

Supplementary Materials: Statistical Mechanics of Social Hierarchies: A Mathematical Model for the Evolution of Human Societal Structures

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1. The onset and breakdown of egalitarianism arising from the competition of cognitive and social navigation needs.

In Figure S1 we show the effect, on the expected values of the average and maximum degrees, of changing the size of population. The reader used to studying the thermodynamical limit should not be tempted to do so in this case because we are looking at transitions induced by the change in the population size. The figure shows the approximate behavior as a function of the specific cognitive capacity z .

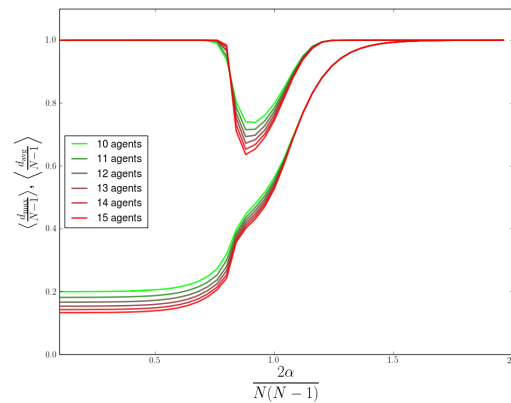


Figure S1. Monte Carlo estimates of the maximum degree and the average degree of the social web representation as a function of $z = 2\alpha/n(n-1)$ for $\beta = 6$. This shows that the scaling of α with $n(n-1)/2$ is a good approximation that gets better for larger values of n .

From the frequencies of the different cases in the empirical record [1] and from the Monte Carlo we extracted the values of the conditional probabilities and standard deviations, shown in Figures S2 and S3.

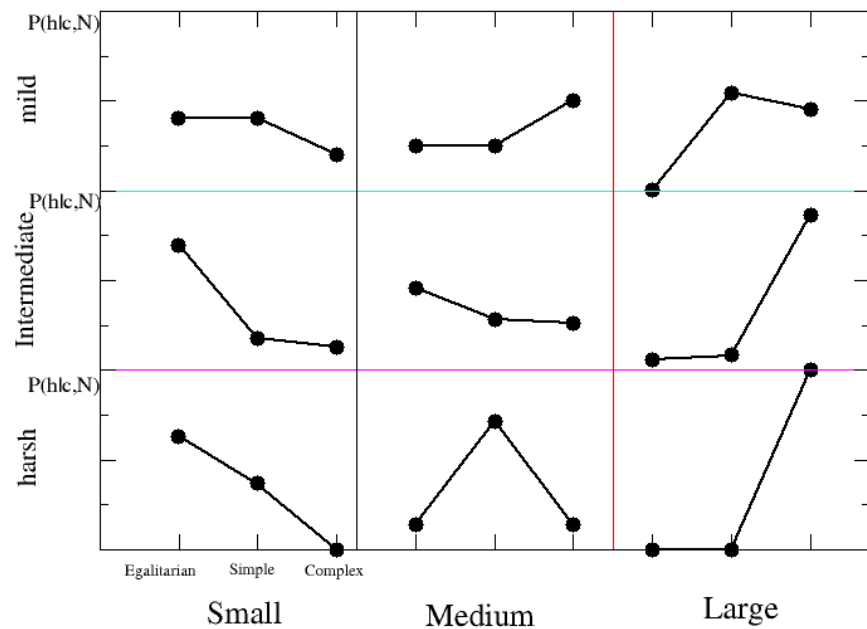


Figure S2. Conditional Probability distributions $P(h|s, c)$ obtained from the Ethnographic Atlas data set.

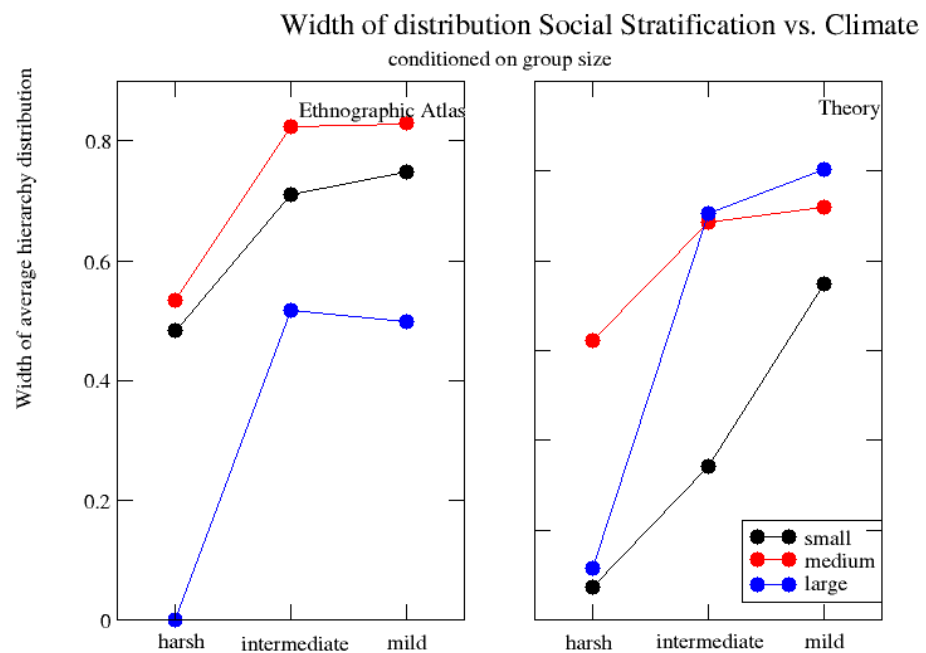


Figure S3. The standard deviation of the distribution of expected hierarchies, or range of values of hierarchy as a function of ecological pressure for small, medium and large groups. Left: From the Ethnographic Atlas data [1]. Right: Theoretical prediction

2. Bibliography

1. Douglas R. White and Burton M and Divale W and Gray P and Korotayev A and Khalturina D, *Standard Cross Cultural Sample Codebook*, https://growthecon.com/assets/Standard_Cross-Cultural_Sample-Codebook.pdf and *EthnographicAtlasWCRevisedByWorldCultures.sav*, 2011

4. Ethnographic Atlas Data

In Table S1 we show the number of data points (ethnographic groups) in the EA relevant for this study. The numerical data extracted from the EA is shown in Table S2 and the corresponding theoretical numbers in Table S3.

Table S1. The number of data points (ethnographic groups) in the EA relevant for this study

↓Stratification	Climate	Number of cultures in Small groups	Number of cultures in Medium groups	Number of cultures in Large groups
1	1	12	1	0
1	2	43	40	2
1	3	4	3	0
2	1	7	5	0
2	2	11	25	3
2	3	4	3	6
3	1	0	1	3
3	2	8	23	31
3	3	2	6	5

4.1. Numerical results

Table S2. The results for the empirical stratification \bar{H}

↓Climate, Group size →	Small	Medium	Large groups
Harsh	0.37	1.0	2.0
Intermediate	0.44	0.81	1.81
Mild	0.80	1.25	1.45

Table S3. The results for the theoretical prediction H_T

↓Climate, Group size →	Small	Medium	Large groups
Harsh	0.01	0.64	1.58
Intermediate	0.13	0.56	1.25
Mild	0.39	0.58	0.77

4.2. Ethnographic data

Table S4. V31. Mean Size of Local Communities

N	Code	Description v31.	Size category
681	0	Missing data (code .)	0
118	1	Fewer than 50	1
107	2	50-99	1
104	3	100-199	2
83	4	200-399	2
60	5	400-1000	2
16	6	1,000 without any town of more than 5,000	3
36	7	Towns of 5,000-50,000 (one or more)	3
62	8	Cities of more than 50,000 (one or more)	3

Table S5. V66. Class Stratification

N	Code	Description v66.	Hierarchy category
182	0	Missing data (code .)	0
533	1	Absence among freemen (O.)	1
206	2	Wealth distinctions (W.)	2
39	3	Elite (based on control of land or other resources (E.))	2
228	4	Dual (hereditary aristocracy) (D.)	3
79	5	Complex (social classes) (C.)	3

Table S6. V95. Climate: Primary Environment (Coded by Frank Moore from Phillips' Comparative Atlas)

N	Code	Description v95.	Climate category
869	0	Not coded	0
11	23	Tundra (northern areas)	1
21	36	Northern coniferous forest	1
8	44	High plateau steppe	1
19	46	Temperate forest (mostly mountainous)	3
3	51	Desert (including arctic)	1
37	52	Desert grasses and shrubs	2
25	54	Temperate grasslands	3
11	55	Mediterranean (dry, deciduous, and evergreen forests)	3
16	56	Temperate woodland	2
5	65	Oases and certain restricted river valleys	3
24	74	Sub-tropical bush	2
27	78	Sub-tropical rain forest	2
64	84	Tropical grassland	2
14	87	Monsoon forest	2
113	88	Tropical rain forest	2

Table S7. EA Data: Cultures, Size, Class Stratification , Climate

	Culture	Size(v31)	Stratification(v66)	Climate(v95)
1	!KUNG	1	1	2
2	ILA	2	2	2
3	NYORO	2	3	2
4	AMBBA	2	1	2
5	KPE	1	2	2
6	FON	3	3	2
7	KISSI	2	1	2
8	BAMBARA	3	3	2
9	YATENGA	3	3	2
10	KATAB	2	1	2
11	KONSO	3	2	3
12	SOMALI	1	2	2
13	WOLOF	3	3	2
14	TEDA	1	3	3
15	BARABRA	1	2	1
16	GHEG	2	1	1
17	NEWENGLAN	3	3	2
18	DUTCH	3	3	2
19	SERBS	3	3	2
20	SYRIANS	3	2	3
21	SINDHI	3	2	2
22	KAZAK	1	3	3
23	GILYAK	1	1	1
24	YAKUT	1	2	1
25	KOREANS	3	3	2
26	LOLO	2	3	3
27	ABOR	2	2	2
28	CHENCHU	1	1	2
29	TAMIL	3	3	2
30	ANDAMANES	1	1	2
31	MERINA	3	3	2
32	GARO	2	2	2
33	LAMET	1	2	2
34	MNONGGAR	2	2	2
35	ATAYAL	2	1	2
36	SAGADA	3	2	2
37	JAVANESE	3	3	2
38	MACASSARE	2	3	2
39	ARANDA	1	1	2
40	KAPAUKU	1	2	2

	Culture	Size(v31)	Stratification(v66)	Climate(v95)
41	WANTOAT	1	1	2
42	TRUKESE	2	1	2
43	TROBRIAND	2	3	2
44	SAMOANS	1	3	2
45	TIKOPIA	2	3	2
46	NABESNA	1	1	1
47	TAREUMIUT	2	2	1
48	TWANA	1	2	1
49	NOMLAKI	2	2	3
50	TENINO	2	2	1
51	OJIBWA	1	1	1
52	HURON	2	2	1
53	HANO	2	1	2
54	CUNA	1	2	2
55	WARRAU	1	1	2
56	MUNDURUCU	1	1	2
57	SIRIONO	1	1	2
58	TUCUNA	2	1	2
59	INCA	3	3	1
60	YAHGAN	1	1	1
61	MATACO	1	1	2
62	TRUMAI	1	1	2
63	DOROBO	1	1	2
64	NAMA	2	2	2
65	LOZI	1	3	2
66	BEMBA	2	3	2
67	KUBA	2	3	2
68	CHAGGA	2	3	2
69	KIKUYU	2	2	2
70	FANG	1	2	2
71	ASHANTI	3	3	2
72	DOGON	2	2	2
73	TALLENSI	2	2	2
74	TIV	2	1	2
75	AZANDE	2	3	2
76	MASAI	1	1	2
77	TIGRINYA	3	3	2
78	SONGHAI	3	3	2
79	SIWANS	3	2	3
80	EGYPTIANS	3	3	3

	Culture	Size(v31)	Stratification(v66)	Climate(v95)
81	RIFFIANS	3	2	3
82	ROMANS	3	3	3
83	IRISH	3	3	2
84	LAPPS	1	2	1
85	HUTSUL	3	2	3
86	PATHAN	2	3	2
87	KHALKA	1	3	2
88	CHUKCHEE	1	2	1
89	YURAK	1	2	1
90	MIAO	2	1	2
91	BURUSHO	2	3	1
92	LEPCHA	2	2	3
93	BENGALI	3	3	2
94	MARIAGOND	1	2	2
95	TODA	1	1	2
96	TANALA	2	3	2
97	VEDDA	1	1	2
98	BURMESE	3	3	2
99	SEMANG	1	1	2
100	ANNAMESE	3	3	2
101	IFUGAO	2	2	2
102	SUBANUN	1	1	2
103	BALINESE	2	3	2
104	ALORESE	2	2	2
105	MURNGIN	1	1	2
106	TIWI	2	1	2
107	WOGEO	1	1	2
108	MAJURO	2	3	2
109	IFALUK	1	1	2
110	KURTATCHI	2	3	2
111	LESU	2	1	2
112	BUNLAP	1	2	2
113	LAU	1	2	2
114	PUKAPUKAN	2	1	2
115	MAORI	2	3	3
116	MARQUESAN	1	3	2
117	COPPERESK	1	1	1
118	KASKA	1	1	1
119	YUROK	1	2	3
120	TUBATULAB	1	1	2

	Culture	Size(v31)	Stratification(v66)	Climate(v95)
121	HAVASUPAI	2	1	2
122	SANPOIL	1	1	3
123	OMAHA	1	1	3
124	CREEK	2	1	3
125	NAVAHO	2	1	2
126	ZUNI	3	1	2
127	AZTEC	3	3	3
128	BARAMACAR	1	1	2
129	TAPIRAPE	2	1	2
130	JIVARO	1	1	1
131	YAGUA	1	1	2
132	AYMARA	2	2	1
133	CAYAPA	2	1	2
134	MAPUCHE	1	2	3
135	BACAIRI	1	1	2
136	NAMBICUAR	1	1	2
137	AWEIKOMA	1	1	2
138	RAMCOCAME	2	1	2
139	MBUTI	2	1	2
140	MBUNDU	2	3	2
141	VENDA	2	3	3
142	NYAKYUSA	2	1	2
143	MENDE	2	3	2
144	YORUBA	3	3	2
145	BIRIFOR	2	1	2
146	MAMBILA	2	1	3
147	MARGI	2	1	2
148	MAMVU	1	1	2
149	SHILLUK	2	3	2
150	LANGO	2	2	2
151	IRAQW	2	2	2
152	MZAB	3	3	3
153	KABYLE	2	1	2
154	TRISTAN	2	1	3
155	WALLOONS	3	3	2
156	CZECHS	3	3	2
157	HEBREWS	3	3	3
158	HAZARA	2	2	2
159	KORYAK	2	2	1
160	YUKAGHIR	1	1	1

	Culture	Size(v31)	Stratification(v66)	Climate(v95)
161	JAPANESE	3	3	2
162	MINCHINES	3	3	2
163	TIBETANS	3	3	1
164	COORG	2	3	2
165	KERALA	3	3	2
166	NICOBARES	1	1	2
167	SINHALESE	3	3	2
168	KACHIN	2	3	3
169	PURUM	1	2	2
170	CAMBODIAN	3	3	2
171	HANUNOO	2	1	2
172	DUSUN	2	2	2
173	DIERI	1	1	2
174	KARIERA	1	1	2
175	KERAKI	1	1	2
176	PONAPEANS	1	3	2
177	YAPESE	1	3	2
178	ULAWANS	2	1	2
179	NASKAPI	1	1	1
180	EYAK	1	2	1
181	ATSUGEWI	1	2	3
182	MIAMI	2	1	2
183	CHEROKEE	2	1	2
184	DELAWARE	2	1	2
185	MARICOPA	1	1	2
186	TAOS	2	1	2
187	HUICHOL	2	2	2
188	CHOCO	1	1	2
189	CARINYA	2	1	2
190	GUAHIBO	1	1	2
191	CUBEO	1	1	2
192	TUNEBO	2	1	2
193	ONA	1	1	1
194	CHOROTI	1	1	2
195	CAMAYURA	2	1	2
196	BOTOCUDO	1	1	2
197	SOTHO	2	3	3
198	YAO	2	1	2
199	YOMBE	2	3	2
200	GANDA	3	3	2

	Culture	Size(v31)	Stratification(v66)	Climate(v95)
201	BETE	2	1	2
202	NUPE	2	3	2
203	CONIAGUI	1	1	2
204	BAYA	2	1	2
205	LUO	2	2	2
206	CHEREMIS	2	2	2
207	NURI	2	2	2
208	AINU	1	1	2
209	OKINAWANS	3	3	2
210	DARD	2	3	2
211	BHIL	1	3	2
212	AKHA	2	1	2
213	PAIWAN	2	3	2
214	WIKMUNKAN	1	1	2
215	ENGA	2	1	2
216	LAKALAI	2	2	2
217	ATTAWAPIS	1	1	1
218	DIEGUENO	2	1	2
219	WASHO	1	1	3
220	PAWNEE	2	3	3
221	COCHITI	2	1	2
222	YUCATECMA	3	3	2
223	WAICA	1	1	2
224	TEHUELCHÉ	1	1	2
225	NGONI	2	3	2
226	WUTE	2	3	2
227	BRAZILIAN	3	3	2
228	BULGARIAN	3	3	2
229	BASSERI	2	2	2
230	KET	1	1	1
231	MINCHIA	2	2	3
232	KHASI	1	3	2
233	SIAMESE	3	2	2
234	PURARI	2	2	2
235	ONOTOA	2	2	2
236	MANUS	2	2	2
237	YUKI	1	2	3
238	NATCHEZ	2	2	2
239	JEMEZ	2	1	2
240	BLACKCARI	3	1	2
241	MAM	3	2	3
242	MISKITO	2	1	2
243	GOAJIRO	1	2	2
244	YABARANA	1	1	2
245	CHIBCHA	3	3	1
246	ALACALUF	1	1	3
247	APINAYE	1	1	2
248	TUPINAMBA	2	2	2