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Strange Stars in the Vector Interaction Enhanced Bag Model

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Abstract: The vector interaction enhanced Bag model (vBag) for dense quark matter extends the commonly used thermodynamic Bag model (tdBag) by incorporating effects of dynamical chiral symmetry breaking (DχSB) and vector repulsion. Motivated by the suggestion that the stability of strange matter is in tension with chiral symmetry breaking (DχSB) we examine the parameter space for its stability in the vBag model in this work. Assuming the chiral transition occurs at sufficiently low density, we determine the stability region of strange matter as a function of the effective Bag constant and the vector coupling. As an astrophysical application, we construct contours of maximum mass $M_{\max}$ and radius at maximum mass $R_{\max}$ in this region of parameter space. We also study the stability of strange stars in the vBag model with maximum mass in the $2M_{\odot}$ range by computing the spectrum of radial oscillations, and comparing to results from the tdBag model, find some notable differences.

Keywords: compact stars; strange matter; mass–radius relations

1. Introduction

It has long been hypothesized [1] that the true ground state of strongly interacting matter in bulk form should comprise of an equal number of up, down and strange quarks, commonly referred to as strange matter [2]. To date, the ab-initio approach to strong interactions cannot rule out this possibility since the regime of cold and dense matter presents challenges to lattice simulations of Quantum Chromodynamics (QCD) [3]. Therefore, studies of bulk strange matter at zero temperature are largely confined to phenomenological models with applications to the astrophysical realm of neutron stars, strangelets in cosmic rays [4] or dark matter [5]. In the case of neutron stars, dense nuclear matter may be in a long-lived metastable state, with a low probability of converting to strange stars. Nevertheless, the astrophysical possibility of strange stars remains sufficiently interesting to explore with newer models of QCD, especially ones with more fidelity to non-perturbative aspects of confinement and dynamical chiral symmetry breaking. In this article, we present a study of the stability of strange matter in the vector interaction enhanced bag model [6] and explore the mass and radius values of strange stars. In the vBag model, the vector repulsion is quantified by a coupling parameter K_v which allows the possibility of massive strange stars (and more generally hybrid stars with quark matter cores [6]) that are consistent with the $2M\odot$ observations inferred from compact binaries [7,8]. This consistency can be achieved in the simple tdBag model with perturbative α_s corrections to the quark pressure, but the applicability of perturbative QCD to the moderate density regime encountered in neutron stars is dubious. We also present results for the radial oscillation modes of strange stars in the vBag model model.

2. Theory

EOS of Strange Matter: The vBag Model

The regime of large baryochemical potential characterizing the cold and dense deconfined quark phase is commonly investigated in the framework of effective models such as the thermodynamic bag model [9] (tdBag) or variants of the Nambu–Jona-Lasinio model [10,11] (NJL). In this context, it is worth pointing out that the original MIT Bag model [12] is a phenomenological approach to describe a confined system of relativistic quarks and gluon fields, i.e., hadrons, while the tdBag is applied to bulk quark matter. In the MIT bag model, the bag constant B can be understood as an effective parameter that stabilizes the hadron by matching the pressure of its constituent free fields at the boundary of the bag to the pressure of the physical (non-perturbative) QCD vacuum. In the tdBag model, which describes an extended Fermi gas of free (or weakly interacting) quarks, the bag constant expresses the energy difference between the physical (non-perturbative) vacuum and the perturbative one. The tdBag model thereby mimics quark confinement but does not include chiral symmetry breaking, a fundamental feature of low-energy QCD. In contrast, the NJL model is designed to include the effects of chiral symmetry breaking in the quark sector (Goldstone phase) but lacks confinement. Despite this difference, in fact, both the tdBag and NJL-like models can be understood as solutions of QCD's in-medium Dyson–Schwinger gap equations within a particular set of approximations [13]. From this perspective, one can make modifications to the tdBag model which brings it into the same class of models as the NJL models. This modification is referred to as the vector-interaction-enhanced (vBag) [14] model. The vBag model accounts in parameterized form for (a) the flavor dependent restoration of chiral symmetry, (b) repulsive vector interactions, and (c) a phenomenological correction to the EoS that describes deconfinement and generally depends on the nuclear EoS, although in this work, the latter is not important since we discuss only strange matter. While formally similar to the tdBag model, the addition of vector repulsion in the vBag model means that it can describe massive hybrid/strange stars that meet the $2M_\odot$ constraint [7,8,15]. The introduction of flavor-dependent chiral bag constants is motivated by fits to the pressure of the chirally restored phase [6]. In addition, a deconfinement bag constant B_{dc} is introduced to lower the energy/particle and thereby favor stable strange matter. The equation of state in the vBag model is then given parameterically by [14]

$$P_q = \sum_{f=u,d,s} P_{vBag,f} + B_{dc} \tag{1}$$

$$\epsilon_q = \sum_{f=u,d,s} \epsilon_{vBag,f} - B_{dc} \tag{2}$$

where $P_{vBag,f}$ and $\epsilon_{vBag,f}$ are the pressure and energy density of a single quark flavor, respectively:

$$P_{vBag,f}\left(\mu_f\right) = P_{FG,f}\left(\mu_f^*\right) + \frac{1}{2}K_v n_{FG,f}^2\left(\mu_f^*\right) - B_{\chi,f} \tag{3}$$

$$\epsilon_{vBag,f}\left(\mu_f\right) = \epsilon_{FG,f}\left(\mu_f^*\right) + \frac{1}{2}K_v n_{FG,f}^2\left(\mu_f^*\right) + B_{\chi,f} \tag{4}$$

The subscript FG refers to the ideal, zero temperature Fermi gas formula for that thermodynamic quantity. K_v parameter in the second term on the right side of the equations is a coupling constant that results from vector interactions, and $B_{\chi,f}$ is the Bag constant of a single flavor. The chemical potential, μ_f^*, allows for a quasiparticle description of dressed quarks consistent with the solution of the vector gap from Dyson–Schwinger studies and is parameterized by the following relation.

$$\mu_f = \mu_f^* + K_v n_{FG,f}\left(\mu_f^*\right) \tag{5}$$

with number density $n_{FG,f}\left(\mu_f^*\right) = n_f(\mu_f)$.

3. Strange Stars in the vBag Model

The vBag equation of state can also be expressed as [16]:

$$P_q = \frac{1}{3}(\epsilon_q - 4\sum_f B_{\chi,f}) + \frac{4}{3}B_{dc} + \frac{K_v}{3}\sum_f n_f^2(\mu_f)\,. \tag{6}$$

which has a non-barotropic form since $n_f(\mu_f)$ encodes composition information. One can define an effective Bag constant in the vBag model, much as in the tdBag model, but apportioned into chiral and deconfinement bag constants with distinct origins.

$$B_{\text{eff}} = \sum_{f=u,d,s} B_{\chi,f} - B_{dc} \tag{7}$$

The EoS becomes

$$P_q = \frac{1}{3}(\epsilon_q - 4B_{\text{eff}}) + \frac{K_v}{3}\sum_f n_f^2(\mu_f)\,. \tag{8}$$

The parameter B_{eff} can be used, in either case of two or three-flavor quark matter in hybrid stars, to manipulate the value of pressure at which the phase transition occurs, while K_v controls the stiffness of the quark matter EoS [17].

Along with the quark EoS, there are also electrons to maintain charge neutrality if the strange quark's large current mass is taken into account. The flexbility of the vBag model parameters that refer to chiral and deconfinement transitions implies that hybrid stars can masquerade as neutron stars [18] and it has been clearly shown that quark matter is not too soft to support maximum masses of $2M_\odot$ or more [6,19], as was the case with the tdBag model. Furthermore, it appears difficult to distinguish two-flavor from three-flavor quark matter even in cases where a phase transition can be said to have occurred, as in the presence of a distinct kink in the mass–radius relation [17]. Thus, high-precision M–R data may still not be enough to draw firm conclusions on the nature of phases and phase transitions in neutron stars, motivating the study of oscillation modes in this paper.

Although hybrid stars are more likely to exist, in this work, we focus on strange stars within the vBag model. Strange stars contain strange quark matter (up, down, and strange quarks) in the entire star, with the exception of a thin nuclear crust [20] (suspended by strong electric fields at the surface of quark matter) or possibly a crust of quark-alphas [21] or strangelets which is globally neutral [22]. Since the nature of the crust is outside the scope of this work, we apply the vBag EoS to model bare strange stars. The stability of three-flavor quark matter over nuclear matter is rather straightforward in the tdBag model, as the argument rests solely on the energy per particle of free or weakly interacting quarks, including the s-quark [9]. For reasonably small values of α_s, m_s, we can find a range of values in B where the energy per particle of three flavor matter (but not two-flavor matter) is lower than nuclear matter. Typically, for models that also include chiral restoration, the mass of the dressed s-quark is too large to justify the argument for absolutely stable three-flavor matter in this way, if only the chiral Bag constant is considered. However, in vBag, we can use the additional free parameter B_{dc} to reduce B_{eff} (compared to chiral models) and lower the energy per particle. Whether this is sufficient to stabilize strange matter depends on the chiral Bag constant for the light flavors $B_\chi^{u/d}$. Specifically, if the magnitude of $B_\chi^{u/d}$ is comparable to the chiral condensate one obtains within NJL-type models, the chiral transition takes place at densities where the energy per baryon is already distinctively larger than 930 MeV, ruling out stable strange matter. This could hold even if the effective bag constant is in a domain where tdBag predicts stable strange matter, due to the fact that the vBag effective bag constant allows to distinguish chiral symmetry breaking and confinement, both acting with opposite signs.

For the purpose of this paper, we are assuming that the chiral transition has to occur at rather small densities so that a window in stable quark matter can still be found, and that the effective bag constant B_{eff} accounts for both chiral and confinement effects, in the same fashion as the Bag constant in tdBag. Nevertheless, we are going beyond the tdBag model in exploring chiral effects because the two stability lines (energy/particle = 930 MeV for 2-flavor and 3-flavor quark matter) in Figure 1 can be understood as setting an upper limit on an effective chiral bag constant. Figure 1 shows contours of constant maximum mass M_{max} and radius at maximum max R_{max} in the (B_{eff}, K_v) parameter space of the vBag model. For strange stars, lower values of B_{eff} and higher value of the vector coupling provide additional pressure, which implies a higher maximum mass. The dotted lines in the figure show constant energy/baryon of 930 MeV for the two and three flavor case, narrowing the allowed parameter space such that there is an upper limit on $B_{\text{eff}} \approx 70$ MeV/fm^3 and a lower limit $K_v \approx 2$ if we demand consistency with the $2M_\odot$ constraint on the maximum mass. While we do not claim that these heavy stars are strange stars, it is useful to recall that the maximum mass sets strong constraints on the strange matter EOS as well. For example, the confirmation of a $2.4M_\odot$ neutron star (e.g., the Black Widow pulsar) would rule out the possibility of absolutely stable strange matter in the vBag model.

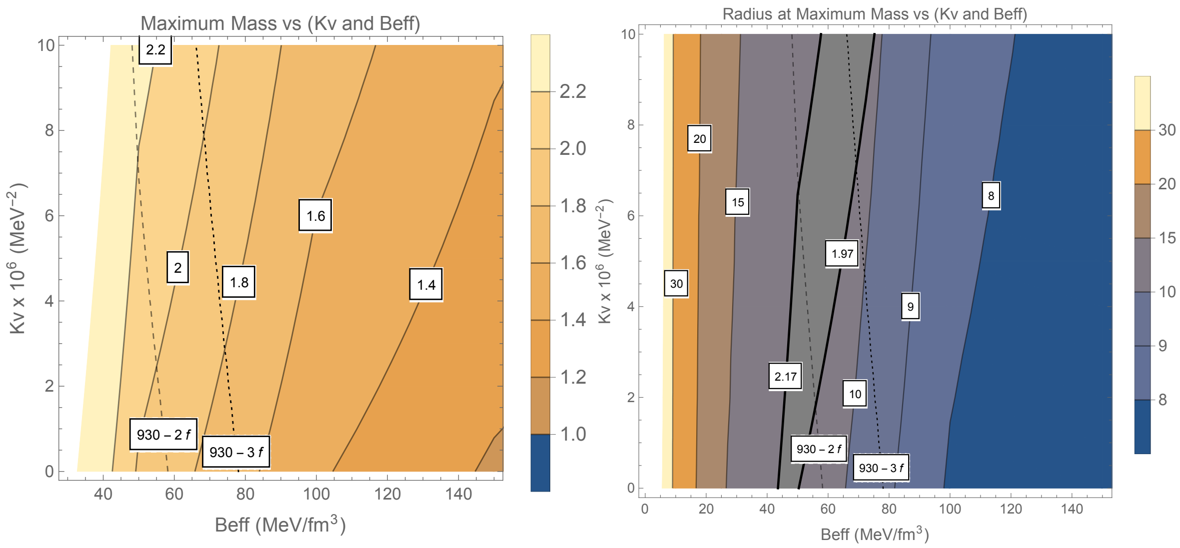

Figure 1. Contours of constant maximum mass (**left panel**) and of constant radius at maximum mass (**right panel**) in the vBag parameter model space. Also shown are the two-flavor and three-flavor lines of constant energy/baryon = 930 MeV (light dotted lines in both panels) and contours corresponding to values of maximum mass (right panel only) of two heavy neutron stars—J1614-2230 at $1.97M_\odot$ from pulsar observations [8] and the neutron star merger event GW170817 at $2.17M_\odot$ [23]. The apparent degeneracy of mass (or radius) in this parameter space is due to the fact that both parameters B_{eff} and K_v can be chosen independently to effectively soften or stiffen the equation of state beyond a desired baryon density that still admits stable strange matter.

Figure 2 shows the mass–radius plot of strange stars for typical parameters of K_v and B_{eff} used in this work. The maximum mass M_{max} is significant as it is the point where the radial mode turns unstable, unless one adopts different adiabatic indices for the perturbation and the background EoS.

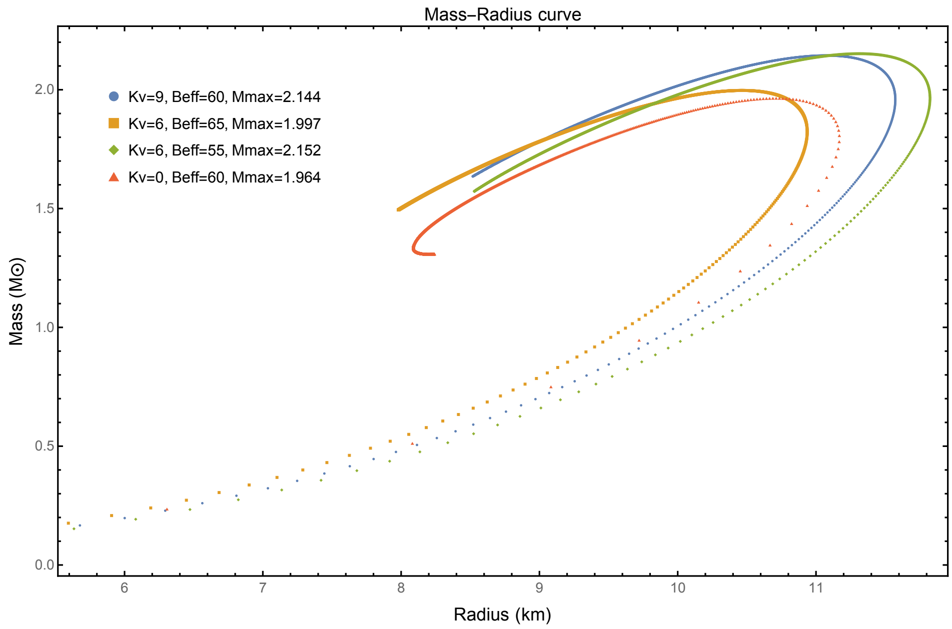

Figure 2. The mass–radius plot for strange stars in the vBag model ($K_v \neq 0$), compared to the tdBag model (K_v = 0). Parameters K_v and $B_{\rm eff}$ have been chosen to correspond to maximum mass values that can approach or exceed $2M_\odot$.

4. Stability against Radial Oscillations

Having described the EOS for strange stars in the vBag model, we now explore radial oscillations, which are of general interest to the dynamics of compact stars [24,25]. The radial stability of self-gravitating gaseous spheres in general relativity was explored first by Chandrasekhar [26] and subsequently applied to neutron stars and strange stars to determine the mass–radius configuration that can be realistically supported by a given equation of state [27,28]. In this section, we obtain (numerically) the frequency of the first two principal radial oscillation modes for different choices of parameters in the vBag model. The corresponding mass and radius of the densest star that is stable against radial oscillations is also determined. We aim to ascertain any differences in the trend of mode frequencies between the vBag and the tdBag models.

The equations determining the eigenfrequency of the radial oscillation mode are [28,29]:

$$\begin{aligned}
\frac{\mathrm{d}\xi}{\mathrm{d}r} =& -\frac{1}{r}\left(3\xi + \frac{\eta}{\gamma}\right) - \frac{\mathrm{d}P}{\mathrm{d}r}\frac{\xi}{(P+\varepsilon)} \\
\frac{\mathrm{d}\eta}{\mathrm{d}r} =& \xi\left\{\frac{\omega^2}{c^2}\mathrm{e}^{\lambda-v}\left(\frac{P+\varepsilon}{P}\right)r - \frac{4}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\right. \\
& \left. -\frac{8\pi G}{c^4}\mathrm{e}^{\lambda}(P+\varepsilon)r + \left(\frac{\mathrm{d}P}{\mathrm{d}r}\right)^2\frac{r}{P(P+\varepsilon)}\right\} \\
& +\eta\left[-\frac{\mathrm{d}P}{\mathrm{d}r}\frac{\varepsilon}{P(P+\varepsilon)} - \frac{4\pi G}{c^4}(P+\varepsilon)r\mathrm{e}^{\lambda}\right]
\end{aligned} \tag{9}$$

where $\xi = \Delta r/r$ is related to the Lagrangian displacement Δr of the radial perturbation, and $\eta = \Delta P/P$ is related to the Lagrangian displacement Δr of the associate pressure perturbation. To solve these equations consistently, an equation of state $P = P(\epsilon)$ is needed, which in our case comes from the vBag model of the previous section. The first order form of the oscillation equations presented here has the advantage of not requiring the computation of derivatives of the adiabatic index γ, defined in its relativistic form as $\gamma = \frac{\epsilon+P}{P}\frac{dP}{d\epsilon}$. $\lambda(r), \nu(r)$ in Equation (9) are standard metric functions of the interior Schwarzschild metric of the star, which can be obtained from the solution of the TOV equations [30]:

$$\begin{aligned}\frac{\mathrm{d}m}{\mathrm{d}r} &= \frac{4\pi r^2}{c^2}\epsilon \\ \frac{\mathrm{d}P}{\mathrm{d}r} &= -\frac{P+\epsilon}{c^2}\left(\frac{Gm}{r^2} + \frac{4\pi G}{c^2}Pr\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1} \\ \frac{\mathrm{d}\nu}{\mathrm{d}r} &= -\frac{2}{P+\epsilon}\frac{dP}{dr}\end{aligned} \tag{10}$$

The oscillation eigenfrequency ω can be determined by the unique solution (up to an arbitrary normalization of ξ which is here chosen equal to unity) to these equations that are regular at the center of the star, and satisfy $\Delta P = 0$ at the surface. This implies that $\eta = -3\gamma\xi$ is at the center of the star, and

$$\eta = \xi\left[\left(1-\frac{2GM}{Rc^2}\right)^{-1}\left(-\frac{\omega^2 R^3}{GM} - \frac{GM}{Rc^2}\right) - 4\right]$$

at the surface. The numerical singularity at $r = 0$ in Equations (9) can be removed by a Taylor expansion, and we use a shooting method with corresponding initial values given as $\xi(r \to 0) = 1 + \xi_1 r^2, \eta(r \to 0) = \eta_0 + \eta_1 r^2$ that also satisfy the surface boundary condition to determine the eigenfrequencies ω_0, ω_1, etc. Stability requires $\omega^2 > 0$, and the critically stable configuration (which is also the maximum mass configuration) has $\omega_0^2 = 0$ since it is the fundamental mode and the first to turn unstable.

Significant numerical simplification and speedup are obtained by working in geometrized units $G = c = 1$, and using the reduced enthalpy version [31,32] of the TOV equations Equations (10), as $d\nu(r)/dr = -2dh(r)/dr$, where $h(r)$ is the reduced enthalpy with $dh = dp/(\epsilon + p)$. In particular, since only $d\nu/dr$ can be obtained directly from a solution of the standard form of the TOV equations (Equations (10)), the function $\nu_(r)$ that is required to solve Equations (9) is more easily determined in terms of the central enthalpy h_c.

5. Results and Conclusions

In Table 1 below, we present results for the first two radial oscillation modes of strange stars in the vBag model, with varying parameter choices to highlight the effect of changing B_{eff} and K_v. We also present a comparison against the trends of the more familiar and previously studied tdBag model, finding some differences of note. A check on our numerical result comes from the excellent agreement (to better than 0.2%) with the published result [28] for the frequency of the first two eigenmodes in the tdBag model for B = 60 MeV/fm^3. Compactness and frequency is presented in geometrized units $M^*(\text{km}) = 1.477\ M(M_\odot)$ and $\omega^*(\text{km}^{-1}) = 0.0209\ \omega(\text{kHz})$ with $G = c = 1$.

Table 1. Frequencies of the fundamental (ω_0) radial oscillation mode and the first overtone (ω_1) in the vector interaction enhanced Bag model (vBag) model of strange stars. All configurations are stable against oscillations, i.e., have $\omega^2 > 0$. $M^*(\text{km}) = 1.477\ M(M_\odot)$ and $\omega^*(\text{km}^{-1}) = 0.0209\ \omega(\text{kHz})$.

EOS	Mass	Radius	Compactness R/M^*	Frequency [kHz]		Frequency [$\frac{\omega^*}{\sqrt{M^*/R^3}}$]	
B, B_{eff} [MeV/fm^3] K_v[GeV^{-2}] ρ_c[10^{-3}km^{-2}]	[$M_\odot$]	[km]	[Dimensionless]	ω_0	ω_1	[Dimensionless]	
vBag							
$B_{\text{eff}} = 60.0, K_v = 4, \rho_c = 0.913$	1.902	11.172	3.984	2.114	7.248	0.987	3.384
$B_{\text{eff}} = 60.0, K_v = 9, \rho_c = 0.913$	2.051	11.527	3.802	1.918	6.928	0.904	3.265
$B_{\text{eff}} = 65.0, K_v = 6, \rho_c = 0.886$	1.839	10.934	4.032	2.424	7.763	1.114	3.569
$B_{\text{eff}} = 55.0, K_v = 6, \rho_c = 0.886$	2.068	11.765	3.846	1.808	6.670	0.875	3.328
td-Bag							
$B = 60.0, \rho_c = 1.410$	1.962	10.799	3.731	0.725	6.165	0.317	2.693
$B = 70.0, \rho_c = 1.410$	1.802	10.143	3.817	1.388	6.976	0.576	2.895

5.1. Eigenfrequencies in the vBag and tdBag Models

We focus our discussion on the fundamental mode, as it is the most likely to be excited in a binary merger [33] and is the first mode to become unstable. In both vBag and tdBag models, the eigenfrequency of the fundamental mode increases with increasing (effective) Bag constant, with other variables such as the vector coupling and central density held constant. Increasing the strength of the vector coupling, holding other variables constants, leads to lower frequencies. In both models, we found that increasing the central density for a fixed EOS parameter set decreases the eigenfrequency. Thus, in a 1-parameter cut of the vBag model, we may conclude that stars with higher compactness value R/M^* have higher eigenfrequencies or shorter time period of oscillation.

However, there are also some differences in the trends of the eigenfrequency of the radial modes between the vBag and tdBag models. In the tdBag model, which is a 1-parameter family of EOS, the eigenfrequency increases with increasing compactness R/M^*. In the vBag model which is a 2-parameter family of EOS, the eigenfrequency may either increase or decrease with compactness as both $B_{\rm eff}$ and K_v are varied. This is apparent from comparing parameter sets 2 and 4, and 1 and 3 of the vBag model in Table 1 above. Thus, in the vBag model, one can tune the deformability of the star independently of compactness. Another difference between the two models regarding the frequency is that the vBag model produces significantly higher frequencies than in the tdBag model for configurations close to $2M_\odot$. For example, the vBag model applied to a star with $M = 2.051\ M_\odot$ (maximum mass of $M = 2.144\ M_\odot$) gives $\omega_0 = 1.918$ kHz, while the tdBag model for $M = 1.962\ M_\odot$ (maximum mass of $M = 1.964\ M_\odot$) gives $\omega_0 = 0.725$ kHz. This difference arises on account of the sharp decrease in frequency as the maximum mass configuration is approached, which is barely below $2M_\odot$ for the tdBag model but well above that for the vBag model. Therefore, the radial oscillation frequency is a good indicator of proximity to the maximum mass configuration, which in turn sheds light on the EOS.

5.2. Mode Instability

The frequency of stable radial modes has $\omega^2 > 0$, while $\omega^2 < 0$ signals instability. The fundamental mode ω_0 is the first mode to turn unstable, followed by the first overtone and so on. The critical stellar configuration at which the frequency goes to zero (infinite time period) is also the maximum mass configuration, provided it is assumed that the perturbation obeys the same equation of state as the unperturbed background EOS; in other words, when the Schwarzschild discriminant is zero [34]. Working under this assumption, we confirm numerically that the mode becomes unstable at the maximum mass, and list critically stable mass–radius configurations for typical parameter sets in the vBag model in Table 2 below.

Table 2. Maximum mass and corresponding radius for critically stable stellar configurations in the vBag model of strange stars. All configurations with higher central densities are unstable against oscillations, i.e., have $\omega^2 < 0$.

EOS $B, B_{\rm eff}$ [MeV/fm^3] K_v[GeV^{-2}]	Max. Mass [$M_\odot$]	Radius at Max. Mass [km]	Central Density [10^{-3} km^{-2}]
vBag			
$B_{\rm eff} = 60.0$, $K_v = 4$	2.015	10.717	1.536
$B_{\rm eff} = 60.0$, $K_v = 9$	2.144	11.069	1.458
$B_{\rm eff} = 65.0$, $K_v = 6$	1.997	10.457	1.632
$B_{\rm eff} = 55.0$, $K_v = 6$	2.152	11.307	1.395
td-Bag			
$B = 60.0$	1.964	10.724	1.510
$B = 70.0$	1.818	9.905	1.802

Higher values of maximum mass, beyond $2.2M_\odot$ can be attained in the vBag model as shown in Figure 1, but require very large values of K_v and small values of $B_{\rm eff}$ rendering 3-flavor matter barely absolutely stable. Similarly, radius at the maximum mass of much more than 11 km is disfavored. The minimum compactness of the maximum mass strange star in the tdBag model has been shown to be 3.696 [32]. In some of the configurations supported in the vBag model with absolutely stable quark matter that are consistent with the observation of $2M_{\odot}$ compact stars, the compactness can be even smaller.

6. Discussion and Conclusions

We have explored the stability of strange matter and of strange stars against radial oscillations in the vector interaction enhanced Bag model. The novel aspect of this work, compared to previous such studies, is the use of the vBag model, which includes spontaneous breaking of chiral symmetry as well as a phenomenological description of confinement. While NJL models and an earlier paper on the vBag model generally do not find strange matter [6] to be absolutely stable (due to the large value of the dressed s-quark mass), we assume here that chiral symmetry is at least partially restored for the s-quark and the light quarks at densities of interest, so that $B_{\rm eff}$ lowers the energy/particle sufficiently. Consequently, we find that the upper limit to the chiral Bag constants for 3-flavor quark matter can be higher than for 2 flavors, as chosen in [6].

There are two main results of this work. Our first main result is the stability window for strange matter in the vBag model, as displayed in Figure 1 by the parametric limits on $(B_{\rm eff}, K_v)$ corresponding to fixed energy/particle, which is also consistent with the observation of heavy neutron stars and radius in the 10–12 km range. This reinforces, at least in spirit, the conclusion of more detailed work on hybrid stars [17] that precision mass and radius measurements alone may not be enough to distinguish ordinary neutron stars from those containing large fractions of quark matter. The second main result, following from a dynamical stability analysis against radial oscillations of strange stars, is that the oscillation frequency appears not to have a monotonic behaviour with compactness, as found in past works [28,35] that used variants of the tdBag model. We suspect this is due to the flexbility contained in the vBag model that arises from the distinct scalar and vector part of the quark self-energy, albeit computed within the same non-perturbative framework of the quark's gap equations.

Our result for oscillation frequencies also has a different trend than those typically obtained in pQCD models [36,37]. For example, the scaling relations obtained in [1] and reflected in the the ratio of oscillation frequencies [36] are based on pQCD models (tdBag models+perturbative corrections), while in the vBag model, such simple scaling relations are not found. However, it should be possible to find a domain where the vBag model can mimic the tdBag model with perturbative corrections, and it would be interesting to confirm the occurrence of scaling relationships in that case.

The relevance of oscillation modes to compact star mergers has been well established and is actively studied in numerical simulations [23,38]. While we have studied the oscillations of isolated, non-rotating strange stars, the methods and results outlined here can nevertheless be relevant to study mergers in which one or both component is a hybrid star with a quark matter core [39]. Since hybrid stars can masquerade as neutron stars [18] and phase transitions to quark matter can be effectively camouflaged [17] in mass–radius curves, the unveiling of quark matter in neutron stars may rest on precise observations of dynamical phenomena, such as the radial and non-radial oscillation modes that are excited pre or post-merger.

Author Contributions: All authors contributed equally to the computations and drafting of the paper. In particular, M.S. studied the stability of strange matter in the vBag model. T.K. developed the equation of state for the vBag model and its application to compact stars. P.J. computed the radial oscillation modes for strange stars.

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Abbreviations

The following abbreviations are used in this manuscript:

EOS	Equation of State
DχSB	Dynamical chiral symmetry breaking
QCD	Quantum Chromodynamics
FG	Fermi Gas
TOV	Tolman-Oppenheimer-Volkov

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