



# Article Decision Support Models and Methodologies for Fire Suppression

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Abstract: Wildfires are recurrent natural events that have been increasing in frequency and severity in recent decades. They threaten human lives and damage ecosystems and infrastructure, leading to high recovery costs. To address the issue of wildfires, several activities must be managed and coordinated in order to develop a suitable response that is both effective and affordable. This includes actions taken before (mitigation, prevention, and preparedness), during (response), and after the event (recovery). Considering the available resources and the safety of the involved personnel is a key aspect. This article is a review focused on fire suppression, which comprises actions belonging to the preparedness phase (deployment) and the response phase (dispatching) of the wildfire management scheme. It goes through the models and methodologies that, applying operations research and optimization techniques, address the management of resources to address fire suppression. This article presents a review of the studies published after the last review on the topic in 2017, but also includes some interesting papers before that date. It concludes with some classifying tables and a few conclusions about possible future lines of research.

Keywords: fire suppression; wildfires; decision making; optimization; operations research

## 1. Introduction

Natural disasters are inherent occurrences in the evolutionary process of Earth, which may cause the loss of lives or material damage. They are recurring events that humanity has been faced with throughout history. To suitably respond to these unexpected disturbances and recover from them, several strategies have been developed.

Similar to earthquakes or floods, wildfires are important processes affecting ecosystems. A wildfire can be initiated naturally or derived from human activity. If well managed, it does not necessarily result in a disaster. Nevertheless, some key factors may affect the behavior of wildfires, making them more harmful and unmanageable. *Mega-fires* are produced in situations where the conditions are adverse, and first fire suppression efforts fail. Having multiple ignition points hinders the probability of success of these first efforts [1], but also severe weather conditions such as high temperatures or wind speeds. Although these conditions are seldom met during any particular year [2], some models dealing with data on historical large fire events conclude that situations leading to these incidents will inevitably occur [3]. Nonetheless, size is not the only way of measuring the potential hazard of a wildfire, since small events can also be devastating [4] due to their severity.

Recent studies have shown that wildfires have been increasing in frequency and severity in recent decades. Some reasons for this are related to human activity, for example, arson attacks [5] or misuse of fire in certain areas and seasons prone to fire. According to Nagy et al. [6], humans ignited four times as many large fires as lightning, being the dominant source of large fires in the eastern and western U.S. Moreover, an aggressive wildfire suppression policy may lead to a fuel accumulation, which contributes to more intense wildfires [7].



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**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). In addition to the direct interaction of humans with forests, global warming is also causing an increase in wildfires. A rapid raise in temperature is expected to lead to a further escalation in the number of wildfires in the near future [8]. This may be even more alarming due to the fact that forests are more likely to ignite during their period of regeneration [9], which is increasing since wildfires occur more often.

As wildfires become more frequent and devastating, more personnel and resources are put at disposal to act on them, so the fire suppression costs have risen [10]. Furthermore, the wildland–urban interface (WUI) is rapidly enlarging, since house density is growing and thus the number of threatened houses, so fire suppression costs are expected to continue escalating [11]. However, adding more resources to the system may not be the ultimate solution. Acquiring more resources would entail their under-utilization during the majority of the season, or their use under situations in which they are not completely adequate [12]. In this regard, resource scarcity due to limited budgets can be addressed by improving the efficiency of the existing resources [12] and taking advantage of weather, fuel, and topographic changes that create containment opportunities to enhance the effectiveness of fire suppression activities [13]. However, this is not easy to implement. Managers usually work under stressful conditions and time-pressure environments, pushing them sometimes to over-allocate resources relative to values protected, creating inefficiencies [14].

Given the severe consequences of wildfires on ecosystems and human communities, as well as the difficulty and urgency to find solutions, it is not surprising that wildfire managers are always looking for more robust solutions to help them make decisions in such uncertain situations. There are many open problems related to fire management, each requiring a specific solution that may be well determined using operations research (OR) [15]. However, most of these problems are interrelated and thus need an integrated framework in order to address several aspects simultaneously.

Nonetheless, citing Martell [16]: 'OR must be kept in proper perspective and viewed simply as one of many means of improving management, not as an end in itself [...]. OR specialists must ensure they develop decision-making aids that serve the needs of their clients'. It is therefore clear that the successful implementation of OR techniques can only be achieved by a close collaborative effort between operation researchers and fire specialists.

This paper is a review of fire suppression studies, that describes how OR models and methodologies have been used to provide decision support when dealing with the acquisition and allocation of resources (just before the fire) and how to use them (during the fire) to mitigate the hazard of wildfires.

The structure of this review is as follows: Section 2 describes the problem, focusing on wildfire suppression and presenting previous reviews on the topic. Section 3 describes the methodology followed to develop this study and gives an overview of the included papers. Section 4 is the bulk of the review, where all the recent papers about fire suppression are described, discussed, and classified (Tables in Appendices A–C). Some specific features of these models are discussed in Section 5 associating the reviewed models with these features. To finalize, Section 6 comments briefly on the work developed and describes possible future work on applying OR to wildfire suppression. Section 4 is a thorough description of all the reviewed models. Therefore, if the reader just wants a general overview of the latest papers that apply OR to fire suppression, he/she can directly go to Section 5.

## 2. Fire Suppression

The role of fire suppression is to control—and ultimately extinguish—destructive wildfires found by the detection systems [16]. Decisions on how to extinguish a fire are heavily influenced by how the fire grows and develops, depending on weather conditions, terrain features or fuel type, conditions, and attributes. However, they are also influenced by the available resources, and where those are located. Thus, fire suppression management encompasses decisions not only related to directly acting on the fire once it has ignited, but also to arrange all the available resources prior to the beginning of the wildfire.

Martell [16] divides the fire suppression process into four stages: resource acquisition and strategic deployment, resource mobilization, initial attack (IA) dispatching, and extended attack (EA) management.

The first phase includes all the long-term decisions to make before the fire, which will heavily influence the others. The second, related to resource mobilization, deals with how the acquired resources are distributed between the bases, where the resources will await to be dispatched in an initial attack. This distribution can be performed at the beginning of the fire season, but it may change depending on fire occurrences [17].

Initial attack (IA) is an aggressive way of extinguishing the fire with the first resources to arrive. It is focused on arranging the deployed resources, deciding on the strategies to be used and how to implement them to prevent the fire escaping control. If the initial attack fails, an extended attack is needed. Extended attack (EA) comprises two key stages: containment and control. Containment entails the creation of control lines that are expected to hold the fire spread. Control deals with the completion of a control line around the fire, any spot fires, and any other interior areas to be saved as well as the cooling down of any hotspot that may be a threat to the constructed control line.

This fire suppression scheme comprehends decisions corresponding to the preparedness stage of a disaster (long-term decisions in acquisition and deployment of resources to bases) and others corresponding to the response stage (resource mobilization, initial attack, and extended attack). The inherent interrelation between these stages makes it almost impossible to develop a specific plan for one of them exclusively. In this regard, operations research can provide integrated tools that help decision-makers determine alternatives.

To develop a suitable fire suppression strategy, a significant amount of information is needed. The lack of good quality information could be a major limitation for the development of OR models. Thompson et al. [18] review several articles criticizing the lack of robust information regarding fire response and suppression resource performance; they suggest that the first step to improve performance is real-time monitoring and analysis.

To anticipate and manage the extinguishment of the fire as fast and efficiently as possible, its behavior must be predicted, due to its uncertain nature. In this regard, fire growth simulation models are a good forecasting tool. Some of them are: FlamMap (now includes FARSITE), Phoenix, CFES2, or DEVS-FIRE. They are included in this section for completion, since they provide the necessary information to feed the dispatch models; however, simulation tools are out of the scope of this review—for more information read [19].

More static information, but useful for long-term planning, is provided by risk maps and indices that help identify the best strategy [20,21].

All these indices and simulations need empirical and reliable data to work with, so geographic information systems (GISs) and historical data are often used when available.

Previous reviews on fire suppression can be found in the papers by Duff and Tolhurst [22] and Dunn et al. [23], which mention a large number of articles using OR.

Duff and Tolhurst [22] focus on the preparedness and response stages, including detection, dispatch, and tactical fire suppression. The authors also draw attention toward the fact that wildfire behavior can widely differ depending on weather conditions, terrain features, or fuel attributes and state, so it may also be beneficial to rely on simulation models. Simulation models take into account natural factors, or even the firefighting plan itself, considering its influence on the fire behavior.

Dunn et al. [23] mainly center on the response stage. Nevertheless, they also mention the importance of the pre-incident planning to support future decisions on dispatch. This planning should be dynamically adjusted as the wildfire develops, due to the uncertainty these kinds of events imply. It is interesting to mention that this review contains some classifying tables of decision support models, depending on their objectives and methodologies.

Both reviews finalize by recognizing the importance of OR and how it can help making more robust decisions in uncertain situations; acknowledging that plenty of work is still to be conducted before fire managers can benefit from decision support systems that integrate their experience into a more realistic paradigm. The reviews also suggest that cooperation between agencies is needed, at local, regional, and national levels, to develop joint strategies addressing several aspects of fire suppression in a coordinated way, since fire suppression involves many processes highly influenced by one another.

#### 3. Methods

This work reviews all the recent papers applying OR techniques, specifically mathematical programming and heuristics, to optimize fire suppression strategies. To classify the papers, the fire suppression stages described by Martell [16] are grouped according to the classical stages of a disaster: the actions taken before the fire ignites on the one hand (resource acquisition and mobilization) grouped in the stage of preparedness and the actions taken during the event on the other hand (initial attack and extended attack) grouped in the stage of response. The next section is divided into three subsections addressing operations research methodologies applied to fire suppression in the preparedness stage, the response stage, and integrated schemes comprising both stages. In the Appendices A–C there are summary tables describing the main characteristics of each of the reviewed papers.

This review comprehends a total of thirty-six papers. Only six were published more than 10 years ago, the oldest dating from 2003. Twenty of them were published after the last review by Dunn et al. [23] in 2017. More than half of the reviewed papers are written by people conducting their research in the U.S. There are also some studies from Australia, China, Greece, Portugal, Spain, South Korea, or Colombia. Researchers from Brazil, India, Iran, Mexico, or the UK also collaborated in some of the papers. Most of the papers are published in the following journals: Canadian Journal of Forest Research, Environmental Hazards, Forest Science, Fundamental Research, INFOR, or Int. J. Wildland Fire.

There is a considerable amount of literature regarding how mathematical optimization may be used to improve the decision-making process. Despite this, there exist many other ways of approaching the problem; for instance, logistic regression is used to determine the possible failure of initial attack, identifying the key factors for the success of the fire suppression strategy [3,24], or to determine potential fire control locations in pre-fire planning [25]; random forest ensembles are also used to model wildfire risk [26]. Although these kinds of models are out of the scope of this review, it is worth mentioning that not only OR optimization may prove useful for developing fire-suppression strategies and to support the decision-making process. There are many other ways of tackling the problem, not necessarily conflicting with OR methodologies, but complementing them, in the sense that they may provide information to feed the OR models and improve the robustness of the solutions.

### 4. Operations Research for Fire Suppression

## 4.1. Preparedness: Allocating and Deploying Resources

In the fire suppression paradigm, there is much work to be done before the actual fire starts. Planning is key for a good development of the fire suppression strategy and some decisions should be taken in advance: acquisition of material, hiring of personnel, and allocation of resources to bases, or even the decisions on where to locate these bases.

Suarez et al. [27] address the problem of locating temporary operations centers (TOCs), which will serve as coordination centers for deploying resources. They use a methodology based on a time-expanded graph, which is the main contribution of the work, that allows for modeling the dynamics of the wildfire, or the costs of the routes in a dynamic fashion. The set of nodes of the graph is divided between candidate nodes for facilities or TOCs and demand points; the arcs account for transportation costs and time, depending on the quality of the roads. The model is a two-stage stochastic mixed integer linear programming model (MILP). In the first stage, it minimizes the costs of opening TOCs and placement of inventory resources in them. In the second, the costs of distribution and some penalties related to excess and shortage of inventory are also minimized. Stochasticity is applied in the form of scenarios with associated probabilities in the second stage.

Ríos-Mercado [28] contributes to the literature integrating the calculation of different fire behavior indices with an MILP model that determines the optimal deployment of brigades. The first step is calculating the potential risk of fires, based on GIS information of the area. Then, the areas are classified according to their risks and importance. Last, an MILP model developed by Dimopoulou and Giannikos [29] is run to determine the location of several limited resources to maximize the weighted coverage of the demand points.

Another approach on how to allocate resources is based on the response times of the available resources and the area they can cover within that time. Sakellariou et al. [30] predict a burning probability for each fire-prone region and propose an MILP model whose objective is maximizing the covered area. The model selects the optimal location of the fire agency stations and prepositions vehicles, each of which can cover a circle of 31 min radius (maximum time response) considering available road network and realistic travel times, based on the speed limits of the roads and the average velocity of the trucks.

In the same vein, Zeferino [31] addresses the allocation of aerial resources for initial and extended attack, maximizing the expected value of the hazard coverage [32]. In this case, aircrafts are allocated based on their response time, which gives a radius of action. The main contribution of this work is that it explicitly considers redundancy in the allocation of the aerial resources, considering the unavailability of some aircrafts due to maintenance tasks or rest periods. Nevertheless, no attention is paid to the actual time the resources may take to reach each point, but only to the radius of action of the aircraft.

Normally, at the beginning of the season, the resources are deployed to their homebases so as to be prepared for the fire season, considering their optimal allocation for minimizing their movement when needed, as studied in [30,31]. However, due to the stochasticity of fire occurrence, some benefits may be drawn from a system in which relocation is allowed and optimized, providing a more dynamic framework.

Chow and Regan [17] present a static standard p-median formulation that allocates aerial resources to a water source, based on a predetermined demand, minimizing deployment time. This model is then extended into the time dimension to obtain a chanceconstrained dynamic relocation model. The dynamic extension of the model takes into account stochasticity on the day-to-day demand due to weather, and considers relocation if beneficial. To avoid complexity, the authors propose the evaluation of the relocation using a rolling horizon of seven days. The authors acknowledge that the dynamic formulation may be less cost-effective, but achieves better results regarding suppression effectiveness. A shortcoming of the model is that in the demand forecast, the burning indices for all nodes are assumed to be independent of each other, whereas in fact there is a close relationship between the burning index and actual fire occurrence in adjacent nodes.

Addressing relocation matters as well, Wei et al. [33] present a simulation–optimization procedure to share crews and engines between dispatch zones. They address issues related to shift length, but also the effect of resource drawdown policies, which is not previously addressed in the literature. Resource drawdowns are the number of resources that should be held in their homebases for initial attack assignment and are unavailable for use outside their local areas. A level of demand is determined using regression models, calculating available resources as the maximum dispatched historically. The MILP model minimizes resource movement distances as a proxy of costs. A limitation of the model is that the surplus of resources is not deemed beneficial, which may be useful for building additional fire lines; moreover, they do not allow for substitutions between crews and engines to cover demand, while covering a demand with a different resource than requested may be more beneficial than not sending any resources whatsoever.

#### 4.2. Response: Dispatching Resources

Once a fire has started, fire managers need to respond to it, deciding which resources to dispatch, where, and when. The papers in this section mainly address how to optimally dispatch the available resources to contain and control the fire.

From a theoretical point of view, the fire-fighter problem (FFP) has drawn attention from several researchers since proposed in a 1995 conference by Hartnell, B. [34]. It is an NP-complete [35] deterministic discrete-time model for the spread and containment of fire. Although many methods have been applied to solve it [36–40], they mostly analyze the mathematical aspects of the problem and do not deal with real cases.

There exist more complex and realistic methodologies, including completion times, or modeling of the fire spread more accurately. Some of them use existing simulators to predict fire behavior, or to combine it with the fire suppression process, to create an integrated strategy. This is important as fire suppression actions severely affect the behavior of the wildfire, changing its final shape and perimeter [41]. Many strategies for wildfire suppression optimization have been tested using OR methodologies, since different methods and approaches may better characterize some aspects over others. In this section, resource dispatch models have been classified based on these different approaches.

#### 4.2.1. Fire-Line Based

In models of this kind, the containment condition of the fire is that the built fireline is greater than the fire perimeter. They are normally fed with information regarding fire spread rate and the rate at which the resources can build a fireline, in order to contain the fire. The objective is to minimize the sum of all costs and damages, using the Cost plus net value (C+NVC) methodology. This methodology accounts for the pressuppression costs (related to wildfire management prior to a fire season), suppression costs (expenditures related to the direct fire management during fire season), and NVC (net wildfire damages).

Some authors have used the C+NVC methodology in a theoretical framework for wildfire management, but Donovan and Rideout [42] developed an MILP model in which the needed resources are optimized to achieve the minimum value of C+NVC. It is based on a knapsack problem, including a temporal dimension for dispatching several resources to contain a fire—it does not work with multiple fires. The fire perimeter is precomputed using Farsite, and resources have to be dispatched in given time periods to build a fireline faster than the perimeter growth, which is assumed to be completed at the end of the optimization horizon. It assumes that a contained fire will be extinguished—an escaped fire would give rise to infeasibilities in the model. A limitation of this model is that it only determines the mix of resources needed, but does not provide details about the strategy.

Also using the C+NVC function, Hu and Ntaimo [43] develop a stochastic extension of the model in [42], that does not work with escaped fires either. This comprises an integrated simulation–optimization framework that combines fire simulation, resources optimization, and fire suppression simulation in a feedback loop, which may include expert knowledge calibrations between iterations. First, a set of fires is simulated using DEVS-FIRE to determine the fire perimeter. Then, a two-stage MILP model is developed, using several scenarios. In its first stage, pre-suppression plus expected suppression costs and NVC of the burned area are minimized. In the second stage, the suppression costs and a penalty for the uncovered perimeter for each scenario are minimized. The MILP model determines a series of resources that will be dispatched to contain the fire (by having a fireline construction rate faster than the perimeter growth). This fire-suppression strategy is then tuned with a simulation model, in which different attack techniques are coded. This approach may be interesting because it provides a few different strategies to choose from. Nevertheless, this iterative approach may not be operational due to the short decision times.

Also using the C+NVC methodology, Rodríguez-Veiga et al. [44] developed an MILP model that selects the resources needed for forest fire suppression. The formulation addresses maximum flight times and the required rest breaks for air resources and maximum daily operation time for brigades. A fire simulator estimates the growth of the fire perimeter, with no update; with this information, the model aims to dispatch resources that can build a fireline faster than the perimeter growth. If fire containment is not achieved in the optimization horizon, infeasibilities may arise, so a second and simpler model is built to focus on the maximization of the resource performance, only considering time constraints

but not the evolution of the fire. The main contribution of this model is considering several resources with different fireline production rates, combining air and ground resources.

However, as Hu and Ntaimo [43] acknowledge, these methodologies are simplistic, since they do not account for the interaction between fire spread and suppression, and thus tend to overestimate the resources that are needed.

## 4.2.2. Fire Points-Based

A more general approach is considering that there are several fire events, so the resources must be dispatched to a number of locations to cover a set of demand points. Usually, a group of teams have to visit each of the points to address the demand. The fire is considered to be contained when all the fire points have been visited and provided with the necessary resources, or enough time has been spent on them. This problem is based on the vehicle routing problem (VRP), as some authors acknowledge.

Yang et al. [45] built a two-layer emergency logistic system with a single depot and multiple demand sites. The Wangzhengfei fire simulation determines the fire propagation, and then each fire site is prioritized based on its emergency level. In the second layer, a vehicle routing problem (VRP) is solved where the vehicles, starting from their depots, may serve several sites along their routes. Two ways of solving the problem are proposed, depending on the fire spread velocity. For fast propagation, the focus is on extinguishing the fire as soon as possible, which is achieved when the rate of increase in the burned area is null, whereas for slow propagation, an immune clonal algorithm is used to minimize travel times and costs, determining the necessary resources in each fire-point based on fire spread velocity. The main contribution of the study is that, between the mentioned models in this section, it is the only one that considers a bound on the time for the resources to arrive to a node. However, it does not consider the completion times of the tasks in each node.

Wu et al. [46], also using the Wangzhengfei fire simulation scheme, determined fire spread, wherein speed is included in the MIP model to determine completion times at each point in a dynamic fashion. It considers a problem similar to a VRP in which the temporal scope is important. The objective is to find an optimal schedule for dispatching the firefighting teams suitably to extinguish several prioritized fire points depending on the severity of the fire in each of them, including constraints that force the first M points with higher priority levels to be visited first. The problem is considered on an undirected graph, minimizing the total distance traveled by all firefighting teams and assuming that the available resources are sufficient to extinguish the fire.

Wang et al. [47] take the model from Wu et al. [46] and transform it into a multiobjective model, minimizing travel distance as well as also total rescue time as the main objective. The main contribution of this paper is the calculation of the Pareto solution, which may be useful for providing different alternatives for the fire manager to choose from. In this approach, fuzzy logic and the  $\epsilon$ -constraint method are used. However, as the authors acknowledge, the problem is difficult to rapidly solve by commercial software such as CPLEX.

Bodaghi et al. [48] proposed a methodology that determines the sequence of demand points to be visited by the chosen vehicles, minimizing the weighted sum of the completion times of the operations. The model itself is deterministic in nature, but the methodology includes a loop that varies the input parameters in a stochastic fashion using Monte Carlo simulations to create different scenarios. It can be used in any disaster relief operation requiring the transportation of resources. Specifically, the authors test the methodology using real data from a bushfire in Australia. The main contribution is that it integrates sequencing and scheduling of resources, considering uncertainty. It also benefits from GIS information on fast and safe travel routes. Moreover, the completion times are stochastic, based on stochastic time processing and demands at each point.

Shahidi et al. [49] modeled a more complex situation, with a novel approach in which aerial and ground resources are coordinated in order to cover the demand of several points. This demand is modeled as the necessary time spent by ground resources or the amount of water in liters discharged by the aerial resources. Moreover, despite the fact that in previous models each node was attended to by only one vehicle or resource, in this case several ground resources can be combined to cover the demand faster. The main contribution of this approach is thus the coordination of aerial and ground resource operations which makes the model more realistic, proposing a novel VRP that accounts for the refill of the aerial resources. The authors solve the test cases with a new proposed greedy algorithm, since they found CPLEX incapable of solving the problem in real-world scales.

It is interesting to mention that some models [46,48,49] start with a non-linear formulation, and then they are linearized using the Big-M method. Although making the model linear is an advantage, it may be difficult to determine a suitable value of the big M.

This way of modeling and optimizing the fire suppression operations is more realistic in the sense that it considers various resources and several fire points to be attended, instead of considering only a big fire. However, none of the mentioned approaches take into account the interaction between fire growth and suppression; ignoring the fact that fire suppression affects fire behavior oversimplifies the dynamic of large fire management [41] and may lead to a possible overestimation of resources.

Instead of visiting some demand points, Shahparvari et al. [50] determine the scheduling of several tasks that should be completed in order to contain the fire. Each of them is assigned with a certain number of resources as a demand to be covered. The model is bi-objective, minimizing firstly the total time taken to complete all activities and secondly the shortages in resources. The novelty of this study resides in their proposed time-based decomposition approach (greedy), called *Coordination algorithm*, that outperforms both a genetic algorithm and the exact solution approach, as well as the consideration of precedence constraints for operations and time windows.

#### 4.2.3. Grid-Based

In these approaches, the space is discretized into a grid, in which each cell may have different characteristics, accounting for heterogeneity between them. This is of special importance to easily integrate GIS data, which provide realistic information about the landscape. The remarkable feature of these models is that they allow for the interaction between fire spread and suppression, leading to more realistic strategies, and avoiding the overestimation of resources. In this case, the fire behavior is not modeled by a simulator, but integrated in the very optimization model, in a way that the simulation of fire spread and the optimization of the resources are performed simultaneously, affecting one another. The suppression strategy relies on the placement of controls. Controlled cells are cells on which a treatment has been performed to stop or delay the fire spread.

An example of these models is developed by Wei et al. [41]. Fire behavior parameters are calculated using FlamMap and fed into a base MILP model, that tries to stop the fire spread as early as possible, considering the minimum travel time (MTT) of fire. Previous models using the MTT methodology within an MILP model failed to correctly determine the fire arrival time for those cells not in the binding burning path [51]. To overcome this issue, two iterative approaches are developed, running a time correct model. Discretizing the time in short periods makes it possible to limit the number of resources available in each of them. The objective is to minimize the sum of fire loss across all burned cells, assuming that controlling a cell interrupts the fire spread to adjacent cells. It also includes the firefighter's safety concerns regarding fire intensity thresholds. The model is deterministic with respect to weather conditions, fire spread, and availability of suppression resources. The major contribution is that this methodology accounts for the interaction between fire growth and suppression, using the corrected MTT methodology. However, the model is still simplistic because it does not consider completion times, nor the movement of the brigades, and it only limits the number of controls that can be allocated in each period, but permits simultaneity of the controls as long as they do not exceed the number allowed.

Alvelos [52] transformed an optimization problem into a feasibility problem. This includes all the necessary constraints to correctly determine fire arrival times for any objective function, instead of using an iterative scheme such as the one by Wei et al. [41].

The approach is based on the MTT methodology solving the shortest path problem by the Dijkstra algorithm and taking into consideration how fire suppression actions hinder the progress of fire. The author tested several objectives. This approach does not consider completion nor traveling times between controls either, it only defines time instants where the resources become available, to avoid having unlimited resources.

As Alvelos [52] recognizes, obtaining good quality solutions for large real instances is a major challenge for the future. Mendes and Alvelos [53] take the model and solve it using a heuristic iterated local search. One of the major benefits of using this approach is the improvement of the solving times compared with the exact model (CPLEX) used by Alvelos [52], which increases with the grid size.

Belval et al. [54] propose a similar integrated methodology to correctly determine the fire arrival time to each cell. A new feature of this model is that fire intensity is calculated in a spatially dynamic fashion, tracking the binding paths of the fire, instead of taking this information from fire simulators as most of the models do. Based on the intensity, the concept of beneficial fires is introduced, exploring fire management objectives different from just containment. In one of the case studies, the authors examine how fire behavior can be altered rather than just suppressed, using a multiobjective approach. The information on fire spread rates to determine fire arrival times and intensities is deterministic and taken from FlamMap. A major issue remains unaddressed since the resources are assumed to be unlimited, and the timing of the controls is not determined correctly, not addressing the problem of simultaneous controls nor considering completion times before fire arrival.

In order to limit the resources, Belval et al. [55] present a multistage model based on [54]. Introducing stochastic weather trees, resources have to be dispatched attending to non-anticipativity constraints, which allow for a better interaction between fire spread and fire suppression. Stochasticity affects fire spread rates, whereas including stochastic weather trees is a major contribution of the model, as it entails large running times, or even the inability of solving the problem if beneficial fires are considered (the exact algorithm cannot close the gap). This does not solve the problem of simultaneity of controls either, since it only restricts their number within each period.

Although the mentioned grid-based models include some constraints limiting the resources and their availability times, they do it in a simplistic way. Belval et al. [54] only limit the controls in cells not reachable in the response time by forbidding the placement of controls in certain precalculated cells, but assume that resources are unlimited. Wei et al. [41] and Belval et al. [55] limit the number of controls to be placed in each stage to reflect limited resources, but do not include specific time constraints related to fire arrival time or to avoid simultaneity of controls within each period. Something similar occurs in the models by Alvelos [52] and Mendes and Alvelos [53], where resources are made available in certain time instants, avoiding simultaneity, but not considering fire arrival times related to the timing of the controls. Moreover, these models do not impose continuity on the suppression operations and the placing of controls.

Belval and Wei [56] are the first authors that fully address this problem. Given a grid with an ignition point, the model simulates the movement of the fire using the MTT methodology and determines the cells/nodes in which suppression is needed (controls). A brigade, responsible of placing the controls, travels between adjacent nodes, spending time in traveling and also some extra time if a control in a cell is needed. The fire intensity is modeled in a spatially dynamic fashion as in [54], and the time it takes to control a cell depends on it. The added value of this paper is the strategy timing, as tracking the fire arrival and brigade arrival times allows for imposing some feasibility and safety constraints: a control cannot be placed in a cell in a certain moment if it is already burned or if the fire is too close for it to be safe. Moreover, it avoids the simultaneity of controls. However, this level of detail in the model implies that the running times are unaffordable, taking days to solve some cases and running out of memory for others. Moreover, it considers only one brigade and crossing a cell more than once is forbidden; these assumptions are not realistic

since normally several teams are coordinated and the traveling paths are usually roads that can be used many times, as long as it is safe.

A different approach, albeit interesting to mention, is the one by Homchaudhuri et al. [57], who developed a simulation–optimization scheme. Fire spread, based on the Huygens principle, is simulated stochastically in heterogeneous terrain, implementing a wind–slope correction. Given predefined curves, the optimization module determines the starting points and parameters of the curves that the brigades follow to close the perimeter. It considers constant fireline production rate and that the starting point of one crew is the finishing point of another one. Then, fire suppression is simulated to determine the total area burned using the Monte Carlo method, assuming the worst scenario, and discarding solutions deemed unacceptable/infeasible. Fire propagation and suppression affect one another in an iterative way. The objective is to minimize the area enclosed by the curve if the fire is surrounded completely. This value is infinity if the fire escapes the enclosed area. A major limitation reported by the authors is that this is a completely data-driven method, whereas firefighting operations have a strong heuristic component based on expert knowledge, so a method which combines both would be more convenient.

In any case, the mentioned methodologies address detailed space information and entail an improvement over the ones in previous sections. Aside from considering the interaction between fire spread and suppression, these methodologies determine the final shape of the firelines to be constructed, providing a more extensive and realistic suppression plan.

### 4.2.4. POD-Based

Another way of discretizing the space is using potential wildland fire operation delineations (PODs), which are the representation of areas that summarize risks and identify fire management opportunities [58]. This representation provides a tight relationship between the real landscape and the modeled grid, which uses terrain features such as rivers or roads as POD boundaries, grouping in each POD a piece of landscape with similar characteristics. This approach bridges the gap between OR techniques and decision makers, since it makes use of predefined PODs that are normally determined by managers.

This way of modeling the space is used by Wei et al. [59]. They developed an MILP model to aggregate these structures into a response POD (rPOD) for containing large fires—a patch between PODs is created using adjacency relationships, where containment lines are established along the boundaries of the rPOD. Stochasticity is included by weather scenarios, in terms of wind speed and direction as well as fuel moisture. However, they do not consider spread probabilities nor the timing of line construction in relation to fire arrival times. Safety constraints are included to avoid fire suppression in places with flame length over a threshold, which is calculated using FlamMap. This approach, like in [54], also considers beneficial fires, represented by positive conditional net value change (cNVC). Point protection is represented by avoided loss in terms of cNVC. The highlight of this model is that it considers both fire confinement and point protection in a joint manner, leveraging a more accurate space discretization such as PODs. However, it does not impose a limitation on the resources, and models the fire in a very simplistic way using adjacency rules, but not calculating arrival times. Thus, fireline construction times cannot be determined based on fire arrival.

Wei et al. [60] built on [59] to improve the development of rPODs considering fire spread probabilities and spread rates. This allows for determining the order in which the PODs are adhered to the rPOD within a set of periods, estimating fire arrival time to the boundary as the earliest. The time it takes to build a fireline is dependent on flame length. A second model to determine the timing of the suppression strategy is built, which takes as input the rPOD boundaries identified from the previous model and the set of points selected for point protection. This novel second optimization encourages the completion of containment line prior to fire arrival, avoiding firefighters being surrounded or endangered by the fire. The model improves on [59], because it limits the number of crew hours to be

used in each period; however, a major shortcoming is that it does not address fireline or point protection simultaneity within each period.

A limitation of the mentioned models is that a constructed fireline is assumed to hold once built; there is scarce study of system redundancy in fire suppression. Belval and Wei [56] account for fireline quality construction in terms of the needed time for constructing the line for it to hold depending on fire intensity; however, it only forces this time to be enough, but does not consider line breaching.

In this regard, Wei et al. [61] extended the models from [59,60] to evaluate the effectiveness of contingency strategies under randomly generated scenarios through an MILP model. The goal is to study how redundant firelines may reduce uncertainties from stochastic fireline breaching. The minimum travel time algorithm (MTT) is used to dynamically track the fire arrival time to the centroid of each POD. The methodology tests four types of fireline construction to delineate rPODs, evaluating trade-offs between fire loss and suppression effort, finding that no contingency strategy could outperform the others in all random scenarios. Thus, fire managers may select different containment strategies based on their risk preference, resource costs, resource availability, or firefighter safety. Nevertheless, despite PODs being a good way of dividing the space based on terrain features, sometimes they are too large for tracking fire spread using the MTT algorithm.

#### 4.2.5. Other Models

Being faced with uncertainty is one of the challenges that managers face when making dispatch decisions for fire suppression. The models previously mentioned deal with this uncertainty via forecasts, simulations, and stochastic programming. Another way of addressing the uncertainty of fire behavior may be by direct observation. Chan et al. [62] propose an innovative approach for the resource allocation problem during fire suppression. In order to cope with the uncertainty, the authors developed a strategy in three phases called "Firefly". First, a set coverage problem is identified, which maximizes the area explored by a number of deployed drones. Solved by a greedy algorithm, the solution provides information of how the fire is developing and about the utility of the surveilled cells. In some cases, the first phase is not able to develop a plan to watch over all the cells given the available drones, so a second phase estimates the utility of the cells not assessed. Third, a knapsack problem is solved to maximize the utility of the chosen areas where the brigades are going to be dispatched to, modeling the space as a graph.

To this point, we have mentioned a number of papers in which the OR model tries to develop a holistic-containing plan; however, in some other situations, managers only need decision support in very specific tasks. Rodríguez-Veiga et al. [63] propose two linear integer programming models to solve two different decision problems related to the allocation of aerial resources, wherein flying routes should be optimized and monitored to avoid and reduce the risk of collision. The first model is designed to maximize the output per hour of aerial resources flight time, and the second manages the allocation of aerial resources to refueling bases. The first one should be run each time a new aerial resource enters or abandons the extinguishing protocol. It uses stochasticity due to the uncertainty in the efficiency of aerial resources during a wildfire. The second model is executed after the coordinator determines when and where each aircraft would run out of fuel. This one is deterministic due to the nature of the parameters involved.

Despite all the efforts to construct firelines capable of holding the fire, sometimes they are not sufficient and the fire finally escapes, endangering lives, assets, and infrastructure. Although it is out of the scope of this paper, it is important to mention that OR is also useful in optimizing the operations related with asset protection [64,65] and evacuation [66].

### 4.3. Preparedness and Response: Combined Approaches

The previous sections have presented models that deal with the deployment and the dispatch of resources as two independent problems. These two stages of fire suppression are very interrelated, and the daily deployment budget is sensitive to changes in suppression

assumptions [67]; thus, some authors have proposed integrated models. They are generally devoted to initial attack, and although they use fire predictions and consider dispatch, their main concern is to determine the optimal deployment of resources prior to the occurrence of a wildfire.

Haight and Fried [68] present a two-stage stochastic MILP model for the deployment of resources called standard response model (SRM). Scenarios are created using CFES2, which represent the daily number, location, and intensity of the fires. For each fire the standard response required is calculated as the "desired number of resources that can reach the fire within a specified response time". This will measure whether a fire can be contained and how much effort is required to do it, or whether the fire will escape. Two objectives are minimized in the objective function: the number of suppression resources deployed and the expected daily number of fires not receiving a standard response. The authors also create a heuristic approach based on CFES2 to compare the results with the MILP model. The strength of the scenario-based SRM is its tractability and integration of expert knowledge through the definition of standard response. However, the model does not estimate the number of escaped wildfires, nor models fire containment. Moreover, exogenously generated dispatch rules assume a perfect knowledge of the resources needed to define the standard response, which is a very specific assumption that does not hold in reality.

Based on [68], Yohan et al. [69] developed a two-stage model for deployment and dispatch that minimizes the expected number of fires not receiving a predefined response. This response is also defined as the required number of resources that can reach the fire within a maximum time. They apply logic from the scenario optimization (scenarios are created via the fire simulation model from Byungdoo et al. [70]) and the maximal covering location framework. The first stage addresses the assignment of helicopters at the beginning of the fire season and the second determines their daily dispatch, assuming the rest of the resources are located at their bases. The major contribution of the paper is the utilization of GIS information to account for heterogeneities between the different areas to protect, considering priorities given the fire intensity.

Based on [68], Ntaimo et al. [71] improved the model considering that resources can be moved between their bases before a fire occurs. It also considers multiple types of fire-fighting resources with different production rates. The first stage minimizes fixed costs from renting and relocation. The second stage, based on the methodology by Donovan and Rideout [42], minimizes the C+NVC of the burned area for each scenario, considering fires not receiving standard response. In this case, a fire is said to receive a standard response if the sum of all production rates is greater than a certain production rate. The output of the model is the number of contained fires and the expected number of escaped fires. Using rule-based dispatching poses a major improvement over [68], since it relaxes the assumption of the manager's perfect knowledge of the resources needed. The set of scenarios is developed using BehavePlus as a fire simulator, which determines the standard response required by each fire. However, if too many scenarios are used the computing capacity is insufficient, so the authors propose a sampling method in order to solve it.

Another approach is followed by Gallego Arrubla et al. [72], developing a one-stage MILP model. The model includes stochasticity for resource pre-allocation, deployment, and dispatch of dozers. Combining a fire behavior simulator and a wildfire risk model (Texas Wildfire Risk Assessment system) with a probabilistically constrained stochastic MILP, they account for the risk-aversion of the fire manager, integrating expert knowledge. Following the line of Donovan and Rideout [42], they also use the C+NVC to compute the cost associated with fire suppression, determining the number of contained fires and the wildfire risk associated with fires not receiving a standard response, along with the cost derived from damages and losses produced by the fire. Standard response is determined, as in [71], as the minimum standard production rate to be achieved for containment. A limitation of the model is that it only includes one type of resources with constant production rates, instead of combining the production rates from different types of firefighting resources.

A new contribution to the literature on initial attack planning can be found in a paper by Ntaimo et al. [73], who do not consider the standard response required by each fire, but develop an explicit fire growth response model (EFGRM) which accounts for the fire behavior. Combining simulation and the two-stage SRM by Ntaimo et al. [71], the first instant in which fire is contained is determined. In this case, BehavePlus is used, for developing the fire scenarios and to calculate the fire perimeter each half an hour in a period of six hours. This allows for developing a more specific action plan on how to contain the fire and determining how much of the fire perimeter remains unattended. However, it is assumed that no explicit interaction between the fire perimeter and fireline construction exists. The results of the study demonstrate that the response time restriction imposed by the fire manager planning unit has a direct impact on the number of fires that can be contained.

Not considering costs directly, Sakellariou et al. [74] developed a methodology with two modules, aimed at covering the maximum population served within the predefined time frame. The first module is directed at strategic planning. Two scenarios are considered, based on an ideal (10 min) and real (31 min) time response, to maximize the coverage and minimize the number of supply points, selected from candidate facility locations. Once the optimal locations have been determined, the second module assesses the response capabilities of moving vehicles via Dijkstra's algorithm, to find the best routes from the supply to the demand points. A novelty of this paper is that it performs a second computation to determine an alternative route in case natural or artificial barriers arise. As well as for Suarez et al. [27], arrival times are calculated considering the road network quality. The authors propose this second module to be used in real-time operations. The added value of this model lies in the fact that it deals with WUI fires considering operational and strategic efficiency in an integrated framework.

Another objective is explored by Zhou and Erdogan in [1], minimizing the people at risk who need to be evacuated, minimizing also the total expected cost of hiring additional on-duty resources. To address the two objectives, goal programming is used. However, it may pose disadvantages since the assumptions made on goals and the priorities must be made by the decision-maker and are difficult to determine. The MILP model has two stages and addresses fires in the WUI interface as Sakellariou et al. [74]. The main improvement, compared to the previous deployment and dispatch models, is that the fire behavior is simulated within the model, similarly to as in [41,52] or [54–56], accounting for the interaction between the fire and the suppression strategy. It is an integrated model that support decisions in resource acquisition and allocation before the fire starts, and decisions during the fire event regarding resource deployment and dispatch. Due to the growth in model size as the grid enlarges, the authors explore a novel approach to keep the number of variables constant by increasing the size of the grid for the scenarios.

Another approach can be found in Wei et al. [67]'s work, with a simulation–optimization methodology that also models the interaction between fire behavior and suppression. It is a two-stage stochastic model: suppression resources have to first be acquired and then deployed and dispatched for the season, ensuring they are sufficient for suppressing a series of scenarios. A chance-constrained approach is used, creating a deterministic equivalent formulation such that most fires have to be controlled via initial attack. The goal, once the resources have been acquired in the first stage, is trying to put the fires out as soon as possible, considering their specific fire behavior. The perimeter growth is calculated with FARSITE, and suppression is performed by having a fireline construction rate higher than perimeter growth. A limitation of this model is that it does not consider changes in staff levels, or relocation; moreover, resources are assumed to attend one fire per day and then return to their bases.

This model is further improved by Wei et al. [75] by including a post-optimization procedure to assess the solution and refine it to determine final solutions. In addition, it includes endogenously designed dispatch rules into resources acquisition and deployment decisions, which is the main contribution of this paper. However, it still assumes that the

manager could anticipate the fire locations and their features before creating the dispatch plan for each day. Moreover, a major issue remains unresolved and is that resources are limited to be dispatched to only one fire per day as in [67]. Tracking the first hour a fire is contained could allow for releasing the resources engaged in that fire, and redeploying them to other fires in the same fire day.

All these models have proved useful in integrating into a single model two of the fire suppression stages described in [16]. However, as not only two, but all the stages are interrelated, skipping just one of them may result in suboptimal solutions, so further studies to test the utility of multistage systems may be justified [75].

#### 5. Summary and Discussion

The ultimate goal of all the reviewed models is to provide suitable support in decisionmaking for addressing the negative impacts of wildfires. Given, however, that wildfires occur in very different landscapes and jurisdictions with very different interests, the optimization objectives may differ importantly from one another.

Regarding fire suppression goals, Sakellariou et al. in [30] maximize the area covered by the suppression resources available, whereas Sakellariou et al. in [74] and Zeferino in [31] maximize the expected value of hazard coverage. Extending the maximal covering location model, several authors work with the concept of standard response as a term in the objective function, minimizing the number of fires not receiving such a response [68,69,71,72].

Another approach is to minimize the expected area burned: in one of the solution approaches, Alvelos [52] minimizes the area burned and its associated costs. Homchaudhuri et al. [57] minimize the total area burned until the fire is completely suppressed, so do Belval and Wei in [56] plus the least distance traveled, whereas Belval et al. in [55] only minimizes the area impacted by fire and suppression and Belval et al. in [54] minimize the value of the area burned, also accounting for suppression costs. Since the amount of area burned is closely related to the time the suppression operation lasts, several authors use it as a proxy. Shahidi et al. [49] minimize suppression time in terms of the sum of arrival times of aerial and ground teams to the fire points, Bodaghi et al. [48] and Shahparvari et al. [50] minimize the weighted sum of completion times for all demand points and activities, respectively, the latter also minimizing shortages in resources. Wang et al. [47] and Rodríguez-Veiga et al. [63] minimize fire extinguishing rescue time as well as total transport distances. Alvelos [52], in one of the solution approaches, minimizes the earliest instant for containment and the associated costs.

A methodology specifically designed to address costs is the C+NVC methodology used by Donovan and Rideout in [42], that accounts for presuppression and suppression costs, as well as a net value change representing net wildfire damages. Several other authors have leveraged on this methodology such as Hu and Ntaimo [43], Rodríguez-Veiga et al. [44], or Gallego Arrubla et al. [72] and Ntaimo et al. in [71,73], which include fires not receiving a standard response in the net value change to account for fire damages. Wei et al. in [59–61] precompute the C+NVC of each POD maximizing total cNVC, since they consider potential benefits from fire. In [59,60], they also consider point protection. And in [60,61], they also include a term for minimizing the weighted sum of crew hours.

Some other authors do not base their decisions on C+NVC methodology, yet minimize the costs related to fire suppression too, minimizing prefire costs and deployment costs [1,27,45,67]. Suarez et al. [27] also minimizes penalties by shortages and excess of resources. Wei et al. in [33] only minimize daily transport costs. Zhou and Erdogan [1] propose to include a term regarding property loss and another representing the amount of people at risk, to reduce future costs due to the necessity of evacuation operations.

Others cope with the minimization of costs indirectly, minimizing distances [46,63], travel times [17,45–47], or total operational time [47,63].

It may be noted that several of the aforementioned models have multiple objectives; most of them use a weighted sum in the same objective function, mixing very different objectives such as total travel time, costs, area burned, net value change, crew hours, etc. Some of the authors address the minimization of costs while minimizing total travel time [45] or, as Alvelos [52] does in two of its approaches, combining the minimization of cost with the minimization of the earliest instant for containment or the total area burned. Minimizing also the latter, Belval and Wei in [56] include a term for minimizing the distance traveled, and Belval et al. in [54,55] minimize the number of controls. Another approach of dealing with the area burned and losses is consider the total net value change, which is calculated within a POD in [61] which also accounts for the total crew hours. So does Wei et al. [60], considering too the net value change due to successful point protection, as in [59], which do not consider crew hours.

Others deal with the minimization of fires not receiving a standard response while minimizing the number of resources deployed at stations [68], or the total costs of operating helicopters [69]. Ntaimo et al. [71,73] consider the cost of fire not receiving a standard response within the net value change using the C+NVC methodology.

Rodríguez-Veiga et al. [63] maximize water download while minimizing distances between air resources and fronts.

A suppression operation is a process including several and very disparate objectives to consider; thus, it is difficult for the decision-maker to provide suitable weights for such different goals. In this regard, considering all of them in the same objective function may not be the best methodology.

Only two of the revised papers leverage multi-criteria approaches to address the combination of several objectives, different from weighting them in the same objective function. Wang et al. [47] combine extinguishing time and total transport distance using fuzzy logic through the  $\epsilon$ -constraint method to determine a Pareto solution in order to provide several alternatives for the decision-maker. Zhou and Erdogan [1] apply goal programming to study the trade-offs between total expected number of people at-risk and the expected total cost verifying the Pareto efficiency of the solutions; however, as the authors acknowledge, goal programming requires the decision-maker to establish the goals to each objective and these assumptions are not usually easy to make either.

Another key aspect of the models in Section 4.3 but especially in Section 4.2, is how they deal with the fire—the way a fire is said to be contained.

A basic approach is to set demand requirements: Chan et al. [62] cover the number of resources that must be allocated in order to suppress a fire on a certain site. Most of the models in Section 4.2.2 set a level of demand to be covered in terms of time spent [46,47], amount of resources to be allocated to each task [50], or a combination of both [48,49].

Haight and Fried [68] and Yohan et al. [69] use the concept of standard response, defined as the "desired number of resources that can reach the fire within a specified response time". Ntaimo et al. [71] and Gallego Arrubla et al. [72] also use this concept but by comparing fire spread rate with the fireline rate of construction needed for suppression. However, these models assume that dispatchers have a perfect knowledge of the amount of resources needed to contain the fire, which in general is not certain [75]. To overcome this issue, Ntaimo et al. [73] developed a more dynamic approach based on an explicit fire growth response model, determining the percentage of the unattended perimeter.

Models dealing directly with fireline construction in Section 4.2.1 assume the fire to be controlled when total line production of the firefighting resources exceeds the total fire perimeter [42–44]. This assumption can be also found in [67,75]. A similar methodology can be found in [45], where the fire in each fire point is supposed to be contained when the increment in burned area is null. Nevertheless, these models are still limited in the sense that they can determine when or if the fire is contained, but not address the actual strategy of how to build the firelines. Models that address fireline construction rates in a more detailed way can be found in [57], where the fire is contained when it is completely surrounded by the quadratic functions representing the fireline built by each of the teams or in [59,60] where the boundaries of the rPOD are determined as the firelines needed to contain the fire. In the study of Wei et al. [61], as they consider fireline breaching, the fire is controlled when there are no firelines left to breach along rPOD boundaries. These

models, in addition to determining when the fire is contained based on estimated fireline production rates, also define the final shape of the fire and the necessary firelines.

Some other models that also provide fireline shapes as an outcome can be found in Section 4.2.3 [41,54–56], where the space is discretized in cells and controls are placed in strategic locations to stop or delay the spread of fire. These models, in general, assume that the perimeter of the study area is non-flammable, so the fire is said to be contained when all the cells are labeled as either burned, saved, or controlled; Zhou and Erdogan [1] also follow a similar approach. Alvelos [52] and Mendes and Alvelos [53], in one of their approaches, go a step further, and determine that all the perimeter of the fire needs to have controls, not letting the fire reach the landscape's boundary.

Methodologies using the standard response concept consider escaped fires as those not receiving their specific standard response and try to minimize their number [68,69,71,73]. Wei et al. in [67,75] limit the conditional probability of a fire day with escaped fires.

A more strict approach regarding escaped wildfires can be found in [57], which ensures fire does not escape and in [55], which does not constrain the number of suppression nodes in the final stage to ensure the test cases contained the modeled fire. Some other authors consider escaped wildfires out of the scope of their study [42], which would provide infeasible solutions for the model. This problem is addressed in [44]; the authors first develop a model in which resources are assumed to be enough to contain the fire, but as some infeasibilities may arise, they build a second model in which the suppression efforts are maximized so as to minimize the escaped wildfires. Methodologies that also assume that enough resources are available to contain the fire can be found in [46] or [60,61].

To develop a more suitable response to the fire, several authors include fire growth simulators in their methodologies to predict fire behavior: CFES2 [68], Behave/Behave-Plus [28,71–73], Burn-P3 [30], DEVS-FIRE [43], FARSITE [42,67,75], and FSPro [60]. The Wangzengfei model is used in [45–47].

On the other hand, some other authors make use of fire spread concepts to incorporate fire movement into their own models, normally to account for its interaction with fire suppression, which is a very interesting feature of some of the reviewed models. In the grid-based models [1,41,52–56] and in [1], the minimum travel time (MTT) of fire methodology is used. Based on fire spread rates, the spread time between adjacent cells is calculated, simulating the movement of the fire through the shortest paths, which is hindered by the placement of controls. Also in [61], the arrival time of the fire to each POD is determined using the MTT algorithm, considering the delay fireline constructions entail. A less integrated approach can be found in [57] or [67,75], which instead of addressing fire suppression and spread at the same time, use a simulation–optimization scheme.

The fire simulators are normally used for predicting fire movement and its associated parameters. However, they may also be applied to calculate fire intensity to model the fire more accurately, such as FlamMap [55,56] and Fsim [61]. FlamMap is also used in [54] to establish an upper bound for the intensity of fires to consider them beneficial. Some authors use the fire intensity information provided by simulators to determine non-safe situations for firemen ([59] FlamMap, [60] Flep-Gen).

These safety requirements are usually demanded by fire managers and often overlooked by OR researchers. Nevertheless, some efforts have been made to consider manager choices with regard to the risk level he/she is willing to take [72] or to avoid engagement in certain locations under perilous conditions regarding flame length thresholds [59,60]. The manager can also determine the number of resources of each type that must be allocated to a fire [44] and the standard response needed in terms of resources' demand [68,69] or in terms of needed line production rate [71,73]. Their expertise can also be included considering only those fronts that are selected by the coordinator for attack [63] or by giving different weights and priorities to the various areas to protect [54] and goals to achieve [1,68]. Including expert knowledge can allow for bridging the gap between OR theoretical models and the actual application of the developed strategies. Another safety requirement is related to the timing of the suppression operations. This is a major concern to be addressed, for providing suitable strategies to be applied in real cases. To the best of our knowledge, only two models include constraints related to the continuity of operations and the detailed timing thereof. Belval and Wei [56] model the continuous movement of the brigades, and determine where and when to locate controls, ensuring the firefighting resources are able to escape before the fire arrives, while spending enough time building them. Wei et al. [60] developed a first optimization in which firelines along PODs are built before the fire arrives; once the rPOD and its boundaries are determined, a second optimization phase maximizes the gap between fire arrival time and completion times of the firelines.

Some other models that address the timing of controls/fireline construction but with less stress on firefighters' safety are found in [41,55,61] which locate controls/firelines within periods to avoid locating them in places already burned and limiting the amount of allowed controls/firelines per period, but do not forbid simultaneity within periods. Bodaghi et al. [48] and Shahidi et al. [49] determine the visiting order of the nodes and time spent in each of them to minimize completion times but do not consider fire arrival. Homchaudhuri et al. [57] eliminate solutions in which firelines to be built will lie in places which will burn before the fireline can be placed. The authors of [43] simulate several alternatives for the timing of the fireline construction. Yang et al. [45] determine the visiting order of the fire points based on their priority, considering a time limit, while Alvelos [52] and Mendes and Alvelos [53] limit the instants in which the resources become available.

As observed in Section 4.2.4, most of the models assume that constructed controls and firelines will hold the fire; however, this situation may not be true in those cases with high-intensity fires or adverse weather conditions. Grid-based models consider that placing a control in a cell may delay fire spread; if the delay is set to a high value, it reflects that the control will hold, while low values represent situations in which fire will end up spreading into the cell [1,41,52–55]. Belval and Wei [56] go a step further, and guarantee that the control in a cell will completely stop fire spread through it; given the fire intensity, it is ensured that enough time is spent on the control for it to hold. However, fireline breaching is a stochastic process depending on many factors and [61] is, to the best of our knowledge, the first paper that accounts for this fact. The authors developed a logistic regression to estimate the holding probability of a fireline, using historical data. Based on this breaching possibility, they study several alternative containment strategies.

All the efforts are thus focused on providing realistic models to support decisionmaking for fire suppression, considering all the characteristics aforementioned and including GIS information or stochasticity. However, such amounts of data may lead to very complex models. Regarding grid-based models, the complexity increases with grid size, so difficulties when solving the models with commercial solvers may arise [1,52,54–56]. Alvelos acknowledges in [52] that obtaining good solutions for large instances is a challenge, which is addressed in [53] by using a heuristic iterated local search. To address the growth of the model, Zhou and Erdogan in [1] propose to increase the size of the grid to keep the number of variables constant. Furthermore, some other authors found difficulties regarding running times [47,49,73]. Shahidi et al. in [49] developed a greedy algorithm to overcome this issue. Ntaimo et al. in [73] had problems when a large number of scenarios were involved, so the authors proposed a sampling method.

Decision-making for fire suppression is a very complicated task, to be performed in high-pressure environments within tight timelines. This is why several authors have proposed as future work the implementation of heuristics to speed up the process of obtaining solutions [46,48,54–56,60], and some of them have come to the real implementation of a heuristic to tackle the problem [41,45,49,50,53,57,68]).

## 6. Conclusions

In this review, a number of studies directly addressing the optimization of fire suppression strategies and operations have been presented, wherein some methods and simulation tools are available to supply the necessary information required by the fire suppression models. This review is mainly focused on recent years (after the reviews by Duff and Tolhurst [22] and Dunn et al. [23]), but some previous works have also been considered, leading to 36 publications being reviewed, described, and classified.

All the described models are based on optimization techniques from mathematical programming, and although the theoretical study of the fire-fighter problem (FFP) has given rise to interesting discussion, the focus of this review is on the papers discussing procedures that support decision-making in real situations.

This literature review intends to be a compendium of the most recent techniques for wildfire suppression optimization. Showing the methods and results also applied to real cases, it tries to highlight the interest of researchers for contributing to the real implementation of OR methodologies as decision support tools. However, several aspects remain to be addressed properly.

A number of fire growth models have been mentioned for predicting fire behavior, albeit there is little research about forecasting actual fire occurrence. Furthermore, the presented simulators and the models based on the minimum travel time only consider the linear propagation of fire, whereas, in real situations, spotting may occur several kilometers away, resulting in multiple fire ignition points.

Moreover, some of the models are not able to solve real instances because they rely on commercial solvers that do not provide solutions for large instances. In this regard, a future line of research would be the implementation of heuristics, as some authors suggest, to speed up the process. Another key point for the models to be operational should be the improvement of their availability for free use. None of the reviewed papers provide source code. Moreover, some of them do not describe their methodology nor the constraints of the model in detail, so they are not reproducible. Then again, to run these models, landscape, resources availability, and meteorological data are necessary, so updated GIS information or satellite images are needed, which entails close collaboration with the emergency services.

Since the main objective of fire suppression research is ultimately to help real decisionmaking, future efforts may focus on faster, more detailed, and more interpretable models. Close cooperation between decision-makers and modelers is needed. Decision-makers require tools that are more understandable, and models can benefit from the integration of expert knowledge and available data to provide more accurate and useful solutions.

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# Abbreviations

The following abbreviations are used in this manuscript:

C+NVC	Cost plus net value change
EA	Extended attack
EFGRM	Explicit fire growth response model
FFP	Fire-fighter problem
GISs	Geographic information systems
IA	Initial attack
MIP	Mixed integer programming
MILP	Mixed integer linear programming
MTT	Minimum travel time
OR	Operations research
POD	Potential wildland fire operation delineations
SDI	Suppression difficulty index
SRM	Standard response model
TOC	Temporary operations centers
VRP	Vehicle routing problem
WUI	Wildland-urban interface

# Appendix A. Preparedness: Allocating and Deploying Resources

Table A1. List of main characteristics of papers that address resources' preparedness.

Reference	Deterministic/Stochastic	Expert Knowledge	Resources	<b>Objective Function</b>	Multiple Fires	GISs
Suarez et al. (2016) [27]	Stochastic	No	Several, generic	Minimize costs, excess, and shortage of inventory	No	No
Ríos-Mercado (2020) [28]	Deterministic	No	Several, generic	Weighted maximization of number of demand points covered	Yes (demand points)	Yes
Sakellariou et al. (2020) [30]	Stochastic	Yes	Trucks	Weighted maximization of number of demand points covered	-	Yes
Zeferino (2020) [31]	Stochastic	No	Aircrafts	Maximization of expected value for the hazard coverage through IA and EA	-	Yes
Chow and Regan (2011) [17]	Stochastic	No	Aircrafts	Minimize distances of the closest servers in terms of deployment time	No	No
Wei et al. (2016) [33]	Stochastic	No	Crews and engines	Minimize total transport cost for each day	No	Yes

# **Appendix B. Response: Dispatching Resources**

Table A2. List of main characteristics of papers that address the dispatch of resources for response. Fireline-based.

Reference	Deterministic/ Stochastic	Expert Knowledge	Resources	<b>Objective Function</b>	Interaction of fire with Suppression Strategies	Fire Simulator	GISs	Solution Method
Donovan and Rideout (2003) [42]	Deterministic	No	Several, not specified	Minimize all costs and damages (C+NVC)	No	FARSITE	No	Exact (LINGO)
Hu and Ntaimo (2009) [43]	Stochastic	Yes	Crew, dozer, tractor plow	Minimize all costs and damages (C+NVC)	No	DEVS-FIRE	Yes	Exact (CPLEX)
Rodríguez-Veiga et al. (2018) [44]	Deterministic	Yes	Aircrafts, land brigades, engines	Minimize all costs and damages (C+NVC)	No	Not specified	Yes	Exact (Gurobi)

	lable A	A3. List of main c	naracteristics of papers i	that address the dispatch of resou	rces for response. Fire-	points based.		
Reference	Deterministic/ Stochastic	Expert Knowledge	Resources	<b>Objective Function</b>	Interaction of fire with Suppression Strategies	Fire Simulator	GISs	Solution Method
Yang et al. (2019) [45]	Deterministic	No	Each vehicle carries several resources	Minimize total travel time of vehicles and their costs	No	Wangzhenfei model	Yes	Heuristic. Inmunal clone algorithm (for VRP)
Wu et al. (2019) [46]	Deterministic	No	Fire-fighting teams	Minimize total travel distance of firefighting teams	No	Wangzhenfei model	No	Exact (CPLEX)
Wang et al. (2020) [47]	Deterministic	No	Fire-fighting teams	Minimize fire extinguishing rescue time and total transport distance of fire-fighting teams	No	Wangzhenfei model	No	Exact (CPLEX)
Shahidi et al. (2022) [49]	Deterministic	Yes	Aerial and ground resources	Minimize total suppression time	No	-	No	Heuristic. Greedy
Bodaghi et al. (2020) [48]	Stochastic	No	Different resource types in different vehicles	Minimize weighted sum of completion times over all demand points	No	-	Yes	Exact (CPLEX and frequency approach)
Shahparvari et al. (2021) [50]	Deterministic	No	Aircrafts and ground crews	Minimize total time taken to complete all activities and then shortages in resources	No	-	No	Heuristic. Greedy time-based decomposition

Table A3. List of main characteristics of papers that address the dispatch of resources for response. Fire-points based.

Reference	Deterministic/ Stochastic	Expert Knowledge	Resources	Objective Function	Interaction of Fire with Suppression Strategies	Fire Simulator	GISs	Solution Method
Wei et al. (2011) [41]	Deterministic	No	Not specified	Minimize sum of fire loss across all burned cells	Yes. Integrated with MTT	MTT. FlamMap	No	Iterative approach and MILP (solver not specified)
Alvelos (2018) [52]	Deterministic	No	Not specified	Several criteria	Yes. Integrated with MTT	MTT	No	Exact (CPLEX)
Mendes and Alvelos (2022) [53]	Deterministic	No	Not specified	Minimize total number of burned nodes plus a weighted number of assigned resources	Yes. Integrated with MTT	MTT	No	Heuristic. Iterated local search
Belval et al. (2015) [54]	Deterministic	Yes	Not specified	Minimize the value of the area burned and the number of controls (costs)	Yes. Integrated with MTT	MTT. FlamMap	No	Exact (CPLEX)
Belval et al. (2016) [55]	Stochastic	No	Not specified	Minimize cells affected by burning and controls (no weights)	Yes. Integrated with MTT	MTT. FlamMap	No	Exact (CPLEX)
Belval and Wei (2019) [56]	Stochastic	No	Brigades (crews)	Minimizing expected area burned with least distance travelled	Yes. Integrated with MTT	MTT. FlamMap	No	Exact (CPLEX)
Homchaudhuri et al. (2013) [57]	Deterministic/ Stochastic	No	Brigades (crews)	Minimization of PI index	Yes. Simulation- optimization	Own based on Huygens principle	No	Heuristic. Genetic algorithm

<b>Table A4.</b> List of main characteristics of papers that address the dispatch of resources for response. Grid-based.
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	Table	A5. List of main (	characteristics of pa	ipers that address the dispatch of i	resources for response	e. POD-based.		
Reference	Deterministic/ Stochastic	Expert Knowledge	Resources	Objective Function	Interaction of Fire with Suppression Strategies	Fire Simulator	GISs	Solution Method
Wei et al. (2018) [59]	Deterministic	Yes	Not specified	Maximize total cNVC within POD along with reductions in loss of successful point protection	No	FlamMap	Yes	Exact (solver not specified)
Wei et al. (2019) [60]	Deterministic	Yes	Not specified	Maximize total cNVC within POD with reductions due to point protection and minimizing crew hours	No	FSPro/Flep-Gen	Yes	Exact (solver not specified)
Wei et al. (2021) [61]	Stochastic	Yes	Hand crew	Maximizes total cNVC within a POD, includes also weighted total crew hours	Yes. Integrated with MTT	MTT. Fsim	Yes	Exact (solver not specified)

Table A5. List of main characteristics of papers that address the dispatch of resources for response. POD-based.

Table A6. List of main characteristics of papers that address the dispatch of resources for response. Other models.

Reference	Deterministic/ Stochastic	Expert Knowledge	Resources	<b>Objective Function</b>	Interaction of Fire with Suppression Strategies	Fire Simulator	GISs	Solution Method
Chan et al. (2020) [62]	Deterministic	No	Drones and brigades	Maximize utility of allocated resources	Yes. By observation	Observation	No	-
Rodríguez-Veiga et al. (2018) [63]	Deterministic	Yes	Aircrafts, example with helicopters	First model: Maximizing water download while minimizing distances between air resources and fronts. Second model: Operational time	No	Not specfied	No	Exact (Gurobi)

# **Appendix C. Preparedness and Response: Combined Approaches**

Reference	Deterministic/ Stochastic	Expert Knowledge	Resources	Objective Function	Interaction of Fire with Suppression Strategies	Fire Simulator	GISs	Solution Method
Haight and Fried (2007) [68]	Stochastic	Yes	Engines	Minimize weighted sum of resources deployed and the expected number of fires not receiving standard response	No	CFES2	No	Exact (CPLEX)
Yohan et al. (2014) [69]	Stochastic	Yes	Helicopters	Total annual operational costs and weighted total expected number of fires not receiving standard response	No	Korean fire simulation model	Yes	Exact (CPLEX)
Ntaimo et al. (2012) [71]	Stochastic	Yes	Dozers	Minimize total fixed costs and fire damages (C+NVC)	No	Behave/BehavePlus	Yes	Exact (CPLEX)
Gallego Arrubla et al. (2014) [72]	Stochastic	Yes	Dozers	Minimize total fixed costs and fire damages (C+NVC)	No	Behave/BehavePlus	Yes	Exact (CPLEX)
Ntaimo et al. (2013) [73]	Stochastic	Yes	Dozers	Minimize total fixed costs and fire damages (C+NVC)	No	Behave/BehavePlus	Yes	Exact (CPLEX)

Table A7. List of main characteristics of papers that address preparedness and response.

Table A8. List of main characteristics of papers that address preparedness and response. (Continues table from previous page)

Reference	Deterministic/ Stochastic	Expert Knowledge	Resources	<b>Objective Function</b>	Interaction of Fire with Suppression Strategies	Fire Simulator	GISs	Solution Method
Sakellariou et al. (2020) [74]	Deterministic	No	Trucks	Maximum covering of population served within the predefined time frame	No	-	Yes	Exact (Dijkstra)
Zhou and Erdogan (2019) [1]	Stochastic	Yes	Multiple, not specified	Minimize total expected number of people at risk and the expected total operational and fire cost	Yes. Integrated	Not specified	Yes	Exact (Gurobi)
Wei et al. (2015) [67]	Stochastic	No	Hand crew, engines and water tenders	Minimize total budget for stationing all resources	Yes. FARSITE Simulation–optimization		No	MILP combined with iterative approach
Wei et al. (2015) [75]	Stochastic	No	Hand crew, engines, and water tenders	Minimize total budget for stationing all resources	Yes. Simulation–optimization	FARSITE	No	MILP combined with iterative approach

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