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Electromechanical Modelling of a Piezoelectric Stack Energy Harvester †

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Abstract: In this paper, the behavior of a piezoelectric stack generator subject to a direct mechanical force, such as that experienced by pedestrian loads, is characterized and modelled. A lumped element model is developed that describes the electrical output of piezoelectric stack generators for quasi-static, off-resonance and random input conditions which are typical of underfloor or supporting structure energy harvesters. This research demonstrates the ability for accurate predictions of output voltage and current based on the material properties and geometry of the piezoelectric stack. The behavior of real-world random loading patterns is incorporated in the model, pairing mechanical input with interfacing circuitry requirements in order to maximize energy transfer. The developed model facilitates time-efficient design by reducing the simulation time to the order of seconds.

Keywords: energy; harvester; piezo; stack; modelling

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1. Introduction

A significant challenge faced by practical implementations of energy harvesters is matching the electrical output of the harvester with the operating conditions of the circuit to be powered [1,2]. In the case of a piezoelectric stack harvester, this is a multi-variable process that involves a coupled electromechanical system [3]. While behavior near resonance is well studied [4], the modelling of forces that are directly applied at low frequency is relatively unexplored [5–7]. Under such conditions, the material properties and geometry of the stack must be controlled to provide a suitable output voltage, while maximising the total energy conversion. A general approach is to use finite element analysis to model such a coupled system, which requires significant overhead to compute iteratively [8]. This work addresses this issue by enabling time-efficient iteration of the piezoelectric stack parameters. The effect of real-world force input data is also incorporated, which has significant impact on the optimal device parameters due to the dependence of the impedance of the piezoelectric stack on excitation frequency [9]. Without this capability, such behavior may be difficult to model for a real system. The developed model is used to predict the performance of two piezoelectric stack energy harvesters of different configurations, which is validated using controlled testing of direct force application at a low frequency.

2. Materials and Methods

The behaviour of a piezoelectric stack may be described using the coupled constitutive equations, where *S* is the mechanical strain, s^E the compliance at constant electric field (indicated by the superscript *E*), *T* the mechanical stress, *D* the charge density, *d* and d^t the piezoelectric charge constant and its transpose (indicated by the superscript *t*), ε^T the dielectric permittivity at constant stress (indicated by the superscript *T*) and *E* the electric field [10], where the underscores denote matrix form.

$$\underline{S} = \underline{s}^E \cdot \underline{T} + \underline{d}^t \cdot \underline{E} \tag{1}$$

$$\underline{D} = \underline{d} \cdot \underline{T} + \underline{\varepsilon}^T \cdot \underline{E} \tag{2}$$

The compliance, piezoelectric charge constant and the dielectric permittivity are material properties that are determined by the grade of piezoelectric ceramic. The mechanical stress and strain, and consequently the electric field and charge density are controlled by the geometric properties of the piezoelectric stack, and are hence the control parameters for optimisation of device performance. A lumped element model is used to described the coupled behaviour, shown in Figure 1a [11]. The mechanical properties of the stack are depicted by the RLC circuit, and can be expressed as:

$$L = m = \rho A_s L_s \tag{3}$$

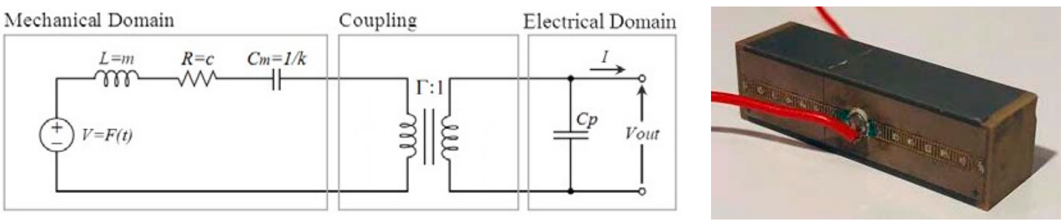
$$R = c = 2\zeta\sqrt{km} \tag{4}$$

$$C_m = \frac{1}{k} = \frac{L_s}{Y A_s} \tag{5}$$

where *m*, *k* and *c* are the equivalent mass, stiffness and damping respectively, *ρ* the density of the piezoelectric material, A_s the cross sectional stack area, L_s the stack length, *ζ* the damping ratio and *Y* the Young's modulus of the piezoelectric material. The input voltage *V* is equivalent to the force input to the stack, *F*(*t*). The capacitance C_P in the electrical domain is the physical capacitance of the piezoelectric stack. Electromechanical coupling is represented by an ideal transformer, with a turns ratio of Γ. The turns ratio is governed by:

$$\Gamma = K_g\sqrt{kC_P} \tag{6}$$

where K_g is the generalised electromechanical coupling factor [12]. The piezoelectric stack operates in d_{33} mode, therefore K_g may be expressed in terms of the material coupling factor, k_{33}.



(**a**) (**b**)

Figure 1. (**a**) The lumped element model of the piezoelectric stack, consisting of mechanical domain, electrical domain and a coupling term, modelled by an ideal transformer. The coupling ratio Γ is described in terms of the material and physical properties of the device, so as to be valid for a piezoelectric stack of arbitrary parameters; (**b**) A multi-layered piezoelectric stack subjected to testing.

$$K_g^2 = \frac{k_{33}^2}{1 - k_{33}^2} \tag{7}$$

For d_{33} operation, the material coupling factor can be expressed in terms of the piezoelectric charge constant, compliance and permittivity [13].

$$k_{33} = \frac{d_{33}}{\sqrt{\varepsilon_{33}^{T} s_{33}^{E}}} \tag{8}$$

By combining Equations (6)–(8); the ratio can be expressed in terms of the piezoelectric stack parameters, where *n* is the number of piezoelectric elements in the stack and *t* is the thickness of each element.

$$\Gamma = \sqrt{\frac{d_{33}^{2} \varepsilon_{33}^{T} A_{s} k n}{(\varepsilon_{33}^{T} s_{33}^{E} - d_{33}^{2}) t}} \tag{9}$$

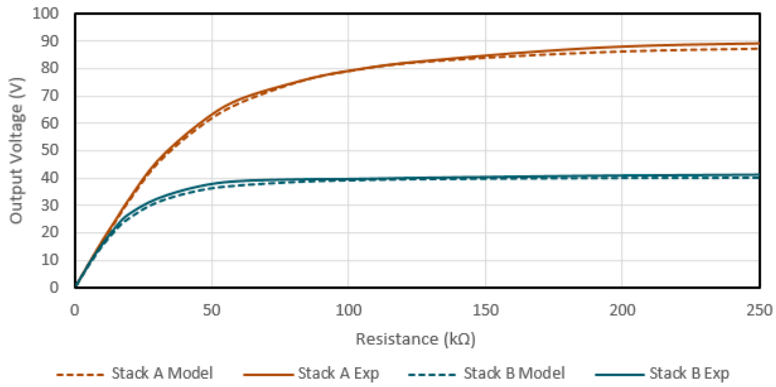


Figure 2. Comparison between modelled (dotted lines) and experimentally obtained (solid lines) output voltage of two different piezoelectric stacks connected to a resistive load, subjected to low frequency directly applied force.

3. Results and Discussion

The lumped element model provides the capability to predict the performance of an arbitrary piezoelectric stack, and integrate the device into circuit simulation software to observe the effects of connecting any interfacing circuitry. To validate the accuracy of the model under low frequency conditions typical of pedestrian loads, two piezoelectric stacks were modelled and tested, see Figure 1b. The properties of the piezoelectric stacks are listed in Table 1. Each stack was subjected to a 1000 N, 4 Hz sinusoidal load and its electrical output characterised when connected to a resistive load. A comparison between the modelled and experimentally obtained output is shown in Figure 2. The model was found to be accurate to within 5% across a wide range of electrical loading conditions.

Table 1. Properties of the piezoelectric stacks.

Property	Piezo Stack A	Piezo Stack B	Units
d33	480	480	$\times 10^{-8}$ m/V
As	7.85	15.4	$\times 10^{-5}$ m^2
t	0.5	0.5	mm
N	98	98	-
Ls	62	62	mm

Additionally, the model was used to evaluate the performance of each stack when subjected to a random input load. The loading pattern was measured by pedestrians walking over a suspended platform, shown in Figure 3. Under this loading condition, stack A produced an average power output of 17.4 mW, while stack B produced 8.6 mW when connected to a 1500 Ω resistive load. At the same electrical loading conditions under 4 Hz sinusoidal input, the average power output of stack A and B were found to be equivalent. This demonstrates a clear performance difference exists between the two stacks that would otherwise be difficult to quantify analytically. Combined with optimisation tools, this model provides a method of simulating the performance of piezoelectric stack energy harvesters when subjected to loading patterns typical of real world applications, being random or stochastic in nature.

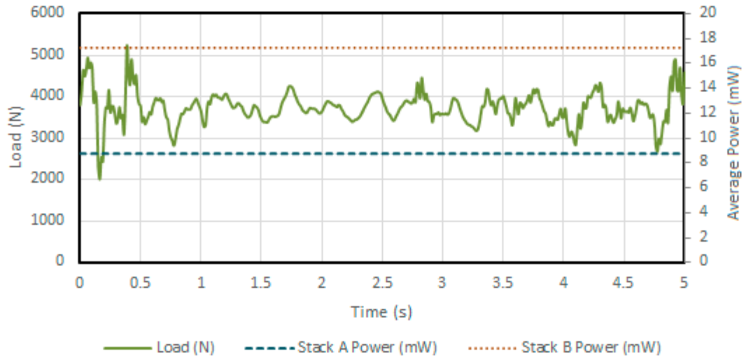


Figure 3. Measured force applied to a platform support with pedestrian traffic, and the output power of each piezoelectric stack.

4. Conclusions

A lumped element circuit model for predicting the performance of piezo stack energy harvesters has been presented. A low frequency (<10 Hz), directly applied force to the piezoelectric stack is modelled and verified, where the frequencies of interest are typical of underfloor or supporting structure energy harvesters. The model was proven to be accurate in simulating the coupled electromechanical behavior of piezoelectric stack harvesters of arbitrary shape and material properties, to within 5% of experimentally obtained values. The work allows the integration of measured loading data and interfacing circuitry to provide an accurate representation of true energy harvester performance. This provides a robust method of optimisation of piezoelectric stack energy harvesters to suit quasi-static, off-resonance and random input conditions.

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