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Article

# Super-Higgs in Superspace

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**Abstract:** We determine the effective gravitational couplings in superspace whose components reproduce the supergravity Higgs effect for the constrained Goldstino multiplet. It reproduces the known Gravitino sector while constraining the off-shell completion. We show that these couplings arise by computing them as quantum corrections. This may be useful for phenomenological studies and model-building. We give an example of its application to multiple Goldstini.

**Keywords:** supersymmetry; Goldstino; superspace

## 1. Introduction

The spontaneous breakdown of global supersymmetry generates a massless Goldstino [1,2], which is well described by the Akulov-Volkov (A-V) effective action [3]. When supersymmetry is made local, the Gravitino "eats" the Goldstino of the A-V action to become massive: The super-Higgs mechanism [4,5].

In terms of superfields, the constrained Goldstino multiplet $\Phi_{NL}$ [6–12] is equivalent to the A-V formulation (see also [13–17]). It is, therefore, natural to extend the description of supergravity with this multiplet, in superspace, to one that can reproduce the super-Higgs mechanism. In this paper we address two issues—first we demonstrate how the Gravitino, Goldstino, and multiple Goldstini obtain a mass. Secondly, by using the Spurion analysis, we write down the most minimal set of *new* terms in superspace that incorporate both supergravity and the Goldstino multiplet in order to reproduce the super-Higgs mechanism of [5,18] at lowest order in $\bar{M}_{Pl}$.

The usual presentation of the super-Higgs mechanism introduces Goldstino and Gravitino self-couplings (or mass terms) by hand and shows that the resulting action is consistent under local supersymmetry [2]. One then applies a shift of the Gravitino to obtain a massive Gravitino action coupled to matter supercurrents. We review this approach in the Appendix A.

Instead, to obtain *additional* insight, we take one step back from this and will address the question of "how" the Gravitino and Goldstino obtain these self-couplings. This approach is strongly analogous to that of the Higgs mechanism presented by Englert and Brout [19] (see also an excellent review of these topics [20]). In this approach we start from a classical action that contains only kinetic terms and current couplings to fields. At this point, both the Gravitino and Goldstino are classically massless. We then generate all the necessary terms from current correlators, which give the self-energies or vacuum polarization amplitudes, and then, by resumming this series, obtain propagators.

What is perhaps surprising is that these terms arise from an interplay of the gravitational cosmological constant that appears in the scalar component of the Anomaly multiplet $\Phi$, i.e., $\langle x \rangle$. We will refer to $\langle x \rangle$ as the "Goldstino condensate" as it is the vev around which the scalar term $G^2$ of the Constrained Goldstino mutliplet $f\Phi_{NL}$ fluctuates. If supersysymmetry is broken, $\Phi$ flows to $\frac{8f}{3}\Phi_{NL}$ in the Infrared Radiation (IR) [12], with $f$ being the supersymmetry breaking order parameter.

We then set the gravitational cosmological constant to cancel the F-term of the Goldstino, a prerequisite for the super-Higgs mechanism. In retrospect this should not be so surprising: in supersymmetry all coefficients should be promoted to background superfields and this includes the cosmological constant (more precisely the Goldstino condensate, these being simply related by factors of $\bar{M}_{Pl}$). It is therefore sensible to suspect that the cosmological constant is contained in a chiral superfield. In this paper we show that the cosmological constant may naturally be contained within the scalar component of $\Phi$ which flows in the Infrared to the Goldstino multiplet $\Phi_{NL}$. Furthermore, we show that all the soft terms associated with the super-Higgs mechanism may be naturally written in terms of superspace couplings involving $\Phi_{NL}$.

Furthermore, this approach emphasizes how the Goldstino couples to general matter supercurrents, both through Goldberger-Treiman type derivative couplings and non-derivative couplings. We will show how these non-derivative couplings are derived also, as an exchange of a Gravitino mode between two supercurrents. There are corrections to our results at higher order in $1/\bar{M}_{Pl}$, equivalent to full nonlinear supergravity, but for most applications including phenomenology do not concern us. These can of course be derived by computing the higher orders in the quantum effective action.

Our action may be useful for analyzing Goldstino couplings to $U(1)_R$ currents and may be naturally extended to the use of $S^{\alpha\dot{\alpha}}$ and $R^{\alpha\dot{\alpha}}$ current multiplets [21]. It may also offer an in principle phenomenological way to determine the correct supercurrent multiplet that describes nature. Furthermore, this setup allows for straightforward analysis of models with many Goldstini [22] and with Pseudo-Goldstini [23,24].

In Section 4, we extend the usual presentation of supersymmetric soft masses in superspace [25], by treating $\Phi_{NL}$ as a Spurion multiplet, to include also terms proportional to the Gravitino mass $m_{3/2}$. We then show that expanding these terms in components, and including previously well-known terms associated with the scalar and Gaugino soft masses $m_0^2$ and $m_{1/2}$, reproduces the super-Higgs mechanism at leading order. While our effective superspace action does not have the full complexity of supergravity, including complicated expressions involving the Kähler metric, we gain a simpler more transparent construction that constrains the off-shell completion. To find these *new* superspace terms that we must add, we take suitable couplings of the Goldstino multiplet $\Phi_{NL}$, the supercurrent multiplet $\mathcal{J}^{\alpha\dot{\alpha}}$, the Graviton multiplet $H^{\alpha\dot{\alpha}}$, $\bar{M}_{Pl}$ and a constant term $\langle x\rangle$, constrained by dimensional analysis. This is reasonable in a weak field expansion in $1/\bar{M}_{Pl}$. For a theory without matter the procedure is straightforward. For a theory with matter, to demonstrate the full shift of the super-Higgs mechanism, the Goldberger-Treiman relation(s) should appear explicitly. These are identified with the superspace terms proportional to $m_0^2$ and $m_{1/2}$ [25], and are then accounted for. We then demonstrate the super-Higgs mechanism with matter supercurrents.

The outline of this paper is as follows: In the next section we will review the constrained Goldstino multiplet $\Phi_{NL}$ and its relation to the Ferrara-Zumino (F-Z) [26] supercurrent multiplet $\mathcal{J}^{\alpha\dot{\alpha}}$. We then compute the quantum corrections in components that reproduces the terms necessary for the super-Higgs mechanism. Next we promote the component terms to a full superspace effective action, by coupling the Goldstino multiplet to the supercurrent multiplet and determine that the superspace formulation of these new terms correctly reproduces the components of the super-Higgs mechanism. In the Appendix A we include a review of [18], which is the component formulation of the super-Higgs mechanism. We adopt two-component spinor notation throughout.

## 2. Nonlinear Susy Coupled to Supergravity

It is well known that in a theory with supersymmetry one can describe the supersymmetric current in terms of a general supermultiplet $\mathcal{J}_\mu$. As a bosonic completion this multiplet also contains the $R$-symmetry and the conformal symmetry currents and satisfies the general relation

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha\Phi, \tag{1}$$

where $\mathcal{J}_{\alpha\dot{\alpha}} = -2\sigma^{\mu}_{\alpha\dot{\alpha}}\mathcal{J}_{\mu}$. The multiplet so formed is called the Ferrara-Zumino supercurrent multiplet [26], which is a real linear multiplet: $D^2 J = \bar{D}^2 J = 0$. The multiplet $X$ on the right-hand side is the Anomaly multiplet. In components the Anomaly multiplet contains the parameters describing the quantum effects leading to non-conservation of the supersymmetry, $R$-symmetry and conformal symmetry currents, supplemented by bosonic degrees of freedom in the form of an auxiliary field in its scalar component. Therefore

$$\Phi = x + \sqrt{2}\theta\xi(x) + \theta\theta F(x) \tag{2}$$

where $x$ is an auxiliary field, $\xi_\alpha = \sigma^{\mu}_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_{\mu}$ and $F(x) = \frac{2}{3}T + i\partial_\mu j^\mu$. If the theory is superconformal, then $\Phi = 0$. The component form of the supercurrent multiplet is given by

$$\begin{aligned} \mathcal{J}_\mu &= j_\mu + \theta^\alpha (S_{\mu\alpha} + \frac{1}{3}\sigma_{\mu\alpha\dot{\alpha}}\bar{\sigma}^{\nu\dot{\alpha}\beta}S_{\nu\beta}) + \frac{i}{2}\theta^2\partial_\mu\bar{x} + \bar{\theta}_{\dot{\alpha}}(\bar{S}^{\dot{\alpha}}_{\mu} + \frac{1}{3}(\epsilon^{\dot{\alpha}\dot{\gamma}}\bar{S}_{\nu\dot{\beta}}\bar{\sigma}^{\nu\dot{\beta}\alpha}\sigma_{\mu\alpha\dot{\gamma}})) \\ &- \frac{i}{2}\bar{\theta}^2\partial_\mu x + \theta^\alpha\sigma^{\nu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}(2T_{\nu\mu} - \frac{2}{3}\eta_{\mu\nu}T - \frac{1}{4}\epsilon_{\nu\mu\rho\sigma}\partial^{[\rho}j^{\sigma]}) + ... \end{aligned} \tag{3}$$

$j_\mu$ is the R-current, $S^{\alpha}_{\mu}$ the supercurrent, $T_{\mu\nu}$ the stress energy tensor, the vacuum expectation value of $x$, denoted $\langle x \rangle$, is in general non-zero and will play the part of a cosmological constant as shown in the remainder of the paper. This $x$ may be equivalent to the auxiliary terms found in [27–30] however from this perspective it appears in the scalar component of $\Phi$.

As discussed in [12] the supercurrent multiplet can also be studied in theories in which supersymmetry is broken spontaneously. Even though the conserved charge of the global broken symmetry does not exist, its conserved current and its commutators with the charge do. This allows one to construct superspace and supermultiplets in the usual way and extends the above formalism to the non-supersymmetric case. In this scenario, the large distance limit of the chiral superfield $X$ is identified with the Goldstino, also including the supersymmetry breaking order parameter $f$, the scale of supersymmetry breaking. To describe the Goldstino supermultiplet we may start from a lefthanded chiral superfield $\bar{D}_{\dot{\alpha}}\Phi = 0$ and apply the constraint $\Phi^2 = 0$. The solution to this equation is given by the nonlinear Goldstino multiplet [12,31]

$$\Phi_{NL} = \frac{G^2}{2F} + \sqrt{2}\theta G + \theta^2 F. \tag{4}$$

The field $G^\alpha$ is the Goldstino and $F$ the auxiliary field. In general one must integrate out $F$, which may be complicated to do in practice, but will lead as $\langle F \rangle = f + ...$, where the ellipses may often be ignored due to terms with higher derivatives. The above result remains valid for both $F$-term and $D$-term supersymmetry breaking as long as the chiral superfield $\Phi$ can be constructed.

In this formalism, the term $\langle x \rangle = \frac{4}{3}\langle G^2 \rangle$ is a Goldstino condensate and has been discussed before in [32,33] (the coefficient appearing here was derived in [12] where in the IR $\Phi \to \frac{8}{3}f\Phi_{NL}$). For $\langle x \rangle$ not to vanish depends on the details of the supersymmetry breaking mechanism. In fact it is curious that one may obtain $\langle x \rangle$ non-zero even when supersymmetry is not broken and $f = 0$ [34].

The ellipses in Equation (3) may be determined from a shift from $y^\mu = x^\mu - i\theta^\alpha\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\beta}}$. Our metric conventions are mostly minus, $\eta_{\mu\nu} = (1, -1, -1, -1)$. The supersymmetry current algebra is

$$\{\bar{Q}_{\dot{\alpha}}, S_{\alpha\mu}\} = \sigma^{\nu}_{\alpha\dot{\alpha}}(2T_{\mu\nu} + i\partial_\nu j_\mu - i\eta_{\mu\nu}\partial^\lambda j_\lambda - \frac{1}{4}\epsilon_{\nu\mu\rho\lambda}\partial^\rho j^\lambda) \tag{5}$$

$$\{Q_\beta, S_{\mu\alpha}\} = 2i\epsilon_{\lambda\beta}(\sigma_{\mu\rho})^{\lambda}_{\alpha}\partial^\rho\bar{x} \tag{6}$$

where the first term of Equation (5) is the conserved symmetric energy tensor and the remaining terms are Schwinger Terms that vanish in the vacuum. It is straightforward to use the definition of the Supercharge, $Q_\alpha = \int d^3x S^0_\alpha$, to relate this expression to the super algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^{\mu}_{\alpha\dot{\alpha}}P_\mu. \tag{7}$$

The simplest nonlinear Goldstino superfield $\Phi_{NL}$ action that breaks supersymmetry spontaneously is given by

$$\mathcal{L} = \int d^4\theta \Phi_{NL}^{\dagger}\Phi_{NL} + \int d^2\theta f^{\dagger}\Phi_{NL} + \int d^2\bar{\theta} f \Phi_{NL}^{\dagger}. \tag{8}$$

As shown in [12] this action, in the absence of other couplings involving the auxiliary field, is equivalent to the full A-V action in components

$$\mathcal{L}_{AV} = -|f|^2 + i\partial_\mu \bar{G}\bar{\sigma}^\mu G + \frac{1}{4|f|^2}\bar{G}^2\partial^2 G^2 - \frac{1}{16|f|^6}G^2\bar{G}^2\partial^2 G^2\partial^2\bar{G}^2. \tag{9}$$

Let us for the moment focus on only the first two terms, which comprise the A-V action, ignoring terms with a higher number of derivatives. The relevant terms in the A-V action are the supersymmetry breaking term and the Goldstino kinetic terms. The supersymmetry breaking term $-|f|^2$ in the action is a cosmological constant. The minimum of the scalar potential is $V_{\text{min}} = +|f|^2$ which is positive definite, and we see that global supersymmetry is broken. The supergravity action will also generate a cosmological constant, but of opposite sign. If these are made to be equal, the overall cosmological constant vanishes. This will appear shortly.

We introduce the linear supergravity action which provides kinetic terms for the Gravitino. The supergravity fields are embedded in a real vector superfield. In Wess-Zumino gauge the components are given by

$$\begin{aligned} H_\mu &= \theta^\alpha \sigma^\nu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}(h_{\mu\nu} - \eta_{\mu\nu}h) + \bar{\theta}^2\theta^\alpha(\psi_{\mu\alpha} + \sigma_{\mu\alpha\dot{\alpha}}\bar{\sigma}^{\rho\dot{\alpha}\beta}\psi_{\rho\beta}) \\ &+ \frac{i}{2}\theta^2 M_\mu - \frac{i}{2}\bar{\theta}^2 M_\mu^{\dagger} + \theta^2\bar{\theta}_{\dot{\alpha}}(\bar{\psi}_\mu^{\dot{\alpha}} + \bar{\sigma}_\mu^{\dot{\alpha}\alpha}\sigma^\rho_{\alpha\dot{\beta}}\bar{\psi}_\rho^{\dot{\beta}}) - \frac{1}{2}\theta^2\bar{\theta}^2 A_\mu \end{aligned} \tag{10}$$

where $H_\mu = \frac{1}{4}\bar{\sigma}_\mu^{\dot{\alpha}\alpha}H_{\alpha\dot{\alpha}}$, $h_{\mu\nu}$ is the linear Graviton, $\psi_\mu^\alpha$ is the Gravitino and $M_\mu, A_\mu$ are auxiliary fields. The kinetic terms of the supergravity action are given by [31]

$$-\int d^4\theta H^\mu E_\mu^{FZ} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\psi_{\mu\alpha}\bar{\sigma}_\nu^{\dot{\alpha}\alpha}\partial_\rho\bar{\psi}_{\sigma\dot{\alpha}} - \bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}_\nu^{\dot{\alpha}\alpha}\partial_\rho\psi_{\sigma\alpha}) - \frac{1}{3}|\partial_\mu M^\mu|^2 + \frac{1}{3}A_\mu A^\mu ... \tag{11}$$

The ellipses include a linearized Graviton kinetic term. $E^{FZ}_{\alpha\dot{\alpha}}$ is defined as

$$E^{FZ}_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\tau}}D^2\bar{D}^{\dot{\tau}}H_{\alpha\dot{\alpha}} + \bar{D}_{\dot{\tau}}D^2\bar{D}_{\dot{\alpha}}H_\alpha^{\dot{\tau}} + D^\gamma\bar{D}^2 D_\alpha H_{\gamma\dot{\alpha}} - 2\partial_{\alpha\dot{\alpha}}\partial^{\gamma\dot{\tau}}H_{\gamma\dot{\tau}}. \tag{12}$$

This gives a kinetic term for the Graviton and Gravitino, but the remaining fields are auxiliary, and therefore not dynamical. The auxiliary field $A_\mu$ integrates out to give $A_\mu = \frac{j_\mu}{\bar{M}_{Pl}} + ...$, with the ellipses denoting higher order terms in $1/F$ and $1/\bar{M}_{Pl}$. The complex field $M_\mu$ plays a role in generating a cosmological constant once we weakly couple the supercurrent mutliplet to linear supergravity.

Now that we have introduced the Goldstino and supergravity actions, we would like to couple the supercurrent multiplet to a linear supergravity muliplet [31]

$$\begin{aligned} \frac{1}{8\bar{M}_{Pl}}\int d^4\theta \mathcal{J}_{\alpha\dot{\alpha}}H^{\alpha\dot{\alpha}} &= \frac{1}{2\bar{M}_{Pl}}(h_{\mu\nu}T^{\mu\nu} + \psi_\mu S^\mu + \bar{\psi}_\mu\bar{S}^\mu - j^\mu A_\mu) \\ &- \frac{1}{4\bar{M}_{Pl}}\partial_\mu M^\mu x - \frac{1}{4\bar{M}_{Pl}}\partial_\mu M^{\dagger\mu}\bar{x}. \end{aligned} \tag{13}$$

Also $\bar{M}_{Pl} = M_{pl}/8\pi$ is the reduced Planck mass. It is useful here to recall that the supersymmetry current multiplet $J^{\alpha\dot{\alpha}}$ contains at order $\theta^\beta$ a contribution from fermionic matter and a term proportional to the supersymmetry breaking term,

$$S^\mu = S^\mu_{\text{matter}} + i\sqrt{2}f\sigma^\mu_{\alpha\dot{\alpha}}\bar{G}^{\dot{\alpha}} \ , \ \bar{S}^{\mu\dot{\alpha}} = \bar{S}^\mu_{\text{matter}} + i\sqrt{2}f^{\dagger}\bar{\sigma}^{\mu\dot{\alpha}\alpha}G_\alpha. \tag{14}$$

We remind the reader that at this point, the Gravitino and Goldstino are massless in the classical action.

## 3. Super-Higgs as an Effective Action

In this section, we focus on the component formalism and explicitly compute the relevant terms in the effective action at tree level to realize the super-Higgs mechanism. We shall find that the necessary terms all arise from couplings to the cosmological constant terms that we introduced in our definition of the supercurrent multiplet.

Our approach is non-standard within the supergravity literature, in that rather than using the local supersymmetry transformations, as outlined in the Appendix A, to generate a self-consistent action, we instead use the approach of evaluating a quantum corrections to a classically massless action. In Higgs type mechanisms the symmetry breaking is often a result of strong dynamics, in which case the fundamental quantity to compute is the self-energy from evaluating the vacuum polarization amplitude. Diagrammatically, at tree level, a single Goldstone mode is exchanged between two gauge bosons to generate a transverse contribution to the gauge boson's mass [19,20]. Our approach is analogous and therefore puts the super-Higgs mechanism on a similarly equal footing to the other forms of symmetry breaking phenomena. To demonstrate this our principle interest is in the evaluation of the supercurrent correlator, which may also have interesting applications beyond the scope of this paper.

All corrections to our approach are higher order in $1/\bar{M}_{Pl}$ and therefore does not necessitate a full nonlinear analysis, although these corrections may also be generated as higher order corrections to the quantum effective action.

The starting point is to interpret the classical action as only the massless kinetic terms of the various fields. All remaining terms will be considered to be interaction terms.

### *3.1. The Cosmological Constant*

First we focus on the overall cosmological constant. This will be found by collecting the relevant contributions to the vacuum energy density by integrating out the auxiliary fields. Varying the combination of Equations (11) and (13) with respect to $M = \partial_\mu M^\mu$ (treating $M$ and $M^\dagger$ as independent fields) we find that

$$M^\dagger = -\frac{3}{4\bar{M}_{Pl}} x \ , \ \ M = -\frac{3}{4\bar{M}_{Pl}} \bar{x}. \tag{15}$$

Substituting this result back in the original action, and integrating out $A^\mu$, gives the terms

$$\mathcal{L}_{\text{aux}} = \frac{3x\bar{x}}{16\bar{M}_{Pl}^2} - \frac{3j_\mu j^\mu}{16\bar{M}_{Pl}^2} \tag{16}$$

These are the relevant parts of the effective action which describes the $\frac{1}{\bar{M}_{Pl}^2}$ back reaction of the supergravity auxiliary fields on matter.

We can see that this mechanism provides a non-vanishing contribution to the vacuum energy density of the form [27,35]

$$\rho_{\text{Vac}} = -\frac{3\langle x\rangle\langle\bar{x}\rangle}{16\bar{M}_{Pl}^2}. \tag{17}$$

Taking the cosmological constant $|f|^2$ from the Goldstino action we may cancel the overall cosmological constant by setting

$$\frac{3\langle x\rangle\langle\bar{x}\rangle}{16\bar{M}_{Pl}^2} = |f|^2, \tag{18}$$

and hence

$$\langle x \rangle = \frac{4}{\sqrt{3}} \bar{M}_{Pl} f. \tag{19}$$

This unambiguously defines the gravitational cosmological constant contribution to the vacuum energy as proportional to $\langle x \rangle$.

It has been shown in [5] that whether one generates a Gravitino mass, or not, depends on whether these terms cancel, which is related to whether we are in Minkowski background or anti de Sitter space. We take them to exactly cancel such that the massive Gravitino does appear. Expanding $x$ around its vev $x \to \langle x \rangle + \delta x$ at this order in $\bar{M}_{Pl}$ there is a contribution to the Goldstino self-coupling (We are careful not to write mass as the Goldstino couples to the Gravitino and in the mass basis there is only a massive Gravitino) coming from Equation (16), this takes the form

$$\mathcal{L}_{\text{aux}} \supset \frac{3(\langle x \rangle + \frac{4}{3}G^2)(\langle \bar{x} \rangle + \frac{4}{3}\bar{G}^2)}{16\bar{M}_{Pl}^2} = \frac{3}{16\bar{M}_{Pl}^2}[\langle x \rangle \langle \bar{x} \rangle + \frac{4}{3}G^2 \langle \bar{x} \rangle + \frac{4}{3} \langle x \rangle \bar{G}^2 + \left(\frac{4}{3}\right)^2 G^2\bar{G}^2], \tag{20}$$

which therefore arises from coupling to $\langle x \rangle$ directly.

### 3.2. The Component Terms

In this section, we will derive the tree level contributions of the effective action and explicitly demonstrate how the Gravitino mass arises. First, to clarify notation, we define the time ordered current correlators

$$\langle T[S^\mu_\alpha(p) S^\nu_\beta(-p)] \rangle = \tilde{\Pi}^{\mu\nu}_{\alpha\beta} \tilde{G}_{3/2}(p^2) \tag{21}$$

$$\langle T[S^\mu_\alpha(x) S^\nu_\beta(0)] \rangle = \Pi^{\mu\nu}_{\alpha\beta} G_{3/2}(x^2) \tag{22}$$

which are related by a Fourier Transform. In analogy with gauge theories, these are the linear response functions or vacuum polarization amplitude of the Gravitino.

Next, considering the classical action to have only massless kinetic terms, and introducing the linear current to field couplings Equation (13) it is straightforward to see that the effective action generated from linear supergravity will give

$$\mathcal{L}' \supset \frac{i}{8M_{pl}^2}[\tilde{G}_{3/2}(0)\psi^\alpha_\mu(\sigma^{\mu\nu})_\alpha^{\ \beta}\psi_{\beta\nu} + \tilde{\bar{G}}_{3/2}(0)\bar{\psi}_{\dot{\alpha}\mu}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}}\bar{\psi}^{\dot{\beta}}_\nu] \tag{23}$$

The terms comprise the Gravitino self-energy term (at zero momentum). It may be thought of as the effect of the matter, more specifically the $\langle x \rangle$ term, on the gravity sector. The explicit evaluation of this term is found in the next subsection and demonstrates how the Gravitino self-couplings arise in this formalism. The result is a shifted pole at

$$m_{3/2} = \frac{i\tilde{G}_{3/2}(0)}{8M_{Pl}^2} = \frac{3\langle x \rangle}{16\bar{M}_{Pl}^2} = \frac{f}{\sqrt{3}\bar{M}_{Pl}} \tag{24}$$

where the last equality is found only after cancelling the overall cosmological constant. Without matter, we already have the relevant terms to achieve the super-Higgs mechanism and the overall action is locally invariant. One may attempt a geometric sum of mass insertions to obtain the massive Gravitino propagator found in [18].

Upon inclusion of matter the action is however not yet invariant under local supersymmetry. For this to happen, it is necessary to generate a term of the form $\frac{1}{\bar{M}_{Pl}} G^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{S}^{\dot{\alpha}}_{\mu\text{mattter}}$. To obtain this term we look to compute

$$\mathcal{L}' \supset \frac{i}{8\bar{M}_{Pl}^2}(S^\alpha_\mu \tilde{D}^{\mu\nu}_{\alpha\beta}(0) S^{\nu\beta} + \bar{S}^\mu_{\dot{\alpha}} \tilde{\bar{D}}^{\dot{\alpha}\dot{\beta}}_{\mu\nu}(0) \bar{S}^\nu_{\dot{\beta}}), \tag{25}$$

these terms comprise the reaction of the Gravitino, back onto the matter sector, where the two point function is given by (We wish to point out that this is not $\langle\bar{\psi}^{\mu}_{\dot{\alpha}}(x)\psi^{\nu}_{\alpha}(y)\rangle$).

$$\langle\psi^{\mu}_{\alpha}(x)\psi^{\nu}_{\beta}(y)\rangle = \int \frac{d^4p}{(2\pi)^4}\tilde{D}^{\mu\nu}_{\alpha\beta}(p^2)e^{-ip.(x-y)}. \tag{26}$$

In the zero-momentum limit, the relevant pieces are given by [18]

$$\tilde{D}^{\mu\nu}_{\alpha\beta}(p^2) = \frac{2i}{3}\frac{m_{3/2}}{p^2-m^2_{3/2}}\left(\eta^{\mu\nu}\epsilon_{\alpha\beta}+i(\epsilon\sigma^{\mu\nu})_{\alpha\beta}\right). \tag{27}$$

Equivalently, one may instead use the local supersymmetry transformations to find the terms that must be added to make the action invariant. Finally, collecting the terms, one recovers the effective contributions

$$\begin{aligned}\mathcal{L}_{\text{eff}} = -i&\left(m_{3/2}\psi^{\alpha}_{\mu}(\sigma^{\mu\nu})_{\alpha}^{\beta}\psi_{\nu\beta}+m^{\dagger}_{3/2}\bar{\psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}}\bar{\psi}^{\dot{\beta}}_{\nu}\right)\\ &-\frac{f^2}{3\bar{M}^2_{Pl}m_{3/2}}G^{\alpha}G_{\alpha}-\frac{(f^{\dagger})^2}{3\bar{M}^2_{Pl}m^{\dagger}_{3/2}}\bar{G}_{\dot{\alpha}}\bar{G}^{\dot{\alpha}}\\ +\frac{if^{\dagger}\sqrt{2}}{12\bar{M}^2_{Pl}m^{\dagger}_{3/2}}&G^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_{\mu\text{matter}}+\frac{if\sqrt{2}}{12\bar{M}^2_{Pl}m_{3/2}}\bar{G}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}S_{\alpha\mu\text{matter}}.\end{aligned} \tag{28}$$

From these we see that the cosmological constant factor $\frac{\langle x\rangle}{4\bar{M}^2_{Pl}}$ determines the Gravitino mass $m_{3/2}$. It is only after we require the vanishing cosmological constant, in the form of Equation (18), that we find the relation $m_{3/2}=\frac{f}{\sqrt{3}\bar{M}_{Pl}}$.

Finally, we note that the local supersymmetry transformations are also *modified* by the presence of a non-vanishing $\langle x\rangle$, these become

$$\delta\psi_{\mu\alpha} = -\left(2\bar{M}_{Pl}\partial_{\mu}\epsilon_{\alpha}+i\frac{\langle x\rangle}{4\bar{M}_{Pl}}\sigma_{\mu\alpha\dot{\alpha}}\epsilon^{\dot{\alpha}}\right). \tag{29}$$

Taking Equation (28) and also including the Goldberger Trieman relations one then has all the necessary terms to apply the shifts Equation (36) and obtain the massive Gravitino action coupled to matter supercurrents Equation (40). However, we will defer this until we have obtained these very same terms from effective superspace terms, in the next section.

## 4. The Superspace Terms

In the previous section we demonstrated that certain terms needed for the super-Higgs mechanism are generated in an effective action that combines linear supergravity and the Goldstino multiplet. In this section we will promote the effective terms in components to full superspace terms in the action. One may write the supersymmetry breaking soft masses in superspace [23,25] using a standard Spurion analysis in which the Goldstino multiplet $\Phi_{NL}$ acts as the Spurion multiplet. The sfermion and Gaugino masses $m_0^2$ and $m_\lambda$ are used as coefficients in these superspace terms though they may be computed from tree [36,37] or loop level diagrams or may be encoded in current correlators when one addresses the problem of supersymmetry breaking with strong coupling (e.g., [38–41]). These soft mass terms are reproduced in Equations (37) and (38).

Our contribution is to introduce the additional superspace terms proportional to $m_{3/2}$ and show that the collection, in components, do indeed satisfy the super-Higgs mechanism. Our starting point of this superspace effective action is to assume all the relevant pieces of Section 2. In particular we also assume the vanishing overall cosmological constant Equation (18). Next we introduce a *new* term, we add to the supergravity action

$$-\frac{1}{64}\int d^4\theta\left(\frac{m_{3/2}\Phi_{NL}}{f}+\frac{m^{\dagger}_{3/2}\Phi^{\dagger}_{NL}}{f^{\dagger}}\right)\left[\bar{D}_{\dot{\alpha}}D_{\alpha}H^{\beta\dot{\beta}}\bar{D}^{\dot{\alpha}}D^{\alpha}H_{\beta\dot{\beta}}-\frac{4}{3}(\bar{D}_{\dot{\alpha}}D_{\alpha}H^{\alpha\dot{\alpha}})^2\right] \tag{30}$$

which in components gives a Gravitino self-coupling

$$-i\left(m_{3/2}\psi^{\alpha}_{\mu}(\sigma^{\mu\nu})_{\alpha}^{\beta}\psi_{\nu\beta}+m^{\dagger}_{3/2}\bar{\psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}}\bar{\psi}^{\dot{\beta}}_{\nu}\right)+\ldots \tag{31}$$

The second superspace term was introduced to remove terms of the form $\psi^2$. The ellipses denote some higher order terms or derivative couplings, which we will ignore. Different off-shell completions will generate different couplings at this order, including couplings between the Goldstino and the off-shell supergravity fields, which may be interesting to catalogue but deviate too far from the present discussion.

Next we introduce the *new* superspace term

$$\frac{9}{784}\int d^4\theta\left(\frac{m_{3/2}\Phi_{NL}}{f}+\frac{m^{\dagger}_{3/2}\Phi^{\dagger}_{NL}}{f^{\dagger}}\right)\frac{\mathcal{J}_{\alpha\dot{\alpha}}\mathcal{J}^{\alpha\dot{\alpha}}}{|f|^2}, \tag{32}$$

which introduces the Goldstino couplings

$$\frac{i}{2\sqrt{6}\bar{M}_{Pl}}(G^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_{\mu\text{matter}}+\bar{G}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}S_{\alpha\mu\text{matter}})-m_{3/2}\bar{G}_{\dot{\alpha}}\bar{G}^{\dot{\alpha}}-m^{\dagger}_{3/2}G^{\alpha}G_{\alpha}. \tag{33}$$

There are also some higher order terms, mostly suppressed by higher orders in $1/F$.

Collecting the relevant terms, one obtains

$$\begin{aligned}\mathcal{L}=&-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\psi_{\mu\alpha}\bar{\sigma}^{\dot{\alpha}\alpha}_{\nu}\partial_{\rho}\bar{\psi}_{\sigma\dot{\alpha}}-\bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}_{\nu}\partial_{\rho}\psi_{\sigma\alpha})-i\left(m_{3/2}\psi^{\alpha}_{\mu}(\sigma^{\mu\nu})_{\alpha}^{\beta}\psi_{\nu\beta}+m^{\dagger}_{3/2}\bar{\psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}}\bar{\psi}^{\dot{\beta}}_{\nu}\right)\\&+\frac{i}{2\sqrt{6}\bar{M}_{Pl}}(G^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_{\mu\text{matter}}+\bar{G}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}S_{\alpha\mu\text{matter}})-m_{3/2}\bar{G}_{\dot{\alpha}}\bar{G}^{\dot{\alpha}}-m^{\dagger}_{3/2}G^{\alpha}G_{\alpha}\\&+\frac{1}{2\bar{M}_{Pl}}(\psi_{\mu}S^{\mu}+\bar{\psi}_{\mu}\bar{S}^{\mu})+\frac{1}{2}iG^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}\bar{G}^{\dot{\alpha}}+\frac{1}{2}i\bar{G}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}\partial_{\mu}G_{\alpha}+\ldots\end{aligned} \tag{34}$$

### 4.1. The Case of No Matter

We consider first the case of vanishing matter contributions, $S^{\mu}_{\alpha\text{matter}}=0$. The component Lagrangian Equation (34) reduces to

$$\begin{aligned}\mathcal{L}=&-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\psi_{\mu\alpha}\bar{\sigma}^{\dot{\alpha}\alpha}_{\nu}\partial_{\rho}\bar{\psi}_{\sigma\dot{\alpha}}-\bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}_{\nu}\partial_{\rho}\psi_{\sigma\alpha})-i\left(m_{3/2}\psi^{\alpha}_{\mu}(\sigma^{\mu\nu})_{\alpha}^{\beta}\psi_{\nu\beta}+m^{\dagger}_{3/2}\bar{\psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}}\bar{\psi}^{\dot{\beta}}_{\nu}\right)\\&-m_{3/2}\bar{G}_{\dot{\alpha}}\bar{G}^{\dot{\alpha}}-m^{\dagger}_{3/2}G^{\alpha}G_{\alpha}+\frac{i}{\sqrt{2}\bar{M}_{Pl}}(F\psi^{\alpha}_{\mu}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{G}^{\dot{\alpha}}+F^{\dagger}\bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}G_{\alpha})\\&+\frac{1}{2}iG^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}\bar{G}^{\dot{\alpha}}+\frac{1}{2}i\bar{G}_{\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}\partial_{\mu}G_{\alpha}+\ldots\end{aligned} \tag{35}$$

In this case, we can realize the super-Higgs mechanism by applying the shifts [42]

$$\Psi_{\mu\alpha}=\psi_{\mu\alpha}-\frac{i}{\sqrt{6}}\sigma_{\mu\alpha\dot{\alpha}}\bar{G}^{\dot{\alpha}}-\sqrt{\frac{2}{3}}\frac{\partial_{\mu}G_{\alpha}}{m_{\frac{3}{2}}}\ ,\ \bar{\Psi}^{\dot{\alpha}}_{\mu}=\bar{\psi}^{\dot{\alpha}}_{\mu}-\frac{i}{\sqrt{6}}\bar{\sigma}^{\dot{\alpha}\alpha}_{\mu}G_{\alpha}-\sqrt{\frac{2}{3}}\frac{\partial_{\mu}\bar{G}^{\dot{\alpha}}}{m^{\dagger}_{\frac{3}{2}}}, \tag{36}$$

then one reproduces

$$\mathcal{L} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\Psi_\mu^\alpha \sigma_{\nu\alpha\dot{\alpha}}\partial_\rho \bar{\Psi}_\sigma^{\dot{\alpha}} - \bar{\Psi}_{\mu\dot{\alpha}}\bar{\sigma}_\nu^{\dot{\alpha}\alpha}\partial_\rho \Psi_{\sigma\alpha}) - i\left(m_{3/2}\Psi_\mu^\alpha(\sigma^{\mu\nu})_\alpha^\beta \Psi_{\nu\beta} + m_{3/2}^\dagger \bar{\Psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}}\bar{\Psi}_\nu^{\dot{\beta}}\right)$$

the Lagrangian of a massive Gravitino Weyl spinor $\Psi_\mu^\alpha$.

### *4.2. Coupling to Matter*

To couple the massive Gravitino to the matter supercurrent requires that all components of the shift of Equation (36) couple to the matter supercurrent. After integration by parts, one of these shifted terms is the Goldberger-Treiman relation. These terms are the most useful for phenomenology [43]. In the superspace formalism, the Goldberger-Treiman terms appear in components in their non-derivative form [23,25] from superspace terms such as

$$\int d^2\theta \frac{m_\lambda}{2f}\Phi_{NL}W^\alpha W_\alpha + \int d^2\bar{\theta}\frac{m_\lambda^\dagger}{2f^\dagger}\Phi_{NL}^\dagger \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} \tag{37}$$

and

$$\int d^4\theta \frac{m_0^2}{|f|^2}\Phi_{NL}^\dagger \Phi_{NL}\Phi^\dagger \Phi \tag{38}$$

where $\Phi$ represents some generic matter chiral superfield and $W^\alpha$ is the superfield strength tensor. $m_\lambda$ and $m_0$ are the soft supersymmetry breaking masses of Gauginos and scalars. Expanding out these terms in superspace, first give the soft breaking mass terms and at linear order in the Goldstino, give the Golberger Treiman relations. After collecting the components and use of the equation of motion, this will supply

$$\frac{G^\alpha}{\sqrt{2}f}\partial_\mu S^\mu_{\alpha\text{matter}} + \frac{\bar{G}_{\dot{\alpha}}}{\sqrt{2}f^\dagger}\partial_\mu \bar{S}^{\mu\dot{\alpha}}_{\text{matter}}. \tag{39}$$

Including these terms with the action given by Equation (34) and applying the shifts of Equation (36) we find

$$\mathcal{L} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\Psi_\mu^\alpha \sigma_{\nu\alpha\dot{\alpha}}\partial_\rho \bar{\Psi}_\sigma^{\dot{\alpha}} - \bar{\Psi}_{\mu\dot{\alpha}}\bar{\sigma}_\nu^{\dot{\alpha}\alpha}\partial_\rho \Psi_{\sigma\alpha}) - i\left(m_{3/2}\Psi_\mu^\alpha(\sigma^{\mu\nu})_\alpha^\beta \Psi_{\nu\beta} + m_{3/2}^\dagger \bar{\Psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}}\bar{\Psi}_\nu^{\dot{\beta}}\right)$$
$$+ \frac{1}{2\bar{M}_{Pl}}(\Psi_\mu^\alpha(S_\alpha^\mu)_{\text{matter}} + \bar{\Psi}_{\mu\dot{\alpha}}(S^{\mu\dot{\alpha}})_{\text{matter}}) + \ldots \tag{40}$$

The Lagrangian of a Majorana fermion coupled to matter.

We note here that the coefficients of the superspace terms added above are chosen by hand to ensure detailed cancellations. They are therefore not fixed and should be understood as coming from a higher energy completion. These terms should therefore be fixed by a string theory completion at the Planck scale, and we understood the above action as an effective low energy description of this completion. What the detailed high energy completion is we leave to further work.

## 5. Applications

In this section, we wish to demonstrate some of the uses that an effective action of the super-Higgs mechanism in superspace may have. To demonstrate this we first write down the effective action which, in components, will reproduce the linearized action that correctly describes the theory of multiple Goldstini [22].

### *Goldstini*

As a simple application, we would like to write an action whose components naturally reproduce the effect of multiple supersymmetry breaking sectors, and hence, multiple Goldstini. We use an index

$i$ running from 1 to $N$, to label the supersymmetry breaking sectors, with $F_i$ F-terms and $\Phi_i$ chiral superfields. This is the interaction basis $\eta_i$. The mass basis we will label $\{G, \zeta_a\}$. We define the chiral superfields

$$\Phi_{iNL} = \frac{\eta_i^2}{2F_i} + \sqrt{2}\theta\eta_i + \theta^2 F_i \tag{41}$$

from which the components of the supercurrent may be found

$$S^\alpha_{\mu\text{total}} = S^\alpha_{\mu\text{matter}} + \sum_i i\sqrt{2}F_i\sigma_{\mu\alpha\dot{\beta}}\bar{\eta}_i^{\dot{\beta}} = S^\alpha_{\mu\text{matter}} + i\sqrt{2}f_{eff}\sigma_{\mu\alpha\dot{\beta}}\bar{G}^{\dot{\beta}}. \tag{42}$$

It is straightforward to see that the uneaten Goldstino do not appear in the supercurrent, which is a simple application of

$$G^\alpha = \frac{1}{f_{eff}}\sum_i F_i\eta_i^\alpha \tag{43}$$

where $f_{eff}^2$ is the sum of the $f_i^2$. We now introduce the $N$ Goldstino multiplet Lagrangian

$$\sum_i \left(\int d^4\theta\Phi_{iNL}X^\dagger_{iNL} + \int d^2\theta f_i^\dagger\Phi_{iNL} + \int d^2\bar{\theta} f_i\Phi^\dagger_{iNL}\right). \tag{44}$$

The current supermultiplet to supergravity coupling is modified

$$\frac{1}{\bar{M}_{Pl}}\int d^4\theta J_{\alpha\dot{\alpha}}H^{\alpha\dot{\alpha}} \supset -\frac{1}{4\bar{M}_{Pl}}\partial_\mu M^\mu\sum_i(x_i + \langle x_i\rangle) - \frac{1}{4\bar{M}_{Pl}}\partial^\mu M^\dagger_\mu\sum_i(\bar{x}_i + \langle\bar{x}_i\rangle) \tag{45}$$

which after integrating out $\partial_\mu M^\mu$, using $x_i = \frac{4}{3}\eta_i^2$ and setting $C = \sum_i\langle x_i\rangle$, which is sum of all possible contributions, therefore leads to

$$\frac{3(\sum_i\frac{4}{3}\eta_i^2 + C)(\sum_j\frac{4}{3}\bar{\eta}_j^2 + \bar{C})}{16\bar{M}_{Pl}^2} = \sum_i(m^\dagger_{3/2}\eta_i^2 + m_{3/2}\bar{\eta}_i^2) + \frac{3|C|^2}{16\bar{M}_{Pl}^2} + \sum_{i,j}\frac{1}{3\bar{M}_{Pl}^2}\eta_i^2\bar{\eta}_j^2 \tag{46}$$

which are the uneaten Goldstini masses and a contribution to the Goldstino self-coupling. Once the overall cosmological constant is assumed to vanish $C = \frac{4}{\sqrt{3}}f_{eff}\bar{M}_{Pl}$, this sets $m_{3/2} = C/4\bar{M}_{Pl}^2 = f_{eff}/\sqrt{3}\bar{M}_{Pl}$. this also fixes the masses of the other $\zeta_i$ Goldstini which are proportional to C (and not on their respective $F_i$ as one might naively think from an effective theory approach). It is the vanishing of the overall cosmological constant that sets $m_a = 2m_{3/2}$ (the factor of 2 arising as the Goldstini above are two-component spinors that make a Majorana fermion): this seems to be the most intuitive argument for the Goldstini mass formula.

Let us now compute the effective action. In fact no new computation is necessary: After applying Equation (42) will reproduce exactly the same terms as found in Equation (28). Importantly, the uneaten Goldstini do not appear.

It is interesting to ask what may be learned about multiple Goldstini coupling to supersymmetric standard model matter. The Goldstini obey analogue Goldberger-Treiman type couplings from a natural extension of Equations (37) and (38) and these have already been explored [22]. However, interestingly the Goldstino to matter coupling of the form $\eta^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{S}^{\dot{\beta}}_\mu/\bar{M}_{Pl}$ only appears for the true Goldstino $G_\alpha$ and not for the Goldstini.

It is also interesting to consider locating different supersymmetry breaking sectors, and therefore different $\Phi_{NL}$, in spatially separated regions of a higher dimensional space, such as on different "branes". If there were different values for the condensate in these different regions then at leading order at least, one may be able to achieve a splitting between the Goldstini masses: we leave this for future research.

## 6. Discussion

In this paper, we attempted to derive an effective action, in superspace, that manifestly respects global supersymmetry and whose components reproduce the super-Higgs mechanism. The coefficients of these superspace terms appear to be chosen by hand to reproduce the necessary components but we demonstrated that these coefficients arise from explicitly computing the quantum corrections in components. After using this choice of coefficients, the components respect the necessary modified local supersymmetry transformations. It is perhaps unfortunate that local supersymmetry does not seem to fix the coefficients at the level of superfields but suggest that we should interpret our action as an effective one, in any case.

Still we think this setup is useful as it achieves the super-Higgs mechanism of the Goldstino multiplet and more interestingly, through the use of the supercurrent multiplet. Additionally, we have outlined how an effective action may be written that reproduces the results of multiple Goldstini [22].

There is much interest in the literature on the constrained Goldstino superfield. In [44] a supergravity setup is explored which leads to an explicit solution to the nilpotent Goldstino superfield constraint $\Phi^2 = 0$. A similar result in the context of $\mathcal{N} = 2$ supergravity is investigated in [45], where the constrained linear superfield approach allows for de Sitter vacua in these models (this general approach is reviewed in [46]). These investigations led to the discovery of new nilpotent superfields in supergravity models, which are still a subject of active research [47]. More recently, constrained superfields have been used in models of Goldstone boson inflation [48]. There is no doubt that the field of constrained Goldstone superfields is an active research direction, and one that will yield many new surprises in the future.

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## Appendix A. The Super-Higgs Mechanism: A Review

In this section, we review the super-Higgs mechanism following closely the appendix of [18]. We choose to review this appendix for two principle reasons: first it makes apparent the importance of couplings to the supercurrent $S^\mu_\alpha$, for example the term Equation (A12), and secondly because we wish to connect the above work directly with phenomenology and observation.

In two-component spinor notation, if supersymmetry is broken by an F-term vacuum expectation value, then the Goldstino must transform as $\delta_\epsilon \chi = \sqrt{2} F \epsilon$ and $\delta_\epsilon \bar{\chi} = \sqrt{2} F^\dagger \bar{\epsilon}$ where $(\epsilon, \bar{\epsilon})$ are supersymmetry transformation parameters and $\sqrt{2}$ is a convention. Treating supersymmetry as a global symmetry, Noether's theorem leads to a conserved supercurrent

$$\delta\mathcal{L} = (\partial_\mu \epsilon^\alpha) S^\mu_\alpha + (\partial_\mu \bar{\epsilon}_{\dot{\alpha}}) \bar{S}^{\mu\dot{\alpha}} = 0. \tag{A1}$$

Integrating by parts one finds the variation of the action

$$\delta\mathcal{S} = -\int d^4x \left[ \epsilon^\alpha (\partial_\mu S^\mu_\alpha) + \bar{\epsilon}_{\dot{\alpha}} (\partial_\mu \bar{S}^{\mu\dot{\alpha}}) \right] = 0. \tag{A2}$$

The action may be determined

$$S_{GT} = \int d^4x \left[ \frac{\chi^\alpha}{\sqrt{2}F} \partial_\mu S^\mu_\alpha + \frac{\bar{\chi}_{\dot{\alpha}}}{\sqrt{2}F^\dagger} \partial_\mu \bar{S}^{\mu\dot{\alpha}} \right]. \tag{A3}$$

This gives the familiar Goldberger-Treiman relation and a kinetic term for the Goldstino. The supercurrent should contain general matter contributions and a term proportional to the vev:

$$S^{\mu}_{\alpha} = S^{\mu}_{\alpha\text{matter}} + i\sqrt{2}F\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} \;\; , \;\; \bar{S}^{\mu\dot{\alpha}} = \bar{S}^{\mu\dot{\alpha}}_{\text{matter}} + i\sqrt{2}F^{\dagger}\bar{\sigma}^{\mu\dot{\alpha}\alpha}\chi_{\alpha}. \tag{A4}$$

Invariance of the action under supersymmetry transformations implies the canonically normalized Goldstino kinetic terms

$$\frac{1}{2}i\chi^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}\bar{\chi}^{\dot{\alpha}} + \frac{1}{2}i\bar{\chi}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}\partial_{\mu}\chi_{\alpha}, \tag{A5}$$

these being related to each other by an integration by parts. The reader can verify that varying the above kinetic contribution with respect to $\chi^{\alpha}$ and $\bar{\chi}^{\dot{\alpha}}$ independently will lead to the constraints $\partial_{\mu}S^{\mu\alpha} = 0$ and $\partial_{\mu}\bar{S}^{\mu\dot{\alpha}} = 0$, as required in Equation (A2), thus avoiding double counting the kinetic terms as would result by substituting Equation (A4) in Equation (A3) directly.

We now introduce the Gravitino

$$S_{kin} = -\frac{1}{2}\int d^4x\epsilon^{\mu\nu\rho\sigma}(\psi^{\alpha}_{\mu}\sigma_{\nu\alpha\dot{\alpha}}\partial_{\rho}\bar{\psi}^{\dot{\alpha}}_{\sigma} - \bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}_{\nu}\partial_{\rho}\psi_{\sigma\alpha}) \tag{A6}$$

and consider weakly gauging gravity by the introduction of a Gravitino that couples to the supercurrent

$$S_{\text{int1}} = \int d^4x\frac{1}{2\bar{M}_{Pl}}[\psi^{\alpha}_{\mu}S^{\mu}_{\alpha} + \bar{\psi}_{\dot{\alpha}\mu}\bar{S}^{\dot{\alpha}\mu}]. \tag{A7}$$

This term naturally leads to

$$\frac{iF}{\sqrt{2}\bar{M}_{Pl}}\psi^{\alpha}_{\mu}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} + \frac{iF^{\dagger}}{\sqrt{2}\bar{M}_{Pl}}\bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}\chi_{\alpha}, \tag{A8}$$

Additionally, one must introduce the term

$$\mathcal{S}_{\text{int2}} = \int d^4x\frac{i}{2\sqrt{6}\bar{M}_{Pl}}[\chi^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{S}^{\dot{\alpha}}_{\mu} + \bar{\chi}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}S_{\alpha\mu}], \tag{A9}$$

to obtain

$$-\frac{2F^{\dagger}}{\sqrt{3}\bar{M}_{Pl}}\chi^{\alpha}\chi_{\alpha} - \frac{2F}{\sqrt{3}\bar{M}_{Pl}}\bar{\chi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}. \tag{A10}$$

which is the first source of what will later become the Gravitino mass of the super-Higgs mechanism. There is also a contribution from coupling directly to the cosmological constant to give the overall mass

$$-\frac{F^{\dagger}}{\sqrt{3}\bar{M}_{Pl}}\chi^{\alpha}\chi_{\alpha} - \frac{F}{\sqrt{3}\bar{M}_{Pl}}\bar{\chi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}. \tag{A11}$$

In addition, it generates a non-derivative coupling between the Goldstino and the supercurrent

$$\frac{i}{2\sqrt{6}\bar{M}_{Pl}}\bar{\chi}^{\dot{\alpha}}\sigma^{\mu}_{\alpha\dot{\alpha}}(S^{\alpha}_{\mu})_{\text{matter}} + \frac{i}{2\sqrt{6}\bar{M}_{Pl}}\chi^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}(\bar{S}^{\dot{\alpha}}_{\mu})_{\text{matter}}. \tag{A12}$$

In [18], it was commented that it may seem surprising to add this new term Equation (A12), but that there is no contradiction as this new term vanishes when $M_{Pl} \to \infty$. It is, therefore, interesting to see that it arises *quite* naturally after one computes the effective action Equation (23) or instead from Equation (32).

To preserve local supersymmetry invariance under the *modified* [5] transformations

$$\delta\psi_{\mu\alpha} = -\bar{M}_{Pl}\left(2\partial_\mu\epsilon_\alpha + im_{\frac{3}{2}}\sigma_{\mu\alpha\dot{\alpha}}\epsilon^{\dot{\alpha}}\right) \tag{A13}$$

$$\delta\chi_\alpha = \sqrt{2}F\epsilon_\alpha \tag{A14}$$

one must add a Gravitino self-coupling term

$$S_{m_{3/2}} = -i\int d^4x\left(m_{3/2}\psi^\alpha_\mu(\sigma^{\mu\nu})_\alpha^{\ \beta}\psi_{\nu\beta} + m^\dagger_{3/2}\bar{\psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}}\bar{\psi}^{\dot{\beta}}_\nu\right), \tag{A15}$$

provided that

$$m_{3/2} = \frac{F}{\sqrt{3}\bar{M}_{Pl}}. \tag{A16}$$

Gathering all the terms together the overall Lagrangian is therefore

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\psi^\alpha_\mu\sigma_{\nu\alpha\dot{\alpha}}\partial_\rho\bar{\psi}^{\dot{\alpha}}_\sigma - \bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}_\nu\partial_\rho\psi_{\sigma\alpha}) - i\left(m_{3/2}\psi^\alpha_\mu(\sigma^{\mu\nu})_\alpha^{\ \beta}\psi_{\nu\beta} + m^\dagger_{3/2}\bar{\psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}}\bar{\psi}^{\dot{\beta}}_\nu\right)\\
&+\frac{i}{2}\chi^\alpha\partial_\mu\sigma^\mu_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} + \frac{i}{2}\bar{\chi}_{\dot{\alpha}}\partial_\mu\bar{\sigma}^{\dot{\alpha}\alpha}\chi_\alpha - \frac{F^\dagger}{\sqrt{3}\bar{M}_{Pl}}\chi^\alpha\chi_\alpha - \frac{F}{\sqrt{3}\bar{M}_{Pl}}\bar{\chi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}\\
&+\frac{1}{2\sqrt{6}\bar{M}_{Pl}}i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}(S^\alpha_\mu)_{\text{matter}} + \frac{1}{2\sqrt{6}\bar{M}_{Pl}}i\chi^\alpha\sigma^\mu_{\alpha\dot{\alpha}}(\bar{S}^{\dot{\alpha}}_\mu)_{\text{matter}}\\
&+\frac{iF}{\sqrt{2}\bar{M}_{Pl}}\psi^\alpha_\mu\sigma^\mu_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} + \frac{iF^\dagger}{\sqrt{2}\bar{M}_{Pl}}\bar{\psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}\chi_\alpha\\
&+\frac{1}{2\bar{M}_{Pl}}(\psi^\alpha_\mu(S^\mu_\alpha)_{\text{matter}} + \bar{\psi}_{\mu\dot{\alpha}}(S^{\mu\dot{\alpha}})_{\text{matter}})\\
&+\frac{\chi^\alpha}{\sqrt{2}F}\partial_\mu S^\mu_{\alpha\text{matter}} + \frac{\bar{\chi}_{\dot{\alpha}}}{\sqrt{2}F^\dagger}\partial_\mu\bar{S}^{\mu\dot{\alpha}}_{\text{matter}}.\end{aligned} \tag{A17}$$

The super-Higgs mechanism is realized by applying the shift

$$\Psi_{\mu\alpha} \to \psi_{\mu\alpha} - \frac{i}{\sqrt{6}}\sigma_{\mu\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} - \sqrt{\frac{2}{3}}\frac{1}{m_{\frac{3}{2}}}\partial_\mu\chi_\alpha \tag{A18}$$

$$\bar{\Psi}^{\dot{\alpha}}_\mu \to \bar{\psi}^{\dot{\alpha}}_\mu - \frac{i}{\sqrt{6}}\bar{\sigma}^{\alpha\dot{\alpha}}_\mu\chi_\alpha - \sqrt{\frac{2}{3}}\frac{1}{m^\dagger_{\frac{3}{2}}}\partial_\mu\bar{\chi}^{\dot{\alpha}} \tag{A19}$$

so that the Gravitino eats the Goldstino degrees of freedom and the Lagrangian becomes that of the massive Gravitino coupled to matter

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\Psi^\alpha_\mu\sigma_{\nu\alpha\dot{\alpha}}\partial_\rho\bar{\Psi}^{\dot{\alpha}}_\sigma - \bar{\Psi}_{\mu\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}_\nu\partial_\rho\Psi_{\sigma\alpha}) - i\left(m_{3/2}\Psi^\alpha_\mu(\sigma^{\mu\nu})_\alpha^{\ \beta}\Psi_{\nu\beta} + m^\dagger_{3/2}\bar{\Psi}_{\mu\dot{\alpha}}(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}}\bar{\Psi}^{\dot{\beta}}_\nu\right)\\
&+\frac{1}{2\bar{M}_{Pl}}(\Psi^\alpha_\mu(S^\mu_\alpha)_{\text{matter}} + \bar{\Psi}_{\mu\dot{\alpha}}(S^{\mu\dot{\alpha}})_{\text{matter}}),\end{aligned} \tag{A20}$$

with the Gravitino now carrying the right degrees of freedom for the self-interaction term to be correctly identified with the Gravitino mass.

This review of the super-Higgs mechanism generates the same Lagrangian as that of Deser-Zumino [5], which is a combination of the A-V action [3] plus linear supergravity, with one addition: In their paper they additionally comment on the two cosmological constants $-\frac{f^2}{2}e + ce$, which are set to cancel. where $e \equiv det(e^a_\mu)$ and $e^a_\mu$ is the Vielbein of the Graviton. In this review and in [18] this is implicitly assumed.

## Appendix B. Computing the Current Correlator

Here we demonstrate the evaluation of the time ordered current correlator appearing in Equation (23)

$$\langle T[S^\mu_\alpha(x)S^\nu_\beta(0)]\rangle \,. \tag{A21}$$

We wish to write this correlator in terms of the super algebra. To do this we take first the definition

$$\langle T[S^\mu_\alpha(x)S^\nu_\beta(y)]\rangle = \theta(x^0-y^0)\,\langle S^\mu_\alpha(x)S^\nu_\beta(y)\rangle - \theta(y^0-x^0)\,\langle S^\nu_\beta(y)S^\mu_\alpha(x)\rangle\,, \tag{A22}$$

where $\theta(x^0-y^0)$ is the Heaviside function and the minus sign between the two terms on the right-hand side is standard for fermionic operators. Using the standard manipulation

$$\partial_\mu\,\langle T[S^\mu_\alpha(x)S^\nu_\beta(y)]\rangle = \delta(x^0-y^0)\,\langle\{S^0_\alpha(x),S^\nu_\beta(y)\}\rangle + \langle T[\partial_\mu S^\mu_\beta(x)S^\nu_\alpha(y)]\rangle \tag{A23}$$

and then integrating by parts

$$-\,y^\rho\partial_\mu\,\langle T[S^\mu_\alpha(x)S^\nu_\beta(y)]\rangle = (\partial_\mu y^\rho)\,\langle T[S^\mu_\alpha(x)S^\nu_\beta(y)]\rangle \tag{A24}$$

with $y^\rho$ as a four-vector, one obtains

$$\langle T[S^\rho_\alpha(x)S^\nu_\beta(y)]\rangle = -y^\rho\left(\delta(x^0-y^0)\,\langle\{S^0_\alpha(x),S^\nu_\beta(y)\}\rangle + \langle T[\partial_\mu S^\mu_\beta(x)S^\nu_\alpha(y)]\rangle\right). \tag{A25}$$

Using $\partial_\mu S^\mu = 0$ to remove the second term then $\int d^3x S^0_\alpha(x) = Q_\alpha$ gives

$$\int d^4x\,\langle T[S^\rho_\alpha(x)S^\nu_\beta(y)]\rangle = -y^\rho\,\langle\{Q_\alpha,S^\nu_\beta(y)\}\rangle\,. \tag{A26}$$

Inserting

$$\{Q_\beta,S_{\mu\alpha}\} = 2i\epsilon_{\lambda\beta}(\sigma_{\mu\rho})^\lambda_\alpha\partial^\rho\bar{x} \tag{A27}$$

integrating by parts, we find

$$\int d^4x\,\langle T[S^{\rho\alpha}(x)S^{\nu\beta}(y)]\rangle = (\partial^\sigma y_\rho)2i\epsilon_{\lambda\alpha}(\sigma_{\nu\sigma})^\lambda_\beta\bar{x}. \tag{A28}$$

Thus, we see that the constant $\langle x\rangle$ will contribute to the current correlator. We mention that if one instead evaluates the sum rule for $\{\bar{Q}_{\dot\alpha},S^\mu_\alpha(x)\}$ then one may show the massless Goldstino pole [2,49].

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