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Article

Socio-Economic Sciences: Beyond Quantum *Math*-like Formalisms

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Abstract: Since the beginning of the 21st century, a new interdisciplinary research movement has started, which aims at developing quantum *math*-like (or simply *quantum-like*) models to provide an explanation for a variety of socio-economic processes and human behaviour. By making use of mainly the probabilistic aspects of quantum theory, this research movement has led to many important results in the areas of decision-making and finance. In this article, we introduce a novel and more exhaustive approach, to analyze the socio-economic processes and activities, than the pure quantum math-like modelling approach, by taking into account the *physical* foundations of quantum theory. We also provide a plausibility argument for its exhaustiveness in terms of what we can expect from such an approach, when it is applied to, for example, a generic socio-economic decision process.

Keywords: expected utility; subjective-objective probability; quantum probability; wave function; ensemble; quantum foundations

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1. Introduction

History shows that multiple touch points have occurred, over many years, between the natural and social sciences, which resulted in applying formalisms, concepts, and ideas from mathematics, physics, and even biology in economics and finance. From as early as the beginning of the 20th century, we did see Bachelier's [1] work linking the Brownian motion to stock markets and option pricing for the first time. Any economist will of course know about the seminal contribution of the work of eminent mathematical physicist John von Neumann and economist Oskar Morgenstern [2], who laid the foundations of game theory. We can also mention the contributions of Nicolae Georgescu-Roegen [3] who is best remembered for relating the law of entropy to the economic process. In decision-making, many ideas from the exact sciences have entered several of the formalisms. As an example, capacities and the Choquet integral [4] were used in an attempt to modelling decisions under risk [5,6]. The fundamental contributions of luminaries like Gérard Debreu [7] and Kenneth Arrow [8] have made economics to become a very formal science.

Over the last 25 years or so, the application of statistical mechanics to economics and finance has also entered the research scene. 'Econophysics' [9] is interested in using, amongst others, the methods of analysis from statistical mechanics to uncover patterns in economic and financial data and, in doing so, has provided for explanations of certain social science-based phenomena. Some of this work has also contributed to building economics models with a bottom-up approach. For instance, the work by Belal Baaquie [10] on how a potential-like function can be defined as the sum of elementary supply and demand functions in microeconomics has led to the introduction of a formulation similar to the Hamiltonian formulation from physics right within the foundations of the elementary microeconomic theory.

More recently, we have seen the birth of a research movement which aims at making use of the quantum formalism in the socio-economic sciences. This movement has led to the so-called *quantum-like* models for describing and analyzing the decision-making activities and cognitive processes [11,12], which play an important role both in psychology and in the axiomatic foundations of many economics and finance models [13]. These research efforts seem to be based on the tacit hypothesis that the mathematical-statistical language of quantum physics can work in an *analogous* manner in the socio-economic domain.

Following [14], we prefer to regard such developments as representing the use of quantum mathematics—a synthesis of linear algebra, operator algebras, infinite-dimensional linear vector spaces, and the theory of probability—in the socio-economic sciences. Hence, it can be said that the quantum-like models provide a quantum mathematically analogous (QMA), rather than a quantum physically analogous (QPA), explanation and analysis of the processes occurring in the socio-economic world.

The main purpose of this article is to explain what we mean by a QPA approach and how it could be more exhaustive than the QMA approach. In Section 2, we mention some of the highlights of the QMA approach (in the decision-making and economics contexts). Then, in Section 3, we recall some of the main *physical* foundations of quantum theory. Section 4 gives a plausibility argument for our assertion that the QPA approach will lead to a more exhaustive analysis of the socio-economic activities than the pure QMA approach. We do this by stating some expectations from the QPA approach with reference to a generic socio-economic decision-making process.

2. QMA Approach in the Socio-Economic Sciences: Some Highlights

The topic of decision-making plays a crucial role in many areas of the social sciences. The emphasis here is on the process of decision-making. Understanding the process can aid in helping to predict decision outcomes. We note that the context in which the process is set can also differ. Sociology, for instance, may focus on the decision-making formats societies make as a whole, whilst behavioural psychology may target decision-making of individuals within a lab setting. Economics has provided for a rigorous axiomatic structure which formalizes decision-making to a great extent at a representative individual level. Three key axiomatic frameworks are the von Neumann-Morgenstern expected utility model [2], which has an underlying probability measure (objective probability); the Anscombe-Aumann expected utility model [15], which uses a mixture of objective/subjective probabilities; and, the Savage model [16], which uses purely subjective probability in the formulation of expected utility.

The academic community has grappled for many years in determining what may be a workable approximative framework which can emulate human decision-making. As we already hinted to above, a distinguishing ingredient in many of the key modelling formalisms is the 'type' of probability which is being used, either subjective, objective, or a mixture of subjective/objective probability. The key difference between an objective versus a subjective probability measure can be traced back to the simple and intuitive notion of a 'belief' to occur or not. As an example, the objective probability of me crashing my car into a wall, can, to some extent, be actuarially defined by looking at my past driving history and the age group I belong to. I may have a subjective probability of crashing my car—which is simply my belief that I will crash my car. To date, there is no doubt that the Kolmogorovian probability measure, which is a normalized probability measure defined within a σ-algebra of events (themselves expressed with sets), has been a 'workhorse' within many social science formalisms where uncertainty or risk are an inherent part of the problem at hand. Why can it then be the case that the so-called 'quantum probability' also has relevance within social science?

We may trace back the origins of this use of probability to the continued quest to render decision-making formalisms as accommodating as possible against well-tested decision-making paradoxes. In effect, many decision-making paradoxes in the decision sciences have been observed to occur multiple times and in a variety of lab settings. Prominent examples include the Allais and Ellsberg paradoxes. Over the years, the economic theory community tackled these paradoxes and several other ones. We can think of the Choquet expected utility we already hinted to above. We can also mention the very well-known prospect theory by Kahneman and Tversky [17] and the smooth ambiguity preference model by Klibanoff et al. [18], amongst others. The existence of a multitude of such decision-making paradoxes commands the use of more generalized probability measures. One of those is, in effect, quantum probability.

There are many articles that have made use of the mathematical language of quantum theory to explain the decision-making paradoxes. For example, in [19], the Allais and Ellsberg paradoxes are given a quantum mathematical explanation by using the so-called 'Projective Expected Utility Model', which is the quantum extension of [15]. Formal models are also proposed to explain the disjunction effect by making use of the quantum mathematical idea of interference [20].

Several authors have given arguments in favour of the quantum-like approach in economics and social sciences. For example, in [13], the advantage of using the quantum mathematical formalism is pointed out in terms of the availability of multiple options for the basis sets within a decision-making context. In [21], the authors suggest that the idea of quantization is possible in economics because of 'an information-theoretic basis' for the dynamics in an economic system. They argue that it can be derived from an information flow process ([21], p. 20): "These dynamics have a direct link to the formalism of quantum mechanics in that the equation of motion follows from the extremization of the Fisher information which is isomorphic to the Schrödinger equation". This argument indeed brings us closer to the quantum theory, more precisely to its Bohmian approach. For the Fisher information measure, as it turns out, it finds similarity with the idea of the quantum potential [22], a hallmark concept of the Bohmian quantum theory [23].

An outstanding example, where the QMA analysis has been used in a sophisticated way, resides within the work of Belal Baaquie, who can be seen as the main precursor of quantum-like research in economics and finance. A deep comment made by that author revolved around the fact that when the Black-Scholes Hamiltonian was derived from the theory of financial derivatives, it will be non-Hermitian. As was then remarked, the non-Hermiticity of this Hamiltonian is intimately related to the needed absence of arbitrage. The theory of financial derivatives is very difficult to formulate when arbitrage is allowed. The solutions to the Black-Scholes PDE would be very difficult to fathom in that case. This observation shows the extent of the QMA approach as opposed to the QPA approach in finance. Another example is the work by Li and Zhang [24]. They obtain a potential function, after which they write a call option as a Schrödinger equation. The authors then show they can consider sophisticated volatility functions in the option pricing set up, and solutions for the option price can be found. It needs noting that it can be an important achievement when option prices can be found with complex volatility functions.

In the introduction of the paper, we distinguished between the QMA and QPA approaches. We may claim that at this point in time, the quantum-like community mostly follows the QMA approach. The endeavour to wanting to bring the QPA approach as a new orientation within this community is an important one. We may almost make the argument that the QPA approach englobes the QMA approach, and hence the quantum-like applications could therefore be more powerful.

3. A Brief Account of the Physical Foundations of Quantum Theory

Before justifying the assertion that the QPA approach is more exhaustive than the QMA approach, we recall that primarily, the quantum theory is a *physical* theory of nature. Therefore, the QPA approach will essentially require taking into account the physical foundations of the quantum theory, which do not seem to have been incorporated in the pure QMA approach. We think that these foundations manifest themselves mainly in the intricate and inseparable connection of quantum theory with the previous (classical) theories from physics and with the way in which the experiments are conducted in the quantum domain. The quantum physical foundations also have to do with the variety of perspectives the theory allows on the *physical* nature of reality.

Being one of the stages of the developments in the many-centuries-old discipline of physics, the seeds and the roots of the formulation of quantum theory lie in the preceding classical physics frameworks and paradigms. These include, for example, the Newtonian mechanics, the Lagrangian and the Hamiltonian-Jacobian analytical mechanics, statistical mechanics and thermodynamics, optics and electrodynamics, and wave theories and field theories [25].

As is the case of any theory of physics, the quantum *theoretical developments* have been concomitant with the quantum *experimental findings*. These findings pertained mainly to the observations of the spectrum of the blackbody radiation and the emission-absorption spectra from different materials. The explanation of the blackbody spectrum required Max Planck's introduction of the universal constant, the *quantum of action*, denoted by 'h' in his celebrated 1900 derivation. By modifying the Rayleigh-Jeans result, Planck derived the correct expression for the intensity distribution function of cavity electromagnetic radiation, at a given temperature, across a range of wavelengths. Planck's 'quantum' hypothesis was used by Albert Einstein in 1905 to explain the experimentally observed properties of the photo-electric effect. Understanding the origin of spectral lines required probing into the microscopic structure of matter and developing theoretical models to explain the spectroscopic and interferometric behaviour of atoms, molecules, and nuclei. The founding theoretical and experimental efforts in the quantum domain were due to physicists such as Rutherford, Bohr, and Sommerfeld, and these investigations were continued further by the pioneering contributions of several legends (e.g., de Broglie, Heisenberg, Born, Jordan, Schrödinger, Dirac, Pauli) resulting in the development of the modern quantum theory in the 1920s. For details, see, for example, chp. 13 of [25].

After almost 100 years, even today, the ultimate general aim of most of the quantum experimental set-ups is the same: although much more modified and sophisticated than their previous versions, these set-ups are built to draw probabilistic inferences in terms of the spectroscopic or interferometric intensity distributions only [26]. The usual practice is to work with an ensemble of *identically prepared identical systems* for the determination of the expectation values—that is, the ensemble averages—of single (e.g., $\langle \hat{A} \rangle$) or correlated (e.g., $\langle \hat{A} \cdot \hat{B} \rangle$) physical properties of micro-systems in different situations. See, for example, [27]. Thus, the statistical aspects of quantum theory are in accord with the physical behaviour of micro-systems, as demonstrated in the quantum experiments.

Despite this fact, the experimental observations neither force a purely statistical interpretation for the wave function Ψ of the Schrödinger equation, nor do they prevent it from possessing other (non-statistical) properties [28]. Indeed, the quantum theoretical framework admits a variety of interpretations (Copenhagen, Pilot Wave, Many Worlds, GRW etc.) of the wave function, thereby allowing for different (and debatable) perspectives on the physical nature of reality. For the comparison of these interpretations, see, for example, [29]. These perspectives, among other things, differ from each other in terms of their stances on the issue of whether the (conventional) quantum theory provides a complete description of reality (or it does not), and also in terms of their stances on the role of *observer* (or a lack thereof) in the quantum framework.

Moreover, the quantum paradigm seems to imply a worldview—different from (and more general than) that provided by the classical paradigms—based on the new tenet of *nonlocality* [30], in addition to the older one of *causality*. Additionally, it has given rise to mainly two (and somewhat related) foundational issues: the *measurement* problem and the problem of *quantum-to-classical* transition [31]. The former refers, among other things, to the questions of whether the wave function collapse occurs during the measurement process, and if yes, how. It is best expressed in terms of the notorious "Schrödinger cat paradox". For a detailed discussion, see [29]. The latter, also known as the problem of the appearance of a classical world within quantum theory, is usually investigated by using the well-known decoherence programme. For specific applications of the programme, see, for example, [32].

These physical foundations remind us that quantum theory is much more than just a stand-alone, purely mathematical or statistical toolbox. Thus, by a QPA approach, we mean a way of describing and analyzing the processes and activities taking place in the socio-economic world by, as far as possible, precisely developing the socio-economic analogues of the quantum physical foundations.

4. What We Can Expect from the QPA Approach

With this background, we now turn to our assertion that the QPA approach will lead to a more exhaustive analysis of the socio-economic activities than the pure QMA approach. Here we provide a plausibility argument for this assertion. For this, we describe a generic socio-economic decision-making process in a particular way and consider some of the essential topics from quantum physics. Then, with reference to the process and in terms of those topics, we briefly explain how the QPA approach can be explored for the socio-economic sciences. Our discussion in this section is restricted only to the topics of identical systems, ensemble, and the interpretation of wave function. The elaboration on how our approach can be applied to specific socio-economic situations is beyond the scope of the present article. A detailed account will be given elsewhere.

A typical socio-economic decision-making context (e.g., buying or selling a financial instrument or a product) presents the decision-maker with certain alternatives. The corresponding *process*, say $\mathcal{D}$, of decision-making may be understood as evolving

- *from* the decision-maker's desire to participate in the socio-economic decision-making context,
- *via* the decision-maker's initial preferences (possibly depending on her/his past experiences and cognitive biases),
- *to* the actualization of a particular choice by the decision-maker, subject to the influence of the relevant socio-economic condition on her/him.

Keeping in mind this qualitative understanding of $\mathcal{D}$, the working of the QPA approach could be explained as follows.

We begin with the topic of 'identical systems'. One of the central notions in any theory of physics is that of a physical system, whose state is subject to dynamical evolution. Physicists use different ways to categorize physical systems, such as: a *simple* or a *complex* system; a *single-particle* or a *multi-particle* system; a *matter* system or a *field* system, and so forth. Thus, the foremost expectation from the QPA approach to a social or economics situation is to be able to clearly specify *what the system is in question*. Once the system is identified in a given socio-economic problem, the QPA approach is expected to explain what it could mean by an *ensemble* of such *identically prepared identical* systems. Without having the QPA socio-economic meanings of these terms, we think it will not be possible to theoretically propose and also conduct QPA socio-economic experiments.

Our example above clearly points to the 'decision-maker' as the system in question, for it is the decision-maker who is undergoing the process of decision-making. Further, we propose that two decision-makers are said to be identically prepared identical systems—in the QPA sense—if both of them, in spite of the possible differences in their respective idiosyncrasies and cognitive biases, are participating in the *same* socio-economic decision-making context. An *ensemble* of such decision-makers would then represent the *diversity* in the behavioural decision-making. Of course, the QPA approach will be expected to provide appropriate (i.e., as per the socio-economic context) models, so that such a diversity manifests itself in terms of the variety in the decision-makers' *quantified* initial preferences and the actualized choices.

Now, given that the system in question is the decision-maker, it is the state of the decision-maker that is subject to dynamical evolution. Thus, the next question is: which dynamical law would the QPA approach assume for the evolution of the state of the decision-maker? Obviously, the QPA approach would hypothesize Schrödinger-like dynamics for the decision-maker's state evolution and would also require that a function $\Psi[\mathcal{D}]$, analogous to the quantum wave function, be associated with the decision-maker. This requirement leads to the next issue: that of the interpretation of $\Psi[\mathcal{D}]$. As mentioned in the previous section, there are several interpretations of the wave function. If a Schrödinger-like equation is used in the socio-economic domain, then employing the QPA approach would mean choosing a particular quantum interpretation for its solution $\Psi[\mathcal{D}]$ out of those interpretations and also developing its precise socio-economic analogue.

For example, if the pure statistical interpretation for $\Psi[\mathcal{D}]$ is chosen, then $\Psi[\mathcal{D}]$ should be identified as the probability amplitude for the decision-making process $\mathcal{D}$. To be more precise, with reference to our example above, it should be the amplitude that the decision-maker, when presented with certain alternatives within a given socio-economic context, *starts from* the desire for participating in that context and *arrives at* a choice corresponding to the context. Moreover, in the QPA approach, such an identification would make mandatory a critical examination of how $\Psi[\mathcal{D}]$ may obey the laws of combining amplitudes (in the different socio-economic analogues of quantum experiments) corresponding to those alternatives [33]. The alternatives presented to the decision-maker would, therefore, be expected to play the role of the socio-economic analogues of the quantum *base* (or *basis*) states. This way, as a legitimate quantum-like wave function, $\Psi[\mathcal{D}]$ will be required to admit socio-economically meaningful representations with respect to different (e.g., position-like, momentum-like etc.) decision bases. Additionally, the square of its modulus, that is, $|\Psi[\mathcal{D}]|^2$, should give the distribution of the probability density of decision-making over different base-choices.

If, instead, the statistical interpretation of the wave function is taken as secondary and a dynamical role of the wave function is assumed as primary—such as that proposed in the de Broglie-Bohm quantum theory—then the QPA approach will be expected to provide a causally transparent and ontologically clear analysis of the socio-economic activities, apart from being able to predict the decision-making probability distributions in different contexts. In this case, the amplitude ($R[\mathcal{D}]$) of the function $\Psi[\mathcal{D}]$ will be expected to give rise to the socio-economic analogue of the quantum potential energy ($Q[\mathcal{D}] \propto \nabla^2 R/R$, see ch. 3 of [28]). Additionally, the notion of *space-time trajectory* of a particle—which is routinely used in the classical description of physical processes, but generally disregarded in the conventional quantum interpretation—will become ontologically meaningful. Therefore, in the 'Bohmian' QPA approach, there will have to be an analogous socio-economic notion of the trajectories of the decision-makers from the ensemble, belonging to a suitably defined *decision space* of quantified preferences/choices, in analogy with the physical space of particle positions. Such trajectories (evolving from the initial preferences to the final actualized choices) will be governed by the decision force, $F[\mathcal{D}]$, supposed to be acting on the decision-maker during the process $\mathcal{D}$. It will be given by $-\nabla(V[\mathcal{D}] + Q[\mathcal{D}])$, where $V[\mathcal{D}]$ is the classical potential energy-like function. Of course, the QPA approach will be expected to provide a socio-economically sensible interpretation of these potentials.

The above discussion makes it apparent that a QPA analysis would make us more answerable, in terms of rigour and consistency, by allowing a much wider scope for investigation of the socio-economic activities, than the pure QMA analysis. It is in this sense that we think that the QPA approach will prove to be more exhaustive than the QMA approach.

5. Discussion and Conclusions

In this article, we highlighted some developments that make use of the mathematics of quantum theory in the socio-economic sciences. We called such developments as representing the quantum mathematically analogous (QMA) way of describing and analyzing the socio-economic activities. Then, by providing a brief account of some of the main physical foundations of quantum theory, we introduced a new approach of the "quantum physically analogous (QPA)" description and analysis of the socio-economic activities. Using a generic decision-making process and by specifying what we could expect from the

QPA approach (with reference to the topics such as identical systems, ensemble, and the interpretation of the wave function), in Section 4 we provided a plausibility argument for the exhaustiveness of the QPA approach. This list of "expectations from the QPA approach" is far from being complete. Here, we add two more (and perhaps, the most challenging) items in the list, based on our discussion at the end of Section 3.

There it was mentioned that, besides inheriting the idea of causality from its "classical" predecessors, the quantum paradigm seems to imply a more general worldview in terms of the additional notion of nonlocality. Whether one chooses the conventional or a non-conventional interpretation of quantum theory, it would be interesting to see how a QPA approach could manifest the analogous socio-economic form of nonlocality; how a QPA socio-economic Bell-type inequality could be derived; and also, whether it is violated in appropriately designed QPA socio-economic experiments. Again, such tasks will first of all demand a proper QPA meaning of identical systems in social or economic contexts, as mentioned in Section 4.

The QPA approach can also be explored to see what it could mean by the analogous socio-economic version of the measurement problem. In that case, it would be intriguing to have, for example, a QPA socio-economic description of the "Schrödinger cat paradox". Additionally, a proper QPA approach can be expected to lead to the analogous socio-economic version of the problem of quantum-to-classical transition. For this, we think a truly successful QPA approach will essentially have to provide some room for accommodating the emergence of *classical physically analogous* (CPA) behaviour of the socio-economic systems.

The established QMA approach in the socio-economic sciences is cleaner and relatively more straightforward than the proposed QPA approach. This may be because the former bypasses the intricacies pertaining to and the issues surrounding the physical foundations of quantum theory. However, our discussions imply that, due to this bypassing, both, the physical insight offered by the QMA approach into the workings of the socio-economic world and its strength to provide rigorous analogical reasoning connecting the two fields (quantum theory and the socio-economic sciences), are limited. We think that in the QPA approach, these problems will be naturally addressed.

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References

1. Bachelier, L. Théorie de la spéculation. In *Annales Scientifiques de l'Ecole Normale Supérieure*; Société Mathématique de France: Paris, France, 1900; Volume 17, pp. 21–86.
2. von Neumann, J.; Morgenstern, O. *Theory of Games and Economic Behavior*; Princeton University Press: Princeton, NJ, USA, 2007.
3. Georgescu-Roegen, N. *Analytical Economics: Issues and Problems*; Harvard University Press: Cambridge, MA, USA, 1966.
4. Choquet, G. Theory of capacities. *Annales de l'Institut Fourier* **1955**, *5*, 131–295. [CrossRef]
5. Chateauneuf, A.; Jaffray, J.Y. Some characterizations of lower probabilities and other monotone capacities through the use of Mobius inversion. *Math. Soc. Sci.* **1989**, *17*, 263–283. [CrossRef]
6. Chateauneuf, A. Modelling attitudes towards uncertainty and risk through the use of Choquet integral. *Ann. Oper. Res.* **1994**, *52*, 3–20. [CrossRef]
7. Debreu, G. *Theory of Value*; Yale University Press: New Haven, CT, USA, 1959.
8. Chichilnisky, G. *Markets, Information and Uncertainty: Essays in Economic Theory in Honour of Kenneth J. Arrow*; Cambridge University Press: Cambridge, UK, 2008.
9. Mantegna, R.; Stanley, E. *Introduction to Econophysics: Correlations and Complexity in Finance*; Cambridge University Press: Cambridge, UK, 1999.
10. Baaquie, B. Statistical microeconomics. *Phys. A* **2013**, *392*, 4400–4416. [CrossRef]

11. Khrennikov, A. *Ubiquitous Quantum Structure: From Psychology to Finance*; Springer: Berlin, Germany, 2010.
12. Busemeyer, J.; Bruza, P. *Quantum Models of Cognition and Decision*; Cambridge University Press: Cambridge, UK, 2012.
13. Chichilnisky, G. *A Topological Characterization of the Space of Events and Frameworks of* $\mathbb{R}^n$; Working Paper; Stanford University: Stanford, CA, USA, 2015.
14. Baaquie B. *Quantum Field Theory for Economics and Finance*; Cambridge University Press: Cambridge, UK, 2018.
15. Anscombe, F.; Aumann, R. A definition of subjective probability. *Ann. Math. Stat.* **1963**, *34*, 199–205. [CrossRef]
16. Savage, L. *The Foundations of Statistics*; J. Wiley: New York, NY, USA, 1954.
17. Kahneman, D.; Tversky, A. Prospect theory: An analysis of decision under risk. *Econometrica* **1979**, *2*, 263–292. [CrossRef]
18. Klibanoff, P.; Marinacci, M.; Mukerji, S. A smooth model of decision-making under ambiguity. *Econometrica* **2005**, *73*, 1849–1892. [CrossRef]
19. La Mura, P. Projective Expected Utility. *J. Math. Psychol.* **2009**, *53*, 408–414. [CrossRef]
20. Busemeyer, J.R.; Wang, Z.; Townsend, J.T. Quantum dynamics of human decision-making. *J. Math. Psychol.* **2006**, *50*, 220. [CrossRef]
21. Hawkins, R.J.; Frieden, B. R. Quantization in financial economics: An information theoretic approach. In *The Palgrave Handbook of Quantum Models in Social Science*; Haven, E., Khrennikov, A., Eds.; Springer: Berlin, Germany, 2017
22. Reginatto, M. Derivation of the equations of non relativistic quantum mechanics using the principle of minimum Fisher information. *Phys. Rev. A* **1998**, *58*, 1775–1778. [CrossRef]
23. Bohm, D.; Hiley, B. *The Undivided Universe: An Ontological Interpretation of Quantum Theory*; Routledge: New York, NY, USA, 1993.
24. Li, Y.; Zhang, J.E. Option pricing with Well-Titchmarsh theory. *Quant. Financ.* **2014**, *4*, 457–464. [CrossRef]
25. Modinos, A. *From Aristotle To Schrödinger: The Curiosity of Physics*; Springer: Cham, Switzerland, 2014.
26. Beck, M. *Quantum Mechanics: Theory and Experiment*; Oxford University Press: New York, NY, USA, 2012.
27. Athalye, V.; Roy, S.S.; Mahesh, T.S. Investigation of the Leggett-Garg Inequality for Precessing Nuclear Spins. *Phys. Rev. Lett.* **2011**, *107*, 130402. [CrossRef] [PubMed]
28. Holland, P. *The Quantum Theory of Motion*; Cambridge University Press: Cambridge, UK, 1993.
29. Dürr, D.; Lazarovici, D. *Understanding Quantum Mechanics: The World According to Modern Quantum Foundations*; Springer: Cham, Switzerland, 2020.
30. Vaidman, L. Quantum Nonlocality. *Entropy* **2019**, *21*, 447. [CrossRef] [PubMed]
31. Schlosshauer, M. *Decoherence and the Quantum to Classical Transition*; Springer: Berlin, Germany, 2007.
32. Athalye, V.; Kumar, A. Decoherence of superposition of molecular chiral states due to Rayleigh scattering. *J. Phys. B At. Mol. Opt. Phys.* **2006**, *39*, 2633. [CrossRef]
33. Feynman, R.; Leighton, R.; Sands, M. *The Feynman Lectures on Physics: Volume III*; Basic Books: New York, NY, USA, 2010.