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Article

Accounting for Sustainability: A Dissenting Opinion

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Abstract: Discounted-utilitarian welfare, the commonest social objective studied by economists, is the basis for the theory of green accounting in terms of social utility. Sustainability is a different type of social objective. Consequently, green accounting as derived in many empirical models is not appropriate for studying sustainability. Maximin is a consistent foundation for the analysis of sustainability, both weak and strong, that provides conceptually correct accounting prices. These prices are not yet practicable for real economies, however, and must await further advances. Sustainable development is a generalization of the notion of sustainability and can be analyzed using a generalization of maximin.

Keywords: sustainability; sustainable development; present value; welfare; maximin; green accounting

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1. Introduction

In recent decades, the sustainability of modern societies has been an enduring pre-occupation of scholars from several disciplines. Although economists have responded to the challenge of defining and studying the issue since at least the energy crisis of the 1970s, a popular intellectual watershed was the World Commission on Environment and Development [1] (The Brundtland Report), which famously couched the problem in terms of balancing the "needs" of present and future generations: sustainable development "meets the needs of the present without compromising the ability of future generations to meet their own needs".

Economic examinations have continued to try to put precision to the notion of balance. The dominant response, *green* or *comprehensive* accounting, is an adaptation of discounted-utilitarian

welfare. In this context, mathematical manipulation has shown that comprehensive investment, a linear aggregate of changes in all stocks that contribute to welfare, exactly measures the current change in welfare. Comprehensive investment is frequently called genuine savings because it is an extension of the notion of savings in the national accounts.

Some have criticized genuine savings as an empirical statistic. Difficulties of estimation, however, are not the underlying problem. The present paper argues that there is a fundamental, theoretical incompatibility in using discounted-utilitarian welfare as the starting point in the definition of sustainability. Genuine savings is a measure of short-term outcomes and not of whether a society is being sustained. The measure of sustainable value proposed herein is based on maximin, which was introduced to economic theory by Solow [2].

Depending on the constraints and parameters introduced into the sustainability problem, maximin is consistent with so-called *non-neoclassical* perspectives on sustainability, including strong sustainability. It is a generalization of the notion of a steady state, with a set of invariance properties. It yields a theoretical perspective by which sustainability and the issues involved in accounting for sustainability can be understood more fully.

This paper presents an overview, with reference to a few works considered to be representative of key points of view. It is not a survey or a derivation of results. Initially, a brisk but critical examination reviews the implications of the use of discounted-utilitarian social welfare functions. It is argued that such a social welfare function cannot be a basis for discussing sustainability. Then maximin is introduced as expressing an alternative value for the analysis of sustainability. A criticism of maximin, that it may leave a society in a poverty trap, is addressed through a generalization that is called sustainable development and that explicitly represents the trade-offs involved in present sacrifice for long-term growth in social value.

2. The Theory of Green Accounting

In a discounted-utilitarian model, the formula for net present value provides the foundation for green or comprehensive national accounting. Let the present time be represented by t, and let the quantities consumed of N final goods be represented by $q_i(t), i = 1,..., N$. The set of goods includes both marketed or traded goods and non-traded goods, including environmental services. Also, let there be M capital stocks, $K_j(t)$, $j = 1, ..., M$, traded as well as non-traded, including environmental stocks. Social utility at time t is a function of the levels of (some or all of) these goods and can be written $U_t = \upsilon[q_1(t), ..., q_N(t), K_1(t), ..., K_M(t)]$.

Social welfare at t is expressed as an integral, the present value of future social utility. Since the attained level of welfare depends on the currently available capital stocks, it can also be expressed as a function of those stocks, $W[K_1(t), ..., K_M(t)] = W_t$:

$$\int_t^{\infty} U_s e^{-\rho(s-t)} ds = W[K_1(t),...,K_M(t)] \tag{1}$$

In the integral on the left-hand side, the utility-discount rate ρ is called the rate of time preference. As a preference of the society it is primitive, not subject to economic analysis. Under a set of postulates made by Koopmans [3] it is constant, as in Equation (1). The present value, social welfare, is the expression of *value* in the model.

Wherever the function $W[K_1(t), ..., K_M(t)]$ is differentiable, the two sides of Equation (1) can be differentiated to yield that

$$\rho W_t - U_t = \frac{dW(K_1(t),...,K_M(t))}{dt} = \sum_{j=1}^{M} \frac{\partial W_t}{\partial K_j(t)} \frac{dK_j(t)}{dt} \tag{2}$$

The value of the marginal unit of capital good j (its *shadow price*) is its contribution to total value, $\partial W[K_1(t), ..., K_M(t)]/\partial K_j(t)$. Shadow prices for consumption goods are given by the partial derivatives of the utility function υ.

Rearranging Equation (2) gives

$$\rho W_t = U_t + \sum_{j=1}^{M} \frac{\partial W_t}{\partial K_j(t)} \frac{dK_j(t)}{dt} \tag{3}$$

Equation (3) is a fundamental accounting identity. On the left-hand side is the return on the total value of the economy, ρW_t, which represents net national income expressed in terms of utility. On the right-hand side is the sum of the utility from consumption and the value of comprehensive net investment in period t. It represents net national expenditure or product in utility terms. *Net, not gross*, product and income are relevant for comprehensive accounting [4]. The simple differentiations indicate that the components of green accounting pertain to the current instant only.

In a perfectly competitive economy, the "invisible hand" leads the participants in the market to make choices that maximize total social welfare by implementing the shadow prices as the market prices. Much of theory has been developed for an optimal economy. Accounting can be done using the shadow prices of the various goods that contribute to utility and welfare [5].

In reality, a number of market failures render the economy non-competitive. As a result, market prices are not the signals that induce individuals to make the correct decisions (in terms of maximizing welfare) to produce or to consume. Moreover, the non-traded goods do not have market prices. The implementation of correct prices would give actors an incentive to more appropriate use of all social resources. Green accounting aims to provide the theoretical basis for defining prices for environmental and other non-traded goods, be they consumption goods (amenities such as beautiful scenery or "dis-amenities" such as pollution) or capital goods (such the ability of the environment to absorb pollution), to be used in defining net expenditure and net income. Equation (3) is the theoretic foundation for determining the properties of the ideal system of national accounts using the shadow prices of stocks and flows at an optimum where welfare is maximized.

For traded goods, the market provides paths of prices through time. These prices may or may not be close to the goods' shadow prices. Some economists, e.g., Arrow, Dasgupta and Mäler [6], have begun the enormous task of adapting the theory to an imperfect economy, based on a re-interpretation of Equations (1) to (3) in which welfare is not being maximized. Green accounting is analogous to conventional accounting except that the shadow prices of non-priced and ill priced goods are estimated by cost-benefit techniques rather than observed directly. In practice it is not possible to be completely comprehensive in the sense of including all goods that affect welfare. Also, any estimates that are made involve substantial error (in statistical terms, wide confidence bands).

Market prices provide a point of departure from which green accounting seeks to determine appropriate prices for the most important non-traded goods and to extend the national accounts to

include these neglected contributors to welfare. Arrow *et al*. [6] indicate through a series of examples that finding the shadow prices for many goods must be done at a micro level, rather than at the macro level on which the present discussion has been carried out. The examples express the conviction that the practical obstacles can be overcome with time and effort.

3. The Misleading Indicator of Sustainability

To now there has been no mention of whether the society is sustainable and, if so, whether it is being sustained. The discussion has been in terms of discounted-utilitarian value given the social institutions in place. It utilizes a convenient, well understood methodology based on present value and is often supported by a vague notion that the invisible hand of the market (or perhaps the more visible hand of a government) does a good job in promoting social welfare.

Many authors interpret the *sustaining* of the society to mean that current welfare is not decreasing, so that $dW_t / dt \geq 0$, and so by Equation (2) that total current, net investment $\sum_{i=1}^{N} (\partial W_t / \partial K_i)(dK_i / dt)$ is non-negative. Welfare is considered to be a better measure of well being at a given time than utility, essentially because it takes the future into account in a consistent way. However, Pezzey and Toman [7] point out that, if welfare is the economy's inter-temporal value, no room is left for a concern about *sustaining* welfare. Indeed, if the rate of discount, ρ, is constant, a sustained path is not attained in an optimization model. Making W_t the expression of value of the society may eventually lead to serious decline. Simple examples rooted in the study of sustainability are presented by Asheim [8], Pezzey and Toman [7] and Martinet [9].

In view of the possibility of decline, Neumayer [10] suggests supplementing the discounted-utilitarian model with a constraint that $dW_s / ds \geq 0$ for all times s from the current time t onward. In Neumayer's chapter on the measurement of (weak) sustainability, however, the constraint does not appear in the theoretical analysis. Indeed, it is common for the constraint not to be explicitly imposed. In a footnote, Arrow *et al.* [6] state that it is enough to check the sign of dW_t/dt, *i.e.*, to check whether the condition holds at the current date only. On the contrary, imposing the constraint at all future dates would affect the interpretation of the model and the empirical estimates of the shadow prices.

Furthermore, a constraint is not an objective but an impediment to the achievement of an objective. If sustainability is the value being studied, it should not be introduced through an impediment but instead be the mathematical objective of the economy. Popular concern with sustainability is rooted in a perception, right or wrong, that the activities of present industrial societies are not sustainable; it is felt that a *new* concept of social value is required and that measuring sustainability must be founded on that new value.

The view of sustaining the economy that is adopted herein is expressed by Solow [11]: "If sustainability means anything more than a vague emotional commitment, it must require that something be conserved for the very long run. It is very important to understand what that something is: I think it has to be a generalized capacity to produce economic well-being." In this statement Solow is not only looking to the distant future, but is pointing to a conservation or invariance property. The invariance property applies to one or a few variables related to what one wishes to be conserved, rather than to all variables as in a steady state. It is, indeed, vital to permit some variables—such as ecological variables—to evolve while well-being is sustained.

In this section of the paper it is argued that there is no presently available accounting statistic that is a valid indicator of sustainability or sustainment. This claim applies even if all sources of well-being are explicitly recognized by being included in comprehensive social welfare.

There is, however, a false or *misleading* indicator of sustainment that is based on the theory of green accounting presented above, namely, the total of comprehensive net investment in the economy. Non-negative comprehensive net investment (genuine savings) may or may not be desirable, but is no indicator of sustainability or of whether the society is being sustained.

The point is of practical significance because genuine savings is being used by many to measure the sustainability of various countries or regions. For example, an influential paper by Hamilton and Clemens [12] presents a model of welfare maximization and derives the expression for genuine savings, which is estimated for many countries and used as an indicator of sustainability of each. This indicator appropriately measures the *current change in welfare*, $dW[K_1(t), ..., K_M(t)]/dt$, by recognizing explicitly the contributions of all assets, including the environment, education and other stocks that are not consistently treated in the national accounts. It is a theoretic extension of the national accounts. It pertains to the current time period, however, and is not an estimate of *long-term sustainability*.

For the sake of simplicity in examining this misleading indicator, suppose that, on a finite interval of time, $(t, t + \theta)$, beginning at the present time t, welfare $W[K_1(s), ..., K_M(s)]$ remains constant: $dW_s/ds = 0$ at any time $s \in (t, t+\theta)$. Outside the interval it is assumed not to be constant. According to Equation (2), throughout the interval $(t, t + \theta)$,

$$\rho W_s - U_s = dW_s / ds = 0$$

A rearrangement and a further differentiation with respect to time yield that

$$dU_s / ds = \rho\, dW_s / ds = 0$$

If welfare is constant, then utility is constant.

(It is not, however, possible to argue in the opposite direction. If utility is constant on the interval, a straightforward series of manipulations shows that net investment dW_s/ds is increasing or decreasing at rate ρ on $(t, t + \theta)$ and that welfare W_s is increasing or decreasing toward its level at $t + \theta$.)

Since by supposition $dW_s/ds = 0$ when $s \in (t, t+\theta)$, it must also be that comprehensive net investment or genuine savings, evaluated at the shadow prices $\partial W/\partial K_i$, is zero as well. Since neither utility nor welfare can be measured, the aggregate of net investment is taken to be the signal of whether or not the economy is being sustained.

The conditions hold *no matter how short* the time interval θ. The invariance sought by Solow is curtailed to become what might be termed the short-term stationarity of welfare. After time $t + \theta$ one can, in principle, imagine any time path of utility, welfare and net investment. In particular, all could decline after time $t + \theta$, possibly forever, possibly to zero. The possibility of future decline, even when current net investment is non-negative, undermines the many empirical models that evaluate sustainability in terms of genuine savings over periods of up to a few decades.

Any path of an economy having net investment of zero, evaluated at the prices $\partial W/\partial K_i$ that are pertinent to that path, on a short interval beginning with the present has a constant level of welfare and of utility *on that interval*. But it is not usually interesting to consider whether some measure of welfare is maintained for a short period of time. Welfare may decline because of a business cycle, for example.

A business cycle is a short-term problem for macro policy but hardly a threat to the sustainability of a society.

Suppose that utility is cardinal (measureable) and fluctuates about a rising trend: $U_t = [1 - \alpha \sin t] + Bt$, $0 < B < \alpha < 1$. The change in utility, $\dot{U}_t = \alpha \cos t + B$, is negative on regular intervals. Welfare is

$$W_t = \int_t^{\infty} [(1 - \alpha \sin s) + Bs] e^{-\rho(s-t)} ds = (1/\rho) - \alpha(\cos t + \rho \sin t)/(1 + \rho^2) + B(1 + \rho t)/\rho^2 \quad (4)$$

From Equation (4), the change in welfare is

$$\dot{W}_t = B/\rho + \alpha(\sin t - \rho \cos t)/(1 + \rho^2)$$

If $B < \alpha\rho^2/(1 + \rho^2)$ (if there is a very slow but still positive trend), there are regularly repeated intervals on which welfare decreases, including the present ($t = 0$). Welfare is never less than $1/\rho - \alpha/(1 + \rho^2) + B/\rho^2$ and eventually grows like Bt/ρ. The level of utility never falls below $\min_t [1 - \alpha \sin t + Bt] > 0$. To maintain non-decreasing values of utility or welfare, the society must "throw away" utility over some intervals.

When welfare is falling, investment is falling and in the above example genuine savings gives an incorrect signal. For only one path, on which welfare and utility are maintained *forever*, are the current prices the right ones to use in the formula. The constraint $\dot{U}_t \geq 0$ or $\dot{W}_t \geq 0$ forever is too stringent.

The idea of sustainability has arisen largely because of concerns about changes in environmental variables—in economic terminology, changes in environmental capital. To economists, a key part of the problem is that environmental capital typically is not mediated in markets. Comprehensive capital is specified to include all environmental stocks relevant to welfare.

However, getting the correct signal is not solely a matter of including all the relevant capital goods in the statistic. The statistic is a sum of quantities multiplied by prices. Pezzey [13] argues that incorporating the environment using the shadow prices for maximizing welfare does not imply sustainability. Pezzey thereby puts his finger on the subtle difference between green and sustainability accounting. Shadow prices for problems with different objectives are, generally speaking, different. *The prices, too, must be 'right' in order for the statistics to be valid measures of sustainability.* It is argued below that the prices are not right when they emanate from a discounted-utilitarian welfare function. Therefore, there is no justification in economic theory to claim that information about comprehensive net investment at observable, current prices can provide any information about sustainability. Pezzey and Toman [7] observe that "maximizing a discounted-utility integral has nothing to do with sustainability, and gives a complete and unique prescription for the time paths of every decision that ever has to be made." Combining the two in one model is inconsistent.

4. Sustainability

Hamilton and Clemens [12] and Neumayer [10] list practical problems with measuring prices for genuine savings. Finding prices for a sufficient number of important goods is, however, a surmountable problem. The true problem is deeper. Comprehensive investment or genuine savings is not the right measure because the wrong problem is being studied; the wrong expression of economic

value is being used. The appropriate economic value has to be sustained, as Solow stated, "for the very long run", not for a temporary period of length θ as discussed above.

Before one considers what sustainability means in economic terms, one must first define what it means to sustain something. *To sustain* means *to hold up from below*. The relevant social value is individuals' own perceptions of their well-being, their per-capita utility. What is held up from below by the economy is the minimum level of per-capita utility attained over the indefinite future, $u_{\min} = \min_{s \geq t} u_s$. Dasgupta and Mäler [14] aver that there is nothing sacrosanct about current levels of environmental stocks. Nor is there anything sacrosanct about current levels of utility. The minimum need not occur at the present moment, but at any future time.

This view is consistent with environmentalists' concern that the current level of utility may not be sustainable but that sustaining the society may require a reduction in current consumption and utility. It is also consistent with one of the canonical models for studying economic sustainability. Consider a simple fishery with the natural-growth function $dS/dt = S(1-S)$. Suppose that $S(t) > ½$. In this fishery, the maximin level of harvest is the maximum sustainable yield, ¼. There are many feasible paths on which the harvest can decline to ¼ from some higher level. Compared to the non-decreasing path, it is Pareto superior, and sustainable, to follow one such path.

Defining the social value to be $u_{\min}$ is natural given the subject of analysis: Increasing this minimum increases what is sustained; increasing utility at any time with higher utility does not. Consequently, what is sustained is not sensitive to levels of utility that are greater than what is given by the criterion. This non-sensitivity to the interests of generations who obtain more than the minimum is a departure from some analyses of economic welfare [2]. What is sustained is a lower bound. A lower bound is not sensitive to higher values attained elsewhere.

What is *sustainable*, what can be sustained, is any level of utility up to the maximum level of this minimum over all feasible paths,

$$\bar{u} = \max u_{\min} = \max u \;\; s.t. \;\; u(s) \geq u \; \forall s \geq t \tag{5}$$

The value $\bar{u}$ is the maximin level discussed by Solow [2] and Hartwick [15]. The notion of sustainable utility is subtly different from what Pezzey and Toman [7] seem to define as sustainability and what Neumayer [10] expressly does, *viz*., "the capacity to provide non-declining per-capita utility for infinity". Pezzey and Toman observe that scholars of sustainability reject the idea that individuals should be regarded differently just because of the time they are alive. Neumayer states that "being later in time should be no reason for counting less".

Nor should being earlier in time be reason for counting less. Non-decreasing utility constitutes a bias against the present. When Neumayer [10] allows for "earlier generations to make *voluntary* sacrifices in order that coming generations can enjoy higher levels of utility", he does so by not expressing the dynamic preferences of the economy fully; there must be some value, unexpressed in the utility function, to motivate the voluntary sacrifices. That value should be an explicit part of social value and the social objective.

To Pezzey and Toman and to Neumayer, the hallmark of neoclassical or weak approaches to sustainability is that they postulate a high degree of substitutability of other forms of capital for natural capital. Strong sustainability is characterized by regarding (critical forms of) natural capital as not having substitutes. Current utility may be any function of any subset of the flows q and stocks K. The

maximization in Equation (5) can be done, and analogous results derived, when there are constraints involving flows and stocks [16]. Some of the constraints may involve non-substitutability or limited substitutability among various forms of capital. Critical capital can exist in a maximin program.

At its root sustainability is not a matter of substitutability, limits to substitutability or non-substitutability of manufactured capital for natural capital, nor of the existence of critical capital. It is not a matter of the empirical difficulties of adapting an optimization model to a non-optimal economy. Rather, it is a matter of a careful definition of the social objective and hence of social value. For example, if the social value is to preserve some aggregate of environmental stocks $E_1, \ldots, E_m$, the utility function can take the form $\tilde{u}(E_1,...,E_m)$, or else minimal levels of each stock can be stipulated. Maximin provides a theoretical paradigm for analysis of sustainability that can encompass the main notions of what is important to the very long term, and in which the important constraints can be naturally introduced mathematically. It is not possible to escape from shadow pricing if any aggregate of the form $\tilde{u}(E_1,...,E_m)$ is envisaged.

Both strong and weak sustainability can be expressed in a maximin model through the definition of utility and the constraints. It must be stressed, however, that the long-term problems of strong sustainability are difficult to sort out. Ecologists and environmentalists are currently struggling with concepts such as *environmental health* in order to provide even a set of loose indices, let alone a precise definition [17]. Developing ecological health assessment to a level that can be used in contributing to a definition of sustainability of a society will take a long time. It warrants attention, but accounting must await a consensus among the professionals studying the area.

Still, a maximin program has many of the features of Howarth's [18] articulation of what may be viewed as a philosophical or ethical underpinning of sustainability. Rights, such as political freedoms, were espoused by Rawls [19], who inspired Solow's [2] examination of maximin paths. There is no reason to suppose that Solow did not take Rawlsian justice to be a prerequisite of, a constraint on, an optimal path. Maximin points toward the "operationalization" of sustainability that Howarth seeks. Unlike in Howarth's perception, though, maximin is not based on cost-benefit analysis of the usual sort, which calculates shadow prices for social welfare, W_t.

The notion of rights, which may be considered inviolable, may help to clarify one complaint about maximin: for some dynamic paths maximin may prescribe that some generations give up very great quantities of utility in order to bring the least advantaged up, by a very small amount, to what may be considered to be one of their rights, the maximin level. The prescription is made because it is what sustainability entails. In the original situation the slightly lower level of utility enjoyed by the least advantaged is sustained. Raising the sustained level may well involve a high cost. An analyst who holds that the cost is not worth the benefit has a different value from sustainability. That value should be explicitly stated in the analysis.

5. Some Links to Discounted Utilitarianism

Pezzey and Toman [7] observe that any efficient, dynamic path—including a maximin path—can be expressed as the result of maximizing a generalized expression of welfare, which they call *generalized present value*. Instead of the constant discount rate ρ in the welfare function (1), a variable

instantaneous rate $r(\tau)$ is envisaged in which the discount factor between times t and $s > t$ is $w(s) = \exp[-\int_t^s r(\tau)d\tau]$. Generalized present value is given by

$$\widetilde{W}(K_1(t),...,K_M(t)) = \int_t^\infty U_s w(s)\,ds \tag{6}$$

One must be careful, however, of the interpretation of the generalized present value for sustaining the economy. Unlike the discount rate ρ of Equation (1), the discount factor in this case is not a primitive, not a time preference. The utility weight $w(s)$ and the expression on the left-hand side of Equation (6) are endogenous to the solution for a sustained path and hence to stocks of the problem [20]. Only in a steady state is $r(\tau)$ constant through time.

While it can be expressed as a generalized present value, maximin does not satisfy all of Koopmans's postulates [3]. (For the sake of completeness one notes that these include what Koopmans calls impatience, sensitivity, non-complementarity and stationarity.) The expression of social value in Equation (5) is fundamentally different from Equation (1) in not being an integral or sum of discounted magnitudes. Maximin stands outside the standard, discounting paradigm and the theory that gives rise to the usual accounting methodology.

Pezzey [21] defines what is sustainable to be the maximin level of utility as determined by the current level of capital stocks. He finds (a) that the current level of utility of the economy can be sustained only if it is no greater than the maximin level and (b) that an economy maximizing discounted-utilitarian welfare with a constant discount rate as in Equation (1) cannot sustain the current level of utility if its net investment is negative. The test is "one-sided" in that net investment may be positive but utility may still exceed the maximin level, as in Asheim's example [8].

Pezzey's result (b) is very striking. As Pezzey realizes, however, an application of the one-sided test to an optimal discounted-utilitarian path would have to be motivated by some further value, amounting to promoting sustainability. He ends by asking "why sustainability should be of interest in a present-value maximizing economy". For the discounted-utilitarian optimum, genuine savings must be strictly positive for sustainability. But there is no indication of how relative investment prices, or indeed how investment quantities, diverge from the maximin levels. It is not indicated how a planner can adjust investment levels to sustain utility. Despite the strictness of the inequality, therefore, the one-sided test does not provide a guide for investment policy.

The prices for a discounted-utilitarian optimum are no easier to compute than maximin prices: The canonical models for studying many resource issues are the simple fishery, mentioned above, and the Dasgupta-Heal-Solow-Stiglitz model of an economy with an exhaustible resource and one type of manufactured capital. For both, the maximin solution, including the determination of shadow prices, is much simpler than the discounted-utilitarian solution (cf. [22]).

Moreover, Pezzey does not claim that his result justifies the use of observed statistics in a real, non-optimal economy. Maximin prices are precise, two-sided indicators even for a non-optimal program or for one that does not have sustaining the economy as its objective.

Two hybrid models propose retaining a utilitarian, discounted sum as a component of a criterion of sustainability. Chichilnisky [23] aims to obviate two dictatorships. Utilitarian discounting is considered to be a *dictatorship of the present* because, for any $\rho > 0$, the distant future has vanishingly small weight in the objective. A *dictatorship of the future*, defined by a criterion equal to the limiting

utility in the very far future, neglects the present entirely. For the discount factor $\delta = 1/(1 + \rho)$, she forms a new criterion equal to a weighted average of the two:

$$V_t = \lambda \sum_{s=t}^{\infty} U_s \delta^{-(s-t)} + (1-\lambda) \lim_{s\to\infty} U_s, \ \lambda \in (0,1) \tag{C}$$

The criterion puts real weight on the limit, whereas in economic analysis infinite time is usually used as a purely mathematical device to avoid specifying what happens after finite end points. Alvarez-Cuadrado and Long [24] modify the objective by replacing the limit by the maximin utility:

$$\widetilde{V}_t = \lambda \sum_{s=t}^{\infty} U_s \delta^{-(s-t)} + (1-\lambda)\overline{u}, \ \ \lambda \in (0,1) \tag{ACL}$$

One aim is to adjust the maximin program to be *sensitive* to the interests of all generations.

Figuières, Guyomard and Rotillon [25] summarize some objections to these hybrids. The main problem with Chichilnisky's criterion is that the criticism of utilitarian discounting remains: generations after, say, a few hundred years still have a negligible weight. A large value of the criterion can be obtained by holding one big, medium-term party followed by a very long period of penury in order to recover to produce a high ultimate value. Some fear that this pattern of well-being may be a result of climatic change.

If one is concerned about sensitivity to the interests of all generations, then the *degree* of sensitivity is an issue. In the hybrid criteria, sensitivity is achieved through the infinite, utilitarian sum. Discounting renders the degree of sensitivity to the far future very low. Sensitivity comes at the cost of equity.

In keeping with the definition of what is sustained, maximin eschews sensitivity. In terms of equity, it satisfies the *weak Pareto* principle: an increase in any generation's utility does not decrease the maximin value. It also satisfies *finite anonymity* [25,26]: one can change the timing of the various utility levels a finite number of times and not change the value of the objective.

To escape the dictatorship of the present, Asheim and Mitra [26] investigate the *sustainable discounted-utilitarian social-welfare function*. By design their criterion is insensitive to the interests of the first generation if its utility exceeds the welfare of future generations. They impose the following:

$$W_t^{AM} = \min[W_{t+1}^{AM}, (1-\delta)U_t + \delta W_{t+1}^{AM}] \tag{AM.1}$$

$$W_t^{AM} = U_t \text{ if the path being evaluated has equal utilities for all generations} \tag{AM.2}$$

$$\text{If } x_s \geq y_s \text{ for all } s \geq t\text{, then } W^{AM}(x_t, x_{t+1}, x_{t+2}, ...) \geq W^{AM}(y_t, y_{t+1}, y_{t+2}, ...) \tag{AM.3}$$

$$\lim_{t\to\infty} \delta^t W_t^{AM} = 0 \tag{AM.4}$$

Asheim and Mitra do not claim that their criterion is sensitive to the utilities of all future generations. If there are strong dips in utility the criterion (comparably to maximin) may make current utility equal to the lowest level of future welfare defined by the criterion. Thus it can be insensitive to high levels of utility in the interim. If utility grows steadily from some future time, welfare also can grow steadily, as in the (non-regular) maximin problem studied by Cairns and Tian [27].

Maximin satisfies conditions (AM.2) and (AM.3). If the maximin solution is expressed as a generalized present value as in Equation (6), using the shadow price of equity discussed by Cairns and Long [16] as the discount factor, it satisfies conditions (AM.1) and (AM.4) as well.

6. Accounting for Sustainment

Using the right prices entails using the prices for the right program, the one with the right concept of value and hence the right objective. An accounting methodology for sustainability would be based on the following reasoning. If the sustained level of value is maximized (if the maximin level $\overline{u}(K_1,...,K_n)$ is found and implemented), then at least this level of utility is enjoyed through time. In the solution the sustainability shadow prices $\sigma_i = \partial\overline{u} / \partial K_i$ are affected by any technical and natural constraints imposed in the analysis of the maximin problem (5). Irrespective of what constraints are imposed, (in what is known as a *regular* maximin problem) at any time *t* the value of comprehensive net investment evaluated at the shadow prices $\sigma_i(t)$ is zero [16]:

$$\sum_{i=1}^{M} \sigma_i(t) \frac{dK_i(t)}{dt} = 0 \tag{7}$$

Condition (7), Hartwick's rule of zero net investment in a sustainability problem [15], is the conservation or invariance property of maximin. In the solution this invariance property holds for all time. It does not hold for all time for the prices that emerge from maximizing a welfare integral that happens to be constant over the next short while. In general, in the two programs the shadow prices of capital are not the same. For given levels of the stocks, the interpretation of $\partial\overline{u} / \partial K_i$ is fundamentally different from that of $\partial W / \partial K_i$ [28]. Indeed, in studying several discounted-utilitarian models of varying complexity, Martinet and Rotillon [29] find that what remains invariant changes from model to model, and except in the simplest examples is hard to interpret.

The reason that society perceives the need for a statistic, or for a set of statistics, to study sustainability is that some are concerned and even convinced that the world is not on a sustainable path. If the path is not right, then no matter how comprehensive the array of capital goods studied, the observed and estimated prices of investment goods are not right and the level of comprehensive net investment does not provide a valid signal.

Maximin provides theoretical insights that indicate the enormity of the task of measuring sustainability but also gives some direction to further investigation. The maximin program determines the *maximized* value of sustained income. It must be assured that the economy can be operated optimally to maximize the sustained well-being and that the maximin problem can be solved. It is a difficult mathematical problem over an indefinite future. Where further work is required is in determining suitable accounting prices. The problems of extension to non-optimal economies are daunting. They are, however, no more daunting than finding shadow prices for discounted-utilitarian welfare on either an optimal or a predicted, non-optimal path of the economy as proposed by Arrow *et al*. [6]. A promising direction of research is viability theory, as discussed by Martinet and Doyen [30], which is a generalization of the maximin program for a non-optimal economy.

7. Sustained Development

Maximin satisfies non-dictatorship of the present, non-dictatorship of the future and finite anonymity. It has a satisfying interpretation of what is sustained and what can be sustained that is in harmony with the dictum cited above from the Brundtland Report. Consistently with the definition of sustaining, it expresses a qualitatively different social value from conventional welfare by being

sensitive only to the lowest level of utility. It is the basis for Hartwick's rule for determining the sustainability of a society. It can encompass both strong and weak sustainability and be extended through viability analysis to combinations of the two, as in a study by Martinet [31].

The main criticism of maximin is that, if the criterion is applied in a poor economy, future generations may be mired in a "poverty trap" involving perpetual levels of utility equal to the low, present level. Ironically, poverty may be sustained. The critics invariably recommend economic growth (increasing utility or consumption) to lift the society out of poverty. Sustained development is a phrase that describes growth out of poverty toward a developed state that can be sustained for the very long run.

What is not usually observed in this criticism is that the maximin solution is Pareto efficient. To make the future better off through increased investment and hence growth, the planner must take utility away from the present. But the criticism also implies that in the maximin solution the present generation is at a level of poverty that is so dire that the future must be rescued from it. Such a sacrifice of the interests of an impoverished present may be inconsistent with the dictum from the Brundtland Report, which balances and protects the interests of the present as well as the future.

Although the implications for the early generations of a poor society are not usually stressed, a policy proposal of greater current investment and less consumption has been advanced in several economic models that assume efficiency in maximizing discounted-utilitarian welfare. Beyond the postulated welfare function, the modelers have not specified the extent of sacrifice or the acceptable trade-offs. It is simply considered good if there is growth. However, considering growth to be good inserts another dynamic value into the problem. Expressing that value takes the modelers outside their own, discounted-utilitarian, definition of welfare. Discounted utility is deficient in this context. Consequently, the accounting methodology based on discounted utility is deficient.

For example, Hamilton and Clemens [12] seem to promote genuine savings of at least five percent as being desirable in certain developing countries. Hamilton and Hartwick [32] and Hamilton and Withagen [33] go on to develop the theory of genuine savings in situations where growth may occur.

An instructive recasting of the discussions of [32] and [33] is possible at points where there is sufficient differentiability. The optimality conditions are not used, however, and so the path is not necessarily an optimal discounted-utilitarian path. There is an additional, strong assumption: In general on a non-optimal path, wealth may be a function of time as well as the capital stocks. It is assumed for brevity's sake that this so-called *non-autonomy* of the program can be neglected and that Equations (1) to (3) continue to hold. Alternatively, one can assume that standard accounting practice is followed in the sense that capital gains are neglected in green national accounting.

Let genuine savings, $\sum_{j=1}^{M} \frac{\partial W_t}{\partial K_j(t)} \frac{dK_j(t)}{dt}$, be represented by G_t and net national product or expenditure, $U_t + \sum_{j=1}^{M} \frac{\partial W_t}{\partial K_j(t)} \frac{dK_j(t)}{dt}$, by E_t. Differentiating Equation (3) with respect to time yields that

$$\operatorname{sgn} \dot{E}_t = \operatorname{sgn} \dot{W}_t = \operatorname{sgn} G_t \tag{8}$$

Equation (8) is a net-investment rule in utility terms for a market economy: At any time, each of green NNP and wealth moves in the direction of net investment or genuine savings. Further manipulation of Equation (3) yields that

$$\dot{U}_t = \rho \dot{W}_t - \dot{G}_t = \rho G_t - \dot{G}_t = G_t(\rho - \dot{G}_t / G_t) \tag{9}$$

Equation (9) provides a *short-term* indicator of the rate of change of utility. Each of genuine savings G_t and the difference between the interest rate and the growth of genuine savings, $(\rho - \dot{G}_t / G_t)$, is a component of the growth of current utility. It is curious to propose either of them or their product as a measure of sustainability or of sustainable development in response to a concern that excessive short-term growth may lead to future environmental and hence social problems.

Sustained development, like sustainment, requires a *long-run* solution. Discounted utilitarianism does not necessarily entail growth. Some direction may be found in tentative evidence other authors have found of a preference among consumers for wage or consumption profiles that increase through time [34,35]. An interpretation is that utility is a function of the growth of income as well as its level. If this individual preference ordering can be extended to the entire society, perhaps as an expressed value that favors social *progress*, then some level of sacrifice of the present in the name of growth may be warranted. A preference of the future over the present is implied; the analyst should specify the bias toward the future, or what constitute the "needs" of each generation, according to the preference ordering. This preference ordering is not a conventional welfare function (cf. [31]).

If the preferences are specified, the choice problem for this generalization of a sustained path is a generalization of a maximin problem. Utility may grow toward some limit and be sustained thereafter or else continue to grow indefinitely. Regular growth of utility over time can be considered to be a generalization of a sustained path if the "distribution over time has some definite standard shape" [36]. Suppose, as an example from among a large number of possibilities, that the preferred type of development path is a logistic increase of utility, of the form

$$u_t = \frac{a}{1 + b\exp(-kt)}, a, b, k > 0$$

and that the planner can commit to a path chosen at $t = 0$. Utility grows from $u_0 = a/(1 + b)$ asymptotically toward a at a rate dependent on k. The optimal sustainable-development path is the one for the optimal values of parameters a, b and k according to the preference ordering *and* subject to the environmental and technological constraints. Since maximin maximizes $\bar{u}$ for all $t > 0$, for any given economy it is obvious that $a/(1+b) < \bar{u}$, or less than the "poverty trap" that growth is supposed to avoid. In the logistic program, net investment is always positive.

A general form of the problem, for a vector of parameters α, a preference ordering Γ and a criterion φ, is to

$$\max \Gamma(\alpha) \ s.t. \ u_s \geq \varphi(\alpha, t) \ \forall s \geq t$$

and subject to other constraints.

Shadow prices would not be those from a discounted-utilitarian program but those derived from analyzing the social preferences. The problem is more difficult than the maximin problem. In the context of a sub-optimal society, this problem, too, is a candidate for viability analysis.

8. The Role of Green Accounting: "Responsible Use"

It is important to understand what green accounting can do and what it implies. A workable view is that it is possible to incorporate prices for an increasing number of environmental and other non-priced

or ill priced goods, estimated through cost-benefit analysis, into a consistent accounting methodology. Decisions affecting social welfare, as described in Equation (3), for a non-optimal economy, can be improved using price signals supplied by the market in conjunction with these extensions. The work by Arrow, Dasgupta, Hamilton, Mäler and others is properly viewed as contributing to the understanding of this important subject. Like the conventional national accounts, the accounting methodology should not only be compatible with but built from a disaggregated accounting for microeconomic units.

Contrary to the assumptions and results of many theoretical and empirical studies, however, it is fruitless to use observed prices and accounts to measure sustainability. Trying to measure sustainability using genuine savings is at once too limited and too stringent. It is too stringent in that the condition that welfare be non-decreasing at the current time (or the much stronger condition that it be non-decreasing at all times) is not necessary for the *economy* to be considered sustained at the minimum level of utility attained over the future. It is too limited in that genuine savings relates to the current period only and not to the very long run, which is the focus of sustainability. Economic theory gives no indication of the implications of observed market prices, as adjusted and extended by cost-benefit analysis, for sustaining an economy. The definition of the right prices is subtler than in much of economics: statistics must be based on the right model of value. Deriving sustainability accounts would entail a complete break from the national accounts.

The practical significance of this point is to take the present accounting methodology away from its focus on genuine savings, the misleading indicator of sustainment, and hence away from attempting to measure sustainment altogether.

This change in focus is consistent with the fact that much of what is called "sustainable" practice in policy discussions really involves, not necessarily sustaining society, but a (vague) notion of environmental and social "responsibility". The concept is not defined precisely. In practice, it must be worked out politically. Political decisions implicitly or explicitly incorporate the perceived costs and benefits of particular uses of the environment, rather than a global concern for sustainability or even specifically taking thought for maintaining the society beyond a few generations.

In using accounting statistics for this purpose, even the ones for environmental goods obtained by cost-benefit analysis, one must be aware of what the prices mean. Cost-benefit analysis can, within wide confidence bands, provide price signals that can be the basis for correcting the more egregious market failures. These prices can be presented to individuals, through taxation or other policies, as what they should or must use in their calculations in making their decisions. The use of accounting statistics in informing discussion and practice depends on (a) confidence that cost-benefit analysis can produce prices on which the decisions should be based and (b) the expectation that the decisions of a decentralized economy can effectively provide benefits for current and future generations. It does appear that a decentralized economy is, over the long run, much more effective than a centralized one, even at correcting for market and other failures. No environmentalist praises the record of totalitarian economies in the treatment of the environment.

Still, no statistic is an indicator of whether the society will or will not eventually decline, either because society is not sustainable or because the wrong decisions to sustain it are being made. There is a *conceptual* way of expressing prices that are valid for measuring the sustainability of a region or of the planet. These theoretic prices are based on maximin analysis and for practical purposes viability theory may lead to useful measures as it develops. The research agenda is to determine sustainability

or sustainment prices to complement the comprehensive physical measures that are being compiled by non-economists.

Furthermore, these two techniques may eventually be the foundation of price indicators for quantifying sustainable development according to preferences that are explicit in the trade-offs among the interests of the present generation and future generations.

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Conflict of Interest

The author declares no conflict of interest.

References and Notes

1. World Commission on Environment and Development. *Our Common Future*; Oxford University Press: Oxford, UK, 1987.
2. Solow, R.M. Intergenerational equity and exhaustible resources. *Rev. Econ. Stud.* **1974**, *41*, 9-45.
3. Koopmans, T.C. Stationary ordinal utility and impatience. *Econometrica* **1960**, *28*, 287-309.
4. Weitzman, M.L. On the welfare significance of national product in a dynamic economy. *Q. J. Econ.* **1976**, *90*, 156-162.
5. Asheim, G.B.; Weitzman, M.L. Does NNP growth indicate welfare improvement? *Econ. Lett.* **2001**, *73*, 233-239.
6. Arrow, K.J.; Dasgupta, P.; Mäler, K.-G. Evaluating projects and assessing sustainable development in imperfect economies. *Environ. Resour. Econ.* **2003**, *26*, 647-685.
7. Pezzey, J.C.V.; Toman, M.A. Progress and Problems in the Economics of Sustainability. In *International Yearbook of Environmental and Resource Economics 2002-3*; Tietenberg, T., Folmer, H., Eds.; Edward Elgar: Cheltenham, UK, 2002; pp. 165-232.
8. Asheim, G.B. Net national product as an indicator of sustainability. *Scand. J. Econ.* **1994**, *96*, 257-265.
9. Martinet, V. A step beside the maximin path: Can we sustain the economy by following Hartwick's investment rule? *Ecol. Econ.* **2007**, *64*, 103-108.
10. Neumayer, E. *Weak versus Strong Sustainability: Exploring the Limits of Two Opposing Paradigms*, 2nd ed.; Edward Elgar: Cheltenham, UK, 2003.
11. Solow, R.M. An almost practical step toward sustainability. *Resour. Policy* **1993**, *19*, 162-172.
12. Hamilton, K.; Clemens, M. Genuine savings rates in developing countries. *World Bank Econ. Rev.* **1999**, *13*, 333-356.
13. Pezzey, J.C.V. Sustainability policy and environmental policy. *Scand. J. Econ.* **2004**, *106*, 339-359.
14. Dasgupta, P.; Mäler, K.-G. The Environment and Emerging Development Issues. In *Cost—Benefit Analysis*, 2nd ed.; Layard, R., Glaister, S., Eds.; Cambridge University Press: Cambridge, UK, 1994. Originally published in *World Bank Annual Conference on Development Econ.*, 1990.
15. Hartwick, J.M. Intergenerational equity and the investing of rents from exhaustible resources. *Am. Econ. Rev.* **1977**, *66*, 972-974.

16. Cairns, R.D.; Long, N.V. Maximin: A direct approach to sustainability. *Environ. Dev. Econ.* **2006**, *11*, 275-300.
17. Jorgensen, S.E.; Costanza, R.; Xu, F.-L. *Handbook of Ecological Indicators for Ecosystem Health*; Taylor and Francis Group: Boca Raton, LA, USA, 2005.
18. Howarth, R.B. Towards an operational sustainability criterion. *Ecol. Econ.* **2007**, *63*, 656-663.
19. Rawls, J. *A Theory of Justice*; Harvard University Press: Cambridge, MA, USA, 1971.
20. Withagen, C.; Asheim, G.B. Characterizing sustainability: The converse of Hartwick's rule. *J. Econ. Dyn. Control* **1998**, *23*, 159-165.
21. Pezzey, J.C.V. One-sided sustainability tests with amenities and changes in technology, trade and population. *J. Environ. Econ. Manag.* **2004**, *48*, 613-631.
22. Benchekroun, H.; Withagen, C. The optimal depletion of exhaustible resources: A complete characterization. *Resour. Energy Econ.* **2011**, *33*, 612-636.
23. Chichilnisky, G. An axiomatic approach to sustainable development. *Soc. Choice Welf.* **1996**, *13*, 219-248.
24. Alvarez-Cuadrado, F.; Long, N.V. A mixed Bentham-Rawls criterion for intergenerational equity. *J. Environ. Econ. Manag.* **2009**, *58*, 154-168.
25. Figuières, C.; Guyomard, H.; Rotillon, G. Sustainable development: Between moral injunctions and natural constraints. *Sustainability* **2010**, *2*, 3608-3622.
26. Asheim, G.B.; Mitra, T. Sustainability and discounted utilitarianism in models of economic growth. *Math. Soc. Sci.* **2010**, *59*, 148-169.
27. Cairns, R.D.; Tian, H. Sustained development of a society with a renewable resource. *J. Econ. Dyn. Control* **2010**, *34*, 1048-1061.
28. Cairns, R.D. Value and Income. *Ecol. Econ.* **2008**, *66*, 417-424.
29. Martinet, V.; Rotillon, G. Invariance in growth theory and sustainable development. *J. Econ. Dyn. Control* **2007**, *31*, 2827-2846.
30. Martinet, V.; Doyen, L. Maximin, Viability and Sustainability. In *Proceedings of the 18th Annual European Association of Environmental and Resource Economists Conference*, 29 June–1 July 2011, Rome, Italy.
31. Martinet, V. A characterization of sustainability with indicators. *J. Environ. Econ. Manag.* **2011**, *61*, 183-197.
32. Hamilton, K.; Hartwick, J.M. Investing exhaustible resource rents and the path of consumption. *Can. J. Econ.* **2005**, *38*, 615-621.
33. Hamilton, K.; Withagen, C. Savings growth and the path of utility. *Can. J. Econ.* **2007**, *40*, 703-712.
34. Lowenstein, G.; Sicherman, N. Do workers prefer increasing wage profiles? *J. Labor Econ.* **1991**, *9*, 67-84.
35. Frank, R.; Hutchens, R. Wages, seniority, and the demand for rising consumption profiles. *J. Econ. Behav. Organ.* **1993**, *21*, 251-276.
36. Hicks, J.R. *Value and Capital*; Clarendon Press: Oxford, UK, 1946.