drones

Article

Consensus Control of Large-Scale UAV Swarm Based on Multi-Layer Graph

Taiqi Wang [1], Shuaihe Zhao [1,2], Yuanqing Xia [1,*], Zhenhua Pan [1] and Hanwen Tian [1]

1 School of Automation, Beijing Institute of Technology, Beijing 100081, China
2 Aerospace Shenzhou Aerial Vehicle Ltd., Tianjin 300300, China
* Correspondence: xia_yuanqing@bit.edu.cn

Abstract: An efficient control of large-scale unmanned aerial vehicle (UAV) swarm to establish a complex formation is one of the most challenging tasks. This paper investigates a novel multi-layer topology network and consensus control approach for a large-scale UAV swarm moving under a stable configuration. The proposed topology can make the swarm remain robust in spite of the large number of UAVs. Then a potential function-based controller is developed to control the UAVs in realizing autonomous configuration swarming under the consideration of mutual collision, and the stability of the controller from the individual UAV to the entire swarm system is analyzed by a Lyapunov approach. Afterwards, a yaw angle adjustment approach for the UAVs to reach consensus is developed for the multi-layer swarm, then the direction state of each UAV converges with a fast rate. Finally, simulations are performed on the large-scale UAV swarm system to demonstrate the effectiveness of the proposed scheme.

Keywords: multi-layer graph; potential function; consensus control; UAV swarm

Citation: Wang, T.; Zhao, S.; Xia, Y.; Pan, Z.; Tian, H. Consensus Control of Large-Scale UAV Swarm Based on Multi-Layer Graph. *Drones* **2022**, *6*, 402. https://doi.org/10.3390/drones6120402

Academic Editors: Zhihong Liu, Shihao Yan, Yirui Cong and Kehao Wang

Received: 25 October 2022
Accepted: 2 December 2022
Published: 7 December 2022

1. Introduction

Over the past few decades, the investigation of large-scale swarm has received extensive attention in different fields, such as biology, physics, medicine, sociology, engineering, et al. [1–3]. Swarm refers to a super large-scale isomorphic individual, based on group dynamics and information perception, supported by efficient and safe collaborative interaction between individuals, with the emergence of swarm intelligence as the core, and based on a comprehensive integration of open architecture. It is a complex system with the advantages of invulnerability, adaptive dynamic configuration, functional distribution, and intelligent features. The Boid model is the first model established by Reynold to simulate group behavior [4], and three heuristic rules are introduced in this model, namely separation, cohesion and alignment. On the basis of these three rules, many scholars have conducted in-depth investigation on the swarming movement [5,6]. For example, Olfati-Saber R [7] proposes a theoretical framework of distributed swarm algorithms, and swarm in free space and multiple obstacles avoidance are also considered. Inspired by the above work, Su H et al. [8] revisits the problem of multiagent system in the absence of the above two assumptions.

Combining robot technology with swarm algorithms is one of the hotspots [9,10]. In particular, Enrica Soria et al. [11] published in the journal Nature combines the local principles of potential field methods into an objective function and optimizes those principles with knowledge of the agent's dynamics and environment, resulting in improving drone swarm speed, order and safety. In [12], a multi-layer grouping coordination methodology is proposed to achieve different shape configuration for a large-scale agents. In [13], a new topology approach based on multilevel construction is adopted to present swarm robots of different shapes in the desired region. A novel multi-layer graph is presented by [14] for multi-agent systems to enable scalability of the interaction networks, and the model predictive control method is applied for tracking trajectories; In [15], a multi-layer formation control scheme and a layered distributed finite-time estimator is designed for agents, which impels them to reach the desired positions and velocities according to the the information of agents in their prior layers. In practice, many issues need to be considered in order to implement formation control approaches successfully, such as the avoidance of the obstacles and collisions.

The artificial potential field (APF) models provides an effective solution for practical applications, which attracts the agent to the target and repulses it for avoidance, and can be executed quickly and provides a viable solution [16]. In [17], a rotating potential field is introduced, which makes the UAVs can escape from the oscillations and ensures that the follower-leader maintains the desired angles and distances. Based on the APF approach in [18], a novel automatic vehicles motion planning and tracking framework is presented, and the effectiveness is validated in real experiment. In [19], an adaptive synchronized tracking control based on the neural network is applied to boat by combining with APF and robust $H\infty$ methods, and the artificial potential method is used to guarantee the boat maintaining desired distance with obstacles. In [20], different forms of potential field functions are used for repulsion, velocity alignment and interaction with walls and obstacles, and the proposed model is validated on a self-organized swarm of 30 drones.

Consensus control of multiagent systems is also a hotspot now, which means all the agents in the system converge to the same state by the specific control law. In [21], a distributed active anti-disturbance cooperative control method with a finite-time disturbance observer is proposed to achieve the consensus in finite time for the agents. In [22], the consensus control problem is investigated under an event-triggered mean-square consensus control law for a class of discrete time-varying stochastic multi-agent system. There are three approaches proposed by [23] for consensus control of the multi-agent systems on directed graphs, and some correlative examples are presented to validate the effectiveness. In [24], the synergistic trajectory tracking problem of UAVs formation is investigated, both the position tracking to the desired position and the attitude tracking to the command attitude signal are achieved with the stability analysis and simulations validation.

The main challenges that impede the solving of the configuration and consensus problem for the swarm are the large-sclae of the community and the chronological order of configuration and consensus. Therefore, we have carried out the following research to solve these problems. In order to improve the scalability of the network topology under the large size of the swarm situation, based on the concept of [14], a multi-layer network graph model is proposed for the large-scale UAV swarm, which allows the configuration to be more adjustable and robust. After the configuration of the swarm is completed, to make each UAV in the swarm reach an agreement, a multi-layer recursive consensus control concept is designed for the UAV swarm, so that the yaw angles of UAVs in each layer tend to be consistent.

The remainder of the paper is organized as follows. Section 2 describes some preliminaries and formulates the problem to be investigated in this paper. In Section 3, the multi-layer UAVs swarm configuration control strategy and the consensus concept are proposed. The effectiveness of the proposed methodologies is illustrated by numerical analysis in Section 4. Finally, the results of our work are briefly summarized in Section 5.

2. Preliminaries and Problem Statements

2.1. Graph Theory

In this subsection, some introductions of the graph theory are listed. Firstly, we define undirected graph $\mathcal{G} = (\nu, \varepsilon)$ as the interaction topology which consists of $\nu = (1, 2, \ldots, n)$ a list of vertices , whose elements represent individual UAV in the swarm, and $\varepsilon \subseteq ((i,j) : i,j \in \nu, i \neq j;)$ a list of edges, containing unordered pairs of vertices. An edge $(i,j) \in \varepsilon$ of the undirected graph $\mathcal{G}$ means that the UAV i and UAV j can exchange information. For the undirected graph $\mathcal{G}$, the adjacency matrix is given by $A = [a_{ij}] \in R^{N \times N}$ with $a_{ij} = 0 \Leftrightarrow (i, j) \in \varepsilon$, $a_{ij} = a_{ji}$. The neighboring set of agent is denoted in [25]:

$$N_i = \{j \in \nu : a_{ij} \neq 0\} = \{j \in \nu : (i, j) \in \varepsilon\} \tag{1}$$

2.2. Problem Descriptions

The consistency problem aims at designing appropriate protocols such that the group of UAVs can reach consensus, exploiting only local information exchange among neighbors and unreliable information exchange and dynamically changing interaction topologies. In this paper, our target is to regulate the entire swarm (from each agent to multilayer) move at a same velocity (the same magnitude and direction)and maintain constant distances between the same agent layers. Based on the vicsek model, our hypothesis implies that each agent in the same layer adjusts its velocity by adding to it a weighted average of the differences of its velocity with the others. Then the potential function is necessary to proposed for maintaining a constant distance of each UAV and each layer such that their potentials become minima. In the next section, we will describe the control strategy for our multi-layer grouping swarm specifically.

For brevity, two assumptions is given as follows

Assumption 1. *We assume the Large-scale UAVs swarm consisting n UAVs with the same dynamic characteristics flying in a same altitude space. Therefore, the working environment of each UAV can be consider a two-dimensional space.*

Assumption 2. *In the case of controlling large-scale swarm, we assume each UAV as a point mass, which means the influence of the size and shape of each UAV can be ignored.*

3. Multi-Layer Consensus Control Architecture

3.1. Multi-Layer Graph Model

The proposed multi-layer UAVs swarm model is a multipartite network, which is composed of a series of similar layer structures. In each layer, a certain number of subgroups form a higher layer network by the corresponding control law. Note that each layer is strictly follows the same network characteristics, such as position distribution, potential function, velocity consistency, and so on. Based on the above rules, a hierarchical network structure is constructed.

When the multi-layer structure is considered, the first layer characterized by the position-based interaction forms a primary formation configuration. We assume that the whole swarm includes n UAVs, l layers, and there can only be N_o neighbors from the independent UAVs to each subgroups. We assume that n is divisible by $N_o + 1$. Therefore, the undirect graph of the first layer can be defined as $\mathcal{G}_1 = (\nu_1, \varepsilon_1)$, where $\nu_1 = (1, 2, \ldots, n/(N_o + 1))$ and $\varepsilon_1 \subseteq ((i, j) : i, j \in \nu_1, i \neq j;)$; The second layer undirected graph consists of the first layer, which is defined as $\mathcal{G}_2 = (\nu_2, \varepsilon_2)$, where $\nu_2 = \left(1, 2, \ldots, n/(N_o + 1)^2\right)$ and second list of edges $\varepsilon_2 \subseteq ((i, j) : i, j \in \nu_2, i \neq j;)$; Based on the above rules, we denote $\mathcal{G}_m = (\nu_m, \varepsilon_m)$, $m = 1, 2, \ldots l$ as the interaction network topology to characterize the underlying information flow among the UAVs in the mth layer, where $\nu_m = \left(1, 2, \ldots, n/(N_o + 1)^m\right)$ and $\varepsilon_m \subseteq ((i, j) : i, j \in \nu_m, i \neq j;)$. Then the neighboring set from the first layer to the mth layer can be denoted as $N_i^1, N_i^2, ..., N_i^m$.

3.2. Swarm Configuration

Based on the above conclusions, in order to realize the multi-layer grouping configuration of the whole swarm, firstly we define the dynamic of each UAV as follows:

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = u_i \end{cases} \quad i = 1, 2, ..., n \tag{2}$$

From single agent to multi-agent system, the dynamic protocol of the UAV swarm in each layer is described as follows:

$$\begin{cases} first\ layer \begin{cases} \dot{x}_i^1 = v_i^1 \\ \dot{v}_i^1 = u_i^1 = f_{sum}^1 - k_1 \dot{x}_i^1 \end{cases} & i \in \nu_1 \\ second\ layer \begin{cases} \dot{x}_i^2 = v_i^2 \\ \dot{v}_i^2 = u_i^2 = f_{sum}^2 - k_1 \dot{x}_i^2 \end{cases} & i \in \nu_2 \\ \vdots \\ mth\ layer \begin{cases} \dot{x}_i^m = v_i^m \\ \dot{v}_i^m = u_i^m = f_{sum}^m - k_1 \dot{x}_i^m \end{cases} & i \in \nu_m \end{cases} \tag{3}$$

where $x_i^1, v_i^1 \in R^n$ are respectively the position and velocity of each UAV in the subgroup of first layer, $u_i^1 \in R^n$ is the control input acting on it, f_{sum}^1 is the resultant force contains obstacle avoidance force and collision avoidance force between UAVs. For the second layer, $x_i^2, v_i^2 \in R^{k_2}$ and $u_i^2 \in R^{k_2}$ are respectively the position, velocity and the control input of the UAV in the subgroup of second layer, where $k_2 = n/(N_o + 1)$ is the element number of the ν_2; and f_{sum}^2 is the resultant force contains not only mutual forces from each UAV but also has potential field force from other subgroups in the second layer. Silimarly, $x_i^m, v_i^m \in R^{k_m}$ and $u_i^m \in R^{k_m}$ are respectively the position, velocity and the control input of the UAV in the subgroup of mth layer, where $k_m = n/(N_o + 1)^{(m-1)}$ is the element number of the ν_m; and the resultant force f_{sum}^m contains the mutual forces from each UAV in the whole global and the potential field forces from the second layer to the mth layer. Furthermore, k_1 is a positive constant for damping action.

By analyzing the dynamic model of the UAV (2), we design the corresponding control law to make UAVs reach their desired configuration. Two forces will be engendered based on the designed potential functions to drive all the UAVs move into the desired position and avoid mutual collisions.

The mathematical expression of potential function is as follows

$$V_{ij}(d_{ij}) = \begin{cases} -\xi \frac{d_{ij}}{r_0} \ln(\frac{d_{ij}}{r_0}) + \frac{d_{ij}}{r_0} & x_i \in N_i^1 \\ 0 & otherwise \end{cases} \tag{4}$$

where ξ is the positive control coefficient, $d_{ij} = \|x_i - x_j\|$ is the distance between agent i and agent j, r_0 is the desired radius between each UAV.

Differentiating (3) with respect to d_{ij} yields a potential force as

$$f_{ij} = -\nabla V_{ij}(d_{ij}) = \begin{cases} \xi \ln(\frac{d_{ij}}{r_0}) & x_i \in N_i^1 \\ 0 & otherwise \end{cases} \tag{5}$$

In another case, when UAV i and UAV j are not well-defined neighbors, both can be regarded as obstacles to each other. Therefore, another potential function to avoid obstacles is necessary to proposed as follows

$$V_o(d_{io}) = \begin{cases} \eta(r_0 - d_{io}) \frac{x_i - x_j}{d_{io}} & d_{io} < r_0 \\ 0 & d_{io} \geq r_0 \end{cases} \tag{6}$$

where η is the positive control gain, x_o is the position of the obstacle o. $d_{io} = \|x_i - x_o\|$ is the distance between UAV i and the obstacles.

Then we define the set of the obstacles as

$$O_i = \{j \notin N_i | d_{io} < r_0\} \tag{7}$$

The corresponding force obtained from V_o is

$$f_{io} = -\nabla V_o(d_{io}) = \begin{cases} \frac{\eta}{{d_{io}}^2} r_0(x_i - x_o) & d_{io} < r_0 \\ 0 & d_{io} \geq r_0 \end{cases} \tag{8}$$

Based on the above two forces, the resultant force f^1_{sum} for the first layer is expressed as follows

$$f^1_{sum} = \sum_{j \in N_i^1} f_{ij} + \sum_{o \in O_i} f_{io} \tag{9}$$

the control input can be described as follows

$$\begin{aligned} u_i^1 &= \sum_{j \in N_i^1} f_{ij} + \sum_{o \in O_i} f_{io} - k_1 \dot{x}_i^1 \\ &= -\sum_{j \in N_i^1} \nabla V_{ij}(d_{ij}) - \sum_{o \in O_i} \nabla V_o(d_{io}) - k_1 \dot{x}_i^1 \end{aligned} \tag{10}$$

For the second layer, in addition to the mutual force between the individual UAV, the swarm are also affected by the potential field force between the subgroups. We define the potential function of the second layer as follows

$$V_{ij}^2(d_{ij}^2) = \begin{cases} -\xi \frac{d_{ij}^2}{r_0^2} \ln(\frac{d_{ij}^2}{r_0^2}) + \frac{d_{ij}^2}{r_0^2} & x_i^2 \in N_i^2 \\ 0 & otherwise \end{cases} \tag{11}$$

where $d_{ij}^2 = \left\| x_i^2 - x_j^2 \right\|$, r_0^2 is the desired distance of the second layer. Then the corresponding potentional force is expressed as follows

$$f_{ij}^2 = -\nabla V_{ij}^2(d_{ij}^2) \tag{12}$$

At the same time, each UAV in the swarm has gathered within a fixed area, then the force to avoid obstacles disappears. Therefore, resultant force f^2_{sum} are combined as follows

$$f^2_{sum} = \sum_{j \in N_i^1} f_{ij} + \sum_{j \in N_i^2} f_{ij}^2 \tag{13}$$

The control law u_i^2 of the second layer can be describe as follows

$$\begin{aligned} u_i^2 &= \sum_{j \in N_i^1} f_{ij} + \sum_{j \in N_i^2} f_{ij}^2 - k_1 \dot{x}_i^2 \\ &= -\sum_{j \in N_i^1} \nabla V_{ij}(d_{ij}) - \sum_{j \in N_i^1} \nabla V_{ij}^2(d_{ij}^2) - k_1 \dot{x}_i^2 \end{aligned} \tag{14}$$

For the mth layer, we assume it as the last layer of the whole swarm, then each UAV in mth layer is subject to global forces. The potential function is described as follows

$$V_{ij}^m(d_{ij}^m) = \begin{cases} -\xi \frac{d_{ij}^m}{r_0^m} \ln(\frac{d_{ij}^m}{r_0^m}) + \frac{d_{ij}^m}{r_0^m} & x_i^m \in N_i^m \\ 0 & otherwise \end{cases} \tag{15}$$

where $d_{ij}^m = \left\| x_i^m - x_j^m \right\|$, r_0^m is the desired distance of the second layer. Then the corresponding potentional force is expressed as follows

$$f_{ij}^m = -\nabla V_{ij}^m(d_{ij}^m) \tag{16}$$

Different from (13), the resultant force f_{sum}^{m} combines all the forces from the first layer to the mth layer in the global, and the form is as follows

$$f_{sum}^{m}=\sum_{j\in N_i^1} f_{ij}+\sum_{j\in N_i^2} f_{ij}^2+\ldots+\sum_{j\in N_i^{m-1}} f_{ij}^{m-1}+\sum_{j\in N_i^m} f_{ij}^m \tag{17}$$

The global control law u_i^m is list as follows

$$\begin{aligned} u_i^m &= \sum_{j\in N_i^1} f_{ij}+\sum_{j\in N_i^2} f_{ij}^2+\ldots+\sum_{j\in N_i^{m-1}} f_{ij}^{m-1}+\sum_{j\in N_i^m} f_{ij}^m-k_1\dot{x}_i^m \\ &= -\sum_{j\in N_i^1}\nabla V_{ij}(d_{ij})-\sum_{j\in N_i^2}\nabla V_{ij}^2(d_{ij}^2)-\ldots-\sum_{j\in N_i^{m-1}}\nabla V_{ij}^{m-1}(d_{ij}^{m-1}) \\ &\quad -\sum_{j\in N_i^m}\nabla V_{ij}^m(d_{ij}^m)-k_1\dot{x}_i^m \end{aligned} \tag{18}$$

The control law of the entire UAV swarm are completed. Furthermore, the stability of the configuration needs to be analyzed.

Theorem 1. *Consider a subgroup of n UAVs with dynamics (2), under the control law (10), each UAV can stay at a desired position and the forces and velocity converge to zero finally.*

Proof of Theorem 1. Define a Lyapunov function candidate as

$$V_1=\sum_{j\in N_i^1} V_{ij}(d_{ij})+\sum_{o\in O_i} V_o(d_{io})+\frac{1}{2}{\dot{x}_i^1}^T\dot{x}_i^1 \tag{19}$$

From the above conclusion we can get V_1 is non-negative. Differentiating (19) with respect to time and combining with (2), (3) and (10), we have

$$\begin{aligned} \dot{V}_1 &= {\dot{x}_i^1}^T\Big(\sum_{j\in N_i^1}\nabla V_{ij}(d_{ij})+\sum_{o\in O_i}\nabla V_o(d_{io})+\ddot{x}_i^1\Big) \\ &= {\dot{x}_i^1}^T(-f_{sum}^1+u_i^1) \\ &= -k_1{\dot{x}_i^1}^T\dot{x}_i^1 \\ &\le 0 \end{aligned} \tag{20}$$

Thus the energy of each UAV i $(i=1,2,...,n)$ monotonically decreasing. From the analysis we can conclude that the velocity of UAVs eventually converge as the same. □

Theorem 2. *For the entire swarm with n agents, under the global control law (18), all the UAVs can arrive at the desired positions, the potential forces from the first layer to the mth layer and velocity converge to zero finally.*

Proof of Theorem 2. Define the global Lyapunov function candidate as

$$\begin{aligned} V_m &= \sum_{i=1}^{n}\Big(\sum_{j\in N_i^1} V_{ij}(d_{ij})+\sum_{j\in N_i^2} V_{ij}^2(d_{ij}^2)+\ldots+\sum_{j\in N_i^{m-1}} V_{ij}^{m-1}(d_{ij}^{m-1}) \\ &\quad +\sum_{j\in N_i^m} V_{ij}^m(d_{ij}^m)+\tfrac{1}{2}{\dot{x}_i^m}^T\dot{x}_i^m\Big) \end{aligned} \tag{21}$$

From the (5), (11) and (15) we can get V_m is non-negative.

Differentiating (21) with respect to time and combining with (3) and (18), we have

$$
\begin{aligned}
\dot{V}_m = & \sum_{i=1}^{n} \dot{x}_i^{mT} (\sum_{j \in N_i^1} \nabla V_{ij}(d_{ij}) + \sum_{j \in N_i^2} \nabla V_{ij}^2(d_{ij}^2) \\
& + \ldots + \sum_{j \in N_i^{m-1}} \nabla V_{ij}^{m-1}(d_{ij}^{m-1}) + \sum_{j \in N_i^m} \nabla V_{ij}^m(d_{ij}^m) + \ddot{x}_i^m) \\
= & \sum_{i=1}^{n} \dot{x}_i^{mT} (-f_{sum}^m + u_i^m) \\
= & -k_1 \sum_{i=1}^{n} \dot{x}_i^{mT} \dot{x}_i^m \\
\leq & \ 0
\end{aligned}
\tag{22}
$$

Therefore, the total potential energy can approach the minimumwe and $\dot{x}_i^m \to 0$ as $t \to \infty$ for all the UAVs in the swarm, and so is $\ddot{x}_i^m$. As a result, the multi-layer configuration of the swarm is constructed. □

3.3. Consensus Strategy

In this subsection, all the UAVs in the swarm have formed a fixed formation configuration based on the potential function control law. However, the yaw angle ψ of each UAV is still arbitrarily uncertain. In order to keep the flight states of all the UAVs in consensus, a yaw angle adjustment method based on the concept of the Vicsek model [26] is proposed for the multi-layer UAVs swarm.

For simplicity, we relabel each UAV in different layers. The edge of the first layer is $\varepsilon_1 \subseteq ((i_1, j_1) : i_1, j_1 \in \nu_1, i_1 \neq j_1;)$. For the second layer, $\varepsilon_2 \subseteq ((i_2, j_2) : i_2, j_2 \in \nu_2, i_2 \neq j_2;)$. For the mth layer, the edge is labeled as $\varepsilon_m \subseteq ((i_m, j_m) : i_m, j_m \in \nu_m, i_m \neq j_m;)$. Then the UAVs in each subgroup make corresponding updates according to the states of the previous subgroups, and finally achieve consensus.

For the first layer, the UAVs yaw angle adjustment strategy is as follows

$$
\psi_{i_1}(t+1) = \arctan \frac{\sum_{j_1=1}^{N_o+1} \sin \psi_{j_1}(t)}{\sum_{j_1=1}^{N_o+1} \cos \psi_{j_1}(t)}
\tag{23}
$$

The attitude of the UAV i_1 can be updated according to the attitude of all the UAVs in the same subgroup. Therefore, the consensus of the first layer is achieved.

For the second layer, the UAVs yaw angle are adjusted by the following approach

$$
\begin{aligned}
\psi_{i_2}(t+1) &= \arctan \frac{\sum_{j_2=1}^{N_o+1} \sin \psi_{j_2}(t)}{\sum_{j_2=1}^{N_o+1} \cos \psi_{j_2}(t)} \\
&= \arctan \frac{\sum_{j_2=1}^{N_o+1} \sin(\frac{1}{N_o+1} \sum_{j_1=1}^{N_o+1} \psi_{j_1}(t))}{\sum_{j_2=1}^{N_o+1} \cos(\frac{1}{N_o+1} \sum_{j_1=1}^{N_o+1} \psi_{j_1}(t))}
\end{aligned}
\tag{24}
$$

Therefore, ψ_{i_2} is obtained from the average of the yaw angles of the individual UAVs in all the subgroups for the first layer.

Based on the above strategy, the UAVs yaw angle adjustment strategy for the mth layer is as follows

$$\psi_{i_m}(t+1) = \arctan\frac{\sum\limits_{j_m=1}^{N_o+1}\sin\psi_{j_2}(t)}{\sum\limits_{j_m=1}^{N_o+1}\cos\psi_{j_2}(t)} = \arctan\frac{\sum\limits_{j_m=1}^{N_o+1}\sin(\frac{1}{N_o+1}\sum\limits_{j_{m-1}=1}^{N_o+1}\psi_{j_{m-1}}(t))}{\sum\limits_{j_m=1}^{N_o+1}\cos(\frac{1}{N_o+1}\sum\limits_{j_{m-1}=1}^{N_o+1}\psi_{j_{m-1}}(t))} \tag{25}$$

In the above, we describe the consensus strategy between different layers, then all the UAVs in the swarm achieve consensus eventually. For the specific example of the UAVs swarm, as shown in Figure 1, assume $n = 9$, $N_o = 2$, there are nine UAVs labeled as $A_1, \ldots, A_9$, then the whole swarm can be combined as three first layer subgroups named as $\mathcal{G}_1^1$, $\mathcal{G}_1^2$, $\mathcal{G}_1^3$,which constitute a second layer $\mathcal{G}_2$. Futhermore, $\mathcal{G}_1^1$ is composed of A_1, A_2, A_3, $\mathcal{G}_1^2$ and $\mathcal{G}_1^3$ are consisted of A_4, A_5, A_6, A_7, A_8, A_9, respectively. Then we set the communication between $\mathcal{G}_1^1$ and $\mathcal{G}_1^2$ are connected by A_2 and A_4, $\mathcal{G}_1^2$ and $\mathcal{G}_1^3$ are connected by A_6 and A_8, $\mathcal{G}_1^1$ and $\mathcal{G}_1^3$ are connected by A_3 and A_7. Firstly, the UAV swarm achieves the desired configuration through the forces between the UAV individuals and between the same layers. Taking A_2 as an example, A_2 is subjected to the forces of A_1 and A_3, namely $f_{A_2A_1}$ and $f_{A_2A_3}$, A_2 is also subject to $f^2_{\mathcal{G}_1^1\mathcal{G}_1^2}$ and $f^2_{\mathcal{G}_1^1\mathcal{G}_1^3}$, which are the components between $\mathcal{G}_1^1$ and $\mathcal{G}_1^2$ and between $\mathcal{G}_1^1$ and $\mathcal{G}_1^3$, respectively. After the whole swarm reaches the desired configuration, the resultant force of A_2 is zero. Furthermore, let $\mathcal{G}_1^1$, $\mathcal{G}_1^2$and $\mathcal{G}_1^3$ achieve intra-group consensus through the yaw angle adjustment stragety (23), and reach the same yaw angle ψ_{A_1}, ψ_{A_4} and ψ_{A_7} respectively. For the second layer, $\mathcal{G}_2$ achieve the intra-group consistent yaw angle from the average of the ψ_{A_1}, ψ_{A_4} and ψ_{A_7}. Based on this rule, the entire swarm achieves consensus eventually.

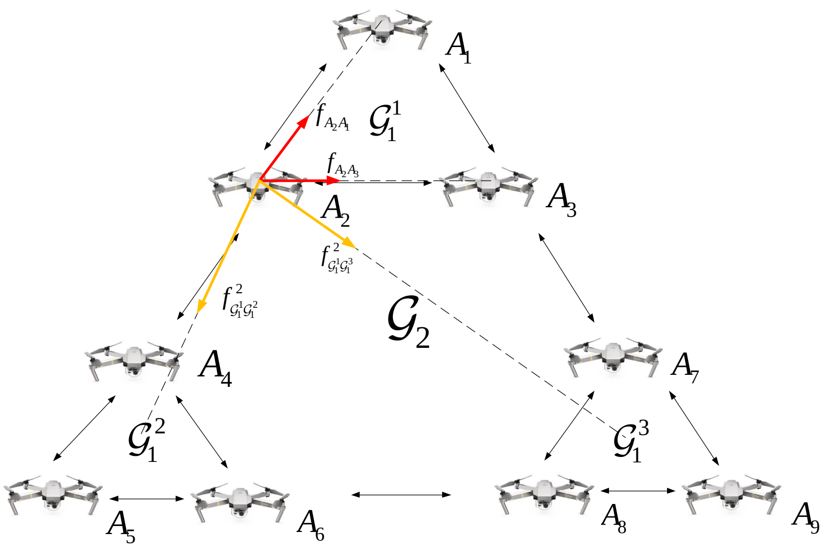

Figure 1. Communication topology with n = 9, N_o = 2.

4. Simulation Study

To illustrate the effectiveness of the proposed multi-layer topology and the consensus algorithm, corresponding simulation results under different conditions are presented in this section. For the multi-layer UAVs swarm, we consider a group of networked UAVs with $n = 27$, $N_o = 2$, which contains two layer subgroups. The control parameters are chosen as $r_0 = 2$ m, $r_0^2 = 4$ m, $\xi = 20$, $\eta = 5$.

4.1. Swarm Configuration

For the proposed multi-layer UAVs system, all the UAVs are randomly distributed in a fixed working area. Firstly, based on the adjacent principle, under the control law (10), all the neighbors in the UAVs swarm are assigned to establish the multi-layer network topology. All the UAVs move towards their desired location and keep the desired distances with their neighbours under the forces between the UAV individuals and the forces between layers at the same level, the repulsive forces makes UAVs avoid collisions and keep the desired distance between them, while the force between layers makes the UAV swarm achieve the desired configuration, then the resultant force from the artificial potential converges to zero. In different situations, the number of UAVs in the whole system and the number of their neighbors can be seted arbitrarily, so as to adjust the structure of the entire network topology. When all the UAVs complete the assignment of neighbors, the first layer topology within a set of subgroups is constructed, under the swarm configuration control law (14), all the subgroups can be treated as independent individuals, then the neighbors are assigned to these subgroups and the second layer network topology is established. Based on this rule, the subgroups will adjust their position and form a higher level group, until all the UAVs achieve the desired configuration. Here, we take 27 UAVs as an example to illustrate the effectiveness of control laws. As shown in Figure 2, in the initial state, the distance between UAV individuals is arbitrary, within the range of 25 m, then after the UAVs start to communicate with each other, in a short iteration step, the UAVs at any position converge quickly. When the step reaches around 150, the distance between UAVs is within 2 m, and the expected configuration is basically achieved. As a result, all UAVs in the swarm of each layer tend to be at the desired distance with the high formation configuration results after 800 steps.

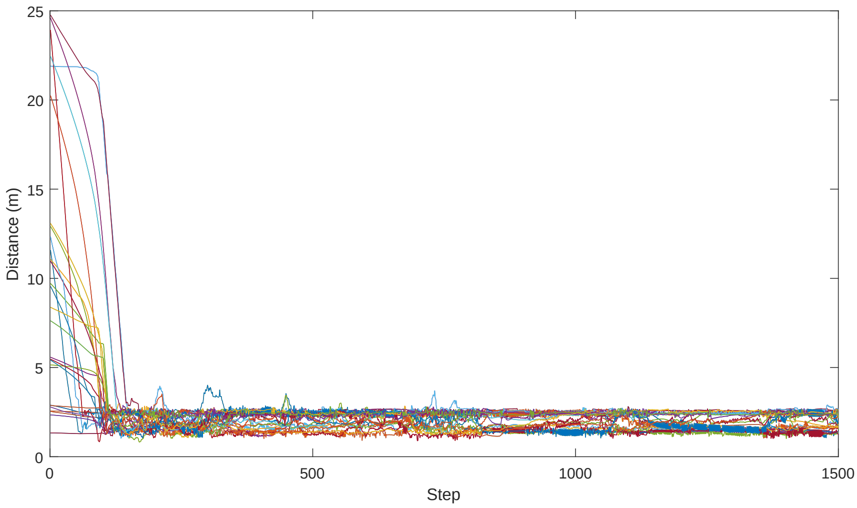

Figure 2. The distance between all neighbor UAVs in the configuration.

4.2. Consensus Control

When the swarm have achieved the desired configuration, the proposed consensus control approach will adjust the attitude of all the UAVs to achieve consensus. Figure 3 shows the process of achieving consensus from the initial yaw angle states. After the system completes the desired configuration, the UAVs have arbitrary yaw angles. Then, under the control law (23), the three UAVs in each group in the first layer adjust the yaw angles to reach a consistent state, and then control law (24) enables the unification of the yaw angle of UAVs between layers. It can be seen that when step = 150, the yaw angle of UAV basically reaches 3 degree. After step = 500, the swarm completes the unification of yaw angle. As a result, all the UAVs realize the motion consensus according to the proposed recursive consensus control concept, while maintaining the desired swarm configuration and moving in the same direction.

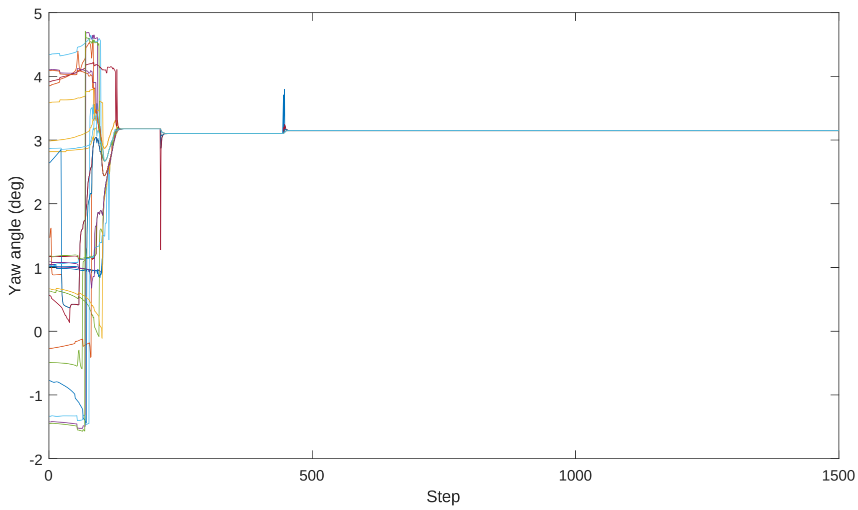

Figure 3. The yaw angle of each UAV in the configuration.

5. Conclusions

The current paper proposed a multilayer framework based on the multi-layer concept to deal with the multiagent problem with arbitrary number of UAVs. The primary contribution is that the designed multi-layer structure can be used to form the desired configuration and keep consensus under the context of large-scale UAVs swarm with Assumption 1 and Assumption 2, rather than moving into random positions. A potential function-based multi-layer controller is developed to drive all the UAVs to achieve the desired configuration precisely without collisions. Then all the UAVs reach an agreement through the consensus algorithm. The stability of the system is proved by the Lyapunov approach. The simulation studies demonstrated the effectiveness of the proposed methods for the UAVs swarm. In our future work, the trajectory tracking and the obstacle avoidance of the large-scale UAVs swarm will be investigated under the Active Disturbance Rejection Control approach.

Author Contributions: Conceptualization, formal analysis, investigation, writing, T.W.; Methodology, pre-processing, writing, S.Z.; Simulation validation, writing-review Z.P.; Supervision, Y.X.; Writing-review, H.T. All authors have read and agreed to the published version of the manuscript.

Funding: This paper is funded by National Natural Science Foundation of China (62203050) and National Natural Science Foundation of China (61720106010).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors would like to thank all coordinators and supervisors involved and the anonymous reviewers for their detailed comments that helped to improve the quality of this article.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

Abbreviations

The following abbreviations are used in this manuscript:

UAV	Unmanned Aerial Vehicle
APF	Artificial Potential Field

References

1. Nasir, M.H.; Khan, S.A.; Khan, M.M.; Fatima, M. Swarm Intelligence inspired Intrusion Detection Systems—A systematic literature review. *Comput. Netw.* **2022**, *205*, 108708. [CrossRef]
2. Rosenberg, L.; Willcox, G.; Askay, D.; Metcalf, L.; Harris, E. Amplifying the social intelligence of teams through human swarming. In Proceedings of the 2018 First International Conference on Artificial Intelligence for Industries (AI4I), San Francisco, CA, USA, 26–28 September 2018; pp. 23–26.
3. Kiebert, L.; Joordens, M. Autonomous robotic fish for a swarm environment. In Proceedings of the 2016 11th System of Systems Engineering Conference (SoSE), Waurn Ponds, Australia, 12–16 June 2016; pp. 1–6.
4. Reynolds, C.W. Flocks, herds and schools: A distributed behavioral model. In Proceedings of the 14th Annual Conference on Computer Graphics and Interactive Techniques, Los Angeles, CA, USA, 27–31 July 1987; pp. 25–34.
5. Cucker, F.; Smale, S. Emergent behavior in flocks. *IEEE Trans. Autom. Control.* **2007**, *52*, 852–862. [CrossRef]
6. Saska, M.; Vakula, J.; Přeućil, L. Swarms of micro aerial vehicles stabilized under a visual relative localization. In Proceedings of the 2014 IEEE International Conference on Robotics and Automation (ICRA), Hong Kong Convention and Exhibition Center, Hong Kong, China, 31 May–7 June 2014; pp. 3570–3575.
7. Olfati-Saber, R. Flocking for multi-agent dynamic systems: Algorithms and theory. *IEEE Trans. Autom. Control.* **2006**, *51*, 401–420. [CrossRef]
8. Su, H.; Wang, X.; Lin, Z. Flocking of multi-agents with a virtual leader. *IEEE Trans. Autom. Control.* **2009**, *54*, 293–307. [CrossRef]
9. Fu, X.; Pan, J.; Wang, H.; Gao, X. A formation maintenance and reconstruction method of UAV swarm based on distributed control. *Aerosp. Sci. Technol.* **2020**, *104*, 1270–9638. [CrossRef]
10. Wu, Y.; Gou, J.; Hu, X.; Huang, Y. A new consensus theory-based method for formation control and obstacle avoidance of UAVs. *Aerosp. Sci. Technol.* **2020**, *107*, 1270–9638. [CrossRef]
11. Soria, E.; Schiano, F.; Floreano, D. Predictive control of aerial swarms in cluttered environments. *Nat. Mach. Intell.* **2021**, *3*, 545–554. [CrossRef]
12. Haghighi, R.; Cheah, C.C. Multi-group coordination control for robot swarms. *Automatica* **2012**, *48*, 2526–2534. [CrossRef]
13. Yan, X.; Chen, J.; Sun, D. Multilevel-based topology design and shape control of robot swarms. *Automatica* **2012**, *48*, 3122–3127. [CrossRef]
14. Pan, Z.; Sun, Z.; Deng, H.; Li, D. A Multilayer Graph for Multiagent Formation and Trajectory Tracking Control Based on MPC Algorithm. *IEEE Trans. Cybern.* **2021**, *52*, 13586–13597. [CrossRef]
15. Li, D.; Ge, S.S.; He, W.; Ma, G.; Xie, L. Multilayer formation control of multi-agent systems. *IEEE Trans. Cybern.* **2019**, *109*, 108558. [CrossRef]
16. Liu, X.; Ge, S.S.; Goh, C.H. Formation potential field for trajectory tracking control of multi-agents in constrained space. *Int. J. Control* **2017**, *90*, 2137–2151. [CrossRef]
17. Pan, Z.; Zhang, C.; Xia, Y.; Xiong, H.; Shao, X. An Improved Artificial Potential Field Method for Path Planning and Formation Control of the Multi-UAV Systems. *IEEE Trans. Circuits Syst. II Express Briefs* **2021**, *69*, 1129–1133. [CrossRef]
18. Huang, Y.; Ding, H.; Zhang, Y.; Wang, H.; Cao, D.; Xu, N.; Hu, C. A motion planning and tracking framework for autonomous vehicles based on artificial potential field elaborated resistance network approach. *IEEE Trans. Ind. Electron.* **2019**, *67*, 1376–1386. [CrossRef]
19. Wen, G.; Ge, S. S.; Tu, F.; Choo, Y.S. Artificial Potential-Based Adaptive H_∞ Synchronized Tracking Control for Accommodation Vessel. *IEEE Trans. Ind. Electron.* **2017**, *64*, 5640–5647. [CrossRef]
20. Vásárhelyi, G.; Virágh, C.; Somorjai, G.; Nepusz, T.; Eiben, A.E.; Vicsek, T. Optimized flocking of autonomous drones in confined environments. *Sci. Robot.* **2018**, *3*, eaat3536. [CrossRef]
21. Wang, X.; Li, S.; Yu, X.; Yang, J. Distributed active anti-disturbance consensus for leader-follower higher-order multi-agent systems with mismatched disturbances. *IEEE Trans. Autom. Control* **2016**, *62*, 5795–5801. [CrossRef]
22. Ma, L.; Wang, Z.; Lam, H.K. Event-triggered mean-square consensus control for time-varying stochastic multi-agent system with sensor saturations. *IEEE Trans. Autom. Control* **2016**, *62*, 3524–3531. [CrossRef]
23. Zhang, H.; Lewis, F.L.; Qu, Z. Lyapunov, adaptive, and optimal design techniques for cooperative systems on directed communication graphs. *IEEE Trans. Ind. Electron.* **2011**, *59*, 3026–3041. [CrossRef]
24. Zou, Y.; Meng, Z. Coordinated trajectory tracking of multiple vertical take-off and landing UAVs. *IEEE Trans. Ind. Electron.* **2019**, *99*, 33–40. [CrossRef]
25. Olfati-Saber, R.; Murray, R.M. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans. Autom. Control.* **2004**, *49*, 1520–1533. [CrossRef]
26. Vicsek, T.; Czirók, A.; Ben-Jacob, E.; Cohen, I.; Shochet, O. Novel type of phase transition in a system of self-driven particles. *Phys. Rev. Lett.* **1995**, *75*, 1226. [CrossRef] [PubMed]