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Editorial

Mathematical Methods in Applied Sciences

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1. Introduction

In this editorial, we introduce "Mathematical Methods in Applied Sciences", a Special Issue of *Axioms* comprising 17 articles. These articles delve into various mathematical methods and emerging trends in applied sciences, spanning from theoretical explorations to practical applications. While covering diverse topics, particular emphasis is placed on fields such as mathematical methods and analysis, statistical modeling, natural language processing, neural networks, inverse problems, numerical methods, and fuzzy systems. The primary objective of this Special Issue is to provide a platform for scientists and researchers to showcase their work in optimization, optimal control theory, biomathematical studies, population dynamics, network problems, and reinforcement learning, as well as machine learning and deep learning, thereby enhancing our understanding of the world.

2. Overview of the Published Papers

This Special Issue contains 17 papers that were accepted for publication after a rigorous review process.

In contribution 1, A. Khanfar et al. present an analytic solution to the Stefan problem, a mathematical model describing the phase change of a material with a moving boundary, considering nonlinear temperature-dependent thermal parameters and a heat source term. The authors establish the existence and uniqueness of the solution in scenarios both with and without a heat source. They then determine lower and upper bounds for solutions of the problem under different conditions. Remarkably, the lower bounds closely align with numerical solutions, suggesting their utility as approximate analytic solutions.

In contribution 2, K. S. Sultan et al. present the derivation of L-moments for several distributions, including logistic, generalized logistic, doubly truncated logistic, and doubly truncated generalized logistic distributions. They introduce new axioms and identities, including recurrence relations specific to L-moments derived from these distributions. They also establish general recurrence relations applicable to L-moments derived from any distribution.

In contribution 3, M. A. Zaitri et al. introduce an analytical solution for the time-optimal control problem during the induction phase of anesthesia, aligning closely with results obtained via the shooting method. The authors employ a pharmacokinetic/pharmacodynamic (PK/PD) model for propofol infusion, proposed by Bailey and Haddad in 2005. The study evaluates this solution by comparing it with the existing literature using the Pontryagin minimum principle and numerical simulations in MATLAB. The results indicate a similarity between the newly proposed analytical method and the shooting method, with the advantage of the former being independence from unknown initial conditions for the adjoint variables.

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In contribution 4, X. Hu and H. Ren address reliability estimation using the inverse Weibull distribution with intuitionistic fuzzy lifetime data. They extend fuzzy set theory concepts to derive intuitionistic fuzzy conditional density, likelihood function, and conditional expectation. Both maximum likelihood and Bayesian estimations, employing the EM algorithm and gamma priors, respectively, are explored. Monte Carlo simulations favor Bayesian estimation, validated by real data, offering precise reliability estimates for intuitionistic fuzzy lifetime data in scientific analysis.

In contribution 5, H. S. Bakouch et al. explore count data analysis using the two-parameter Bernoulli–Poisson–Lindley distribution, obtained through convolution of Bernoulli and Poisson–Lindley distributions. Statistical properties such as moments, survival functions, and parameter inference via maximum likelihood are investigated. Simulation exercises assess estimation effectiveness, followed by application to real datasets. Additionally, a flexible count data regression model is constructed based on the proposed distribution, illustrated with practical examples.

In contribution 6, L. C. Chen and K. H. Chang introduce a corpus assessment method crucial for Natural Language Processing (NLP), especially pertinent in contexts like COVID-19 information retrieval. Traditional approaches based on single parameters, such as keyness value, are deemed inadequate. To address this limitation, the authors propose an extended Analytic Hierarchy Process (AHP)-based approach, considering multiple parameters (keyness, frequency, and range) simultaneously. Empirical validation using COVID-19 research articles confirms the effectiveness of this approach, offering improvements in refining corpus data, multi-parameter consideration, and integration of expert evaluation.

In contribution 7, G. Singh et al. propose a new Laplace variational iterative method for solving (2+1)-D and (3+1)-D Burgers equations, employing a combination of modified variational iteration method and Laplace transform. This method transforms the differential problem into algebraic equations via Laplace transform, and iteratively solves them using the modified variational iterative approach. The technique enables both numerical and analytical solutions for the Burgers equations, validated through three specific examples, demonstrating its effectiveness.

In contribution 8, F. Al Basir et al. devise an integrated pest management model for crop pest control, utilizing periodic application of biopesticide and chemical pesticides. Theoretical analysis yields a periodic solution for pest eradication, ensuring boundedness of system variables. Optimization aims to find the most effective pesticide concentration and application frequency. Employing Floquet theory and small amplitude perturbation method, the study establishes local and global stability of pest eradication periodic solution. Numerical comparisons validate integrated pest management's superiority over single controls, with analytical results illustrated through simulations.

In contribution 9, A. F. Jameel et al. propose a novel approach to solve and analyze two-point fuzzy boundary value problems in fractional ordinary differential equations (FFOBVPs). FFOBVPs describe complex phenomena with uncertainty, making exact or close analytical solutions challenging, particularly for nonlinear problems. The study extends the optimal homotopy asymptotic method (OHAM) to handle FFOBVPs, incorporating fuzzy sets theory and fractional calculus principles. Fuzzification and defuzzification transform fuzzy problems into solvable crisp ones. The method's efficiency and accuracy are demonstrated through solving and analyzing linear and nonlinear FFOBVPs at various fractional derivatives, showcasing its viability for fuzzy analysis.

In contribution 10, R. Alotaibi et al. investigate constant-stress accelerated life tests with test units following the XLindley distribution, employing maximum likelihood and Bayesian estimation methods based on progressively Type-II censored samples. They derive point and interval estimations of model parameters and reliability indices under normal operating conditions. Bayesian estimates are calculated using the Markov chain Monte Carlo algorithm with the squared error loss function. A performance simulation illustrates the proposed methodology, with application to two real-life accelerated life test cases. Numerical outcomes suggest the superiority of the Bayesian estimation method, particularly in evaluating XLindley parameters and reliability measures, under constant-stress accelerated life tests with progressively Type-II censoring.

In contribution 11, A. J. Fernández presents guaranteed-coverage and expected-coverage tolerance limits for Weibull models, addressing situations where extreme sample values are censored or disregarded due to data collection restrictions or outliers. Both unconditional and conditional tolerance bounds are discussed, particularly when the smallest observations are discarded. The paper also explores determining the minimum sample sizes for setting Weibull tolerance limits from trimmed data with fixed numbers or proportions of trimmed observations. Step-by-step procedures for optimal sampling plans are outlined, with numerical examples provided for illustration.

In contribution 12, L. Han et al. address the challenge of solving the dynamic Sylvester equation (DSE) in noisy environments using neural networks. While the original zeroing neural network (OZNN) performs well in noise-free settings, it struggles in noisy conditions. An integral-enhanced zeroing neural network (IEZNN) improves noise handling, but lacks speed. To overcome these limitations, an accelerated double-integral zeroing neural network (ADIZNN) is proposed, designed to resist linear noise and accelerate convergence. Theoretical proofs confirm the convergence and robustness of the ADIZNN, while simulation experiments demonstrate its superior convergence rate and noise resistance compared to OZNN and IEZNN. Additionally, chaos control experiments with a sine function memristor chaotic system highlight the ADIZNN's superior performance in terms of accuracy and error reduction.

In contribution 13, A. Freitas et al. investigate the relationship between meteorological variables and dengue transmission during the 2019 outbreak in the Dominican Republic. Using generalized linear mixed modeling, they analyze weekly dengue incidence rates, finding that temperature and rainfall impact outbreaks with a delay of 2–5 weeks, conducive to mosquito breeding conditions. The study employs a backward-type selection method to identify influential variables, noting variations in lag correlations across provinces. These findings provide critical insights for healthcare authorities to prepare and manage dengue outbreaks effectively.

In contribution 14, D. Karaoulanis et al. highlight the significance of fractional derivatives in modeling anomalous diffusion in brain tissue, linked to diseases like Alzheimer's, multiple sclerosis, and Parkinson's. The accumulation of proteins in axons and discrete swellings contribute to neurodiseases. To model voltage propagation in axons, a fractional cable geometry is adopted, although the absence of a fractional differential geometry based on well-known fractional derivatives poses questions. The Λ-fractional derivative (Λ-FD) is introduced as the unique fractional derivative generating differential geometry for modeling the human neural system's intricate parts. Examples are provided to draw meaningful conclusions, aiding medical and bioengineering scientists in combating brain diseases.

In contribution 15, S. Pakzad et al. emphasize the significance of surface quality in wooden product manufacturing, necessitating a comprehensive understanding of cutting parameters' impacts on wood. Response surface methodology is employed to design experiments and analyze the effects of feed rate, spindle speed, step over, and depth of cut on beech wood surface quality. Mathematical models are derived for the parameters and surface roughness. Optimal machining parameters are determined to enhance surface quality, reducing roughness by up to 4.2 μm. Notably, the feed rate exhibits the most significant impact on surface quality among the machining parameters.

In contribution 16, M. S. Concha-Aracena et al. introduce a theorem demonstrating the generation of density functions from moments of the standard normal distribution, leading to a family of models. Different random variable domains are achieved through transformations, exemplified by transforming the second-order moment to create the Alpha-Unit (AU) distribution, characterized by a single parameter α (AU(α)$\in$[0, 1]). Properties of the AU distribution are presented, along with estimation methods for the α parameter. Monte Carlo simulations confirm the statistical consistency and robustness of the estimators. Real-world applications demonstrate the competitiveness of the AU model, especially for data with a range greater than 0.4 and extremely heavy asymmetric tail, compared to other commonly used unit models.

In contribution 17, A. Hussain et al. delve into biomedical image reconstruction, particularly focusing on functional near infra-red spectroscopy (fNIRs), a non-invasive imaging technology using near infra-red light. Image reconstruction involves solving both forward and backward problems to deduce the image's optical properties from measured boundary data. Researchers employ various numerical methods to tackle these challenges. This study highlights the latest advancements in numerical methods for solving forward and backward problems in fNIRs, offering insights into physical interpretations, state-of-the-art numerical techniques, and toolbox descriptions. A comprehensive discussion on numerical solution approaches for the inverse problem in fNIRs is provided, shedding light on this evolving field.

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List of Contributions

1. Khanfer, A.; Bougoffa, L.; Bougouffa, S. A Nonclassical Stefan Problem with Nonlinear Thermal Parameters of General Order and Heat Source Term. *Axioms* **2024**, *13*, 14. https://doi.org/10.3390/axioms13010014.
2. Sultan, K.S.; AL-Shamari, N.R. Some Axioms and Identities of L-Moments from Logistic Distribution with Generalizations. *Axioms* **2023**, *12*, 928. https://doi.org/10.3390/axioms12100928.
3. Zaitri, M.A.; Silva, C.J.; Torres, D.F.M. An Analytic Method to Determine the Optimal Time for the Induction Phase of Anesthesia. *Axioms* **2023**, *12*, 867. https://doi.org/10.3390/axioms12090867.
4. Hu, X.; Ren, H. Reliability Estimation of Inverse Weibull Distribution Based on Intuitionistic Fuzzy Lifetime Data. *Axioms* **2023**, *12*, 838. https://doi.org/10.3390/axioms12090838.
5. Bakouch, H.S.; Chesneau, C.; Maya, R.; Irshad, M.R.; Aswathy, S.; Qarmalah, N. A Flexible Dispersed Count Model Based on Bernoulli Poisson–Lindley Convolution and Its Regression Model. *Axioms* **2023**, *12*, 813. https://doi.org/10.3390/axioms12090813.
6. Chen, L.-C.; Chang, K.-H. An Extended AHP-Based Corpus Assessment Approach for Handling Keyword Ranking of NLP: An Example of COVID-19 Corpus Data. *Axioms* **2023**, *12*, 740. https://doi.org/10.3390/axioms12080740.
7. Singh, G.; Singh, I.; AlDerea, A.M.; Alanzi, A.M.; Khalifa, H.A.E.-W. Solutions of (2+1)-D & (3+1)-D Burgers Equations by New Laplace Variational Iteration Technique. *Axioms* **2023**, *12*, 647. https://doi.org/10.3390/axioms12070647.
8. Al Basir, F.; Chowdhury, J.; Torres, D.F.M. Dynamics of a Double-Impulsive Control Model of Integrated Pest Management Using Perturbation Methods and Floquet Theory. *Axioms* **2023**, *12*, 391. https://doi.org/10.3390/axioms12040391.
9. Jameel, A.F.; Jawad Hashim, D.; Anakira, N.; Ababneh, O.; Qazza, A.; Alomari, A.-K.; Al Kalbani, K.S. Application of the Optimal Homotopy Asymptotic Approach for Solving Two-Point Fuzzy Ordinary Differential Equations of Fractional Order Arising in Physics. *Axioms* **2023**, *12*, 387. https://doi.org/10.3390/axioms12040387.

10. Alotaibi, R.; Nassar, M.; Elshahhat, A. Reliability Estimation under Normal Operating Conditions for Progressively Type-II XLindley Censored Data. *Axioms* **2023**, *12*, 352. https://doi.org/10.3390/axioms12040352.
11. Fernández, A.J. Tolerance Limits and Sample-Size Determination Using Weibull Trimmed Data. *Axioms* **2023**, *12*, 351. https://doi.org/10.3390/axioms12040351.
12. Han, L.; He, Y.; Liao, B.; Hua, C. An Accelerated Double-Integral ZNN with Resisting Linear Noise for Dynamic Sylvester Equation Solving and Its Application to the Control of the SFM Chaotic System. *Axioms* **2023**, *12*, 287. https://doi.org/10.3390/axioms12030287.
13. Freitas, A.; Rodrigues, H.S.; Martins, N.; Iutis, A.; Robert, M.A.; Herrera, D.; Colomé-Hidalgo, M. Multiplicative Mixed-Effects Modelling of Dengue Incidence: An Analysis of the 2019 Outbreak in the Dominican Republic. *Axioms* **2023**, *12*, 150. https://doi.org/10.3390/axioms12020150.
14. Karaoulanis, D.; Lazopoulos, A.K.; Lazopoulou, N.; Lazopoulos, K. On Λ-Fractional Derivative and Human Neural Network. *Axioms* **2023**, *12*, 136. https://doi.org/10.3390/axioms12020136.
15. Pakzad, S.; Pedrammehr, S.; Hejazian, M. A Study on the Beech Wood Machining Parameters Optimization Using Response Surface Methodology. *Axioms* **2023**, *12*, 39. https://doi.org/10.3390/axioms12010039.
16. Concha-Aracena, M.S.; Barrios-Blanco, L.; Elal-Olivero, D.; Ferreira da Silva, P.H.; Nascimento, D.C.d. Extending Normality: A Case of Unit Distribution Generated from the Moments of the Standard Normal Distribution. *Axioms* **2022**, *11*, 666. https://doi.org/10.3390/axioms11120666.
17. Hussain, A.; Faye, I.; Muthuvalu, M.S.; Tang, T.B.; Zafar, M. Advancements in Numerical Methods for Forward and Inverse Problems in Functional near Infra-Red Spectroscopy: A Review. *Axioms* **2023**, *12*, 326. https://doi.org/10.3390/axioms12040326.