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Decentralized Multi-Performance Fuzzy Control for Nonlinear Large-Scale Descriptor Systems

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Abstract: This article addresses the decentralized multi-performance (MP) fuzzy control problem of nonlinear large-scale descriptor (LSD) systems. The considered LSD system contains several subsystems with nonlinear interconnection and external disturbances, and the Takagi–Sugeno fuzzy model (TSFM) is adopted to represent each nonlinear subsystem. Based on the proportional-plus-derivative state feedback (PDSF) scheme, we aim to design a decentralized MP fuzzy controller that guarantees the stabilization, mixed H_∞, and passivity performance control (MHPPC), and the guaranteed cost control (GCC) performance of the closed-loop Takagi–Sugeno LSD (TSLSD) systems. Furthermore, we introduce the Lyapunov stability theory and the free-weighting matrix scheme to analyze the stability of the TSLSD system. The proposed sufficient conditions can be transformed as linear matrix inequality (LMI) forms through Schur's complement, which can be easily solved with the LMI Toolbox. Finally, to illustrate the proposed approach, two examples and simulation results are presented.

Keywords: decentralized proportional-plus-derivative state feedback control; multi-performance fuzzy control; Takagi–Sugeno large-scale descriptor systems

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1. Introduction

Currently, LS systems have been widely researched due to their applications in various fields such as nuclear reactors, mobile robotics, transportation systems, and power systems [1,2]. It can be known that the current complex dynamical systems are usually composed of strong interconnections and high dimensionality. The LS system provides an efficient method to describe the above complex systems [3]. To solve the control problem of LS systems, the decentralized control scheme has been investigated, which can reduce the dimensionality of the system based on the local information of subsystems [4]. Over the past few decades, decentralized control has been investigated as a branch of control theory, and a number of theoretical results have been proposed for LS systems [5,6].

On the other hand, many works have been given to deal with nonlinear control in the past few decades. In these works, TSFM shows its importance because it can represent nonlinear systems by fuzzy sets and fuzzy reasoning, and many useful linear control methods can be developed for its control problems [7,8]. Recently, the stability analysis of nonlinear LS systems has been extensively studied using TSFM, and many important results have been presented on such systems. For instance, in [9], adaptive decentralized fuzzy dynamic surface control for switched nonlinear LS systems was carried out. Decentralized tracking control for networked LS systems was investigated in [10]. The decentralized event-triggered online adaptive control of unknown LS systems was considered in [11].

On another research front, descriptor systems can describe the dynamic model more accurately and completely than a state–space system [12]. However, the descriptor matrix may affect the structure and behavior of the system. If there exists perturbation in the descriptor matrix, it may cause the system to become unstable. If the descriptor matrix is defined as a singular matrix, the impulse-free and regular problems of the system must be considered [13]; otherwise, the system will not be stable. Recently, descriptor systems have been extended to the area of the LS system and TSFM. To mention a few, disturbance estimation for the discrete-time IT-2 TSLSD system has been investigated in [14]. The authors in [15] presented the passive decentralized fuzzy control for TSLSD systems.

It is well-known that the PDSF is a useful and important method for descriptor systems [16]. In [17], the impulse behavior issue was eliminated through the PDSF controller. Then, the problem of impulse-free and regular can be easily solved by using the PDSF method. Additionally, the PDSF method is simpler than the current existing feedback methods when designing the controller for descriptor systems [18]. Recently, several important works based on the PDSF method have been reported [19,20]. However, the stability conditions proposed by the PDSF method are often formulated as bilinear matrix inequalities. This paper will convert the stability conditions into the LMI form through Schur's complement, and the stability conditions can be solved by using the LMI Toolbox.

In this paper, the GCC is considered in the controller design process. The essential idea of GCC is providing an upper bound on a given performance index, and thus the system is asymptotically stable while the system performance degradation is guaranteed to be less than this bound. Some important results on the GCC were studied in [21,22]. Furthermore, some important works based on disturbance attenuation problems have been reported, such as H_∞ and passivity control [23,24]. In [25,26], MHPPC was proposed. By defining the weighting parameters, a trade-off between H_∞ and passive performance can be made.

This paper deals with the decentralized MP control for LSD systems. Different from the conventional approaches, the PDSF scheme is considered to design the fuzzy controller. The main contributions of this paper are summarized below.

(1) The problem of decentralized PDSF control for TSLSD systems is studied with MHPPC for the first time;
(2) The GCC is introduced in Section 2. Based on the GCC, the system performance degradation is guaranteed to be less than this bound;
(3) The MP control for TSLSD systems is studied subject to the MHPPC and GCC;
(4) The example section shows the effectiveness of the proposed methods, techniques, and procedures.

The structure of this paper is organized as follows: The TSLSD system and the decentralized controller with the PDSF scheme are described in Section 2. In addition, some important definitions and lemmas are also introduced. The stability condition for the closed-loop TSLSD system with the decentralized MP control is presented in Section 3. Numerical examples are given for illustration in Section 4. Finally, Section 5 concludes this paper.

Notations: $He\{\boldsymbol{\eta}\}$ denotes the shorthand notation for $\boldsymbol{\eta} + \boldsymbol{\eta}^{\mathrm{T}}$. $\mathbf{I}_n$ is the identity matrix with $n \times n$ dimension. $\Re^n$ represents the n-dimensional Euclidean space. The symbol $*$ represents the symmetric item in block matrices. $\mathbf{E}^{\mathrm{T}}$ and $\mathbf{E}^{-1}$ stand for matrix transposition and matrix inversion. $\det(\mathbf{E})$ is the determinant of a matrix $\mathbf{E}$. $diag\{\mathbf{E}\}$ denotes the diagonal matrix $\mathbf{E}$.

2. System Description and PDSF Control Method

Consider an LSD system with external disturbance, which is composed of N nonlinear subsystems (Figure 1). The $i-$th interconnected subsystem can be described by the following TSFM:

Plant Rule. $\Re_i^l$: IF $z_{i1}(t)$ is X_{i1}^l and, $\cdots$, $z_{ig}(t)$ is X_{ig}^l, THEN

$$\mathbf{E}_{il}\dot{x}_i(t) = \mathbf{A}_{il}x_i(t) + \mathbf{B}_{il}u_i(t) + \mathbf{G}_{il}v_i(t) + \sum_{k=1,k\neq i}^{N} \overline{\mathbf{A}}_{ikl}x_k(t) \tag{1a}$$

$$y_i(t) = \mathbf{C}_{il}x_i(t) + \mathbf{D}_{il}v_i(t) \tag{1b}$$

where $x_i(t) \in \Re^{n_{xi}}$ denotes the system state of the subsystem. $u_i(t) \in \Re^{n_{ui}}$ denotes the control input of the subsystem. $v_i(t) \in \Re^{n_{vi}}$ is the i-th external disturbance. $X^l_{i\phi}(\phi = 1, 2, \ldots, g)$ are the fuzzy sets of the i-th subsystem. $z_{i1}(t), z_{i2}(t), \ldots, z_{ig}(t)$ are the premise variables of the i-th subsystem. r_i is the number of inference rules of the i-th subsystem. $\Re^l_i$ denotes the l-th fuzzy membership rules. The matrices $\mathbf{A}_{il}$, $\mathbf{B}_{il}$, $\mathbf{C}_{il}$, $\mathbf{D}_{il}$ and $\mathbf{G}_{il}$ are known real matrices with appropriate dimensions of the l-th fuzzy subsystem. $\mathbf{E}_{il}$ values are the descriptor matrices, if necessary, singular. $\overline{\mathbf{A}}_{ikl}$ is the interconnection matrix between the i-th and the k-th nonlinear subsystems.

$$\sum_{l=1}^{r_i} \mu_{il}\mathbf{E}_{il}\dot{x}_i(t) = \sum_{l=1}^{r_i} \mu_{il}\left\{\mathbf{A}_{il}x_i(t) + \mathbf{B}_{il}u_i(t) + \sum_{k=1,k\neq i}^{N} \overline{\mathbf{A}}_{ikl}(\mu_i)x_k(t) + \mathbf{G}_{il}v_i(t)\right\} \tag{2a}$$

$$y_i(t) = \sum_{l=1}^{r_i} \mu_{il}\{\mathbf{C}_{il}x_i(t) + \mathbf{D}_{il}v_i(t)\} \tag{2b}$$

where

$$\mu_{il}(z_i(t)) = \frac{\prod_{\phi=1}^{g} \mu_{il\phi}(z_{i\phi}(t))}{\sum_{\varsigma=1}^{r_i}\prod_{\phi=1}^{g} \mu_{i\varsigma\phi}(z_{i\phi}(t))} \geq 0 \tag{3}$$

with $\mu_{il\phi}(z_{i\phi}(t))$ is the grade of membership of $z_{i\phi}(t)$ in $X^l_{i\phi}$ and $\sum_{l=1}^{r_i} \mu_{il}(z_i(t)) = 1$. For notational simplicity, μ_{il} represents the abbreviation of $\mu_{il}(z_i(t))$ in the description presented in Figure 1.

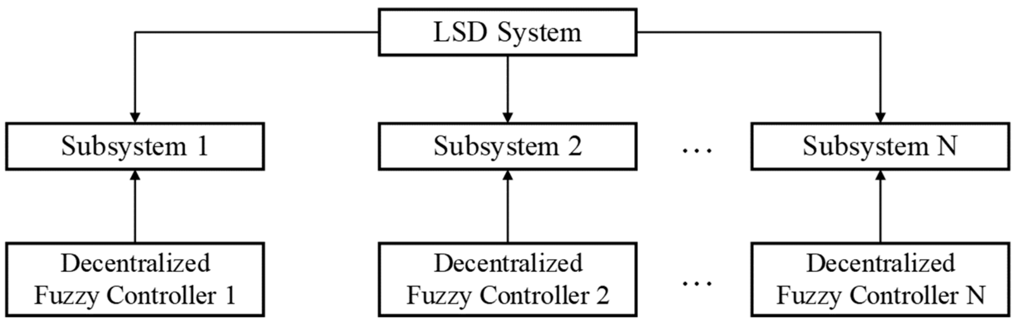

Figure 1. Decentralized control of a nonlinear LSD system.

Given the TSLSD system (2) in each region, we propose a decentralized MP controller of the following form:

$$u_i(t) = -\sum_{l=1}^{r_i} \mu_{il}\mathbf{F}_{dil}\dot{x}_i(t) + \sum_{l=1}^{r_i} \mu_{il}\mathbf{F}_{sil}x_i(t) \tag{4}$$

where $\mathbf{F}_{dil}$ and $\mathbf{F}_{sil}$ are the control gain matrices.

Applying the decentralized MP controller (4) to System (2a), the resulting closed-loop fuzzy subsystem can be cast into the following form:

$$\mathbf{E}_i(\mu_i)\dot{x}_i = \mathbf{A}_i(\mu_i)x_i(t) + \sum_{k=1,k\neq i}^{N} \overline{\mathbf{A}}_{ik}(\mu_i)x_k(t) + \mathbf{G}_i(\mu_i)v_i(t) \tag{5a}$$

$$y_i(t) = \mathbf{C}_i(\mu_i)x_i(t) + \mathbf{D}_i(\mu_i)v_i(t) \tag{5b}$$

where
$$\begin{cases} \mathbf{E}_i(\mu_i) = \sum_{l=1}^{r_i}\sum_{j=1}^{r_i}\mu_{il}\mu_{ij}\left\{\mathbf{E}_{il}+\mathbf{B}_{il}\mathbf{F}_{dij}\right\}, & \mathbf{C}_i(\mu_i) = \sum_{l=1}^{r_i}\mu_{il}\mathbf{C}_{il} \\ \mathbf{A}_i(\mu_i) = \sum_{l=1}^{r_i}\sum_{j=1}^{r_i}\mu_{il}\mu_{ij}\left\{\mathbf{A}_{il}+\mathbf{B}_{il}\mathbf{F}_{sij}\right\}, & \mathbf{D}_i(\mu_i) = \sum_{l=1}^{r_i}\mu_{il}\mathbf{D}_{il} \\ \bar{\mathbf{A}}_{ik}(\mu_i) = \sum_{l=1}^{r_i}\mu_{il}\bar{\mathbf{A}}_{ikl}, & \mathbf{G}_i(\mu_i) = \sum_{l=1}^{r_i}\mu_{il}\mathbf{G}_{il} \end{cases} \quad (6)$$

Now, we present some definitions and lemmas to obtain the main results.

Definition 1 [27]. *For all terminal time $t_p > 0$ and non-zero $v_i(t) \in L_2[0, \ \infty)$, if under zero conditions and exists a scalar $\gamma_i > 0$ such the following inequality holds, the TSLSD system (5) is considered to have MHPPC with $\gamma_i > 0$.*

$$\int_0^{t_p} -\gamma_i^{-1}\theta_i y_i^{\mathrm{T}}(t)y_i(t)ds + \int_0^{t_p} 2(1-\theta_i)y_i^{\mathrm{T}}(t)v_i(t)ds \geq -\gamma_i\int_0^{t_p} v_i^{\mathrm{T}}(s)v_i(s)ds \quad (7)$$

where $\theta_i \in [0, \ 1]$ represents a weighting parameter that defines the trade-off between the H_∞ and passivity performance.

Given a set of positive definite matrices $\mathbf{Z}_{i1}$, $\mathbf{Z}_{i2}$, and $\mathbf{R}_{i1}$, one can define the following cost function:

$$\mathbf{J}_{ic} = \int_0^{\infty}\left\{x_i^{\mathrm{T}}(t)\mathbf{Z}_{i1}x_i(t) + \dot{x}_i^{\mathrm{T}}(t)\mathbf{Z}_{i2}\dot{x}_i(t) + u_i^{\mathrm{T}}(t)\mathbf{R}_{i1}u_i(t)\right\}dt \quad (8)$$

Based on the cost function (8), the GCC is defined as follows:

Definition 2 [28]. *Consider System (5). System (5) is asymptotically stable if there exists a PDSF controller (4) and a positive scalar $\mathbf{J}_{i0}$, such that for all derivative matrices, $\mathbf{E}_i(\mu_i)$ in System (5) is invertible, and (4) is considered to be a GCC for System (5) if the cost function (8) satisfies $\mathbf{J}_i \leq \mathbf{J}_{i0}$.*

Definition 3. *If the following problem has a solution, then the minimization of output energy λ_i implies the minimization of the GCC (8) for System (5).*

$$\mathbf{J}_{ic} < \min\left(\lambda_i\right) \quad (9)$$

Lemma 1 [15]. *The following inequality holds for every real vector ζ, ρ, and any matrix $Z > 0$.*

$$2\zeta^{\mathrm{T}}\rho \leq \zeta^{\mathrm{T}}Z\zeta + \rho^{\mathrm{T}}Z^{-1}\rho \quad (10)$$

where Z is the definite positive matrix.

Lemma 2 [15]. *The following results can be obtained for any positive semidefinite symmetric matrix $\mathbf{X}$, two positive integers r, r_0 satisfying $r \geq r_0 \geq 1$.*

$$\left(\sum_{k=r_0}^{r} x(k)\right)^{\mathrm{T}}\mathbf{X}\left(\sum_{k=r_0}^{r} x(k)\right) \leq (r-r_0+1)\sum_{k=r_0}^{r} x^{\mathrm{T}}(k)\mathbf{X}x(k) \quad (11)$$

3. Decentralized MP Control of TSLSD System

In this section, a decentralized MP controller is designed for the TSLSD system (2). Referring to the MHPPC proposed in Definition 1, the following theorem can be obtained:

Theorem 1. *For given scalars γ_i, $\theta_i \in [0,\quad 1]$, if there exists the positive definite matrix $\mathbf{Q}_i$, free-weighting matrices $\mathbf{L}_1$, $\mathbf{L}_2$, $\mathbf{L}_3$, and controller gains $\mathbf{F}_{sil}$, $\mathbf{F}_{sij}$, $\mathbf{F}_{dil}$, and $\mathbf{F}_{dij}$, such that the following conditions hold for all $i = \{1,\ 2, \cdots,\ N\}$ and $l = \{1,\ 2, \cdots,\ r_i\}$:*

$$\Theta_{ill} < 0\,,\ \text{for } l = 1\cdots r_i \tag{12}$$

$$\Theta_{ilj} + \Theta_{ijl} < 0\,,\ \text{for } l < j = 1\cdots r_i \tag{13}$$

where

$$\Theta_{ill} = \begin{bmatrix} \mathbf{X}_{l11} & * & * & * & * & * \\ \mathbf{X}_{l21} & \mathbf{X}_{l22} & * & * & * & * \\ \mathbf{X}_{l31} & -\mathbf{G}_{il}^{\mathrm{T}} & \mathbf{X}_{l33} & * & * & * \\ \sqrt{\theta_i}\mathbf{C}_{il} & 0 & \sqrt{\theta_i}\mathbf{D}_{il} & -\gamma_i\mathbf{I} & * & * \\ \mathbf{L}_{i3}^{\mathrm{T}} & 0 & 0 & 0 & -\mathbf{I}/2 & * \\ \widetilde{\mathbf{A}}_{ki}\mathbf{Q}_i & 0 & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix},$$

$$\Theta_{ilj} = \begin{bmatrix} \mathbf{X}_{l11} & * & * & * & * & * \\ \mathbf{X}_{j21} & \mathbf{X}_{j22} & * & * & * & * \\ \mathbf{X}_{l31} & -\mathbf{G}_{il}^{\mathrm{T}} & \mathbf{X}_{l33} & * & * & * \\ \sqrt{\theta_i}\mathbf{C}_{il} & 0 & \sqrt{\theta_i}\mathbf{D}_{il} & -\gamma_i\mathbf{I} & * & * \\ \mathbf{L}_{i3}^{\mathrm{T}} & 0 & 0 & 0 & -\mathbf{I}/2 & * \\ \widetilde{\mathbf{A}}_{ki}\mathbf{Q}_i & 0 & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix},$$

$$\mathbf{L}_{i1}^{\mathrm{T}} = -\mathbf{Q}_i\mathbf{S}_{i1}\mathbf{S}_{i2}^{-1},\ \mathbf{L}_{i2}^{\mathrm{T}} = -\mathbf{S}_{i2}^{-1},\ \mathbf{L}_{i3}^{\mathrm{T}} = \mathbf{S}_{i1}^{\mathrm{T}}\mathbf{Q}_i,\ \mathbf{L}_{i4} = He(\mathbf{L}_{i1}) + 2\mathbf{L}_{i3}\mathbf{L}_{i3}^{\mathrm{T}} + 2\widetilde{\Phi}_{ki},\ \mathbf{L}_{i5} = \mathbf{L}_{i2}^{\mathrm{T}} + \mathbf{L}_{i3}^{\mathrm{T}},$$

$$\mathbf{Q}_i = \mathbf{P}_i^{-1},\ \mathbf{X}_{l11} = He(\mathbf{L}_{i1}),\ \mathbf{X}_{l21} = \mathbf{L}_{i5} - \mathbf{A}_{il}\mathbf{Q}_i + \mathbf{E}_{il}\mathbf{L}_{i1} - \mathbf{B}_{il}\mathbf{K}_{1il}, \mathbf{X}_{j21} = \mathbf{L}_{i5} - \mathbf{A}_{il}\mathbf{Q}_i + \mathbf{E}_{il}\mathbf{L}_{i1} - \mathbf{B}_{il}\mathbf{K}_{1ij},$$

$$\mathbf{X}_{l22} = \mathbf{I} + He(\mathbf{E}_{il}\mathbf{L}_{i2} + \mathbf{B}_{il}\mathbf{K}_{2il}), \mathbf{X}_{j22} = \mathbf{I} + He\big(\mathbf{E}_{il}\mathbf{L}_{i2} + \mathbf{B}_{il}\mathbf{K}_{2ij}\big),\ \mathbf{X}_{l31} = -(1-\theta_i)\mathbf{C}_{il},$$

$$\mathbf{X}_{l33} = -\gamma_i\mathbf{I} - 2(1-\theta_i)\mathbf{D}_{il}^{\mathrm{T}},\ \mathbf{K}_{1il} = \mathbf{F}_{sil}\mathbf{Q}_i - \mathbf{F}_{dil}\mathbf{L}_{i1}, \mathbf{K}_{2il} = \mathbf{F}_{dil}\mathbf{L}_{i2},\ \mathbf{K}_{1ij} = \mathbf{F}_{sij}\mathbf{Q}_i - \mathbf{F}_{dij}\mathbf{L}_{i1},$$

$$\mathbf{K}_{2ij} = \mathbf{F}_{dij}\mathbf{L}_{i2},\ \mathbf{X}_{66} = -2(N-1)^{-1}\varepsilon,\ \varepsilon = diag\underbrace{\left[\mathbf{I}_{nxi}\cdots\mathbf{I}_{nxi}\right]}_{N-1},\ \widetilde{\mathbf{A}}_{ki} = \underbrace{\left[\overline{\mathbf{A}}_{1i}^{\mathrm{T}}\cdots\overline{\mathbf{A}}_{ki,k\neq i}^{\mathrm{T}}\cdots\overline{\mathbf{A}}_{Ni}^{\mathrm{T}}\right]}_{N-1}^{\mathrm{T}}.$$

Then, System (5) is stable and also satisfies MHPPC.

Proof 1. Choose the following Lyapunov function for the TSLSD system (5):

$$\sum_{i=1}^{N} V_i(x_i(t)) = \sum_{i=1}^{N} x_i^{\mathrm{T}}(t)\mathbf{P}_i x_i(t) \tag{14}$$

where $\mathbf{P}_i$ is the positive definite matrix.

The following equation can be obtained according to (5a) with $\mathbf{S}_{i1}$ and $\mathbf{S}_{i2}$:

$$\begin{aligned} \sum_{i=1}^{N}\mathbf{\Lambda}_i \triangleq 2\Big[x_i^{\mathrm{T}}(t)\mathbf{S}_{i1} + \dot{x}_i^{\mathrm{T}}(t)\mathbf{S}_{i2}\Big] \times \Big[-\mathbf{E}_i(\mu_i)\dot{x}_i + \mathbf{A}_i(\mu_i)x_i(t)\Big] \\ + \sum_{k=1,k\neq i}^{N}\overline{\mathbf{A}}_{ik}(\mu_i)x_k(t) + \mathbf{G}_i(\mu_i)v_i(t)] \equiv 0 \end{aligned} \tag{15}$$

where $\mathbf{S}_{i1}$ and $\mathbf{S}_{i2}$ are the free-weighting matrices with appropriate dimensions.

Bring (15) into the derivative of (14).

$$\sum_{i=1}^{N}\dot{V}_i(x_i(t)) = \sum_{i=1}^{N}\Big\{2\dot{x}_i^{\mathrm{T}}(t)\mathbf{P}_i x_i(t) + 2\big[x_i^{\mathrm{T}}(t)\mathbf{S}_{i1} + \dot{x}_i^{\mathrm{T}}(t)\mathbf{S}_{i2}\big] \times\Big[-\mathbf{E}_i(\mu_i)\dot{x}_i + \mathbf{A}_i(\mu_i)x_i(t) + \sum_{k=1,k\neq i}^{N}\bar{\mathbf{A}}_{ik}(\mu_i)x_k(t) + \mathbf{G}_i(\mu_i)v_i(t)\Big]\Big\} \tag{16}$$

According to [15], the Lyapunov function (16) can be rewritten as follows by using Lemma 1 and Lemma 2:

$$\sum_{i=1}^{N}\dot{V}_i(x_i(t)) \leq \sum_{i=1}^{N}\tilde{x}_i^{\mathrm{T}}\Xi_i\tilde{x}_i \tag{17}$$

where $\tilde{x}_i^{\mathrm{T}} = \begin{bmatrix} x_i(t) & \dot{x}_i(t) & v_i(t)\end{bmatrix}^{\mathrm{T}}$, $\Phi_{ki} = (N-1)\sum_{k=1,k\neq i}^{N} x_i^{\mathrm{T}}(t)\overline{\mathbf{A}}_{ki}^{\mathrm{T}}\overline{\mathbf{A}}_{ki}x_i(t)$,

$$\Xi_i = \begin{bmatrix} He(\mathbf{S}_{i1}\mathbf{A}_i(\mu_i)) + \mathbf{S}_{i1}\mathbf{S}_{i1}^{\mathrm{T}} + 2\Phi_{ki} & * & * \\ \mathbf{P}_i + \mathbf{S}_{i2}\mathbf{A}_i(\mu_i) - \mathbf{E}_i^{\mathrm{T}}(\mu_i)\mathbf{S}_{i1}^{\mathrm{T}} & -He(\mathbf{S}_{i2}\mathbf{E}_i(\mu_i)) + \mathbf{S}_{i2}\mathbf{S}_{i2}^{\mathrm{T}} & * \\ \mathbf{G}_i^{\mathrm{T}}(\mu_i)\mathbf{S}_{i1}^{\mathrm{T}} & \mathbf{G}_i^{\mathrm{T}}(\mu_i)\mathbf{S}_{i2}^{\mathrm{T}} & 0 \end{bmatrix}.$$

Left-multiply and right-multiply the Ξ_i by $\begin{bmatrix} \mathbf{Q}_i & \mathbf{L}_{i1}^{\mathrm{T}} & 0 \\ 0 & \mathbf{L}_{i2}^{\mathrm{T}} & 0 \\ 0 & 0 & \mathbf{I}\end{bmatrix}$ and its transpose, and it yields:

$$\begin{bmatrix} \mathbf{L}_{i4} & * & * \\ \mathbf{L}_{i5} + \mathbf{E}_i(\mu_i)\mathbf{L}_{i1} - \mathbf{A}_i(\mu_i)\mathbf{Q}_i & He(\mathbf{E}_i(\mu_i)\mathbf{L}_{i2}) + \mathbf{I} & * \\ 0 & -\mathbf{G}_i^{\mathrm{T}}(\mu_i) & 0 \end{bmatrix} \tag{18}$$

where $\widetilde{\Phi}_{ki} = (N-1)\sum_{k=1,k\neq i}^{N} x_i^{\mathrm{T}}(t)\mathbf{Q}_i^{\mathrm{T}}\overline{\mathbf{A}}_{ki}^{\mathrm{T}}\overline{\mathbf{A}}_{ki}\mathbf{Q}_i x_i(t)$ and

$$\begin{cases} \mathbf{L}_{i1}^{\mathrm{T}} = -\mathbf{Q}_i\mathbf{S}_{i1}\mathbf{S}_{i2}^{-1}, & \mathbf{L}_{i4} = He(\mathbf{L}_{i1}) + 2\mathbf{L}_{i3}\mathbf{L}_{i3}^{\mathrm{T}} + 2\widetilde{\Phi}_{ki}, \\ \mathbf{L}_{i2}^{\mathrm{T}} = -\mathbf{S}_{i2}^{-1}, & \mathbf{L}_{i5} = \mathbf{L}_{i2}^{\mathrm{T}} + \mathbf{L}_{i3}^{\mathrm{T}}, \\ \mathbf{L}_{i3}^{\mathrm{T}} = \mathbf{S}_{i1}^{\mathrm{T}}\mathbf{Q}_i, & \mathbf{Q}_i = \mathbf{P}_i^{-1} \end{cases}.$$

Substituting (6) into (18) and considering the fuzzy membership functions, then

$$\sum_{i=1}^{N}\dot{V}_i(x_i(t)) \leq \sum_{l=1}^{r_i}\mu_{il}^2\tilde{x}^{\mathrm{T}}\widetilde{\Xi}_{ill}\tilde{x} + \sum_{l<j}^{r}\mu_{il}\mu_{ij}\tilde{x}^{\mathrm{T}}\Big\{\widetilde{\Xi}_{ilj} + \widetilde{\Xi}_{ijl}\Big\}\tilde{x} \tag{19}$$

where

$$\widetilde{\Xi}_{ill} = \begin{bmatrix} \mathbf{L}_{i4} & * & * \\ \mathbf{Z}_{l1} & \mathbf{Z}_{l2} & * \\ 0 & -\mathbf{G}_{il}^{\mathrm{T}} & 0 \end{bmatrix} \tag{20}$$

$$\widetilde{\Xi}_{ilj} = \begin{bmatrix} \mathbf{L}_{i4} & * & * \\ \mathbf{Z}_{j1} & \mathbf{Z}_{j2} & * \\ 0 & -\mathbf{G}_{il}^{\mathrm{T}} & 0 \end{bmatrix} \tag{21}$$

$\mathbf{Z}_{l1} = \mathbf{L}_{i5} - (\mathbf{A}_{il} + \mathbf{B}_{il}\mathbf{F}_{sil})\mathbf{Q}_i + (\mathbf{E}_{il} + \mathbf{B}_{il}\mathbf{F}_{dil})\mathbf{L}_{i1}$, $\mathbf{Z}_{j1} = \mathbf{L}_{i5} - \left(\mathbf{A}_{il} + \mathbf{B}_{il}\mathbf{F}_{sij}\right)\mathbf{Q}_i + \left(\mathbf{E}_{il} + \mathbf{B}_{il}\mathbf{F}_{dij}\right)\mathbf{L}_{i1}$,

$$\mathbf{Z}_{l2} = \mathbf{I} + He((\mathbf{E}_{il} + \mathbf{B}_{il}\mathbf{F}_{dil})\mathbf{L}_2) \text{ and } \mathbf{Z}_{j2} = \mathbf{I} + He\Big(\Big(\mathbf{E}_{il} + \mathbf{B}_{il}\mathbf{F}_{dij}\Big)\mathbf{L}_2\Big).$$

Based on Definition 1, the following cost function can be defined with zero initial condition:

$$\begin{aligned}\mathbf{J}_{im} &= \int_0^{t_p}\Big(\gamma_i^{-1}\theta_i y_i^{\mathrm{T}}(t)y_i(t) - 2(1-\theta_i)y_i^{\mathrm{T}}(t)v_i(t) - \gamma_i v_i^{\mathrm{T}}(t)v_i(t)\Big)dt \\ &= \int_0^{t_p}\Big(\gamma_i^{-1}\theta_i y_i^{\mathrm{T}}(t)y_i(t) - 2(1-\theta_i)y_i^{\mathrm{T}}(t)v_i(t) - \gamma_i v_i^{\mathrm{T}}(t)v_i(t) \\ &\qquad + \sum_{i=1}^{N}\dot{V}_i(x_i(t))dt - V_i\big(x_i(t_p)\big)\Big)\end{aligned} \tag{22}$$

$$\leq \int_0^{t_p}\Psi_i dt \tag{23}$$

where

$$\Psi_i = \gamma_i^{-1}\theta_i y_i^{\mathrm{T}}(t)y_i(t) - 2(1-\theta_i)y_i^{\mathrm{T}}(t)v_i(t) - \gamma_i v_i^{\mathrm{T}}(t)v_i(t) + \sum_{i=1}^{N}\dot{V}_i(x_i(t)) \tag{24}$$

Substituting (2b) and (19) into (24), then

$$\Psi_i = \sum_{l=1}^{r_i}\mu_{il}^2\widetilde{x}^{\mathrm{T}}\widetilde{\Xi}_{ill}\widetilde{x} + \sum_{l<j}^{r}\mu_{il}\mu_{ij}\widetilde{x}^{\mathrm{T}}\Big\{\widetilde{\Xi}_{ilj} + \Xi_{ijl}\Big\}\widetilde{x} \tag{25}$$

where

$$\widetilde{\Xi}_{ill} = \begin{bmatrix}\mathbf{L}_{i4} & * & * & * \\ \mathbf{Z}_{l1} & He((\mathbf{E}_{il} + \mathbf{B}_{il}\mathbf{F}_{dil})\mathbf{L}_2) + \mathbf{I} & * & * \\ \mathbf{X}_{l31} & -\mathbf{G}_{il}^{\mathrm{T}} & \mathbf{X}_{l33} & * \\ \sqrt{\theta_i}\mathbf{C}_{il} & 0 & \sqrt{\theta_i}\mathbf{D}_{il} & -\gamma_i\mathbf{I}\end{bmatrix} \tag{26}$$

$$\widetilde{\Xi}_{ilj} = \begin{bmatrix}\mathbf{L}_{i4} & * & * & * \\ \mathbf{Z}_{j1} & He\Big(\Big(\mathbf{E}_{il} + \mathbf{B}_{il}\mathbf{F}_{dij}\Big)\mathbf{L}_2\Big) + \mathbf{I} & * & * \\ \mathbf{X}_{l31} & -\mathbf{G}_{il}^{\mathrm{T}} & \mathbf{X}_{l33} & * \\ \sqrt{\theta_i}\mathbf{C}_{il} & 0 & \sqrt{\theta_i}\mathbf{D}_{il} & -\gamma_i\mathbf{I}\end{bmatrix} \tag{27}$$

$$\mathbf{X}_{l31} = -(1-\theta_i)\mathbf{C}_{il} \text{ and } \mathbf{X}_{l33} = -\gamma_i\mathbf{I} - 2(1-\theta_i)\mathbf{D}_{il}^{\mathrm{T}}.$$

The matrices (26) and (27) can be written as follows with $\mathbf{K}_{1il} = \mathbf{F}_{sil}\mathbf{Q}_i - \mathbf{F}_{dil}\mathbf{L}_1$, $\mathbf{K}_{2il} = \mathbf{F}_{dil}\mathbf{L}_2$, $\mathbf{K}_{1ij} = \mathbf{F}_{sij}\mathbf{Q}_i - \mathbf{F}_{dij}\mathbf{L}_1$ and $\mathbf{K}_{2ij} = \mathbf{F}_{dij}\mathbf{L}_2$:

$$\widetilde{\Xi}_{ill} = \begin{bmatrix}\mathbf{L}_{i4} & * & * & * \\ \mathbf{X}_{l21} & \mathbf{X}_{l22} & * & * \\ \mathbf{X}_{l31} & -\mathbf{G}_{il}^{\mathrm{T}} & \mathbf{X}_{l33} & * \\ \sqrt{\theta_i}\mathbf{C}_{il} & 0 & \sqrt{\theta_i}\mathbf{D}_{il} & -\gamma_i\mathbf{I}\end{bmatrix} \tag{28}$$

$$\widetilde{\Xi}_{ill} = \begin{bmatrix}\mathbf{L}_{i4} & * & * & * \\ \mathbf{X}_{j21} & \mathbf{X}_{j22} & * & * \\ \mathbf{X}_{l31} & -\mathbf{G}_{il}^{\mathrm{T}} & \mathbf{X}_{l33} & * \\ \sqrt{\theta_i}\mathbf{C}_{il} & 0 & \sqrt{\theta_i}\mathbf{D}_{il} & -\gamma_i\mathbf{I}\end{bmatrix} \tag{29}$$

where

$$\mathbf{X}_{l21} = \mathbf{L}_{i5} - \mathbf{A}_{il}\mathbf{Q}_i + \mathbf{E}_{il}\mathbf{L}_{i1} - \mathbf{B}_{il}\mathbf{K}_{1il},$$

$$\mathbf{X}_{j21} = \mathbf{L}_{i5} - \mathbf{A}_{il}\mathbf{Q}_i + \mathbf{E}_{il}\mathbf{L}_{i1} - \mathbf{B}_{il}\mathbf{K}_{1ij},$$
$$\mathbf{X}_{l22} = \mathbf{I} + He(\mathbf{E}_{il}\mathbf{L}_{i2} + \mathbf{B}_{il}\mathbf{K}_{2il}),$$
$$\mathbf{X}_{j22} = \mathbf{I} + He\big(\mathbf{E}_{il}\mathbf{L}_{i2} + \mathbf{B}_{il}\mathbf{K}_{2ij}\big).$$

By using the Schur complement, the conditions (28) and (29) can be rewritten as follows:

$$\widetilde{\Xi}_{ill} = \begin{bmatrix} \mathbf{X}_{l11} & * & * & * & * & * \\ \mathbf{X}_{l21} & \mathbf{X}_{l22} & * & * & * & * \\ \mathbf{X}_{l31} & -\mathbf{G}_{il}^{\mathrm{T}} & \mathbf{X}_{l33} & * & * & * \\ \sqrt{\theta_i}\mathbf{C}_{il} & 0 & \sqrt{\theta_i}\mathbf{D}_{il} & -\gamma_i\mathbf{I} & * & * \\ \mathbf{L}_{i3}^{\mathrm{T}} & 0 & 0 & 0 & -\mathbf{I}/2 & * \\ \widetilde{\mathbf{A}}_{ki}\mathbf{Q}_i & 0 & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix} \tag{30}$$

and

$$\widetilde{\Xi}_{ilj} = \begin{bmatrix} \mathbf{X}_{l11} & * & * & * & * & * \\ \mathbf{X}_{j21} & \mathbf{X}_{j22} & * & * & * & * \\ \mathbf{X}_{l31} & -\mathbf{G}_{il}^{\mathrm{T}} & \mathbf{X}_{l33} & * & * & * \\ \sqrt{\theta_i}\mathbf{C}_{il} & 0 & \sqrt{\theta_i}\mathbf{D}_{il} & -\gamma_i\mathbf{I} & * & * \\ \mathbf{L}_{i3}^{\mathrm{T}} & 0 & 0 & 0 & -\mathbf{I}/2 & * \\ \widetilde{\mathbf{A}}_{ki}\mathbf{Q}_i & 0 & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix} \tag{31}$$

where $\mathbf{X}_{l11} = He(\mathbf{L}_{i1})$, $\mathbf{X}_{66} = -2(N-1)^{-1}\varepsilon$.

Notice that (12) and (13) are equivalent to $\widetilde{\Xi}_{ill} < 0$ and $\widetilde{\Xi}_{ilj} + \widetilde{\Xi}_{ijl} < 0$, which implies that $\Psi_i < 0$ from (25). From (23), the $\Psi_i < 0$ means

$$\gamma_i^{-1}\theta_i y_i^{\mathrm{T}}(t)y_i(t) - 2(1-\theta_i)y_i^{\mathrm{T}}(t)v_i(t) - \gamma_i v_i^{\mathrm{T}}(t)v_i(t) + \sum_{i=1}^{N}\dot{V}_i(x_i(t)) < 0 \tag{32}$$

Integrating (32) from 0 to t_p leads to

$$\mathbf{J}_{im} < -V_i\big(x_i(t_p)\big) + V_i(x_i(0)) \tag{33}$$

which implies that $\mathbf{J}_{im} < 0$ because under zero initial condition $V_i(x_i(0)) = 0$ and $V_i\big(x_i(t_p)\big) \geq 0$. Thus, if Condition (33) is satisfied for all terminal times $t_p > 0$, System (5) has MHPPC with γ_i, and this completes the proof. □

In this section, we design a fuzzy controller that satisfies the performance of MHPPC, and in the next section, we will consider the GCC performance for the fuzzy controller to reduce the output cost of the controller. Based on Definition 2 and Definition 3, the following result can be obtained:

Theorem 2. *For the given scalars $\lambda_i > 0$, matrices $\mathbf{R}_{i1}$, $\mathbf{Z}_{i1}$, and $\mathbf{Z}_{i2}$, the positive definite matrix $\mathbf{Q}_i$, free-weighting matrices $\mathbf{L}_1$, $\mathbf{L}_2$, and $\mathbf{L}_3$, and the controller gains $\mathbf{F}_{sil}$, $\mathbf{F}_{sij}$, $\mathbf{F}_{dil}$, and $\mathbf{F}_{dij}$, such that the following conditions hold for all $i = \{1, 2, \cdots, N\}$ and $l = \{1, 2, \cdots, r_i\}$:*

$$\widetilde{\Theta}_{ill} < 0\,,\ \text{for } l = 1\cdots r_i \tag{34}$$

$$\widetilde{\Theta}_{ilj} + \widetilde{\Theta}_{ijl} < 0\,,\ \text{for } l < j = 1\cdots r_i \tag{35}$$

$$\begin{bmatrix} -\min(\lambda_i) & * \\ x_i(0) & -\mathrm{Q}_i \end{bmatrix} < 0 \tag{36}$$

where

$$\widetilde{\Theta}_{ill} = \begin{bmatrix} \mathbf{X}_{l11} & * & * & * & * & * & * \\ \mathbf{X}_{l21} & \mathbf{X}_{l22} & * & * & * & * & * \\ \mathbf{K}_{1il} & \mathbf{K}_{2il} & -\mathbf{R}_{i1}^{-1} & * & * & * & * \\ \mathbf{Q}_i & 0 & 0 & -\mathbf{Z}_{i1}^{-1}/2 & * & * & * \\ \mathbf{L}_{i1} & \mathbf{L}_{i2} & 0 & 0 & -\mathbf{Z}_{i2}^{-1}/2 & * & * \\ \mathbf{L}_{i3}^{\mathrm{T}} & 0 & 0 & 0 & 0 & -\mathbf{I}/2 & * \\ \widetilde{\mathbf{A}}_{ki}\mathbf{Q}_i & 0 & 0 & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix}$$

$$\widetilde{\Theta}_{ilj} = \begin{bmatrix} \mathbf{X}_{l11} & * & * & * & * & * & * \\ \mathbf{X}_{j21} & \mathbf{X}_{j22} & * & * & * & * & * \\ \mathbf{K}_{1ij} & \mathbf{K}_{2ij} & -\mathbf{R}_{i1}^{-1} & * & * & * & * \\ \mathbf{Q}_i & 0 & 0 & -\mathbf{Z}_{i1}^{-1}/2 & * & * & * \\ \mathbf{L}_{i1} & \mathbf{L}_{i2} & 0 & 0 & -\mathbf{Z}_{i2}^{-1}/2 & * & * \\ \mathbf{L}_{i3}^{\mathrm{T}} & 0 & 0 & 0 & 0 & -\mathbf{I}/2 & * \\ \widetilde{\mathbf{A}}_{ki}\mathbf{Q}_i & 0 & 0 & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix}$$

then the decentralized MP controller (4) is considered to be the GCC for System (5).

Proof 2. Let us assume $v_i(t) = 0$; the following Lyapunov function can be obtained by arranging (17):

$$\sum_{i=1}^{N} \dot{V}_c(x_i(t)) \le \sum_{i=1}^{N} \hat{x}_i^{\mathrm{T}} \hat{\Xi}_i \hat{x}_i \tag{37}$$

where $\hat{x}_i^{\mathrm{T}} = \begin{bmatrix} x_i(t) & \dot{x}_i(t) \end{bmatrix}^{\mathrm{T}}$, $\hat{\Xi}_i = \begin{bmatrix} He(\mathbf{S}_{i1}\mathbf{A}_i(\mu_i)) + \mathbf{S}_{i1}\mathbf{S}_{i1}^{\mathrm{T}} + 2\Phi_{ki} & * \\ \mathbf{P}_i - \mathbf{E}_i^{\mathrm{T}}(\mu_i)\mathbf{S}_{i1}^{\mathrm{T}} + \mathbf{S}_{i2}\mathbf{A}_i(\mu_i) & -He(\mathbf{S}_{i2}\mathbf{E}_i(\mu_i)) + \mathbf{S}_{i2}\mathbf{S}_{i2}^{\mathrm{T}} \end{bmatrix}$.

Based on the Lyapunov function (37) and adding the cost function defined by (8) to it gives

$$\begin{aligned} \mathbf{J}_{ic} &= \int_0^{t_p} \Big(x_i^{\mathrm{T}}(t)\mathbf{Z}_{i1}x_i(t) + \dot{x}_i^{\mathrm{T}}(t)\mathbf{Z}_{i2}\dot{x}_i(t) + u^{\mathrm{T}}(t)\mathbf{R}_{i1}u(t) \Big) dt \\ &= \int_0^{t_p} x_i^{\mathrm{T}}(t)\mathbf{Z}_{i1}x_i(t) + x_i^{\mathrm{T}}(t)\mathbf{Z}_{i2}\dot{x}_i(t) \\ &\quad + u_i^{\mathrm{T}}(t)\mathbf{R}_{i1}u_i(t) + \sum_{i=1}^{N} \dot{V}_c(x_i(t))dt - \dot{V}_c(x_i(t_p)) \end{aligned} \tag{38}$$

$$\le \int_0^{t_p} \Psi_{ic} dt \tag{39}$$

where

$$\Psi_{ic} = x_i^{\mathrm{T}}(t)\mathbf{Z}_{i1}x_i(t) + \dot{x}_i^{\mathrm{T}}(t)\mathbf{Z}_{i2}\dot{x}_i(t) + u_i^{\mathrm{T}}(t)\mathbf{R}_{i1}u_i(t) + \sum_{i=1}^{N} \dot{V}_c(x_i(t)) \tag{40}$$

Substituting controllers (4) and (37) into (40), one has

$$\Psi_{ic} = \begin{bmatrix} \mathbf{N}_{i1} & * \\ \mathbf{N}_{i2} & \mathbf{N}_{i3} \end{bmatrix} \tag{41}$$

where

$$\mathbf{N}_{i1} = He(\mathbf{S}_{i1}\mathbf{A}_i(\mu_i)) + \mathbf{S}_{i1}\mathbf{S}_{i1}^{\mathrm{T}} + 2\Phi_{ki} + \mathbf{Z}_{i1} + \mathbf{F}_{sij}^{\mathrm{T}}\mathbf{R}_{i1}\mathbf{F}_{sij},$$

$$\mathbf{N}_{i2} = \mathbf{P}_i - \mathbf{E}_i^{\mathrm{T}}(\mu_i)\mathbf{S}_{i1}^{\mathrm{T}} + \mathbf{S}_{i2}\mathbf{A}_i(\mu_i) - \mathbf{F}_{dij}^{\mathrm{T}}\mathbf{R}_{i1}\mathbf{F}_{sij},$$

$$\mathbf{N}_{i3} = -He(\mathbf{S}_{i2}\mathbf{E}_i(\mu_i)) + \mathbf{S}_{i2}\mathbf{S}_{i2}^{\mathrm{T}} + \mathbf{Z}_{i2} + \mathbf{F}_{dij}^{\mathrm{T}}\mathbf{R}_{i1}\mathbf{F}_{dij}.$$

Left-multiply and right-multiply the (41) with $\begin{bmatrix} \mathbf{Q}_i & \mathbf{L}_{i1}^{\mathrm{T}} \\ 0 & \mathbf{L}_{i2}^{\mathrm{T}} \end{bmatrix}$ and its transpose, and it yields:

$$\begin{bmatrix} \hat{\mathbf{N}}_{i1} & * \\ \hat{\mathbf{N}}_{i2} & \hat{\mathbf{N}}_{i3} \end{bmatrix} \tag{42}$$

where

$$\hat{\mathbf{N}}_{j11} = \mathbf{L}_{i4} + \mathbf{Q}_i \mathbf{Z}_{i1} \mathbf{Q}_i + \mathbf{L}_{i1}^{\mathrm{T}} \mathbf{Z}_{i2} \mathbf{L}_{i1} + \mathbf{K}_{1ij}^{\mathrm{T}} \mathbf{R}_{i1} \mathbf{K}_{1ij}$$

$$\hat{\mathbf{N}}_{j21} = \mathbf{L}_{i5} - \mathbf{A}_i(\mu_i)\mathbf{Q}_i + \mathbf{E}_i(\mu_i)\mathbf{L}_{i1} + \mathbf{L}_{i2}^{\mathrm{T}} \mathbf{Z}_{i2} \mathbf{L}_{i1} - \mathbf{K}_{2ij}^{\mathrm{T}} \mathbf{R}_{i1} \mathbf{K}_{1ij}$$

$$\hat{\mathbf{N}}_{j22} = \mathbf{I} + He(\mathbf{E}_i(\mu_i)\mathbf{L}_{i2}) + \mathbf{L}_{i2}^{\mathrm{T}} \mathbf{Z}_{i2} \mathbf{L}_{i2} + \mathbf{K}_{2ij}^{\mathrm{T}} \mathbf{R}_{i1} \mathbf{K}_{2ij}$$

Substituting (6) into matrix (42) and extracting the membership functions, we have

$$\Psi_{ic} \leq \sum_{l=1}^{r_i} \mu_{il}^2 \tilde{x}^{\mathrm{T}} \hat{\Xi}_{ill} \tilde{x} + \sum_{l<j}^{r} \mu_{il}\mu_{ij} \tilde{x}^{\mathrm{T}} \left\{ \hat{\Xi}_{ilj} + \hat{\Xi}_{ijl} \right\} \tilde{x} \tag{43}$$

where

$$\hat{\Xi}_{ill} = \begin{bmatrix} \hat{\mathbf{N}}_{l11} & * \\ \mathbf{Z}_{l1} + \mathbf{L}_2^{\mathrm{T}} \mathbf{Z}_2 \mathbf{L}_1 - \mathbf{K}_{2il}^{\mathrm{T}} \mathbf{R}_1 \mathbf{K}_{1il} & \mathbf{Z}_{l2} + \mathbf{L}_2^{\mathrm{T}} \mathbf{Z}_2 \mathbf{L}_2 + \mathbf{K}_{2il}^{\mathrm{T}} \mathbf{R}_1 \mathbf{K}_{2il} \end{bmatrix} \tag{44}$$

$$\hat{\Xi}_{ilj} = \begin{bmatrix} \hat{\mathbf{N}}_{j11} & * \\ \mathbf{Z}_{j1} + \mathbf{L}_2^{\mathrm{T}} \mathbf{Z}_2 \mathbf{L}_1 - \mathbf{K}_{2ij}^{\mathrm{T}} \mathbf{R}_1 \mathbf{K}_{1ij} & \mathbf{Z}_{j2} + \mathbf{L}_2^{\mathrm{T}} \mathbf{Z}_2 \mathbf{L}_2 + \mathbf{K}_{2ij}^{\mathrm{T}} \mathbf{R}_1 \mathbf{K}_{2ij} \end{bmatrix} \tag{45}$$

$$\hat{\mathbf{N}}_{l11} = \mathbf{L}_{i4} + \mathbf{Q}_i \mathbf{Z}_{i1} \mathbf{Q}_i + \mathbf{L}_{i1}^{\mathrm{T}} \mathbf{Z}_{i2} \mathbf{L}_{i1} + \mathbf{K}_{1il}^{\mathrm{T}} \mathbf{R}_{i1} \mathbf{K}_{1il}$$

The (44) can be written as follows:

$$\hat{\Xi}_{ill} = \mathbf{\Lambda}_{1ii} + \mathbf{\Lambda}_{2ii} + \mathbf{\Lambda}_{3ii} \tag{46}$$

where

$$\mathbf{\Lambda}_{1ii} = \begin{bmatrix} \mathbf{L}_{i4} & * \\ \mathbf{X}_{l21} & \mathbf{X}_{l22} \end{bmatrix},$$

$$\mathbf{\Lambda}_{2ii} = \begin{bmatrix} \mathbf{K}_{1il}^{\mathrm{T}} \\ -\mathbf{K}_{2il}^{\mathrm{T}} \end{bmatrix} \mathbf{R}_1 \begin{bmatrix} \mathbf{K}_{1il} & -\mathbf{K}_{2il} \end{bmatrix},$$

$$\mathbf{\Lambda}_{3ii} = \begin{bmatrix} \mathbf{Q}_i & \mathbf{L}_{i1}^{\mathrm{T}} \\ 0 & \mathbf{L}_{i2}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{Z}_{i1} & 0 \\ 0 & \mathbf{Z}_{i2} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_i & 0 \\ \mathbf{L}_{i1} & \mathbf{L}_{i2} \end{bmatrix}.$$

Then, by using the Schur complement, the following equation can be obtained:

$$\hat{\Xi}_{ill} = \begin{bmatrix} \mathbf{X}_{l11} & * & * & * & * & * & * \\ \mathbf{X}_{l21} & \mathbf{X}_{l22} & * & * & * & * & * \\ \mathbf{K}_{1il} & -\mathbf{K}_{2il} & -\mathbf{R}_{i1}^{-1} & * & * & * & * \\ \mathbf{Q}_i & 0 & 0 & -\mathbf{Z}_{i1}^{-1} & * & * & * \\ \mathbf{L}_{i1} & \mathbf{L}_{i2} & 0 & 0 & -\mathbf{Z}_{i2}^{-1} & * & * \\ \mathbf{L}_{i3}^{\mathrm{T}} & 0 & 0 & 0 & 0 & -\mathbf{I}/2 & * \\ \widetilde{\mathbf{A}}_{ki}\mathbf{Q}_i & 0 & 0 & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix} \tag{47}$$

Following the same procedure, the following equation can be obtained from (45):

$$\hat{\Xi}_{ilj} = \begin{bmatrix} \mathbf{X}_{l11} & * & * & * & * & * & * \\ \mathbf{X}_{j21} & \mathbf{X}_{j22} & * & * & * & * & * \\ \mathbf{K}_{1ij} & -\mathbf{K}_{2ij} & -\mathbf{R}_{i1}^{-1} & * & * & * & * \\ \mathbf{Q}_i & 0 & 0 & -\mathbf{Z}_{i1}^{-1} & * & * & * \\ \mathbf{L}_{i1} & \mathbf{L}_{i2} & 0 & 0 & -\mathbf{Z}_{i2}^{-1} & * & * \\ \mathbf{L}_{i3}^{\mathrm{T}} & 0 & 0 & 0 & 0 & -\mathbf{I}/2 & * \\ \widetilde{\mathbf{A}}_{ki}\mathbf{Q}_i & 0 & 0 & 0 & 0 & 0 & \mathbf{X}_{66} \end{bmatrix} \tag{48}$$

Notice that (34) and (35) are equivalent to $\hat{\Xi}_{ill} < 0$ and $\hat{\Xi}_{ilj} + \hat{\Xi}_{ijl} < 0$, which implies that $\Psi_{ic} < 0$ from (43). The $\Psi_{ic} < 0$ also means

$$x_i^{\mathrm{T}}(t)\mathbf{Z}_{i1}x_i(t) + \dot{x}_i^{\mathrm{T}}(t)\mathbf{Z}_{i2}\dot{x}_i(t) + u_i^{\mathrm{T}}(t)\mathbf{R}_{i1}u_i(t) + \sum_{i=1}^{N}\dot{V}_c(x_i(t)) < 0 \tag{49}$$

Integrating the (49) from 0 to t_p leads to

$$\mathbf{J}_{ic} < -V_c\big(x_i\big(t_p\big)\big) + V_c(x_i(0)) = \mathbf{J}_{i0} \tag{50}$$

where $\mathbf{J}_{i0} = x_i^{\mathrm{T}}(0)\mathbf{P}_i x_i(0)$.

The above inequality proves that conditions (34) and (35) achieve stability with the bounded cost (8) for System (5) without external disturbance. Next, the Schur complement is utilized (36) to obtain the following inequality:

$$x_i^{\mathrm{T}}(0)\mathbf{P}_i x_i(0) - \min(\lambda_i) < 0 \tag{51}$$

Or

$$x_i^{\mathrm{T}}(0)\mathbf{P}_i x_i(0) < \min(\lambda_i) \tag{52}$$

Based on (52), the following inequality can be found from (50):

$$\mathbf{J}_{ic} < \min(\lambda_i) \tag{53}$$

In Theorems 1 and 2, we consider HMPPC and GCC properties, respectively. In the following proposed theorems, both Theorem 1 and Theorem 2 are considered to design a controller that satisfies MP. □

Theorem 3. *For the given scalars γ_i, $\theta_i \in [0, \ 1]$, $\lambda_i > 0$, $\mathbf{R}_{i1}$ and matrices $\mathbf{Z}_{i1}$ and $\mathbf{Z}_{i2}$, if there exists the positive definite matrix $\mathbf{Q}_i$, free-weighting matrices $\mathbf{L}_1$, $\mathbf{L}_2$, and $\mathbf{L}_3$, and controller gains $\mathbf{F}_{sil}$, $\mathbf{F}_{sij}$, $\mathbf{F}_{dil}$, and $\mathbf{F}_{dij}$, such that the conditions (12), (13), (34), (35) and (36) hold for all $i = \{1, 2, \cdots, N\}$ and $l = \{1, 2, \cdots, r_i\}$. Then, the decentralized MP controller (4) is considered to be the GCC for System (5) and also satisfies MHPPC.*

Proof. If Theorem 3 is satisfied, then Theorem 1 and Theorem 2 are satisfied at the same time, which also means the system satisfies the GCC and MHPPC, and this completes the proof. □

Remark 1. *It should be pointed out that when the description matrix is defined as a singular matrix, the systems will become more complicated than the regular systems. In addition, the obtained results should guarantee the system is not only stable but also to be regular and impulse-free. In [15], the impulse-free and regular problems of the system are discussed in detail with the PDSF scheme.*

4. Simulation Results and Discussions

Simulation 1. *To illustrate the proposed approach, let us consider the following numerical example, which is composed of two interconnected subsystems:*

$$\sum_{l=1}^{r_i} \mu_{il} \mathbf{E}_{il} \dot{x}_i(t) = \sum_{l=1}^{r_i} \mu_{il} \left\{ \mathbf{A}_{il} x_i(t) + \mathbf{B}_{il} u_i(t) + \sum_{k=1, k \neq i}^{N} \overline{\mathbf{A}}_{ikl}(\mu_i) x_k(t) + \mathbf{G}_{il} v_i(t) \right\} \tag{54}$$

$$y_i(t) = \sum_{l=1}^{r_i} \mu_{il} \{ \mathbf{C}_{il} x_i(t) + \mathbf{D}_{il} v_i(t) \} \tag{55}$$

where $i = l = \{1, 2\}$.

Parameters for Subsystem 1:

$$\mathbf{E}_{1l} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{A}_{11} = \begin{bmatrix} -2 & 3 \\ 1.5 & -2.2 \end{bmatrix}, \mathbf{A}_{12} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}, \mathbf{B}_{11} = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}, \mathbf{B}_{12} = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix},$$

$$\overline{\mathbf{A}}_{121} = \begin{bmatrix} 0.5 & 0 \\ 0.8 & 0 \end{bmatrix}, \mathbf{C}_{11} = \mathbf{C}_{12} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \mathbf{D}_{11} = \mathbf{D}_{12} = 1, \mathbf{G}_{11} = \mathbf{G}_{12} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}.$$

Parameters for Subsystem 2:

$$\mathbf{E}_{2l} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{A}_{21} = \begin{bmatrix} -3 & 1 \\ 4 & -2 \end{bmatrix}, \mathbf{A}_{22} = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}, \mathbf{B}_{21} = \mathbf{B}_{22} = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix},$$

$$\overline{\mathbf{A}}_{212} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, \mathbf{C}_{21} = \mathbf{C}_{22} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \mathbf{D}_{21} = \mathbf{D}_{22} = 1, \mathbf{G}_{21} = \mathbf{G}_{22} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}.$$

The performance scalars and matrices are given as follows:

$$\theta_i = 0.5, \mathbf{Z}_{i1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \mathbf{Z}_{i2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \mathbf{R}_{i1} = 0.01.$$

Consider the membership functions in Figure 2. By applying Theorem 3, the minimum allowed $r_1 = 0.3904$, $r_2 = 0.3732$ and corresponding controller gains are

$$\text{For Subsystem 1}: \begin{cases} \mathbf{F}_{s11} = \begin{bmatrix} -151.5869 & -648.3600 \end{bmatrix} \\ \mathbf{F}_{d11} = \begin{bmatrix} 3.3389 & 61.4130 \end{bmatrix} \\ \mathbf{F}_{s12} = \begin{bmatrix} -87.0101 & -364.1830 \end{bmatrix} \\ \mathbf{F}_{d12} = \begin{bmatrix} 2.3792 & 34.3111 \end{bmatrix} \end{cases}$$

and

$$\text{For Subsystem 2}: \begin{cases} \mathbf{F}_{s21} = \begin{bmatrix} -52.0619 & -120.7840 \end{bmatrix} \\ \mathbf{F}_{d21} = \begin{bmatrix} 11.0213 & 39.7035 \end{bmatrix} \\ \mathbf{F}_{s22} = \begin{bmatrix} -52.8868 & -107.1582 \end{bmatrix} \\ \mathbf{F}_{d22} = \begin{bmatrix} 10.6186 & 36.3535 \end{bmatrix} \end{cases}$$

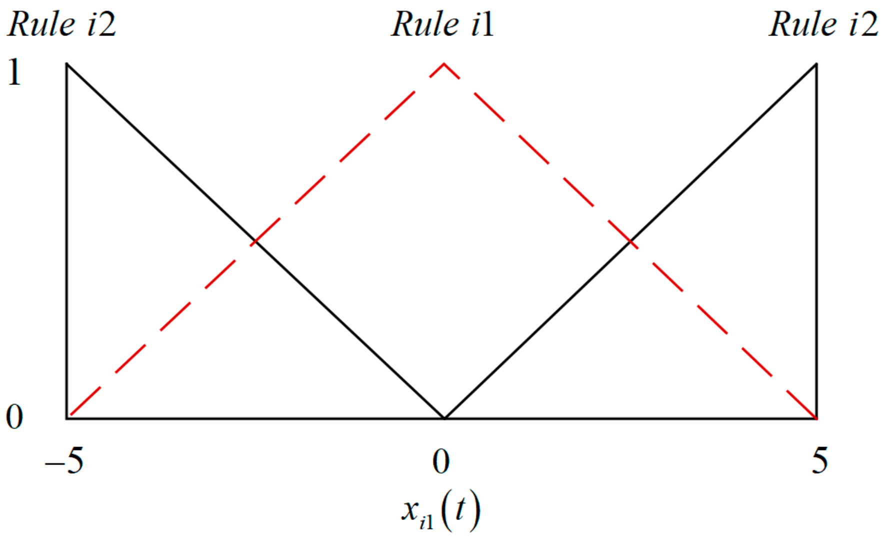

Figure 2. Membership functions for Simulation 1.

Define the external disturbance as $v_i(t) = 0.5\, sin(2t)$ shown in Figure 3.

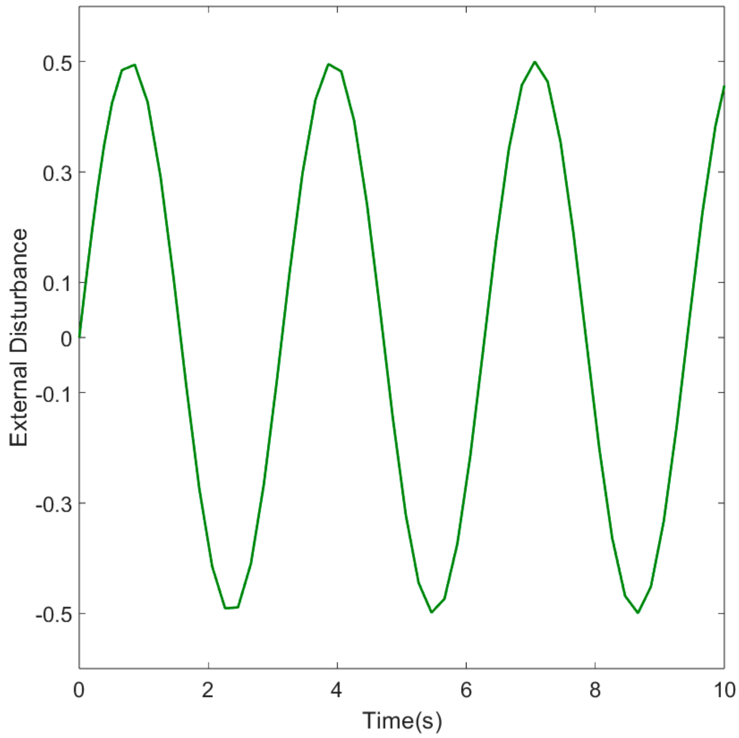

Figure 3. Responses of external disturbance.

Subsequently, with the initial conditions $x_1(0) = \begin{bmatrix} 1.5 & -1 \end{bmatrix}^{\mathrm{T}}$ and $x_2(0) = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix}^{\mathrm{T}}$ for Simulation 1, the simulation results were obtained, which are shown in Figures 4 and 5 with the proposed controller gains. The control signal is shown in Figure 6.

According to (7), if the inequality $\widetilde{\mathbf{J}}_{im} \leq 1$ holds, then the TSLSD system is considered to have MHPPC.

$$\widetilde{\mathbf{J}}_{im} = \frac{\int_0^{t_p} -\gamma_i^{-1}\theta_i y_i^{\mathrm{T}}(t) y_i(t) ds + \int_0^{t_p} 2(1-\theta_i) y_i^{\mathrm{T}}(t) v_i(t) ds}{-\gamma_i \int_0^{t_p} v_i^{\mathrm{T}}(s) v_i(s) ds}$$

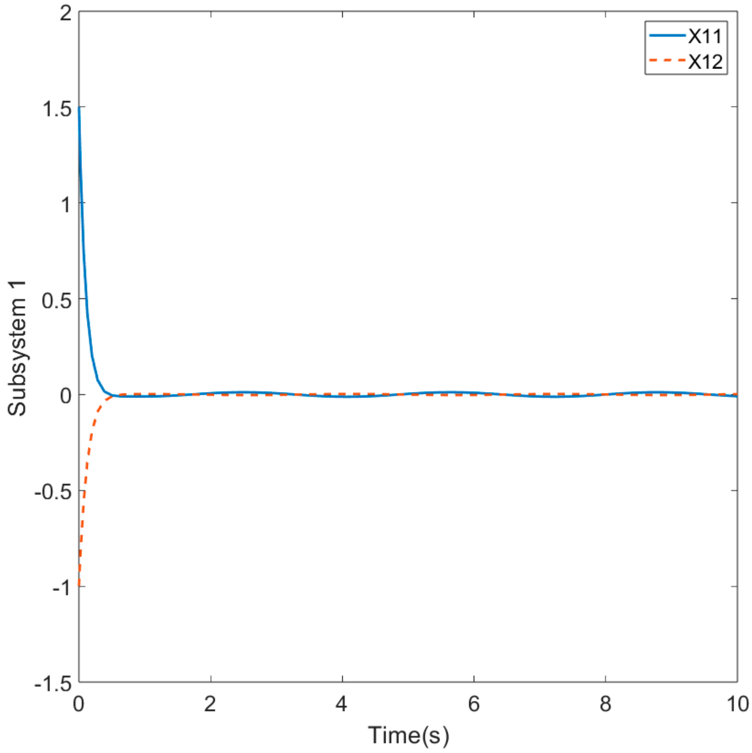

Figure 4. Responses of x_1 for Example 1.

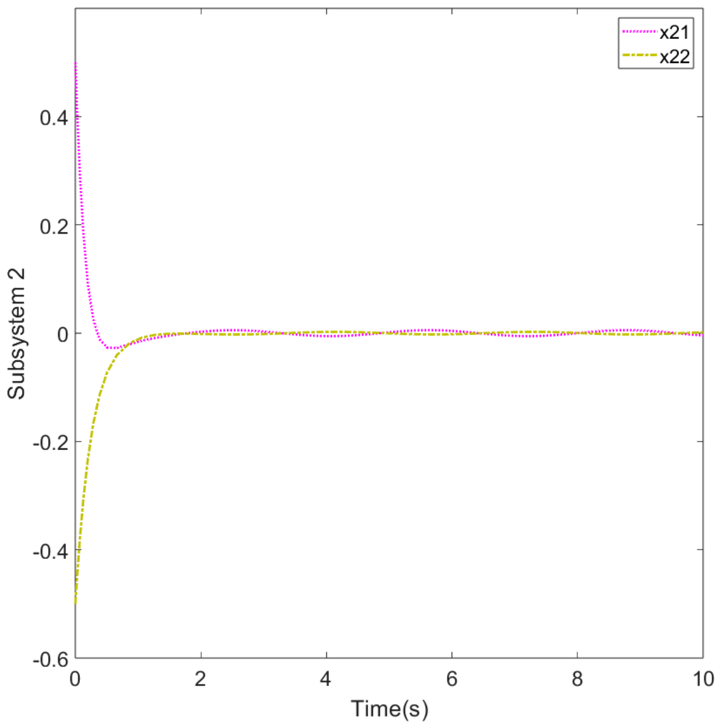

Figure 5. Responses of x_2 for Example 1.

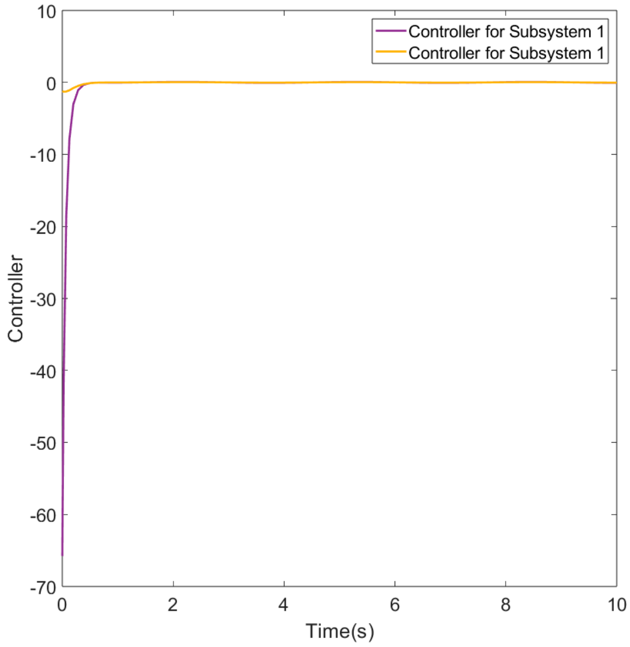

Figure 6. Control signals for Example 1.

The following specific values are used to verify the MHPPC (7) and GCC (9) for Simulation 1 (see Table 1).

Table 1. Specific values of simulation.

Values	$\widetilde{\mathbf{J}}_{im}$	$\boldsymbol{\lambda}_i$	$\mathbf{J}_{ic}$
Subsystem 1	−6.0415	2.8506	1.7369
Subsystem 2	−6.2048	0.2227	0.1812

Simulation 2. *In this section, the following example is borrowed from [29,30]. Consider a double-inverted pendulum system that is connected by a spring (Figure 7).*

$$\dot{x}_{11} = x_{12} \tag{56}$$

$$\dot{x}_{i2} = \frac{m_1 g d}{J_1}\sin(x_{11}) - \frac{k}{J_1}x_{11} + \frac{u_1}{J_1} + \frac{k}{J_1}x_{21} + \frac{v_1}{J_1} \tag{57}$$

$$\dot{x}_{21} = x_{22} \tag{58}$$

$$\dot{x}_{i2} = \frac{m_2 g d}{J_2}\sin(x_{21}) - \frac{k}{J_2}x_{21} + \frac{u_2}{J_2} + \frac{k}{J_2}x_{11} + \frac{v_2}{J_2} \tag{59}$$

where x_{i1} denotes the angular displacement of the $i-$th pendulum from the vertical reference. v_i is the torque disturbance. The torque input u_i can be applied by a servomotor to position each pendulum. The moments of inertia are $J_1 = 2\ \mathrm{kg}\cdot\mathrm{m}^2$ and $J_2 = 2.5\ \mathrm{kg}\cdot\mathrm{m}^2$. The constant of the connecting torsional spring is $k = 2\ \mathrm{N}\cdot\mathrm{m/rad}$. $m_1 = 2\ \mathrm{kg}$ and $m_2 = 2.5\ \mathrm{kg}$ are the masses of two pendulums. The height of the pendulum is $d = 1\ \mathrm{m}$. The gravity acceleration is $g = 9.8\ \mathrm{m/s}^2$. Approximating the subsystems at three points $x_{i1}(t) = (0, \quad \pm 88^\circ)$ (Figure 8), the following fuzzy model can be obtained for LSDS:

Rule 1. IF $x_{i1}(t)$ is $\pm 88^\circ$, THEN

$$\widetilde{\mathbf{E}}_{i1}\dot{x}_i(t) = \widetilde{\mathbf{A}}_{i1}x_i(t) + \widetilde{\mathbf{B}}_{i1}u_i(t) + \sum_{k=1,k\neq i}^{2} \overline{\mathbf{A}}_{ik1}x_k(t) + \mathbf{G}_{i1}v_i(t) \tag{60}$$

$$y_i(t) = \mathbf{C}_{i1}x(t) + \mathbf{D}_{i1}v_i(t) \tag{61}$$

Rule 2. IF $x_{i1}(t)$ is 0, THEN

$$\widetilde{\mathbf{E}}_{i2}\dot{x}_i(t) = \widetilde{\mathbf{A}}_{i2}x_i(t) + \widetilde{\mathbf{B}}_{i2}u_i(t) + \sum_{k=1,k\neq i}^{2} \overline{\mathbf{A}}_{ik2}x_k(t) + \mathbf{G}_{i2}v_i(t) \tag{62}$$

$$y_i(t) = \mathbf{C}_{i2}x(t) + \mathbf{D}_{i2}v_i(t) \tag{63}$$

where

$$\mathbf{A}_{11} = \begin{bmatrix} 0 & 1 \\ -91.19 & -40 \end{bmatrix}, \mathbf{A}_{12} = \begin{bmatrix} 0 & 1 \\ -94.62 & -40 \end{bmatrix}, \mathbf{A}_{11} = \begin{bmatrix} 0 & 1 \\ -86.99 & -40 \end{bmatrix}, \mathbf{A}_{12} = \begin{bmatrix} 0 & 1 \\ -90.42 & -40 \end{bmatrix},$$
$$\mathbf{B}_{1l} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \mathbf{B}_{2l} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, \overline{\mathbf{A}}_{112} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \overline{\mathbf{A}}_{212} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0 \end{bmatrix}, \mathbf{E}_{i1} = \mathbf{E}_{i2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\mathbf{C}_{\mathrm{i}1} = \mathbf{C}_{i2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \mathbf{D}_{1i} = \mathbf{D}_{i2} = 1, \mathbf{G}_{\mathrm{i}1} = \mathbf{G}_{i2} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}.$$

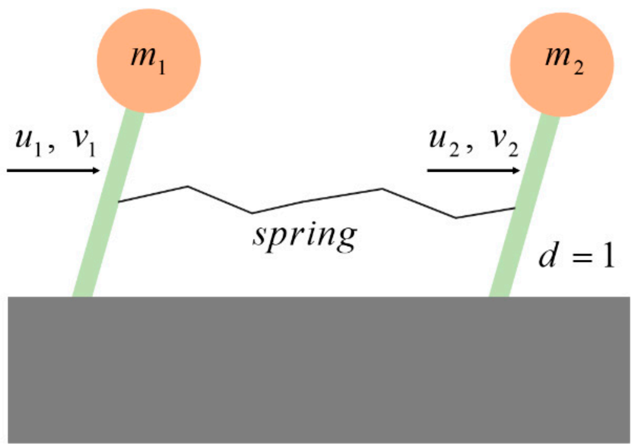

Figure 7. Double-inverted pendulum system.

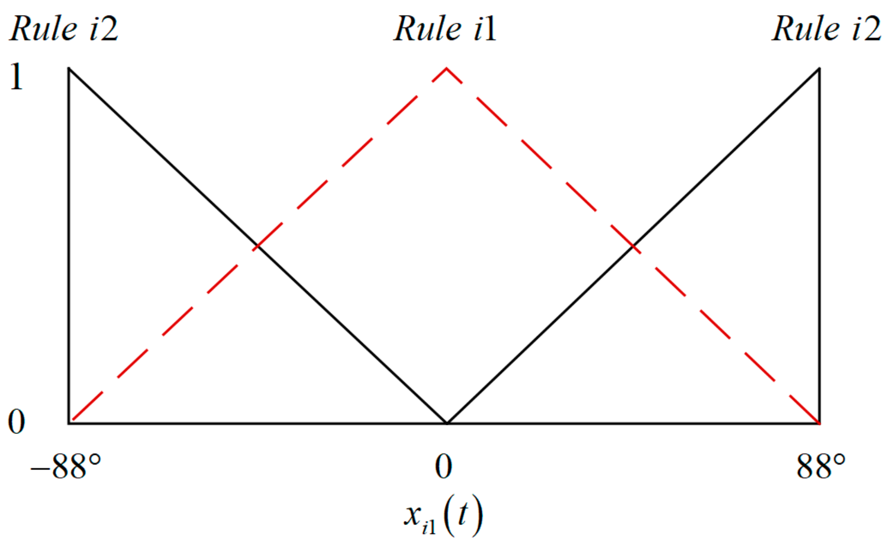

Figure 8. Membership functions for Simulation 2.

The performance scalars and matrices are given as follows:

$$\theta_i = 0.5,\ \mathbf{Z}_{i1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ \mathbf{Z}_{i2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ \mathbf{R}_{i1} = 0.1$$

By using Theorem 3 and solving the corresponding LMIs, we obtain the minimum allowed $r_1 = 0.4995$, $r_2 = 0.4690$ and the controller gains as follows:

$$\text{For Subsystem 1}: \begin{cases} \mathbf{F}_{s11} = \begin{bmatrix} -203.7575 & -87.2880 \end{bmatrix} \\ \mathbf{F}_{d11} = \begin{bmatrix} 0.7300 & 2.4321 \end{bmatrix} \\ \mathbf{F}_{s12} = \begin{bmatrix} 63.0675 & 25.4171 \end{bmatrix} \\ \mathbf{F}_{d12} = \begin{bmatrix} -0.3013 & -0.6113 \end{bmatrix} \end{cases}$$

and

$$\text{For Subsystem 2}: \begin{cases} \mathbf{F}_{s21} = \begin{bmatrix} -163.7991 & -73.4538 \end{bmatrix} \\ \mathbf{F}_{d21} = \begin{bmatrix} 0.5393 & 1.9798 \end{bmatrix} \\ \mathbf{F}_{s22} = \begin{bmatrix} 73.5624 & 31.2760 \end{bmatrix} \\ \mathbf{F}_{d22} = \begin{bmatrix} -0.3153 & -0.7734 \end{bmatrix} \end{cases}$$

Considering the same external disturbance given in Simulation 1 and assuming the initial conditions are $x_1(0) = \begin{bmatrix} 0.3 & 0 \end{bmatrix}^T$ and $x_2(0) = \begin{bmatrix} 0.5 & 0 \end{bmatrix}^T$, the state responses of the system were derived, which are shown in Figures 9 and 10, and the controller signal is shown in Figure 11. The obtained values are listed in Table 2.

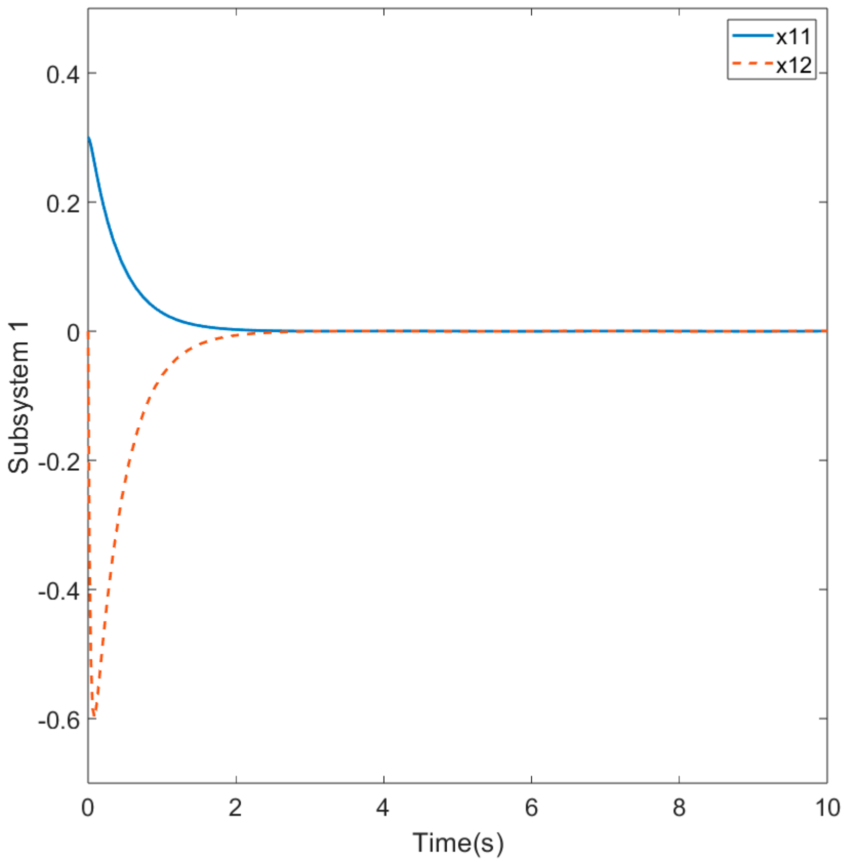

Figure 9. Responses of x_1 for Example 2.

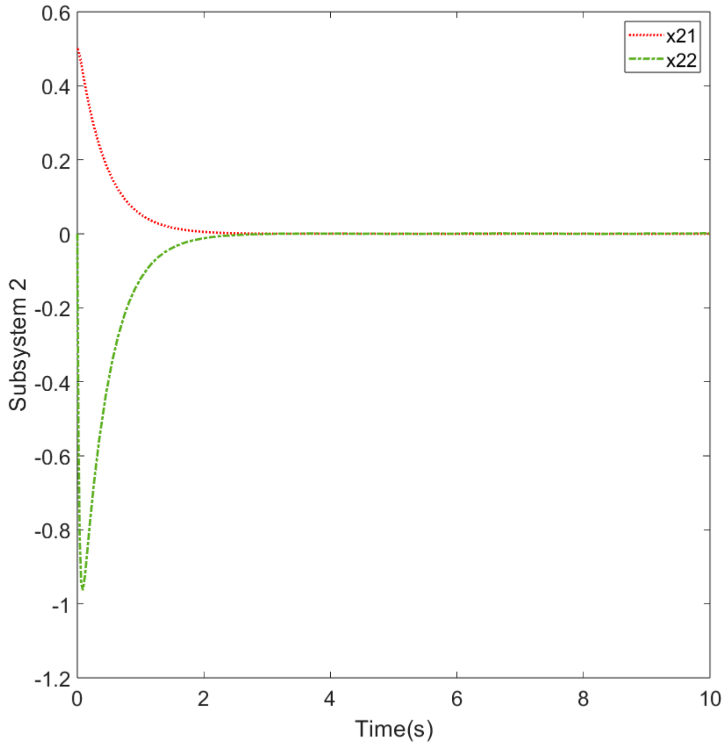

Figure 10. Responses of x_2 for Example 2.

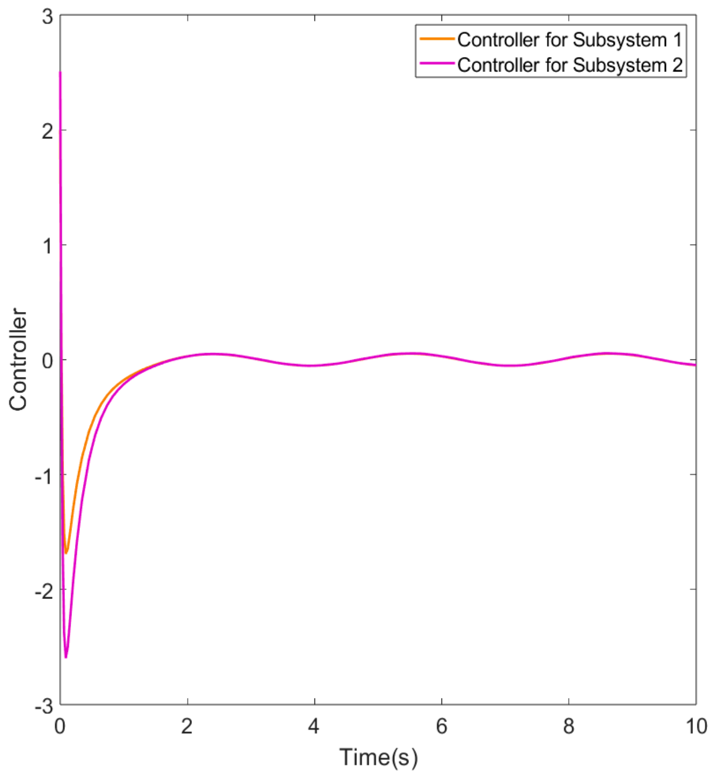

Figure 11. Control signals for Example 2.

From the figures shown above, we can note that all states converge to zero, which means the overall closed-loop fuzzy large-scale system can be controlled by the proposed method. In addition, it is easy to see that the external disturbance is successfully inhibited with MHPPC performance. According to Definition 1 and Definition 2, the values given in Tables 1 and 2 show that all examples satisfy MHPPC and GCC performance. Thus, we demonstrated that the proposed decentralized MP controller makes the nonlinear LSD system asymptotic stable and satisfies MP control.

Table 2. Specific values of simulation.

Values	$\widetilde{J}_{im}$	λ_i	J_{ic}
Subsystem 1	−4.1990	1.1766	0.9294
Subsystem 2	−4.8284	2.9850	2.4077

Simulation environment:

The simulation was conducted in MATLAB, and MATLAB—LMI Toolbox; The computer is equipped with: a 3.60 GHz 64 bit AMD 6-Core Ryzen 5 3500X Processor, WINDOWS 11 Operating System; AMD Radeon RX580 8G.

5. Conclusions

The problem of a decentralized MP control for nonlinear LSD systems was investigated in this paper. It was found that if the descriptor matrix is defined as a singular matrix, the impulse-free and regular problems of the system must be considered; otherwise, the system will be unstable. Different from the existing approaches, the decentralized MP controller was designed with the PDSF scheme. This scheme is more suitable than the state feedback or output feedback schemes because it can easily solve the impulse-free and regular problems. In addition, the stability conditions for TSLSD systems with the MHPPC and GCC were obtained in terms of LMIs. Two numerical examples were given to demonstrate the effectiveness of the proposed method. However, sometimes, we cannot easily obtain the state derivative information from the system. To solve this problem, it is necessary to consider the observer control scheme during the controller design process. In addition, how to construct an accurate mathematical model of real production is another important issue in the control field. As is commonly known, for modeling the system, the nonlinear characteristics, initial values, and some important parameters need to be determined. However, if there is a lack of important parameters or initial values, the constructed system may be quite different from the actual system, which further leads to the inability of the designed controller to effectively control the system. In the literature [31,32], researchers have proposed a mathematical model development method and control method for real production; however, some parts of the initial information are not clear enough. Therefore, considering observer control and how to effectively construct the mathematical model of nonlinear LSD systems is an important topic for future work.

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