Supplementary Information

Surface soil moisture deficit determination using the satellite data

To calculate the soil moisture deficit component, our approach involves defining it as the difference between the soil water content at field capacity (which significantly depends on the soil type) and the actual soil water content calculated using radar satellite data, in particular Sentinel-1.

Soil moisture deficit is the amount of precipitation or irrigation water needed to restore the soil water content back to field capacity. Field capacity is used to characterize the maximum soil water content which is able to reach full vegetation development.

Thus, the moisture deficit in the upper layer of soil (mm) is defined as [S1, S2]:

$$m = W_{FC} - W_{act}, \quad \text{(S1)}$$

where W_{FC} (mm) is soil water content at field capacity, W_{act} (mm) is the actual soil water content;

W_{FC} is defined as water amount that can be held in the soil layer with the depth of h (m) in equilibrium after the maximum wetting from above (due to precipitation or irrigation) and after the excess gravitational water has drained away.

According to [S3], the expression for the water content at field capacity W_{FC} in the soil layer with the depth of h layer will look as follows:

$$W_{FC} = 10 \cdot h \cdot \theta_{FC} \cdot \rho_w, \quad \text{(S2)}$$

where h (m) – soil layer depth; θ_{FC} (%) – volume field capacity, in % to the soil volume; ρ_w (1 t/m^3) – density of water.

The soil hydraulic properties value at standard depths, for example, at the range 0–5 cm or 0–30 cm, is calculated as the weighted average value using numerical integration by the trapezoidal method [S4]:

$$\frac{1}{b-a}\int_a^b f(x)\,dx \approx \frac{1}{b-a}\cdot\frac{1}{2}\sum_{k=1}^{N-1}(x_{k+1}-x_k)(f(x_{k+1})-f(x_k)), \quad \text{(S3)}$$

where N is the number of depths in the interval, x_k is the k-th depth, $f(x_k)$ is the value of the target variable (i.e., soil property) at depth x_k.

Considering relation (S3), the weighted average value of the volume field capacity θ_{FC} (%) for the first standard depth range 0–5 cm is calculated as the arithmetic mean

$$\theta_{FC\_0-5} = \frac{1}{2}\left(\theta_{FC\_0} + \theta_{FC\_5}\right). \quad \text{(S4)}$$

According to [S5], the actual soil water content W_{act} is calculated as

$$W_{act} = 10 \cdot w_{act} \cdot \rho_b \cdot h, \quad \text{(S5)}$$

where w_{act} (%) is actual soil moisture, ρ_b (t/m^3) is the bulk density of the soil, h (m) is the soil layer depth.

The actual soil moisture w_{act} at certain time point for expression (S5) can be obtained from satellite radar data on the soil saturation degree, i.e., is the relative surface soil water content in the layer as the saturation percentage. According to [S6], the degree of soil saturation with soil moisture is connected with dependence:

$$SSM = \frac{w_{act}\cdot\rho_b}{n\cdot\rho_w}, \quad \text{(S6)}$$

where n (in unit fractions) is the porosity, i.e., the ratio of the pore volume to the entire soil volume. From here, it is possible to obtain an expression for the actual moisture of the upper soil layer w_{act} at the moment of satellite imaging. By substituting this into equation (S5), one has:

$$W_{act} = 10 \cdot SMM \cdot n \cdot \rho_w \cdot h. \quad \text{(S7)}$$

Thus, expression (S1) for calculating the deficit of moisture in the surface soil layer in mm, taking into account (S2) and (S7) on the basis of current satellite radar data on the degree of soil saturation, will look like:

$$m = 10 \cdot h \cdot \rho_w(\theta_{FC} - SSM \cdot n), \quad \text{(S8)}$$

where h (m) is the soil layer depth; note that given limited capabilities of satellite radar imagery $h = 0{,}05$ m;

ρ_W (t/m³) is the density of water, 1 t/m³;
θ_{FC} (%) is the weighted average value of the volume field capacity on a given depth range, here $\theta_{FC} = \theta_{FC\_0-5}$ for the first standard depth range 0–5 cm according (S4);
SSM (%) is the soil saturation degree from satellite radar data;
n (in unit fractions) is the porosity, i.e., the ratio of the pore volume to the entire soil volume.

As can be seen from expression (S8), physical sense of the product $SSM \cdot n$ is the actual volumetric soil water capacity at the time of satellite imagery.

To normalize the moisture deficit in the surface layer of the soil with a depth of 0–50 mm and to define its limits, which correspond to different levels of fire danger, one applied a scale of drought levels of the Keetch–Byram type, which contains six ranges (in contrast to the established scale, consisting of four [S7, S8]). As it is known, the Keetch–Byram drought index corresponds to the soil moisture deficit in some soil layers with soil water content at the field capacity $W_{FC\_KBDI} = 203{,}2$ mm.

$KBDI$ is related to PFC (the percentage of actual soil water content to soil water content at field capacity) as follows (if $KBDI$, W_{act}, W_{FC} are measured in mm) [19, 20]:

$$KBDI = 203{,}2(1 - PFC), \quad \text{(S9)}$$

where

$$PFC = W_{act}/W_{FC}. \quad \text{(S10)}$$

In the ratio (S10), W_{act} is measured on a meteorological station for a soil layer of a certain type and a certain depth. For this type and depth, the volumetric field capacity and the corresponding soil water content W_{FC} are set. After inserting expressions (S2) and (S7), taking into account (S10) in the formula (S9), one obtains the Keetch–Byram drought index expression:

$$KBDI = 203{,}2\left(1 - \frac{SSM \cdot n}{\theta_{FC}}\right). \quad \text{(S11)}$$

As can be seen from (S11), such a relationship between the index and the specified ratio takes place regardless of the soil layer depth.

Formally, the drought index $KBDI$ is the soil moisture deficit m_{KBDI} in some soil layers, which is the difference between the soil water content at field capacity and the actual soil water content W_{act_KBDI}:

$$KBDI = m_{KBDI} = W_{FC\_KBDI} - W_{act\_KBDI}. \quad \text{(S12)}$$

Taking into account (S11) and (S12), one obtains the soil moisture deficit expression corresponding to the Keetch–Byram drought index, which is calculated from the moisture deficit in the soil layer 0–50 mm:

$$m_{KBDI} = 203{,}2\left(1 - \frac{SSM \cdot n}{\theta_{FC}}\right). \quad \text{(S13)}$$

Taking into account expression (S8), formula (S13) takes the following form

$$m_{KBDI} = 203{,}2\left(\frac{m}{10 \cdot \rho_W \cdot h \cdot \theta_{FC}}\right). \quad \text{(S14)}$$

Therefore, for the soil moisture deficit (S13), one can use the Keetch–Byram index drought levels scale.

The methodology of converting the actual moisture deficit in soil measured at the actual field capacity at the meteorological stations into the moisture deficit corresponding to the index [S9, S10] was used in the National Fire Danger Rating System [S11].

The advantage of the Keetch–Byram index drought levels scale, to which would be converted the moisture deficit determined for the soil layer of a certain depth by in situ or satellite data, is its significant spread in fire information systems and recognized unambiguous interpretation of drought levels and its relationship with fire danger.

The disadvantage of the classical scale of the Keetch–Byram drought index is its positive values, although the real soil moisture deficit may be negative, which is a sign of soil waterlogging above the field capacity up until to full saturation. Therefore, it is possible to introduce another additional fire danger level—"very low". In addition, in order to comply with the FWI scale, it is proposed to separate the very high fire danger class from the extreme level fire danger class. Thus, the modified fire danger scale according to the Keetch–Byram index values takes the form (Table S1).

Table S1. Fire danger levels according to the Keetch–Byram index.

Index values (mm)	Fire danger levels	Class
<0	Very low	1
0–50	Low	2
50–100	Moderate	3
100–150	High	4
150–175	Very high	5
>175	Extreme	6

The integrated aggregate fire danger indicator is measured on a 100-point scale unified for its partial criteria. Therefore, to normalize the surface soil moisture deficit, with the greater the number the higher fire danger level, use the formula [S12]:

$$\widetilde{m} = \frac{m - m_{min}}{m_{max} - m_{min}} \cdot 100, \quad \text{(S15)}$$

where $\widetilde{m}$ is the normalized surface soil moisture deficit value (reduced to a dimensionless scale from 0 to 100);
m is the moisture deficit, calculated according to (S8);
m_{min} is the minimum surface soil moisture deficit value, which corresponds to the moisture deficit at the highest soil saturation degree ($SSM = 100\%$) and is determined according to (S8): $m_{min} = 10h\rho_w(\theta_{FC} - 100 \cdot n)$;
m_{max} is the maximum surface soil moisture deficit value, which corresponds to the moisture deficit at the lowest soil saturation degree ($SSM = 0\%$) and is determined according to (S8): $m_{max} = 10h\rho_w\theta_{FC}$.

To determine the fire danger levels threshold values according to the normalized soil moisture deficit, we use the following considerations.

Under the optimal conditions for the plant root system soil saturation conditions, when the surface soil moisture deficit $m = 0$ (achieved at $SSM \cdot n = \theta_{FC}$, when soil moisture corresponds field capacity), according to (S8), the moisture deficit $m_{FC} = 0$.

Normalized soil moisture deficit at soil moisture, corresponding to field capacity, is defined as

$$\widetilde{m}_{FC} = 100 - \frac{\theta_{FC}}{n}. \quad \text{(S16)}$$

According to (S14), the limits for the threshold segmentation of the regulated moisture deficit in the surface layer of soil depth from 0 to 5 cm are determined on the basis of satellite data for the six levels of fire danger (Table S2):

Table S2. Fire danger levels according to the normalized surface soil moisture deficit values ranges.

Surface soil moisture deficit values ranges, mm	Normalized surface soil moisture deficit values ranges, scores	Fire danger levels	Class
$m < 0$	$\widetilde{m} < \widetilde{m}_{FC}$	Very low	1
$0 \le m \le m_{50}$	$\widetilde{m}_{FC} \le \widetilde{m} \le \widetilde{m}_{50}$	Low	2
$m_{50} \le m \le m_{100}$	$\widetilde{m}_{50} < \widetilde{m} \le \widetilde{m}_{100}$	Moderate	3
$m_{100} \le m \le m_{150}$	$\widetilde{m}_{100} < \widetilde{m} \le \widetilde{m}_{150}$	High	4
$m_{150} \le m \le m_{175}$	$\widetilde{m}_{150} < \widetilde{m} \le \widetilde{m}_{175}$	Very high	5
$m > m_{175}$	$\widetilde{m} > \widetilde{m}_{175}$	Extreme	6

The numerical subscript at the surface soil moisture deficit m is the Keetch–Byram index threshold values m_{KBDI} relative to which moisture deficit thresholds are calculated in the surface soil layer.

Determination of an improved fire danger index based on the FWI index, taking into account soil moisture deficit

An improved fire danger index FWI_{impr} as a generalized criterion is formed by means of a linear convolution of partial fire danger criteria $\widetilde{FWI}$ and $\widetilde{m}$, normalized to a single scale [0, 100]:

$$FWI_{impr} = k_1 \cdot \widetilde{FWI} + k_2 \cdot \widetilde{m}, \quad \text{(S17)}$$

where the weight coefficients are k_1 and k_2 and satisfy the conditions of

$$k_j \ge 0,\ \sum_{j=1}^{2} k_j = 1\ (j = 1, 2). \quad \text{(S18)}$$

Different methods can be used to determine the coefficients k_1 and k_2, both using machine learning and without it [S12, S13]. If no learning is provided, the weight coefficients can be determined, in particular, directly from the values of the partial criteria in the set of test sites: $\widetilde{FWI}_i,\ \widetilde{m}_i\ i = \overline{1,n}$.

To find the coefficients k1 and k2, one can apply the principal components method [S12, S13] and use the so-called first modified component of the set of unified partial criteria $\widetilde{FWI}_i,\ \widetilde{m}_i\ i = \overline{1,n}$. This modified first component is obtained as a corresponding linear regression across multiple unified partial criteria values in the test sites. According to the results of statistical calculations, the modified first component is determined by relation (S17) where the weight coefficients $k_j\ (j = 1, 2)$ are the squares of the corresponding components of the eigenvector of the covariance matrix of variables $\widetilde{FWI}$ and $\widetilde{m}$ that meet to the largest eigenvalue of this matrix.

The obtained improved fire danger index FWI_{impr} is a numeric rating of the fire intensity from 0 to 100 and is measured on a scale of six levels. Low values FWI_{impr} indicate weather conditions with a low fire danger, while higher values FWI_{impr} indicate weather conditions with an increased fire danger.

References

S1. Kostyakov, A.N. Melioration fundamentals. Selkhozgiz: Moskow, Russia, 1960; 622p. (In Russian)

S2. Vlasova, O.V.; Samoilenko, L.I.; Pidhorodets'ka, L.V.; Kolos, L.M. Corn moisture deficit according to earth remote sensing data. *Land Reclam. Water Manag.* **2009**, *96*, 58–64. (In Ukrainian)

S3. Campbell, G.S.; Campbell, C.S. Water Content and Potential, Measurement. In *Encyclopedia of Soils in the Environment*; Hillel, D., Ed.; Elsevier: Amsterdam, The Netherlands, 2005; pp. 253–257. http://doi.org/10.1016/B0-12-348530-4/00502-6.

S4. Hengl, T.; de Jesus, J.M.; Heuvelink, G.B.M.; Gonzalez, M.R.; Kilibarda, M.; Blagotić, A.; Shangguan, W.; Wright, M.N.; Geng, X.; Bauer-Marschallinger, B.; et al. SoilGrids250m: Global Gridded Soil Information Based on Machine Learning. *PLoS ONE* **2017**, *21*, e0169748. http://doi.org/10.1371/journal.pone.0169748.

S5. Zadorozhny, Y.V.; Smirnova, I.V. Methodical Recommendations for the Implementation of Practical Exercises. In *Agricultural Melioration*. Mykolaiv National Agrarian University: Mykolaiv, Ukraine, 2018; 149p. Available online: http://dspace.mnau.edu.ua/jspui/bitstream/123456789/4355/ 1/Silskohospodarska%20melioratsiia.pdf (accessed on 5 December 2022). (In Ukrainian)

S6. Volkov, V.N.; Narimanyants, E.V. Composition, Physical and Chemical Properties of Soils: Educational Guide. 2007. 48p. Available online: http://window.edu.ru/resource/887/68887/files/rsu664.pdf (accessed on 5 December 2022). (In Russian)

S7. Keetch, J.J.; Byram, G.M. *A Drought Index for Forest fire Control*; Res. Paper SE-38; U.S. Department of Agriculture Forest Service, Southeastern Forest Experiment Station: Asheville, NC, USA, 1968; 32p.

S8. Keetch–Byram Drought Index. Available online: http://en.wikipedia.org/wiki/Keetch%E2%80%93Byram_drought_index (accessed on 5 December 2022).

S9. Garcia-Pratsa, A.; Del Campo, A.; Tarcнsio Fernandes, J.G.; Antonio Molina, J. Development of a Keetch and Byram-Based drought index sensitive to forest management in Mediterranean conditions. *Agric. For. Meteorol.* **2015**, *205*, 40–50.

S10. Burgan, R.E. A method to initialize the Keetch–Byram drought index. *West. J. Appl. For*. **1993**, *8*, 109–115.

S11. National Fire Danger Rating System. Available online: http://www.fs.usda.gov/detail/cibola/landmanagement/resourcemanagement/?cid=stelprdb5368839 (accessed on 5 December 2022).

S12. Ayvazyan, S.; Mkhitaryan, V. *Applied Statistics and Econometrics: In 2 vol. Vol. 1: Probability Theory and Applied Statistics*; Unity: Moscow, Russia, 2001; 656p. (In Russian)

S13. Ayvazyan, S.A. To methodology of population life quality synthetic categories measurement. *Econ. Math. Methods* **2003**, *39*, 33–53. (In Russian)