-tolerant design were presented in [64] for recurrent neural networks and in [65] for hybrid PINNs. The approach based on hybrid PINNs is presented schematically in Figure 10. This last model is an enhanced version of the cumulative damage model based on physics-informed recurrent neural networks [64], also used for fatigue crack growth assessment with a smaller amount of data. This innovative model endeavors to establish a predictive analysis framework for forecasting fatigue crack growth in aircraft window panels. Based on historical flight records and limited inspection observations, the model (i.e., ‘digital twin’) is designed to enhance prognosis and residual strength assessment. Distinct from traditional data-driven models, this approach differs by the utilization of significant training data. Instead, the model’s loss functions are enriched with linear elasticity laws, and the necessary second-order derivatives are computed through automatic differentiation. This approach, from traditional training data methodologies, underscores a reliance on fundamental physical principles, illustrating a more physics-informed approach to predictive modeling, particularly in the context of fatigue crack growth prognosis for critical aircraft components. One of the primary advantages of these models is their ability to compensate for model-form uncertainty in damage estimation and to address incomplete knowledge regarding various unknown factors in the models, as discussed in [66].

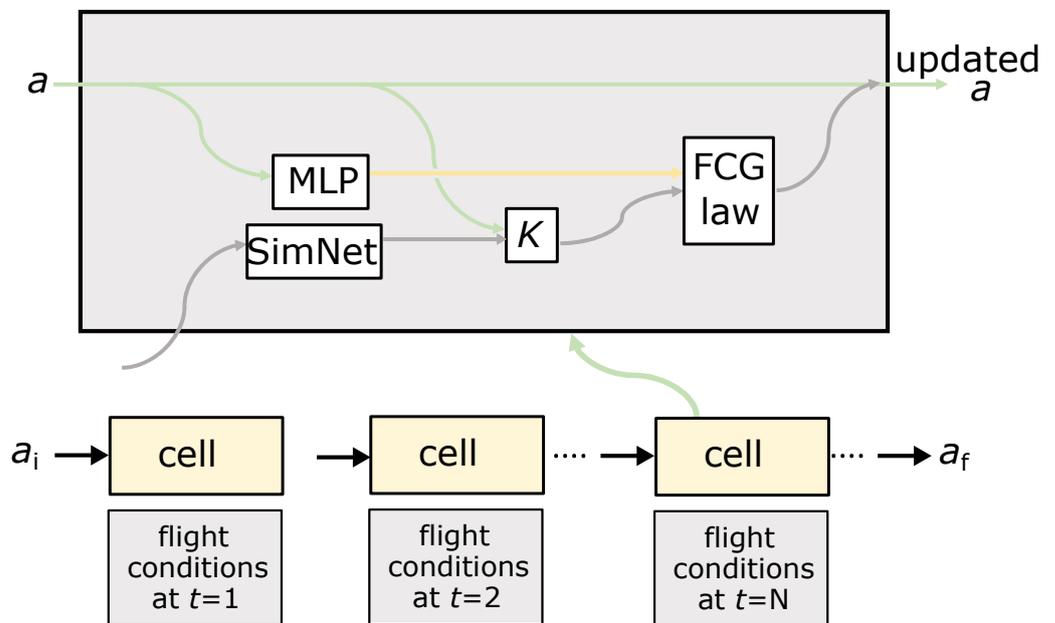


Figure 10. Example of the application of a hybrid physics-informed neural network in fatigue assessment. a —damage (crack length); FCG—fatigue crack growth; K —stress-intensity factor; MLP—multilayer perceptron; SimNet—an AI-accelerated multi-physics simulation framework (NVIDIA); t —time (adapted from Viana et al. [65]).

The model proposed in [65] is based on the NVIDIA framework SimNet™, short for Simulation Network, which is a simulation-based engineering framework that leverages

artificial intelligence and machine learning techniques. This framework is able to integrate various simulation tools and datasets and operates on the principle of collaborative learning, where multiple simulation models interact and learn from each other's experiences, continuously improving their predictive capabilities [51].

5. Conclusions

The integration of digital-twin technology and model updating techniques is enabling a transformative era in aircraft design and the broader aerospace industry. These advancements represent a shift from traditional design paradigms to dynamically updated, data-driven processes. The current capability to continuously monitor aircraft and seamlessly update simulation models ensures that aircraft structures are built to withstand real-world conditions, leading to improved safety, reliability, and sustainability.

Considering damage-tolerant design, these technologies offer a new level of insights into the structural health of aircraft, allowing engineers to accurately assess the effects of damage and develop more accurate predictive models, individualized for each aircraft. By leveraging data to inform maintenance and repair decisions and optimize designs, digital twins and model updating contribute to longer-lasting, safer, and more cost-effective aircraft structures.

Taking into account the fast evolution of diverse machine learning tools applied to diverse engineering simulation problems, disruptive approaches can be developed to support these digital twins. These machine learning tools can handle complex and large-scale data, which can be obtained from structural instrumentation, and integrate these data to update numerical models. These approaches not only accelerate design iterations but also optimize the accuracy of simulations, resulting in more reliable insights into aircraft performance, structural integrity, and maintenance needs. However, these applications require large datasets with significant and reliable data representative of the critical phenomena. In addition, full-field sensors for structural assessment are not available, and real-time sensor data from aircraft in operation present technological challenges due to communication and signal-processing limitations.

Future developments in this field may address these limitations by exploring more streamlined data integration strategies, optimizing computational processes for digital-twin management, and further developing sensor technologies designed for structural assessment and for the harsh environments of aircraft operation.

In essence, the integration of digital twins and model updating techniques based on machine learning offers a paradigm shift in the damage-tolerant design philosophy, enabling more informed, adaptive, and resource-efficient strategies for ensuring the structural integrity and safety of aircraft throughout their operational life.

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Abbreviations

The following abbreviations are used in this manuscript:

AI	artificial intelligence
AIA	Aerospace Industries Association
AIAA	American Institute of Aeronautics and Astronautics
CFD	computational fluid dynamics
CNN	convolutional neural network
DLR	Deutsches Zentrum für Luft- und Raumfahrt (German Aerospace Center)
FCG	fatigue crack growth
FEA	finite-element analysis
GN	graph network
GNN	graph neural network
LASI	Associate Laboratory for Intelligent Systems
LOV	Limit of Validity
ML	machine learning
MLP	multilayer perceptron
PINN	physics-informed neural network
ROMs	reduced-order models
RNN	recurrent neural network
SHM	structural health monitoring
TEMA	Centre for Mechanical Technology and Automation, Univ. Aveiro

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