energies

MDPI

Article

Hierarchically Distributed Charge Control of Plug-In Hybrid Electric Vehicles in a Future Smart Grid

Hanyun Zhou [1], Wei Li [2,*] and Jiekai Shi [2]

[1] College of Information Engineering, Zhejiang University of Technology, Hangzhou 310023, China; zhouhanyun@zjut.edu.cn
[2] College of Information and Electrical Engineering, Hangzhou City University, Hangzhou 310015, China
* Correspondence: liwei@hzcu.edu.cn

Abstract: Plug-in hybrid electric vehicles (PHEVs) are becoming increasingly widespread due to their environmental benefits. However, PHEV penetration can overload distribution systems and increase operational costs. It is a major challenge to find an economically optimal solution under the condition of flattening load demand for systems. To this end, we formulate this problem as a two-layer optimization problem, and propose a hierarchical algorithm to solve it. For the upper layer, we flatten the load demand curve by using the water-filling principle. For the lower layer, we minimize the total cost for all consumers through a consensus-like iterative method in a distributed manner. Technical constraints caused by consumer demand and power limitations are both taken into account. In addition, a moving horizon approach is used to handle the random arrival of PHEVs and the inaccuracy of the forecast base demand. This paper focuses on distributed solutions under a time-varying switching topology so that all PHEV chargers conduct local computation and merely communicate with their neighbors, which is substantially different from the existing works. The advantages of our algorithm include a reduction in computational burden and high adaptability, which clearly has its own significance for the future smart grid. Finally, we demonstrate the advantages of the proposed algorithm in both theory and simulation.

Keywords: PHEVs; hierarchically distributed charge; load demand flatten; operational cost minimization; consensus-like method; water-filling principle

Citation: Zhou, H.; Li, W.; Shi, J. Hierarchically Distributed Charge Control of Plug-In Hybrid Electric Vehicles in a Future Smart Grid. *Energies* **2024**, *17*, 2412. https://doi.org/10.3390/en17102412

Academic Editor: Giovanni Lutzemberger

Received: 14 April 2024
Revised: 30 April 2024
Accepted: 13 May 2024
Published: 17 May 2024

1. Introduction

Electric vehicle (EV) penetration has increased due to its environmental benefits and excellent energy efficiency. Plug-in hybrid electric vehicles (PHEVs) are an excellent choice for urban transport due to their lower fuel consumption, longer driving range, and reduced greenhouse gas emissions. PHEVs use both batteries and internal combustion engines, making them a versatile option for those who want to reduce their carbon footprint while still enjoying the convenience of a traditional vehicle [1].

While the widespread PHEV adoption by consumers brings potential societal and economic benefits, the high-level penetration of PHEVs will impact the safe and reliable operation of the distribution system [2]. Uncoordinated charging of PHEVs will increase electrical loads and, therefore, amplify current peak loads or cause new peaks. PHEV charging can also overload distribution network appliances (e.g., transformers), thereby reducing their service life, resulting in significant voltage deviations from the rated value, and exposing power systems to severe security risks [3]. In a state-of-the-art reference [4], Mansouri et al. proposed a novel approach to controlling and managing energy production based on grid requirements, in which the designed adaptive high-gain observer can estimate the grid energy requirement based on the voltage value at the endpoint of the high-voltage direct current line. The method proposed in [4] leads a new research perspective on information collection and the estimation of electric vehicle loads for smart grid load stabilization.

According to [5], Lopes et al. pointed out that smart metering technologies and communication systems had a significant impact on the stability of a distribution system. The stability of a distribution system can be affected by even 10% penetration of EVs. Therefore, it is crucial to manage and coordinate these technologies carefully to ensure the stability of a distribution system. To this end, the author of [6] addressed this issue by managing the timing and speed of PHEV charging through Demand-Side Management (DSM), resulting in enhanced power supply reliability, reduced energy consumption, and lower supply and demand costs. Moreover, DSM studies for PHEV charging scheduling were classified into time-of-use (TOU) and centralized control. The TOU models presented in [7,8] do not accurately reflect the actual characteristics of a power system's load profile, which can lead to peaking failures. In contrast, refs. [9,10] optimized multiple objectives, including minimizing load variation and charging costs and maximizing EV penetration levels, by employing a centralized control strategy. However, centralized strategies may not be suitable for large-scale future smart grids due to the significant communication and computational overheads required to collect information from all PHEVs.

Based on multi-agent system frameworks, distributed charging control strategies enable the sharing of computational and communication burdens among distributed agents, which may be more suitable for large-scale distribution power systems. A demand response strategy was proposed in [11] to alleviate the potential new load peaks. The authors of [12,13] developed decentralized PHEV charging control schemes to fill the valleys in electric load profiles. However, without considering the energy costs, the algorithms of [12,13] cannot provide incentives for the users to participate in. After considering PHEV users' benefits, reference [14] proposed an optimal charging rate control of PHEVs based on a consensus algorithm aligning each PHEV's interest with the system's benefit. Reference [15] proposed convergent distributed algorithms to calculate a robust price for all users, in which the simulation shows that the proposed method can effectively reduce the monetary expenses for all users in a real-time market. Reference [16] proposed a distributed control strategy for EV charging, which can determine the optimal power allocation to reduce user anxiety. However, references [14–16] designed the objective function only from the user's perspective, either to minimize the charging cost or to minimize user anxiety.

The references [17–19], are more relevant to this work as they address the issues of shifting load demand and reducing energy costs simultaneously. In detail, ref. [17] highlighted the conflict of interest when determining the ideal state of charge curve, as PHEV owners aimed to purchase energy at the lowest possible cost to maximize earnings while also ensuring that the ideal charging did not interfere with their daily driving. The authors of [18] proposed an effective method for addressing the optimal charging control problem for PHEVs in the deregulated electricity market, employing a dynamic programming technique to obtain the optimal solution. It should be worth noting, however, that the implementation of this system required careful consideration to avoid any potential instability. To address this issue, ref. [19] proposed a non-cooperative approach that created an energy charging game to reach the Nash equilibrium, in which each PHEV independently selected the optimal course of action to reduce its energy charging cost. The suggested distributed algorithm can effectively reduce peak demand and overall energy costs. However, the approach proposed in [17–19] is suitable for practical scenarios where PHEV owners arrive or leave randomly. As a result, we feel obliged to provide a distributed algorithm to solve this problem quantitatively, which is suitable for PHEVs' random arrival and departure or the inaccuracy of the forecast base demand.

In this paper, we focus on finding an economically optimal solution under the condition of flattening the load demand in a future smart grid with PHEVs' random penetration. This issue is formulated as a two-layer convex optimization problem, in which the condition of flattening the load demand for the system is considered as an equality constraint of the problem. A time-varying and periodic connectivity topology is employed for the exchange of information between heterogeneous PHEVs. To the best of our knowledge, this approach has not been explored in existing works. The key technical contributions made in this work are summarized as follows:

(1) A two-layer optimization model of load dispatch for PHEV charging control is established in a future smart grid. In detail, this model investigates the power load stability of energy sources and energy cost minimization of PHEV consumers simultaneously, and technical constraints are both taken into account.
(2) A time-varying and periodical connected communication network is considered to model the information exchange among PHEVs, which is substantially different from the existing works. With the expansion of the scale of the future smart grid, this network communication architecture is still able to maintain good robustness.
(3) A consensus-based approach combined with the water-filling method is designed to reach the optimal solution to the two-layer optimization problem. To address the unpredictable arrival of PHEVs and the inaccurate estimate of the base load, the hierarchical algorithm combined with the moving horizon method is proposed, which is also appropriate for engineering practice.

The rest of this paper is organized as follows: In Section 2, the model of the power distribution system and PHEV charging is introduced and the problem formulation is presented. In Section 3, we present the hierarchical algorithm. In Section 4, the convergence and optimality of the proposed algorithm are presented. In Section 5, numerical simulations are given. In Section 6, we conclude this paper.

2. Preliminaries and Problem Formulation

In this section, we outline the architecture of the future smart grid and the model of PHEV charging. Then, the PHEV charging scheduling problem is formulated as a two-layer optimization problem.

2.1. Power Distribution System Modeling

In this paper, a two-layer framework of a future smart grid for coordinated charging of PHEVs is shown in Figure 1. The energy source acts as an energy provider, e.g., a generator connected to the power grid. Each PHEV consumer, connected to the power line, is equipped with a smart meter that has the capability to schedule its energy consumption.

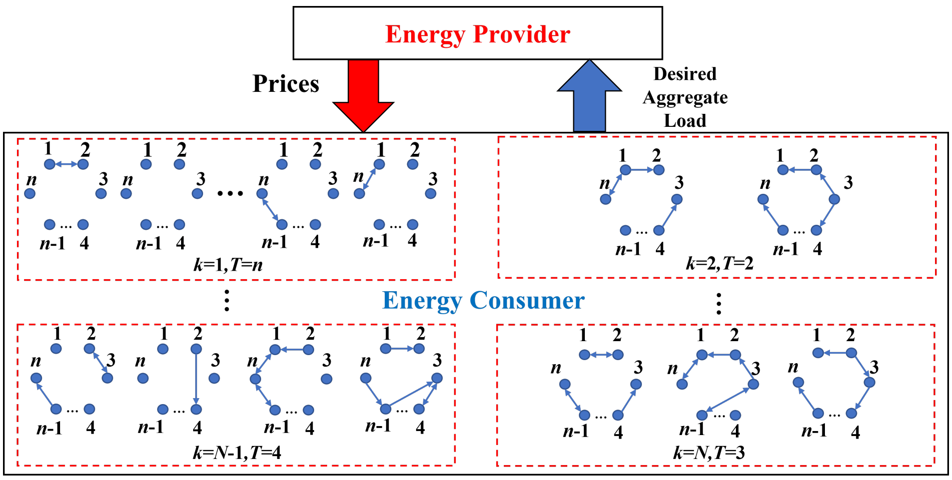

Figure 1. Architecture of future smart grid integrating with energy providers and energy consumers.

Due to recent advancements in smart grid technology, the interactions between smart meters do not have to be manual but can be automatic through a local area network (LAN) [20]. In this paper, we assume that the smart meters are periodically connected to an LAN in a time horizon, rather than being connected all the time. Take $k = N - 1, T = 2$ as an example in Figure 1. There are two time slots in the time horizon $k = N - 1$. Each time slot has only one communication link between n PHEVs, forming a jointly strongly connected communication topology with a period of $T = 2$. This communication mode brings many benefits, such as better robustness, reduced communication load for LAN, longer lifetime of smart meters, and the ability to cope with more complex scenarios where communication interruptions occur due to link interference.

2.2. Dynamic Model of PHEV Charging

Lithium-ion batteries are the preferred choice for PHEVs due to their excellent load characteristics and high energy density. The state of charge (SOC) of the battery in a PHEV is precisely defined as

$$S(k) = \frac{C(k)}{C} \times 100\% \tag{1}$$

The equation for updating the SOC for the i-th PHEV is as follows, where C kWh denotes the battery energy capacity and $C(k)$ denotes the remaining battery energy capacity at time k:

$$S(k+1) = S(k) + \frac{x_i^k \cdot \Delta T}{C_i}\eta \tag{2}$$

where x_i^k is the charging power at time k, and ΔT is the sampling period. The coefficient $\eta \in (0,1)$ is assumed to be constant [21]. Equation (2) is rewritten as

$$S(k+1) = S(k) + a_i x_i^k \tag{3}$$

where

$$a_i = \begin{cases} \frac{\Delta T}{C_i}\eta & \text{if } C_i > 0. \\ 0 & \text{if } C_i = 0. \end{cases} \tag{4}$$

2.3. Problem Formulation

Consider a given set $\mathcal{V}$ of $n = |\mathcal{V}|$ PHEVs, and each household owns a PHEV. The charging horizon of the PHEVs is divided into N time slots. The start and end time slots of the valid scheduling for the PHEVs are denoted by 1 and N, respectively. It is assumed that the forecast base demand $q_i^k, k = 1, \cdots, N$, is known for the household i. The objective of the power load fluctuation minimization problem can be formulated as

$$\min_{d^k} f(d) = \sum_{k=1}^{N} \left(d^k + \sum_{i=1}^{n} q_i^k - \xi \right)^2 \tag{5a}$$

$$\text{s.t.} \begin{cases} \sum_{k=1}^{N} d^k = \sum_{i=1}^{n} b_i \\ 0 \le d^k \le \sum_{i=1}^{n} \overline{x}_i. \end{cases} \tag{5b}$$

The ideal flat power curve is denoted as ξ. The optimization variables are d^k, $k = 1, \cdots, N$, representing the charging energy providing for n PHEVs at time slot k by the energy source. The energy requirement of the i-thPHEV over an N-period charging horizon is represented by b_i. The maximum charger power of the i-th PHEV is denoted by $\overline{x}_i$.

Indeed, $f(d) = 0$ if and only if the aggregate power curve $(d^k + \sum_{i=1}^{n} q_i^k)$ is flat over k. ξ is given by

$$\xi = \frac{1}{N}\left(\sum_{i=1}^{n} b_i + \sum_{k=1}^{N}\sum_{i=1}^{n} q_i^k \right). \tag{6}$$

The lower layer formulates a cost minimization problem to identify the economically optimal solution while adhering to technical constraints. Let x_i denote a charging vector for PHEV i as

$$x_i = [x_i^1, \cdots, x_i^k, \cdots, x_i^N]^{\mathrm{T}}, \tag{7}$$

where x_i^k denotes the i-th PHEV charging power at time slot k. The energy charge for each PHEV i at each time slot k is restricted within the minimum and maximum limits:

$$0 \le x_i^k \le \overline{x}_i, i \in \mathcal{V}, \forall k = 1, \cdots, N. \tag{8}$$

The initial battery level for charging the i-th PHEV is represented by x_i^0. Once the charging process is complete, the battery of each PHEV must reach a predetermined energy target level, denoted as B_i. The energy required to charge the battery of PHEV i can be calculated as follows:

$$b_i = B_i - x_i^0. \tag{9}$$

The total energy requirement of the i-th PHEV is constrained as

$$\sum_{k=1}^{N} x_i^k = b_i, i \in \mathcal{V}. \tag{10}$$

Furthermore, the total energy consumption of n PHEVs at time slot k is the energy d^k provided by the energy source on the upper layer. In this regard, it is required that

$$\sum_{i=1}^{n} x_i^k = d^k, \forall k = 1, \cdots, N. \tag{11}$$

As a result, a feasible energy charging set for each PHEV is defined as

$$\chi_i = \{x_i | \text{ constraints } (8), (10), (11)\}. \tag{12}$$

Flattening the load curve is a preferred design objective for the energy source of the distribution system. However, from the consumers' perspective, scheduling their energy charging process to minimize their total payment at the end of each day is crucial. This provides incentives for them to participate in the charging stage. A cost function $g(x_i^k)$ (dollar) is defined as the cost of purchasing x_i^k units of energy for the i-th PHEV during time slot k from the utility company, adhering to the following assumption.

Assumption 1. *For $\forall i, k = 1, \cdots, N$, $g(x_i^k)$: $R_+ \to R_+$ is strictly convex and twice continuously differentiable with*

$$\frac{d^2 g(x_i^k)}{d(x_i^k)^2} > 0, \ \forall x_i^k \in R_+$$

where $\mathbb{R}_+$ denotes the set of nonnegative real numbers.

The low-layer operational cost minimization problem for each PHEV i can be formulated as

$$\min_{\forall i, x_i \in \chi_i} \sum_{i=1}^{n} \sum_{k=1}^{N} g(x_i^k) \tag{13}$$

Remark 1. *The feasible set constraints for each PHEV consists solely of linear constraints, rendering it both convex and compact. Moreover, the objective function of Equation (13) is strictly convex. Therefore, the problem of minimizing operational costs (13) is also convex and has a unique optimal solution.*

3. Hierarchical Algorithm

This section presents a hierarchical algorithm for achieving the economically optimal solution while flattening the load demand for the distribution system. The objective function is reformulated and the corresponding algorithm is given for the more general case where PHEVs are allowed to arrive randomly.

3.1. Water Filling for the Upper Layer

To solve the DSM problem (5) for the upper layer, a decentralized algorithm based on the water-filling principle is used to flatten the load demand curve. The Lagrangian of (5) can be calculated using the Lagrange multiplier

$$L(d,\lambda) = \sum_{k=1}^{N} \left(d^k + q^k - \xi\right)^2 + 2\lambda\left(\sum_{k=1}^{N} d^k - b\right) \tag{14}$$

where $q^k = \sum_{i=1}^{n} q_i^k$, $b = \sum_{i=1}^{n} b_i$. For convenience, the factor of two is included in (14), and to obtain a result of zero, the Lagrangian is differentiated with respect to d^k:

$$d^k + q^k - \xi + \lambda = 0. \tag{15}$$

Equation (15) can be rewritten as follows by denoting $\delta = \xi - \lambda$, which is independent of k:

$$d^k + q^k = \delta. \tag{16}$$

Let $d_{\max}^k = \sum_{i=1}^{n} \overline{x}_i$. Equation (16) provides the optimality criterion with the introduced constant δ, known as the equipower level. However, it does not take into account the inequality constraint $0 \le d^k \le d_{\max}^k$. It is important to note that after taking the inequality constraint into account, (16) remains valid or

$$d^k = 0 \ \text{ and } \ d^k + q^k \ge \delta, \tag{17}$$

or

$$d^k = d_{\max}^k \ \text{ and } \ d^k + q^k \le \delta. \tag{18}$$

Algorithm 1 presents a decentralized algorithm based on the water-filling principle. The optimal value of δ can be confidently determined using a bi-section approach. The projection operation $\mathcal{D}(\cdot)$ is presented in Equation (19), and ε is a small positive value.

$$\mathcal{D}(d^k) = \begin{cases} d_{\max}^k & \text{if } d^k > d_{\max}^k, \\ d^k & \text{if } 0 \le d^k \le d_{\max}^k, \\ 0 & \text{if } d^k < 0, \end{cases} \tag{19}$$

where $d_{\max}^k = \sum_{i=1}^{n} \overline{x}_i$.

Algorithm 1 Water Filling for the Upper Layer

Input: $\varepsilon, d_{\max}^k, b$ and $q^k, k = 1, \cdots, N$
Output: δ and $d^k, k = 1, \cdots, N$
1. Initialize $\delta_{\min} = \min_k q^k$ and $\delta_{\max} = \max_k q^k + d_{\max}^k$
2. **while** $\delta_{\max} - \delta_{\min} > \varepsilon$ **do**
3. Choose $\delta = (\delta_{\max} + \delta_{\min})/2$
4. Compute $d^k = \mathcal{D}(\delta - q^k), k = 1, \cdots, N$
5. **if** $\sum_{k=1}^{N} d^k > b$ **then**
6. set $\delta_{\max} = \delta$
7. **else if** $\sum_{k=1}^{N} d^k < b$ **then**
8. set $\delta_{\min} = \delta$
9. **end if**
10. **end while**

Remark 2. *In Algorithm 1, we apply a water-filling principle for the upper layer optimization problem. First, the variable $\delta = min_k q^k$ is initialized. Then, δ is systematically raised, $d^k = \delta - q^k$ is calculated and projected to the feasible region $[0, d^k_{max}]$, and $\sum_{k=1}^{N} d^k$ is computed. Finally, δ is gradually increased until it reaches b.*

3.2. Consensus-like Iterative Method for the Lower Layer

Based on equality constraints (10) and (11), the operational cost minimization problem (13) is decoupled as:

$$\min_{\bar{\mathcal{G}},\varsigma_i^k} \sum_{i=1}^{n}\sum_{k=1}^{N} g(\varsigma_i^k) \tag{20a}$$

$$\underline{\varsigma}_i \leq \varsigma_i^k \leq \bar{\varsigma}_i, \forall i,k \tag{20b}$$

$$\sum_{i=1}^{n} \varsigma_i^k = d^k, \forall k \tag{20c}$$

and

$$\min_{\hat{\mathcal{G}},\xi_i^k} \sum_{i=1}^{n}\sum_{k=1}^{N} g(\xi_i^k) \tag{21a}$$

$$\underline{\xi}_i \leq \xi_i^k \leq \bar{\xi}_i, \forall i,k \tag{21b}$$

$$\sum_{k=1}^{N} \xi_i^k = b_i, \forall i \tag{21c}$$

where ς_i^k and ξ_i^k are the optimization variables, $\bar{\varsigma}_i = \bar{\xi}_i = \bar{x}_i$ and $\underline{\varsigma}_i = \underline{\xi}_i = 0$.

Remark 3. *Smart meters and communication technology provide resource scheduling information to each node in each time period [22]. Each node is linked to a virtual node in each time period, and all virtual child nodes form a virtual digraph sequence $\hat{\mathcal{G}}$ as well. This enables a virtual wireless network to exchange information among scheduling periods within a node. Hence, problem (21) is the dual problem of (20).*

To illustrate the implementation process of the proposed method, we will solve (20) as an example. Our previous work [23] proposed a consensus-like iterative method, and the definition of the variables (i.e., λ_i^k, $\phi_i^k(\cdot)$, $\bar{\mathcal{G}}$, and $\mathcal{N}_{i,k}(t)$) can be found in [23]. The consensus-like iterative method to solve (21) is interpreted as follows:

Step 1: Initialization.

Step 2: Let each PHEV i in time k have its own copy of Lagrange multipliers to satisfy (20c), and update the Lagrange multiplier according to the consensus-based iteration (25a), such that all Lagrange multipliers reach consensus.

Step 3: Map the estimated power state ς into the interval $[0, \bar{x}_i^k]$ based on (25b).

Step 4: Due to the fact that the nonlinear projection $\phi_i^k(\cdot)$ may not be a feasible solution for (20), the surplus variable s_i^k is adopted for iteration according to (25c), such that it can be averaged with its neighbors.

Step 5: Rerun Step 2 to Step 4 until the sum of surplus converges to zero. These iterative processes are summarized in Algorithm 1.

Remark 4. *Algorithm 2 guarantees that $s_i^k(t)$ remains non-negative by utilizing the operator $[\cdot]_-$. The algorithm is presented concisely to solve both (20) and its dual problem (21) by adjusting the optimization variables and configuring the network accordingly.*

Algorithm 2 Consensus-like Iteration for the Lower Layer

Initialization:

(1) PHEV selects

$$x_i^k(0) \in [\underline{x}_i, \bar{x}_i] \text{ and } s_i^k(0) \geq 0 \text{ for all } i \text{ and } k \tag{22}$$

such that

$$\mathbf{C}\mathbf{x}(0) + \sum_{i,k} s(0) = d \tag{23}$$

(2) PHEV chooses $\lambda_{i,k}(0)$ such that

$$\mathbf{C}^{\mathrm{T}}_{*N(i-1)+k} \boldsymbol{\lambda}_{i,k}(0) = f\left(x_i^k(0)\right) \tag{24}$$

Update:

$$\lambda_i^k(t+1) = \lambda_i^k(t) + \left[\sum_{j \in \mathcal{N}_{i,k}(t)} a_{i,k}(t)\left(\lambda_j^k(t) - \lambda_i^k(t)\right)\right]_- + \epsilon_{i,k}(t) s_i^k(t) \tag{25a}$$

$$x_i^k(t+1) = \phi_i^k\left(\mathbf{C}^{\mathrm{T}}_{*N(i-1)+k} \lambda_{i,k}(t+1)\right) \tag{25b}$$

$$s_i^k(t+1) = b_{i,k}(t) s_i^k(t) + \sum_{j \in \mathcal{N}_{i,k}(t)} b_{j,k}(t) s_j^k(t) - \left(x_i^k(t+1) - x_i^k(t)\right) \tag{25c}$$

where the parameters of the PHEV network are chosen as follows:

(1) when solving (20):

$\mathbf{C} = \mathbf{1}_n^{\mathrm{T}} \otimes \mathbf{I}_N, \mathbf{x} = \varsigma, \sum_{i,k} s(0) = \sum_{i=1}^n s_i^k(0), d = d^k, \lambda_{i,k} = [\lambda_i^1, \cdots, \lambda_i^N]^{\mathrm{T}}, \mathcal{N}_{i,k}(t) = \mathcal{N}_i^{k+}(t), a_{i,k}(t) = 1/d_i^{k+}(t), b_{i,k}(t) = 1/d_i^{k-}(t), \epsilon_{i,k}(t) = o_i^k b_{i,k}(t), 0 < o_i^k < \ell_i^k$.

(2) when solving (21):

$\mathbf{C} = \mathbf{I}_n \otimes \mathbf{1}_N^{\mathrm{T}}, \mathbf{x} = \boldsymbol{\xi}, \sum_{i,k} s(0) = \sum_{k=1}^N s_i^k(0), d = b_i, \lambda_{i,k} = \left[\lambda_1^k, \cdots, \lambda_n^k\right]^{\mathrm{T}}, \mathcal{N}_{i,k}(t) = \hat{\mathcal{N}}_i^{k+}(t), a_{i,k}(t) = 1/\hat{d}_i^{k+}(t), b_{i,k}(t) = 1/\hat{d}_i^{k-}(t), \epsilon_{i,k}(t) = o_i^k b_{i,k}(t), 0 < o_i^k < \ell_i^k$.

3.3. Hierarchical Algorithm with Moving Horizon

Algorithm 2 considers that all PHEVs start charging at the same time, but this is unrealistic. In this subsection, we explore the general case where PHEVs can arrive randomly and develop an optimal algorithm based on Algorithm 2 and the moving horizon principle.

First, operational cost minimization (13) needs adjustment accordingly as follows

$$\min_{x_i} \sum_{i=1}^{n} \sum_{k=t}^{K(k)-1} g(x_i^k) \tag{26a}$$

$$\text{s.t.} \begin{cases} \sum_{k=t}^{K(k)-1} x_i^k = b_i, i = 1, \cdots, n \\ \sum_{i=1}^{n} x_i^k = d^k, k = t, \cdots, K(k) - 1 \\ 0 \leq x_i^k \leq \bar{x}_i, i = 1, \cdots, n;\ k = t, \cdots, K(k) - 1 \end{cases} \tag{26b}$$

where t means the optimization starts from the present time, $K(k) = \max(K_i), i = 1, \cdots, n$ at time k, and $K_i(K_i \leq N)$ is the charging horizon of the i-th PHEV. If PHEV$_i$ does not arrive, K_i is set to zero. Therefore, $K(k)$ will not change until a new PHEV with a relatively late exit time arrives. Furthermore, $\xi(k)$ is given by

$$\xi(k) = \frac{\sum_{i=1}^n (b_i - b_i(t-1)) + \sum_{k=t}^{K(k)-1} \sum_{i=1}^n q_i^k}{K(k) - t} \tag{27}$$

Algorithm 3 tackles the online optimization problem by minimizing the objective function. Our algorithm is optimal in the sense that for a given k, the objective function in (26) is minimized.

Algorithm 3 Hierarchical Algorithm with Moving Horizon

Input: $\overline{x}_i, K_i$ and $b_i, i = 1, \cdots, n$
Output: $x_i^k, i = 1, \cdots, n$
1. **while** 1 **do**
2. Compute $K(k) = \max(K_i), i = 1, \cdots, n$
3. Perform Algorithms 1 and 2
4. Get $x_i^k, i = 1, \cdots, n$
4. Set $k + 1 = k$
5. **end while**

Remark 5. *Algorithm 3 updates x_i^k at each time slot k. x_i^k can also be updated upon the arrival of another PHEV or a change in the forecast of non-PHEV loads, in order to optimize calculation and communication efficiency.*

4. Convergence and Optimality

Here, we present the convergence and optimality of the proposed hierarchical algorithm.

Theorem 1. *In the power distribution system, as depicted in Figure 1, the solution for the power load fluctuation minimization problem (5) and the energy cost minimization problem (13), given by the hierarchical algorithm, is optimal if and only if the following are satisfied:*

(1) ε is sufficiently small;
(2) Assumption 1 holds;
(3) The topology for PHEVs in the lower layer is jointly strongly connected.

To prove Theorem 1, the following lemmas are needed.

Lemma 1. *The solution given by Algorithm 1 on the upper layer is optimal when ε is sufficiently small.*

Proof. See the proof of Lemma 1 in [24]. □

Lemma 2. *In the power distribution system, the optimal solution to the energy cost minimization problem can be obtained through Algorithm 2 if Assumption 1 holds.*

Proof. The proof of Lemma 2 is divided into two steps.

Step 1: According to Theorem 1 of reference [25] under Assumption 1, one derives that the optimal solution to (20) can be obtained by Algorithm 2 under jointly strongly connected topology. In the same way, one derives that the optimal solution (21) can be obtained by Algorithm 2 under a constructed virtual topology, which is jointly strongly connected as well.

Step 2: In view of the fact that the solution to (20) and the solution to (21) are inconsistent, the penalty-based function is introduced for alternative updating. We reconstruct (20) as:

$$\begin{aligned} &\min_{\overline{\mathcal{G}}, \hat{\varsigma}_i^k(\theta)} \sum_{i=1}^{n} \sum_{k=1}^{N} g\left(\hat{\varsigma}_i^k(\theta)\right) + \frac{\beta}{2} \sum_{i=1}^{n} \sum_{k=1}^{N} \left(\hat{\varsigma}_i^k(\theta) - \hat{\varsigma}_i^{k^*}(\theta - 1)\right)^2 \\ &\text{s.t.} \begin{cases} 0 \le \hat{\varsigma}_i^k(\theta) \le \bar{\varsigma}_i, \forall i, k \\ \sum_{i=1}^{n} \hat{\varsigma}_i^k(\theta) = \tau^k, \forall k \end{cases} \end{aligned} \tag{28}$$

Similarly, we reconstruct (21) as:

$$\begin{aligned} &\min_{\hat{\mathcal{G}},\hat{\xi}_i^k(\theta)} \sum_{i=1}^{n}\sum_{k=1}^{N} g\left(\hat{\xi}_i^k(\theta)\right) + \frac{\beta}{2}\sum_{i=1}^{n}\sum_{k=1}^{N}\left(\hat{\xi}_i^k(\theta) - \xi_i^{k^*}(\theta-1)\right)^2 \\ &\text{s.t.} \begin{cases} 0 \le \hat{\xi}_i^k(\theta) \le \bar{\xi}_i, \forall i,k \\ \sum_{k=1}^{N}\hat{\xi}_i^k(\theta) = \sigma_i, \forall i \end{cases} \end{aligned} \tag{29}$$

where $\xi_i^k(\theta)$ and $\hat{\xi}_i^k(\theta)$ are optimization variables, θ is the number of update iterations, and β is the positive penalty factor.

Compared with (20) and (28) (resp. (21) and (29)), it is obvious that only the objective function is added with the penalty term. As a result, Algorithm 2 is able to solve (28) and (29) but needs to replace the cost function correspondingly. After obtaining the solutions to (28) and (29), the common global optimal solutions to (20) and (21) can be obtained by alternative updating after sufficient iteration.

Steps 1 and 2 establish that (20) and (21) can converge to a common global optimal solution. Since (13) is a convex optimization problem, decoupled as (20) and (21), with a unique optimal solution, we derive that the common optimal solution is the optimal solution to the operational cost minimization problem (13). These establish Lemma 2. □

Here, we give the proof of Theorem 1.

Proof. Lemma 1 proves that the iterative water-filling-based algorithm converges to the unique optimal solution for the power load fluctuation minimization problem (5). Lemma 2 proves that the operational cost minimization problem (13) can achieve the optimal solution through the penalty-based consensus approach. These establish Theorem 1. □

5. Simulation Examples

In this section, we give numerical simulations to illustrate the effectiveness of the proposed algorithm.

5.1. Four-PHEV Simulation with Random Arrival

In this simulation, four PHEVs are taken into account which arrive and exit at random. Tables 1 and 2 provide the simulation parameters. A reasonable sampling interval of 7 samples/h makes full use of the charging characteristics of the battery, reduces the computation of the algorithm, and improves its operation efficiency. The starting time of the PHEV charge is considered to be 1, which corresponds to 18:00 h. There are 168 samples for 24 h. The jointly strongly connected digraph serves as a communication channel for the four PHEVs. The unit for power is kW, and the unit for energy is kWh. Figures 2–8 display the simulation results.

Table 1. Parameters of four PHEVs.

PHEVs	Max Power (kW)	Energy Demand (kW)	Access Time	Exit Time
1	6	25	1	82
2	8.5	35	1	108
3	5.5	30	32	98
4	5	32	45	126

Table 2. Cost function of four PHEVs.

PHEVs	Cost Function (Dollars)	Incremental Cost (Dollars)
1	$\frac{1}{3}(x_i^k+10)^3 + x_i^k - 333.3$	$(x_i^k+10)^2+1$
2	$\frac{2}{3}(x_i^k+4)^3 + 2x_i^k - 42.6$	$2(x_i^k+4)^2+2$
3	$(x_i^k+8)^3 + 3x_i^k - 512$	$3(x_i^k+8)^2+3$
4	$3(x_i^k)^2$	$6x_i^k$

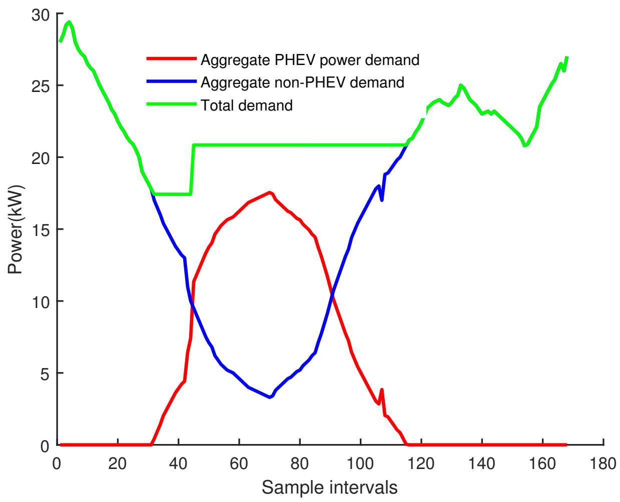

Figure 2. Power curve from Algorithm 3.

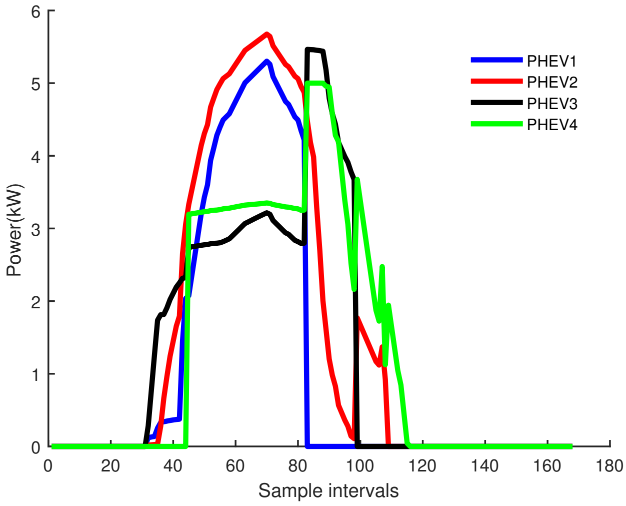

Figure 3. Power allocation by Algorithm 3.

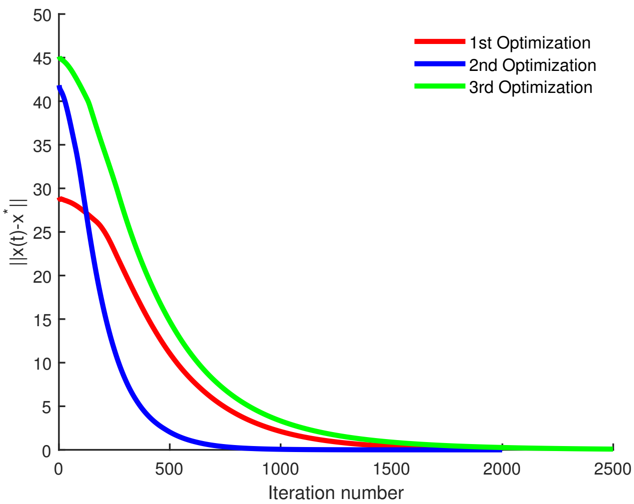

Figure 4. Convergence of Algorithm 3.

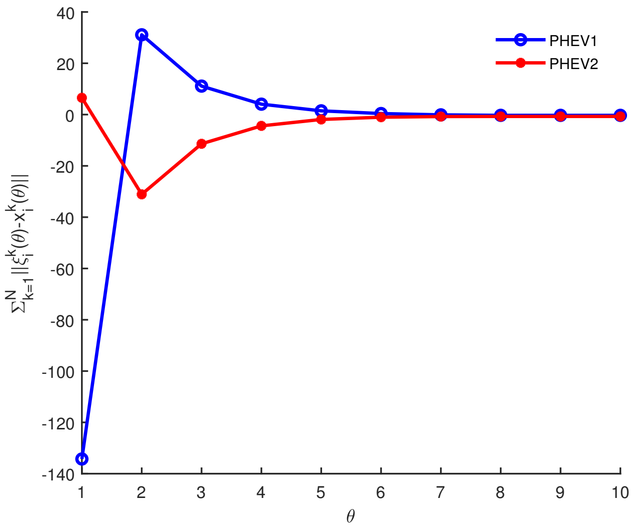

Figure 5. First optimization simulation result.

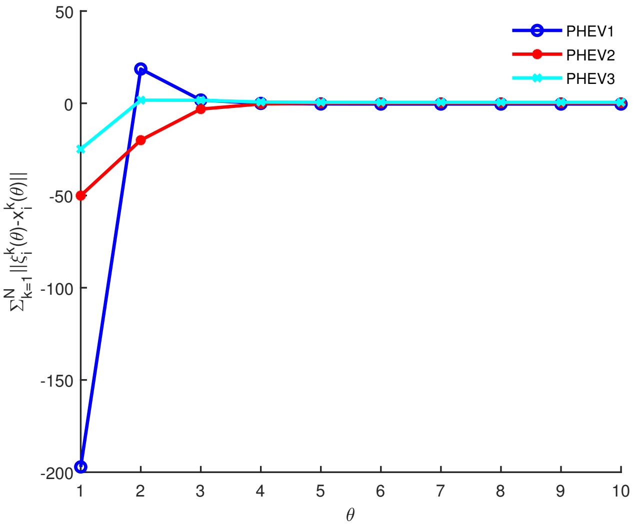

Figure 6. Second optimization simulation result.

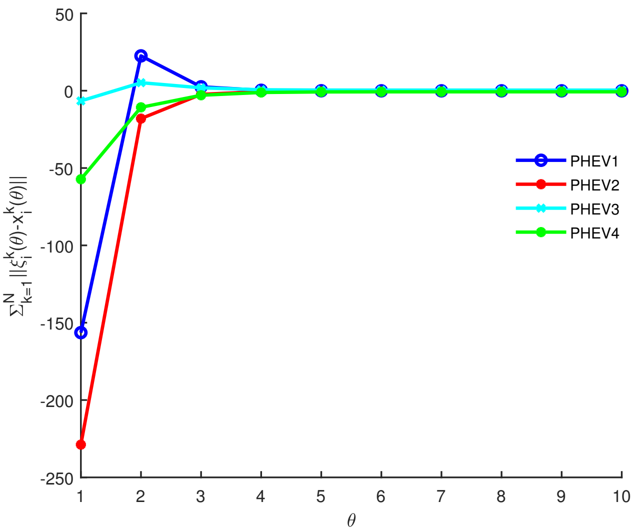

Figure 7. Third optimization simulation result.

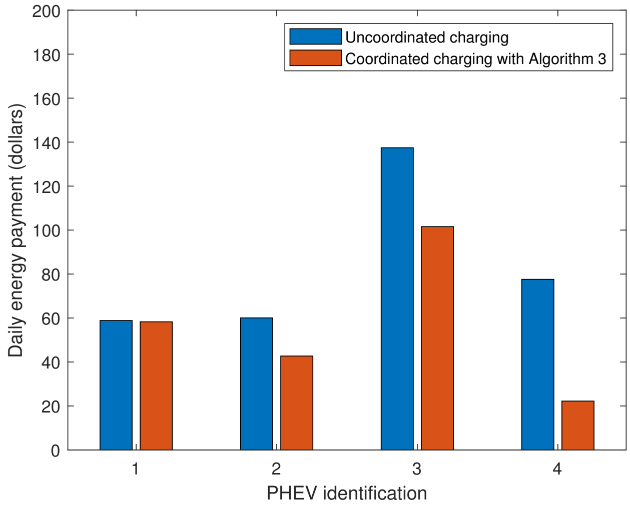

Figure 8. Daily energy cost for the households.

The blue curve in Figure 2 represents the total non-PHEV power demand and is taken from [26]. The total load curve is filled into the valley in Figure 2, which shows that the proposed approach is appropriate for scenarios with random arrival and departure times. Figure 3 depicts the PHEV's charging performance for each time horizon. The state errors for three rolling optimizations, as illustrated in Figures 4–7, show that Algorithm 3 is capable of obtaining a globally optimal solution. According to Figure 8, the coordinated scheduling of charging optimizations lowers consumers' overall costs compared to uncoordinated charging methods (i.e., charging as rapidly and as powerfully as feasible).

5.2. Simulation Using Realistic Data

In this subsection, the realistic non-PHEV household demand curve has a similar pattern to the curve in [11]. It contains 24 h data with 1 min sample times. There are 1440 samples for 24 h. As stated in Assumption 1, the cost function $g(x_i^k)$ (dollar) is defined as the cost of purchasing x_i^k units of energy for the i-th PHEV during time slot k from the energy provider (e.g., the utility company). Moreover, the specific form of the cost function and its incremental cost can be referred to in Table 2. Other parameters of the four PHEV models are given in Table 3. These data are from [27,28]. The simulation results are shown in Figures 9–11.

Table 3. Parameters of four PHEVs.

PHEVs	Max Power (kW)	Battery Capacity (kWh)	Access Time	Exit Time	Energy Demand (kW)
GM Chevy Volt	3.84	16	18:00	06:00	10
Tesla MODEL S	10	60	18:00	09:00	45
Nissan Leaf	6.6	24	23:00	08:00	18
BMW Mini E	11.52	35	24:00	10:00	30

Figure 9 illustrates the power allocated to the Nissan Leaf PHEV using Algorithm 3. From Figure 9, one derives that by dynamically adjusting the charging power and duration for electric vehicle (EV) users, the fluctuation of peak-to-valley load difference can be minimized, resulting in a reduced impact on the stability of the power grid caused by the Nissan Leaf PHEV charging. Figure 10 compares the total power demand curves with and without Algorithm 3. Based on Algorithm 3, the overall load level is reduced and the peak-to-valley difference is minimized, thereby ensuring a more stable operation of the grid. Conversely, when Algorithm 3 is not utilized, there are significant peak loads observed at the 100th and 300th samples, resulting in substantial fluctuations in grid loads. Figure 11 demonstrates the effectiveness of our algorithm in significantly reducing energy costs for each household and providing incentives for their participation in the coordinated charging process.

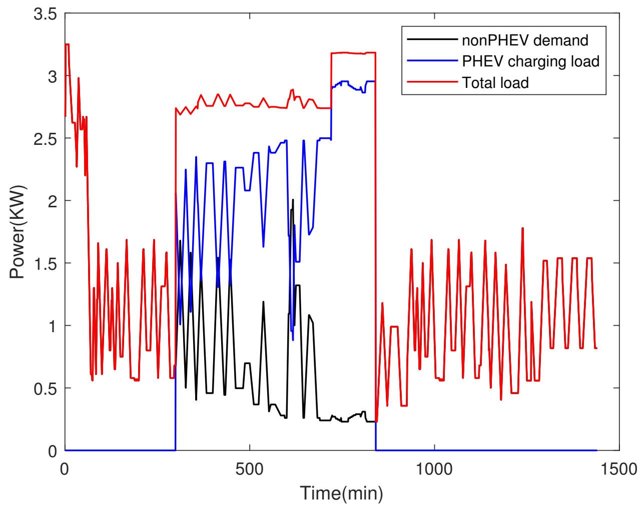

Figure 9. Power curve of Nissan Leaf household.

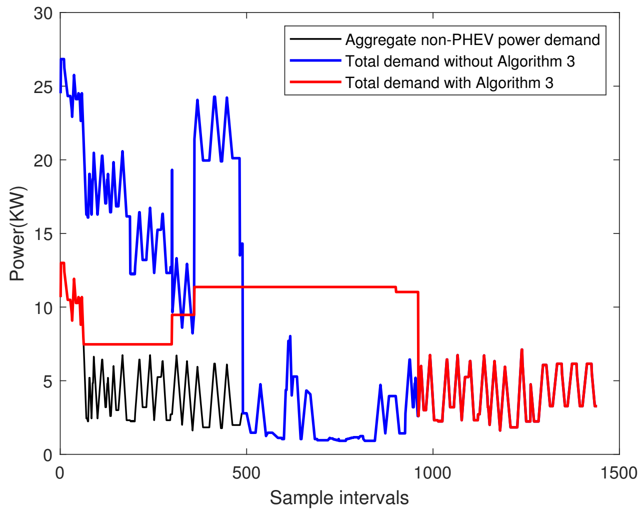

Figure 10. Power curves of distribution system with and without Algorithm 3.

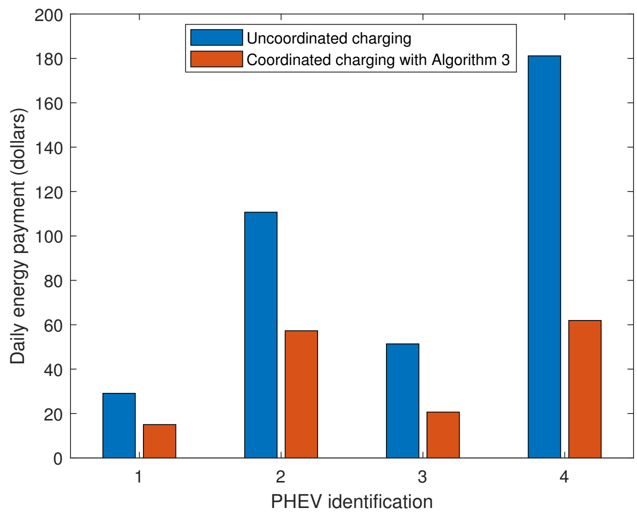

Figure 11. Daily energy costs for households.

6. Conclusions

PHEV penetration offers significant environmental advantages and is increasingly prevalent. This paper addresses the challenge of formulating an economically optimal strategy for flattening the system's load demand by formulating it as a two-layer convex optimization problem and providing a hierarchical approach to solve it. The water-filling method is used to flatten the load–demand curve for the top layer. A distributed and iterative approach is used in the layer below to reduce the overall cost for all users, resembling a consensus. Additionally, a shifting horizon technique is implemented to handle the unpredictable arrival of PHEVs and the inaccurate base demand expansion.

However, it is worth noting that the algorithm proposed in this paper, despite its potential, has some limitations when it comes to practical applications. The distributed nature of the algorithm necessitates the interaction of neighboring nodes through the smart sensor network to gather global information. This reliance on communication between nodes may impact the real-time capabilities of the algorithm in real-world scenarios. As a result, future research should prioritize exploring more practical constraints, such as enhancing the ease of information interaction, achieving fast convergence of algorithms, and allowing PHEVs to inject energy back into the system, which would benefit both PHEV owners and the smart grid further.

Author Contributions: H.Z. conducted theoretical analysis, system implementation, and simulations, and wrote the first and last versions of the draft paper. W.L. contributed to the literature review and reviewed the last version of the draft paper. The proposed approach was discussed with J.S., who verified the first draft paper. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the National Natural Science Foundation of China (No. 62301489) and the Zhejiang Engineering Research Center for Edge Intelligence Technology and Equipment.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding authors.

Acknowledgments: This work presents an extended version of a selected paper in the ICCSSE conference.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Li, Y.L.; Xie, K.G.; Wang, L.F.; Xiang, Y.M. The impact of PHEVs charging and network topology optimization on bulk power system reliability. *Electr. Power Syst. Res.* **2018**, *163*, 85–97. [CrossRef]
2. Hoffmann, F.; Person, J.; Andresen, M. A Multiport Partial Power Processing Converter with Energy Storage Integration for EV Stationary Charging. *IEEE J. Emerg. Sel. Top. Power Electron.* **2022**, *10*, 7950–7962. [CrossRef]
3. Bai, Y.; Qian, Q. Optimal placement of parking of electric vehicles in smart grids, considering their active capacity. *Electr. Power Syst. Res.* **2023**, *220*, 109238. [CrossRef]
4. Adil, M.; Abdelmounime, E.M.; Rachid, L.; Fouad, G. Novel adaptive observer for HVDC transmission line: A new power management approach for renewable energy sources involving Vienna rectifier. *Ifac J. Syst. Control* **2024**, *27*, 100255.
5. Lopes, J.A.; Soares, F.J.; Almeida, P.M.R.; Phanivong, P.K.; Callaway, D.S. Integration of Electric Vehicles in the Electric Power System. *Proc. IEEE* **2011**, *99*, 168–183. [CrossRef]
6. Liu, M.; Phanivong, P.K.; Callaway, D.S. Decentralized Charging Control of Electric Vehicles in Residential Distribution Networks. *IEEE Trans. Control. Syst. Technol.* **2017**, *12*, 266–281. [CrossRef]
7. Finn, P.; Fitzpatrick, C. Demand side management of electric car charging: Benefits for consumer and grid. *Energy* **2012**, *42*, 358–363. [CrossRef]
8. Gao, Y.; Chen, W.; Zhi, W. Research on time-of-use price applying to electric vehicles charging. *IEEE Pes Innov. Smart Grid Technol.* **2012**, *34*, 254–261.
9. Kang, Q.; Feng, S.W.; Zhou, M.C. Optimal Load Scheduling of Plug-In Hybrid Electric Vehicles via Weight-Aggregation Multi-Objective Evolutionary Algorithms. *IEEE Trans. Intell. Transp. Syst.* **2017**, *18*, 2557–2568. [CrossRef]
10. Sortomme, E.; Hindi, M.M.; Macpherson, S. Coordinated charging of plug-in hybrid electric vehicles to minimize distribution system losses. *IEEE Trans. Smart Grid* **2011**, *2*, 198–205. [CrossRef]
11. Shao, S.; Pipattanasomporn, M.; Rahman, S. Demand response as a load shaping tool in an intelligent grid with electric vehicles. *IEEE Trans. Smart Grid* **2011**, *2*, 624–631. [CrossRef]

12. Gan, L.; Topcu, U.; Low, S.H. Optimal decentralized protocol for electric vehicle charging. *IEEE Trans. Power Syst.* **2013**, *28*, 940–951. [CrossRef]
13. Ma, Z.; Callaway, D.S.; Hiskens, I.A. Decentralized Charging Control of Large Populations of Plug-in Electric Vehicles. *IEEE Trans. Control. Syst. Technol.* **2012**, *21*, 108531.
14. Xu, Y. Optimal distributed charging rate control of plug-in electric vehicles for demand management. *IEEE Trans. Control. Syst. Technol.* **2015**, *30*, 1536–1545. [CrossRef]
15. Zazo, J.; Zazo, S.; Macua, S.V. Robust worst-case analysis of demand-side management in smart grids. *IEEE Trans. Smart Grid* **2017**, *8*, 622–673. [CrossRef]
16. Shen, J.; Wang, L.; Zhang, J. Distributed charging control of electric vehicles in pv-based charging stations. In Proceedings of the 2021 IEEE Applied Power Electronics Conference and Exposition (APEC), Phoenix, AZ, USA, Virtual, 14–17 June 2021.
17. Rotering, N.; Ilic, M. Optimal charge control of plug-in hybrid electric vehicles in deregulated electricity markets. *IEEE Trans. Power Syst.* **2010**, *21*, 1021–1029. [CrossRef]
18. Nguyen, H.K.; Song, J.B. Optimal charging and discharging for multiple phevs with demand side management in vehicle-to-building. *Commun. Netw.* **2013**, *14*, 662–671. [CrossRef]
19. Shim, D.; Kim, S.W.; Altmann, J. Strategic management of residential electric services in the competitive market: Deman-oriented perspective. *Energy Environ.* **2017**, *29*, 218–220. [CrossRef]
20. Mohsenian-Rad, A.; Wong, V.W.S.; Jatskevich, J.; Schober, R.; Leon-Garcia, A. Autonomous Demand-Side Management Based on Game-Theoretic Energy Consumption Scheduling for the Future Smart Grid. *IEEE Trans. Smart Grid* **2010**, *3*, 3364–3373. [CrossRef]
21. Krieger, E.M. Effects of Variability and Rate on Battery Charge Storage and Lifespan. Ph.D. Dissertation, Princeton University, Princeton, NJ, USA, 2013.
22. Ma, L.S.; Meng, Z.Q.; Teng, Z.S.; Tang, Q. A measurement error prediction framework for smart meters under extreme natural environment stresses. *Electr. Power Syst. Res.* **2023**, *218*, 109192. [CrossRef]
23. Li, W.; Lin, Z.; Cai, K. Distributed algorithm for a finite time horizon resource allocation over a directed network. *IET Control. Theory Appl.* **2020**, *14*, 122–130. [CrossRef]
24. Mou, Y.; Xing, H.; Lin, Z.; Fu, M. Decentralized Optimal Demand-Side Management for PHEV Charging in a Smart Grid. *IEEE Trans. Smart Grid* **2015**, *6*, 726–736. [CrossRef]
25. Xu, Y.; Han, T.; Cai, K.; Lin, Z.; Yan, G. A distributed algorithm for resource allocation over dynamic digraphs. *IEEE Trans. Signal Process.* **2017**, *65*, 2600–2612. [CrossRef]
26. Vandael, S.; Boucke, N.; Holvoet, T. Decentralized demand side management of plug-in hybrid vehicles in a smart grid. Proceedings of the First International Workshop on Agent Technologies for Energy Systems (ATES 2010) , Toronto, ON, Canada, 10–11 May 2010.
27. Nissan. Nissan Leaf Electric Car Charging. 2014. Available online: http://www.nissanusa.com/electric-cars/leaf/charging-range/charging (accessed on 16 May 2024).
28. Wu, Z. Economic model predictive control of stochastic nonlinear systems. *Aiche J.* **2018**, *31*, 3312–3322. [CrossRef]