



Article

Identification of Fractional Models of an Induction Motor with Errors in Variables

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Abstract: The skin effect in modeling an induction motor can be described by fractional differential equations. The existing methods for identifying the parameters of an induction motor with a rotor skin effect suggest the presence of errors only in the output. The presence of errors in measuring currents and voltages leads to errors in both input and output signals. Applying standard methods, such as the ordinary least squares method, leads to biased estimates in these types of problems. The study proposes a new method for identifying the parameters of an induction motor in the presence of a skin effect. Estimates of parameters were determined based on generalized total least squares. The simulation results obtained showed the high accuracy of the obtained estimates. The results of this research can be applied in the development of predictive diagnostic systems. This study shows that ordinary least squares parameter estimates can lead to incorrect operation of the fault diagnosis system.

Keywords: induction motor; additive noise; skin effect; total least squares; errors-in-variables; fractional derivative



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1. Introduction

Today, methods for identifying the parameters of induction motors based on equivalent circuits are being actively developed. There are a great number of methods for identifying the parameters of induction motor models. An overview of these identification methods is presented in [1,2].

Estimation of induction motor parameters based on ordinary least squares (OLS) and its recursive modifications is considered in [3–6]. The application of the total least squares (TLS) technique and its recursive versions was proposed in articles [7–12].

The use of integer-order dynamical models leads to significant errors between the measured signals and the results obtained from simulations [13]. The greatest discrepancies are high-power squirrel cage induction machines, in which there is a skin effect in the rotor cage bars and in induction and synchronous machines with a solid rotor [14].

Using high integer-order models that more accurately account for the skin effect in the solid rotor of induction motors leads to a great increase in the number of equivalent circuit elements; however, parameter identification of the high integer-order models is very difficult [15].

Another approach to increase the accuracy of the model is to use the fractional order of the models. Such models cannot be described by traditional electrical circuits. The description of such models requires the use of fractional derivatives.

Fractional-order derivatives are widely used to describe various processes and phenomena. Fractional-order differentiation constitutes a valuable instrument for modeling real-world processes. Fractional calculations are widely used for the synthesis of fractional controllers for various electric motors including, among others, induction motors [16–19], direct current (DC) motors [20], and permanent magnet synchronous motors (PMSMs) [21].

The aforementioned research on the topic always assumed that induction motor models were of an integer order.

There are far fewer articles devoted to modeling and identification of induction motors by fractional-order models. In [14,22–24], the researchers represented the skin effect in a solid rotor by means of resistance and inductance with fixed values and fractional-order inductance, depending on the frequency of induced eddy currents.

The authors of [25] proposed a frequency method for identifying models with fractional-order inductance. Identification of models with fractional-order inductance based on the output error (OE) model was considered in [26].

By monitoring the parameters of an induction motor, faults can be diagnosed. The authors of [27] proposed diagnosing a short circuit in a stator winding by reducing stator resistance and proposed determining broken rotor bars by increasing rotor resistance. Similar diagnostic methods can be applied to fractional-order models. The accuracy of diagnostics depends on the accuracy of the estimated parameters. This makes the development of methods for identifying fractional-order models relevant.

This study is the first to consider the identification of fractional models of induction motors with errors in variables. Previously, either fractional output error models [26] or integer models with errors in variables [7–11] were considered. Currents and voltages are always measured with errors, which leads to the fact that it is necessary to identify a system with noise in the input and output signals. Applying standard methods, such as OLS, lead to biased estimates in these types of problems.

For the first time in one article, three fractional orders of an induction motor model with errors in variables were compared for noise immunity.

It was shown that in the presence of current and voltage measurement errors, generalized total least squares allow one to obtain more accurate parameter estimates than ordinary least squares. This study shows that OLS parameter estimates can lead to incorrect operation of the fault diagnosis system.

2. Materials and Methods

2.1. Problem Statement

T-form circuit models are more complex than necessary. They can be transformed into simpler models with no loss of information or accuracy [28]. This configuration has been denoted as the Γ form, from the structure of its two inductances. Modeling a Γ -equivalent circuit will make a locked rotor for $\Omega = 0$.

The equivalent circuit shown in Figure 1 does not describe the skin effects typical of real induction motors. To simulate skin effects, models with non-integer-order derivatives can be presented. Let us replace the resistor R_r and the inductance L_r with the complex resistance Z_r . The equivalent circuit is shown in Figure 2.

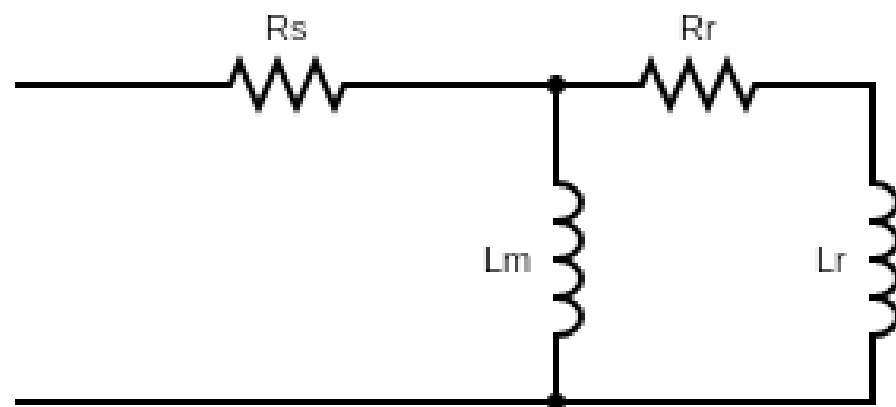


Figure 1. Γ -equivalent circuit model of an induction motor.

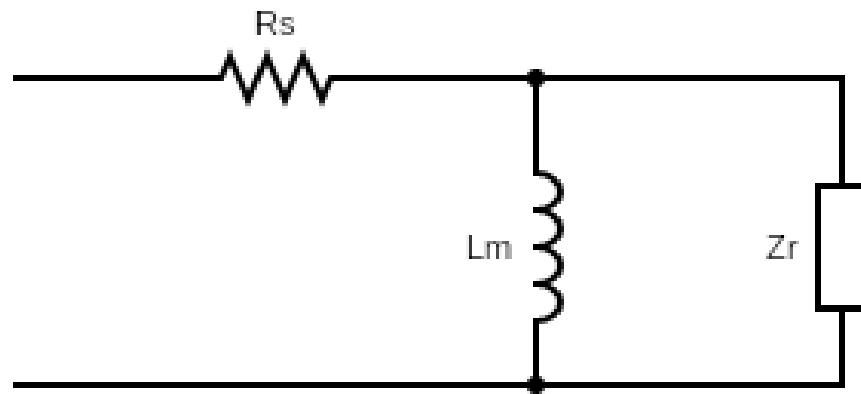


Figure 2. Γ -equivalent circuit model of an induction motor with fractional impedance.

The equivalent impedance of an induction motor is defined as:

$$Z(s) = R_s + \frac{sL_m Z_r(s)}{sL_m + Z_r(s)}, \quad (1)$$

Various models with derivatives of non-integer order are known. $Z_r(s)$ is the one-derivative black box mode [23]:

$$Z_r(s) = \frac{a_0 + s^\alpha}{b_0}, \quad (2)$$

or the two-derivative model [24]:

$$Z_r(s) = \frac{a_0 + s^\alpha a_1 + s^{\alpha+0.5}}{s_0 + s^\alpha b_1}, \quad (3)$$

Furthermore, there is a three-parameter model [14]:

$$Z_r(s) = R_r + sL_r + s^\alpha a_r, \quad (4)$$

The relationship between stator current and stator voltage is defined as:

$$I(s) = \frac{U(s)}{Z(s)}, \quad (5)$$

where $U(s)$ is the stator voltage and $I(s)$ is the stator current.

In real conditions, currents and voltages are always measured with noise:

$$\tilde{i}(t) = i(t) + e_i(t), \tilde{u}(t) = u(t) + e_u(t), \quad (6)$$

where $e_i(t)$ and $e_u(t)$ are additive zero-mean white Gaussian noises corrupting the voltage signal and the current signal, and they are assumed not to be correlated with the voltage signal and the current signal.

The problem with identifying an induction motor is the estimation of the vector of unknown parameters θ from the measured values of the stator current $\tilde{i}(t)$ and stator voltage $\tilde{u}(t)$. The parameter vector of impedance $Z(s)$ (1) for the model $Z_r(s)$ (2) is:

$$\theta = (R_s \quad L_m \quad a_0 \quad b_0 \quad \alpha)^T, \quad (7)$$

For the model $Z_r(s)$ (3), the parameter vector of impedance $Z(s)$ is:

$$\theta = (R_s \quad L_m \quad a_0 \quad a_1 \quad b_0 \quad b_1 \quad \alpha)^T, \quad (8)$$

For the model $Z_r(s)$ (4), the parameter vector of impedance $Z(s)$ is:

$$\theta = (R_s \quad L_m \quad R_r \quad L_r \quad a_r \quad \alpha)^T. \quad (9)$$

2.2. GTLS Algorithm for Identification of Induction Motor

Let us express the value of the impedance in terms of the physical parameters of the equivalent circuits. The impedance value $Z(s)$ (1) for model $Z_r(s)$ (2) is:

$$Z(s) = \frac{L_m s^{1+\alpha} + (L_m a_0 + L_m R_s b_0)s + R_s s^\alpha + R_s a_0}{L_m b_0 s + s^\alpha + a_0}, \quad (10)$$

The impedance value $Z(s)$ (1) for model $Z_r(s)$ (3) is:

$$Z(s) = \frac{L_m s^{1.5+\alpha} + (L_m a_1 + L_m R_s b_1)s^{1+\alpha} + R_s s^{0.5+\alpha} + (L_m a_0 + L_m R_s b_0)s + R_s a_1 s^\alpha + R_s a_0}{L_m b_1 s^{1+\alpha} + s^{0.5+\alpha} + L_m b_0 s + a_1 s^\alpha + a_0}, \quad (11)$$

The impedance value $Z(s)$ (1) for model $Z_r(s)$ (4) is:

$$Z(s) = \frac{L_m L_r s^2 + L_m a_r s^{1+\alpha} + (L_m R_r + L_m R_s + L_r R_s)s + R_s a_1 s^\alpha + R_s R_r}{(L_m + L_r)s + a_r s^\alpha + R_r}. \quad (12)$$

Equation (5) in the time domain for impedances (2)–(4) is defined as:

$$L_m D^{1+\alpha} i(t) + (L_m a_0 + L_m R_s b_0) Di(t) + R_s D^\alpha i(t) + R_s a_0 i(t) = L_m b_0 Du(t) + D^\alpha u(t) + a_0 u(t), \quad (13)$$

$$L_m D^{1.5+\alpha} i(t) + (L_m a_1 + L_m R_s b_1) D^{1+\alpha} i(t) + R_s D^{0.5+\alpha} i(t) + (L_m a_0 + L_m R_s b_0) Di(t) + R_s a_1 D^\alpha i(t) + R_s a_0 i(t) = D^{0.5+\alpha} u(t) + L_m b_1 D^{1+\alpha} u(t) + L_m b_0 Du(t) + a_1 D^\alpha u(t) + a_0 u(t), \quad (14)$$

$$L_m L_r D^2 i(t) + L_m a_r D^{1+\alpha} i(t) + (L_m R_r + L_m R_s + L_r R_s) Di(t) + R_s a_1 D^\alpha i(t) + R_s R_r i(t) = (L_m + L_r) Du(t) + a_r D^\alpha u(t) + R_r u(t). \quad (15)$$

where $D^\alpha i(t)$ is the Grünwald–Letnikov fractional operator, $D^\alpha i(t) = \lim_{h \rightarrow 0} \frac{1}{h^\alpha} \sum_{k=0}^{\lfloor t/h \rfloor} (-1)^k \binom{\alpha}{k} i(t - kh)$,

h is the sampling period, and $\binom{\alpha}{k}$ is Newton's binomial generalized to fractional orders.

Equations (13)–(15) in matrix form are described as:

$$i(t) = \varphi(t) \bar{\theta}, \quad (16)$$

Equation (13) is described as:

$$\begin{aligned} \varphi_i(t) &= (-D^{1+\alpha} i(t) \quad -Di(t) \quad -D^\alpha i(t))^T, \quad \varphi_u(t) = (Du(t) \quad D^\alpha u(t) \quad u(t)), \\ \bar{\theta} &= \frac{1}{R_s a_0} (\theta_i \quad \theta_u)^T, \quad \bar{\theta}_i = (L_m \quad L_m a_0 + L_m R_s b_0 \quad R_s)^T, \quad \bar{\theta}_u = (L_m b_0 \quad 1 \quad a_0). \end{aligned}$$

Equation (14) is described as:

$$\begin{aligned} \varphi(t) &= (-D^{1.5+\alpha} i(t) \quad -D^{1+\alpha} i(t) \quad -D^{0.5+\alpha} i(t) \quad -Di(t) \quad -D^\alpha i(t)), \\ \varphi_u(t) &= (D^{1+\alpha} u(t) \quad D^{0.5+\alpha} u(t) \quad Du(t) \quad D^\alpha u(t) \quad u(t)), \quad \bar{\theta} = \frac{1}{R_s a_0} (\theta_i \quad \theta_u)^T, \\ \bar{\theta}_i &= (L_m \quad L_m a_1 + L_m R_s b_1 \quad R_s \quad L_m a_0 + L_m R_s b_0 \quad R_s a_1)^T, \quad \bar{\theta}_u = (1 \quad L_m b_1 \quad L_m b_0 \quad a_1 \quad a_0), \end{aligned}$$

Equation (15) is described as:

$$\begin{aligned} \varphi_i(t) &= (-D^2 i(t) \quad -D^{1+\alpha} i(t) \quad -Di(t) \quad -D^\alpha i(t))^T, \quad \varphi_u(t) = (Du(t) \quad D^\alpha u(t) \quad u(t)), \quad \bar{\theta} = \\ &= \frac{1}{R_s R_r} (\theta_i \quad \theta_u)^T, \quad \bar{\theta}_i = (L_m L_r \quad L_m a_r \quad L_m R_r + L_m R_s + L_r R_s \quad R_s a_r)^T, \quad \bar{\theta}_u = (L_m + L_r \quad a_r \quad R_r). \end{aligned}$$

For noisy current $\tilde{i}(t)$ and voltage $\tilde{u}(t)$, Equation (16) is described as:

$$\tilde{i}(t) = \tilde{\varphi}(t) \bar{\theta} + \varepsilon(t), \quad (17)$$

where

$$\varepsilon(t) = e_i(t) + \varphi_e(t) \bar{\theta} \text{ and } \varphi_e(t) = \tilde{\varphi}(t) - \varphi(t).$$

Calculating the fractional derivative from noisy data is a serious problem in identifying a fractional system and leads to large errors. Therefore, the signals must be processed by the state variable filter (SVF) proposed in [29]. The SVF is defined by the following equation:

$$L(s) = \left(\frac{\omega}{s + \omega} \right)^\eta, \quad (18)$$

where the order η is an integer chosen such that $\eta > 1.5 + \alpha$ and ω denote the filter cut-off frequency. The choice of the number η is a compromise between filter complexity and filtering quality. However, increasing the order for large η produces a very slight increase in the filtering quality.

The filtered input and output signals $i_f(t)$ and $u_f(t)$ are determined as follows:

$$i_f(t) = L(s)\tilde{i}(t), \quad u_f(t) = L(s)\tilde{u}(t). \quad (19)$$

Using the filtered input and output signals, Equation (17) can be reformulated as:

$$i_f(t) = \varphi_f(t)\bar{\theta} + \varepsilon_f(t). \quad (20)$$

Equation (20), in discrete time, is described as:

$$i_f(t_k) = \varphi_f(t_k)\bar{\theta} + \varepsilon_f(t_k). \quad (21)$$

It is assumed that the fractional order is already known; our goal is to estimate only the fractional differential equation coefficients. We will use generalized total least squares for this. The solving of generalized total least squares is reduced to finding the minimum of the objective function:

$$\min_{\bar{\theta}} \frac{\|\Phi_f \bar{\theta} - I_f\|^2}{\sigma_f^2 + \bar{\theta}^T W \bar{\theta}}, \quad (22)$$

where $I_f = \begin{pmatrix} i_f(t_1) \\ \vdots \\ i_f(t_n) \end{pmatrix}$, $\Phi_f = \begin{pmatrix} \varphi_f(t_1) \\ \vdots \\ \varphi_f(t_N) \end{pmatrix}$, $\|\cdot\| = \|\cdot\|_2$ is the Euclidian norm, W is the

diagonal matrix of noise variances, and $\sigma_f^2 = \frac{1}{N-1} \sum_{i=1}^N (i_f(t_i) - \bar{i}_f)^2$, where $\bar{i}_f = \frac{1}{N} \sum_{i=1}^N i_f(t_i)$.

Total least squares regression assumes that the noise variance is the same in all columns and in the right side. This assumption is not satisfied for Equation (22). Calculating the exact value of the noise variances of each column is a very difficult task. We will assume that the use of the SVF filter makes it possible to achieve an approximately equal signal-to-noise ratio in each column. Then, normalization will make it possible to obtain approximately equal noise variances in each column. The standard deviations for each column can be defined as:

$$\sigma_f^{(j)} = \frac{1}{N-1} \sqrt{\sum_{k=1}^N (\varphi_f^{(j)}(t_k) - \bar{\varphi}_f^{(j)})^2}, \quad (23)$$

where $\bar{\varphi}_f^{(j)} = \frac{1}{N} \sum_{k=1}^N \varphi_f^{(j)}(t_k)$, $\varphi_f^{(j)}$ is j -th column of the matrix Φ_f .

The generalized total least squares problem (22) can be reduced to the total least squares problem [30]:

$$\min_{\bar{\theta}_n} \frac{\|\Phi_{fn} \bar{\theta}_n - I_f\|^2}{1 + \|\bar{\theta}_n\|^2}, \quad (24)$$

where $\Phi_{fn}^{(j)} = \Phi_f^{(j)} \frac{\sigma_I}{\sigma_\phi^{(j)}} \bar{\theta}_n = \frac{\sigma_I}{\sigma_\phi^{(j)}} \bar{\theta}_n$.

The minimum of function (24) can be found as a solution to the biased normal system of equations [31]:

$$(\Phi_{fn}^T \Phi_{fn} - \sigma^2 E) \hat{\theta}_n = \Phi_{fn}^T I_f. \quad (25)$$

$$\hat{\theta}_n = (\Phi_{fn}^T \Phi_{fn} - \sigma^2 E)^{-1} \Phi_{fn}^T I_f, \quad (26)$$

where σ is the minimal singular values of matrices (Φ_{fn}, E_f) and E is the identity matrix.

An augmented symmetric system of equations used to solve total least squares (16) [32] can be expressed as follows:

$$\begin{pmatrix} \sigma E & \Phi_{fn} \\ \Phi_{fn}^T & \sigma E \end{pmatrix} \begin{pmatrix} \sigma^{-1} \varepsilon_{fn} \\ \bar{\theta}_n \end{pmatrix} = \begin{pmatrix} I_f \\ 0 \end{pmatrix}. \quad (27)$$

An inverse change of variable can be performed as follows: $\hat{\theta} = \frac{\sigma_I}{\sigma_\phi^{(j)}} \hat{\bar{\theta}}_n$.

Let us determine the parameter estimates $\hat{\theta}$ from the estimates $\hat{\bar{\theta}}$. For impedance model (2), this would be carried out as follows:

$$\hat{R}_s = \frac{1}{\hat{\bar{\theta}}_6}, \hat{b}_0 = \frac{\hat{\bar{\theta}}_4}{\hat{\bar{\theta}}_1}, \hat{a}_0 = \frac{\hat{\bar{\theta}}_2}{\hat{\bar{\theta}}_1} - \hat{R}_s \hat{b}_0, \hat{L}_m = \hat{\bar{\theta}}_1 \hat{R}_s \hat{b}_0, \quad (28)$$

For impedance model (3), this would be carried out as follows:

$$\hat{R}_s = \frac{1}{\hat{\bar{\theta}}_{10}}, \hat{b}_0 = \frac{\hat{\bar{\theta}}_8}{\hat{\bar{\theta}}_1}, \hat{a}_0 = \frac{1}{\hat{\bar{\theta}}_6 \hat{R}_s}, \hat{a}_1 = \frac{\hat{\bar{\theta}}_5 \hat{a}_0}{\hat{\bar{\theta}}_1}, \hat{L}_m = \frac{\hat{\bar{\theta}}_8 \hat{R}_s \hat{a}_0}{\hat{b}_0}, \hat{b}_1 = \frac{\hat{R}_s \hat{a}_0 - \hat{L}_m \hat{a}_1}{\hat{L}_m \hat{R}_s}. \quad (29)$$

For impedance model (4), this would be carried out as follows:

$$\hat{R}_s = \frac{1}{\hat{\bar{\theta}}_7}, \hat{L}_m = \frac{\hat{\bar{\theta}}_2}{\hat{\bar{\theta}}_6}, \hat{L}_r = \frac{1}{(\hat{\bar{\theta}}_5 / \hat{\bar{\theta}}_2 - 1 / \hat{L}_m)}, \hat{R}_r = \frac{\hat{L}_m + \hat{L}_r}{(\hat{\bar{\theta}}_5 \hat{R}_s)}, \hat{L}_d = \frac{\hat{\bar{\theta}}_2 \hat{R}_s \hat{R}_r}{\hat{L}_m}. \quad (30)$$

If the order of differentiation is unknown, as is often the case in practice, order estimation must be considered along with transfer function coefficient estimation. The use of generalized total least squares is possible when the fractional order is known a priori. This section describes an algorithm for extending the identification method presented above for the case when the order of differentiation is unknown. The algorithm is based on a combination of a generalized least squares method for estimating coefficients and a nonlinear algorithm for optimizing the order of differentiation. The parameter identification problem is presented as a functional minimization. Therefore, the main goal of this approach is to reduce the residual error with respect to α .

$$J(\alpha) = \min_{\alpha} \left(I_f - \frac{\sigma_I}{\sigma_\phi^{(j)}} \Phi_{fn}(\alpha) (\Phi_{fn}^T(\alpha) \Phi_{fn}(\alpha) - \sigma^2 E)^{-1} \Phi_{fn}^T(\alpha) I_f \right)^2. \quad (31)$$

The objective function (31) depends on one parameter. From a priori knowledge, it follows that $\alpha \in [0.7 \ 0.9]$ for the impedance model (2) and $\alpha \in [0.4 \ 0.6]$ for models (2) and (3). The minimum of a function can be found by one of the standard methods for optimizing a function of one variable.

3. Simulation Results

MATLAB was used for simulation. Test cases were compared to the normalized root-mean-square error (NRMSE) of parameter estimation, defined as:

$$\delta \hat{\theta}_i = \sqrt{\frac{(\hat{\theta}_i - \bar{\theta}_i)^2}{\bar{\theta}_i^2}} \cdot 100\%, \quad (32)$$

$$\delta \hat{\theta}_i = \sqrt{\frac{(\hat{\theta}_i - \theta_i)^2}{\theta_i^2}} \cdot 100\%. \quad (33)$$

The parameter estimates obtained on the basis of GTLS from objective functions (22) and (31) were compared with LS estimates defined as:

$$\hat{\theta}_{LS} = \left(\Phi_f^T \Phi_f \right)^{-1} \Phi_f^T I_f, \quad (34)$$

$$J_{LS}(\alpha) = \min_{\alpha} \left(I_f - \Phi_f(\alpha) \left(\Phi_f^T(\alpha) \Phi_f(\alpha) \right)^{-1} \Phi_f^T(\alpha) I_f \right)^2. \quad (35)$$

The FOMCON toolbox [33] was used to model the fractional order of the transfer functions. The sample period was set at $h = 0.0002$ s. The number of data points N in each simulation was 10,000.

Example 1. The impedance model $Z_r(s)$ is described by Equation (2). The true parameter vector is:

$$\theta = (9.52 \quad 0.53 \quad 57.04 \quad 17.04 \quad 0.8)^T, \quad (36)$$

The conductivity function $Y(s)$, is defined as:

$$Y(s) = \frac{1}{Z(s)} = \frac{9.0312s + s^{0.8} + 57.03}{0.53s^{1.8} + 116.2029s + 9.52s^{0.8} + 542.9256}. \quad (37)$$

The ratio of standard deviations “signal-noise ratio” is $SNR = \sigma_u / \sigma_{eu} = \sigma_i / \sigma_{ei} = 10^2$.

Figure 3 shows the Bode diagram of the system defined by (37).

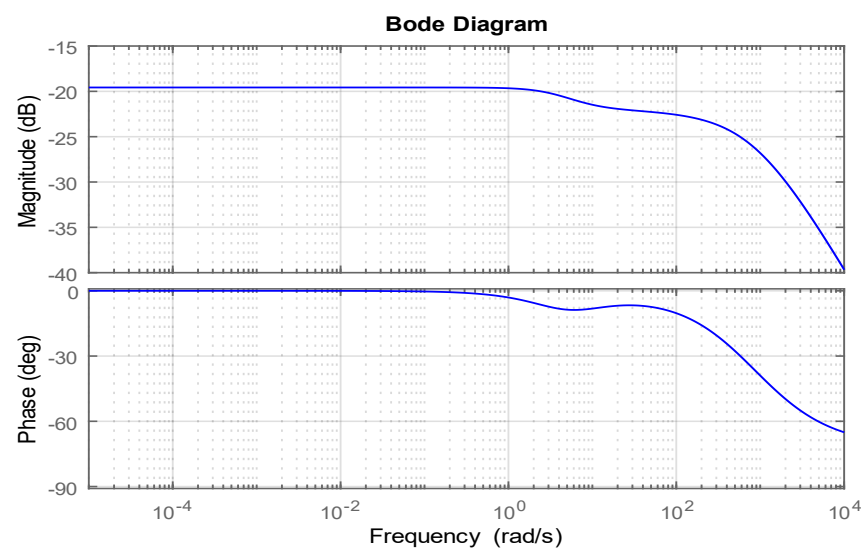


Figure 3. The bode diagrams of the system (37).

Selection of the SVF filters are chosen as:

$$L(s) = \left(\frac{97}{s + 97} \right)^4, \quad (38)$$

Figure 4 shows the objective function (35) of the system (37).

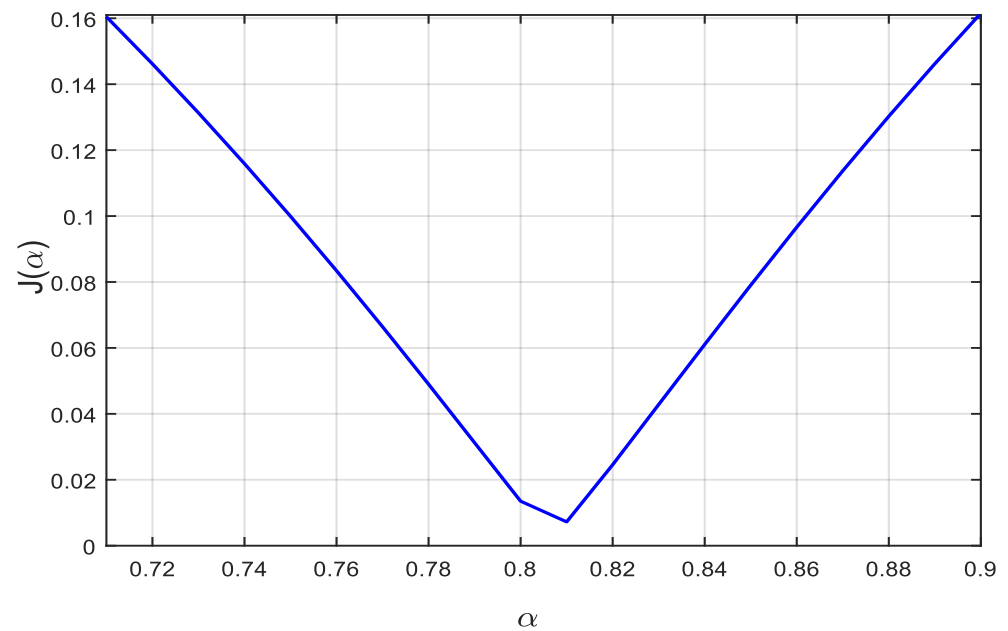


Figure 4. $J_{LS}(\alpha)$ function of the system (37).

Figure 5 shows the objective function (31) of the system (37).

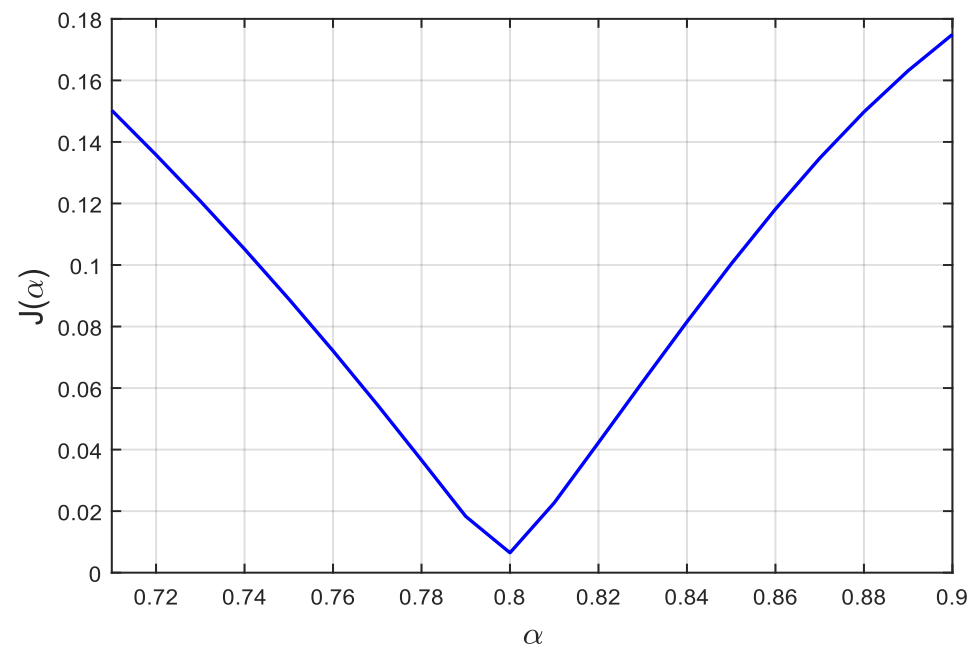


Figure 5. $J(\alpha)$ function of the system (37).

Table 1 shows parameter estimates $\hat{\theta}$ and NRMSE (SNR = 100).

Table 1. The parameter estimates $\hat{\theta}$ and NRMSE (SNR = 100).

$\bar{\theta}$	$\hat{\theta}_{LS}$	$\delta\hat{\theta}_{LS},\%$	$\hat{\theta}_{GTLS}$	$\delta\hat{\theta}_{GTLS},\%$
9.7619×10^{-4}	9.0069×10^{-4}	7.7348	9.9152×10^{-4}	1.5701
0.2140	0.2111	1.3780	0.2207	3.1084
0.0175	0.0183	4.4481	1.7890×10^{-4}	98.9797
0.0166	0.0162	2.5177	0.173	3.8664
0.0018	0.0021	16.6910	3.0271×10^{-4}	83.5653
0.1050	0.1047	0.3104	0.1047	0.3398

Table 2 shows parameter estimates $\hat{\theta}$ and NRMSE (SNR = 100).

Table 2. The parameter estimates $\hat{\theta}$ and NRMSE (SNR = 100).

	θ	$\hat{\theta}_{LS}$	$\delta\hat{\theta}_{LS},\%$	$\hat{\theta}_{GTLS}$	$\delta\hat{\theta}_{GTLS},\%$
R_s	9.52	9.5496	0.3114	9.5525	0.3409
L_m	0.53	0.5368	1.2863	0.5316	0.2928
a_0	57.03	62.4117	9.4336	56.1215	1.5930
b_0	17.04	5.6653	28.4394	17.4248	2.2584
α	0.8	0.8134	1.6750	0.8012	0.150

Example 2. The impedance model $Z_r(s)$ is described by Equation (3). The true parameter vector is

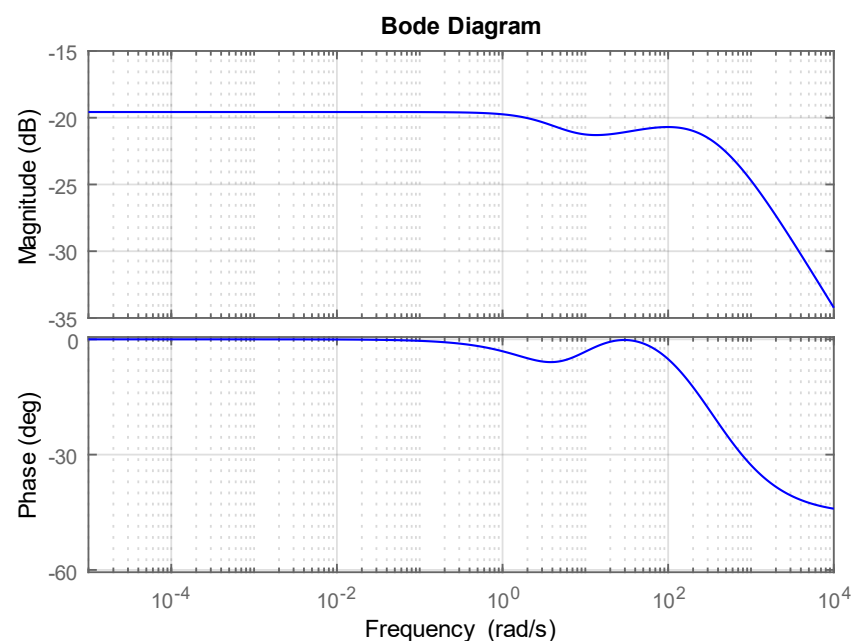
$$\theta = (9.52 \quad 0.53 \quad 57.04 \quad 9.11 \quad 17.04 \quad 0.12 \quad 0.45)^T, \quad (39)$$

The conductivity function $Y(s)$ is defined as

$$Y(s) = \frac{1}{Z(s)} = \frac{s^{1.45} + 9.0312s^{0.95} + 0.0636s + 9.11s^{0.45} + 57.03}{0.53s^{1.95} + 5.4338s^{1.45} + 9.52s^{0.95} + 116.2s + 86.727ss^{0.45} + 542.9256}, \quad (40)$$

The ratio of standard deviations “signal-noise ratio” is $SNR = \sigma_u / \sigma_{eu} = \sigma_i / \sigma_{ei} = 5 \times 10^4$.

Figure 6 shows the Bode diagram of the system defined by (40).

**Figure 6.** The bode diagrams of the system (40).

Selection of the SVF filters are chosen as:

$$L(s) = \left(\frac{510}{s + 510} \right)^4, \quad (41)$$

Table 3 shows parameter estimates $\hat{\theta}$ and NRMSE ($\text{SNR} = 5 \times 10^4$).

Table 3. The parameter estimates $\hat{\theta}$ and NRMSE ($\text{SNR} = 5 \cdot 10^4$).

$\bar{\theta}$	$\hat{\theta}_{LS}$	$\delta \hat{\theta}_{LS}$	$\hat{\theta}_{GTLs}$	$\delta \hat{\theta}_{GTLs}$
9.762×10^{-4}	9.457×10^{-4}	3.1195	9.769×10^{-4}	0.0708
0.0100	9.996×10^{-3}	0.4765	0.0100	0.0238
0.0175	0.0151	13.8781	0.0164	6.4449
0.2140	0.2151	0.5211	0.2151	0.4967
0.1597	0.1590	0.4185	0.1601	0.2315
1.841×10^{-3}	1.791×10^{-3}	2.7572	1.843×10^{-3}	0.7486
1.171×10^{-4}	-9.357×10^{-4}	898.76	3.695×10^{-4}	68.453
0.0166	0.0176	5.6452	0.0167	0.4351
0.0168	0.0167	0.2109	0.0168	0.02053
0.1050	0.1051	6.497×10^{-3}	0.1050	1.2708×10^{-3}

Figure 7 shows the objective function (35) of the system (40).

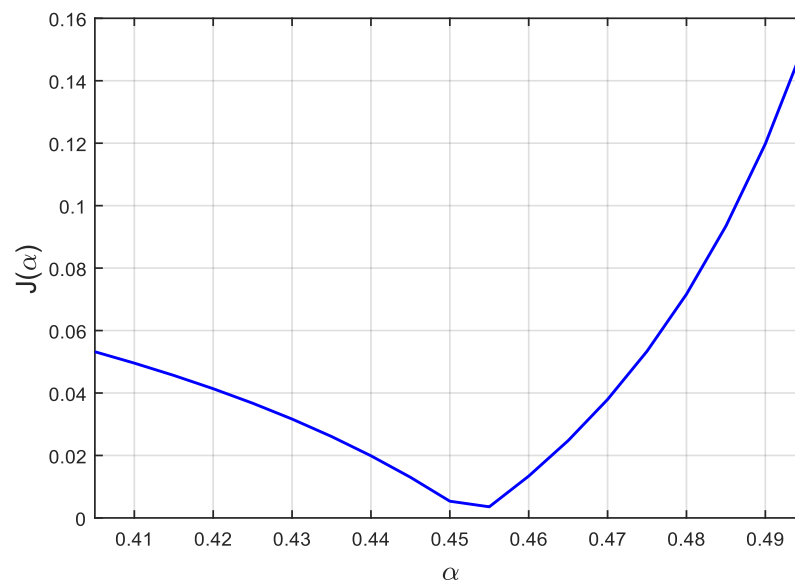


Figure 7. $J_{LS}(\alpha)$ function of the system (40).

Figure 8 shows the objective function (31) of the system (40).

Table 4 shows parameter estimates $\hat{\theta}$ and NRMSE ($\text{SNR} = 5 \times 10^4$).

Table 4. The parameter estimates $\hat{\theta}$ and NRMSE ($\text{SNR} = 5 \cdot 10^4$).

	θ	$\hat{\theta}_{LS}$	$\delta \hat{\theta}_{LS}$	$\hat{\theta}_{GTLs}$	$\delta \hat{\theta}_{GTLs}$
R_s	9.52	9.5206	0.0065	9.5199	0.0049
L_m	0.53	0.5280	0.3726	0.5301	0.0269
a_0	57.04	58.6432	2.8287	57.0057	0.0427
a_1	9.11	9.3285	2.3998	9.1272	0.1888
b_0	17.04	18.5816	9.0469	17.1035	0.3724
b_1	0.12	0.1264	5.3507	0.1172	2.3531
α	0.45	0.445	1.1111	0.449	0.2222

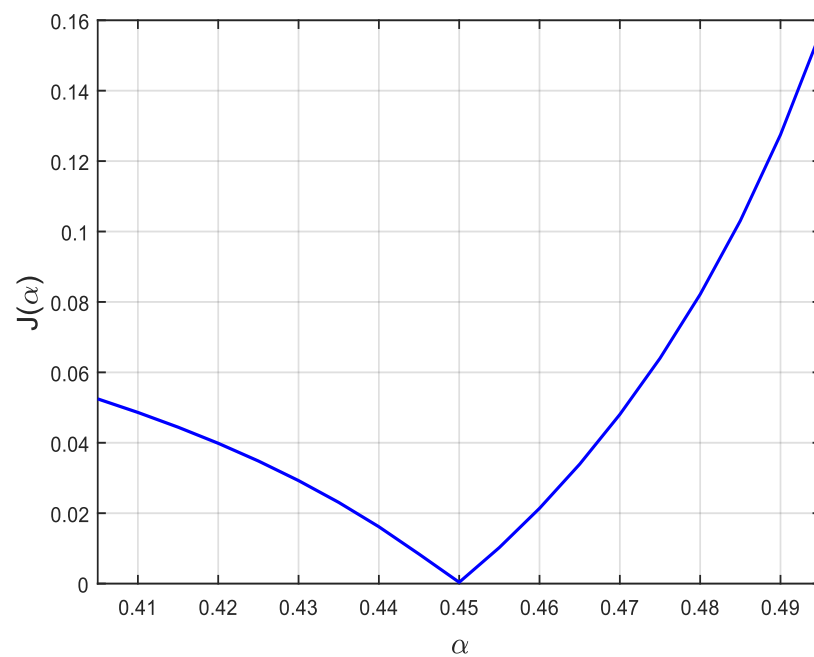


Figure 8. $J(\alpha)$ function of the system (40).

Example 3. The impedance model $Z_r(s)$ is described by Equation (3). The true parameter vector is

$$\theta = (9.52 \quad 0.53 \quad 0.85 \quad 0.0012 \quad 1.303 \quad 0.45)^T, \quad (42)$$

The conductivity function $Y(s)$ is defined as

$$Y(s) = \frac{1}{Z(s)} = \frac{0.5312s + 1.303s^{1.45} + 0.85}{6.36 \cdot 10^4 s^2 + 0.6906s^{1.45} + 5.5075s + 12.405s^{0.45} + 8.092'} \quad (43)$$

The ratio of standard deviations “signal-noise ratio” is $SNR = \sigma_u / \sigma_{eu} = \sigma_i / \sigma_{ei} = 10^3$.

Figure 9 shows the Bode diagram of the system defined by (43).

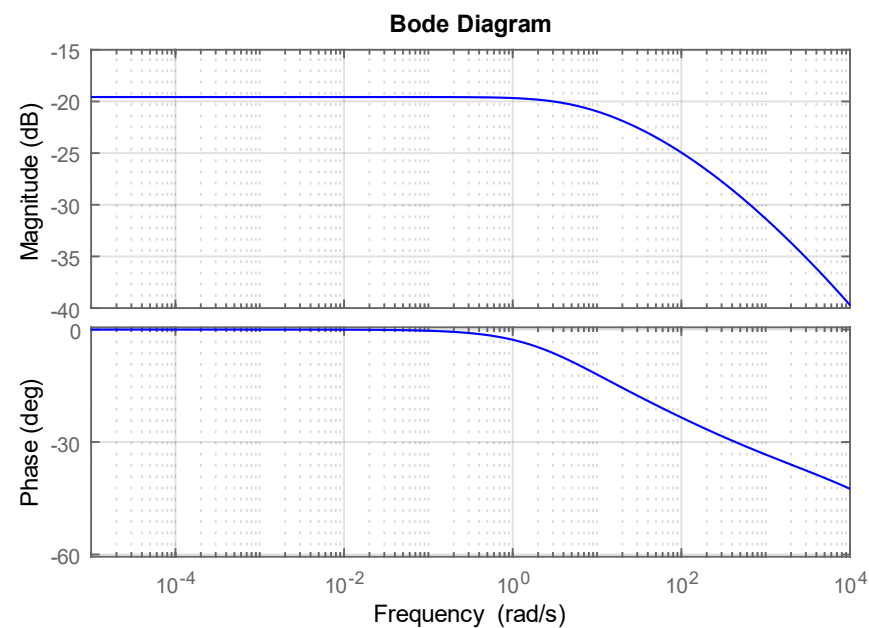


Figure 9. The bode diagrams of the system (43).

Selection of the SVF filters are chosen as:

$$L(s) = \left(\frac{30.7}{s + 30.7} \right)^4, \quad (44)$$

Figure 10 shows the objective function (35) of the system (43).

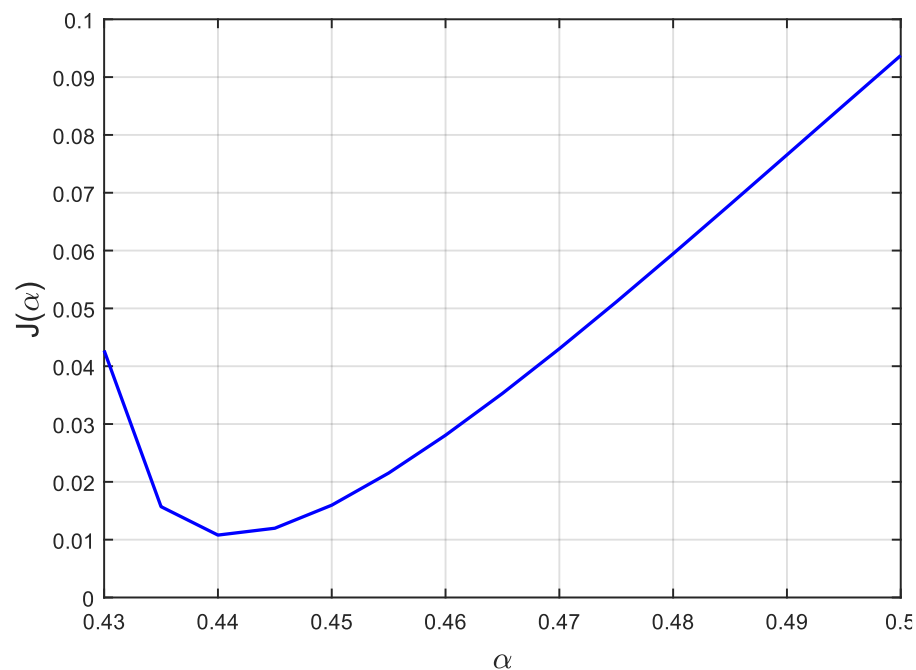


Figure 10. $J_{LS}(\alpha)$ function of the system (43).

Figure 11 shows the objective function (31) of the system (43).

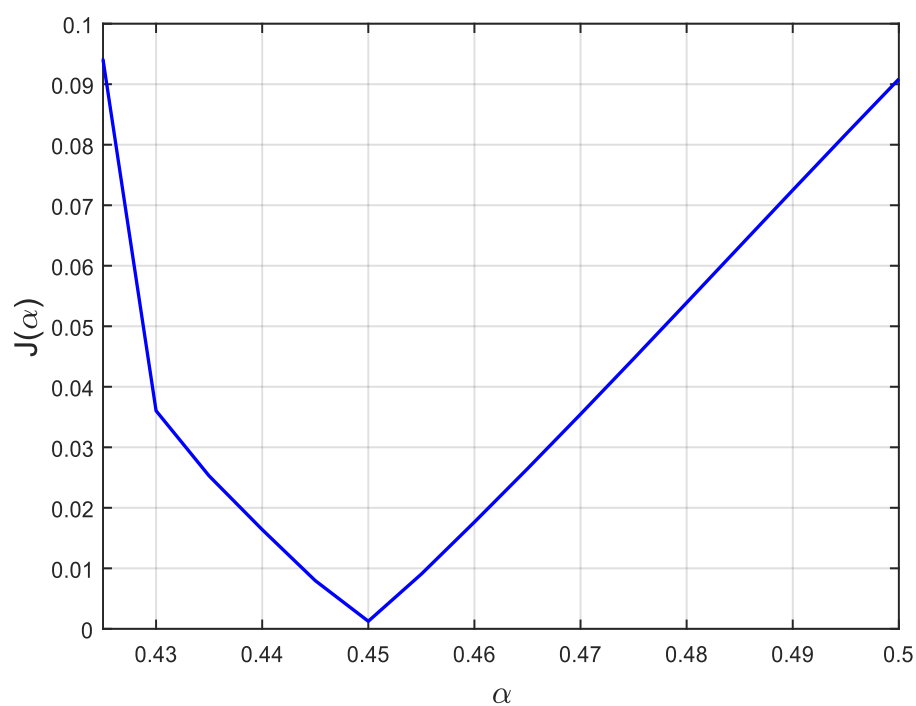


Figure 11. $J(\alpha)$ function of the system (43).

Table 5 shows parameter estimates $\hat{\theta}$ and NRMSE (SNR = 1000).

Table 5. The parameter estimates $\hat{\theta}$ and NRMSE (SNR = 1000).

$\bar{\theta}$	$\hat{\theta}_{LS}$	$\delta\hat{\theta}_{LS},\%$	$\hat{\theta}_{GTLs}$	$\delta\hat{\theta}_{GTLs},\%$
7.859×10^{-5}	2.358×10^{-4}	200.02	7.7041×10^{-5}	1.9783
0.0853	0.0890	4.2894	0.0851	0.2677
0.6806	0.710	4.3891	0.6810	0.0539
1.5329	1.7989	17.3489	1.5425	0.6201
0.0656	0.0669	1.9870	0.0655	0.1878
0.1610	0.1904	18.2696	0.1622	0.7486
0.1050	0.1044	0.5747	0.1049	0.1136

Table 6 shows parameter estimates $\hat{\theta}$ and NRMSE (SNR = 1000).

Table 6. The parameter estimates $\hat{\theta}$ and NRMSE (SNR = 1000).

	θ	$\hat{\theta}_{LS}$	$\delta\hat{\theta}_{LS},\%$	$\hat{\theta}_{GTLs}$	$\delta\hat{\theta}_{GTLs},\%$
R_s	9.52	9.5206	0.3584	9.5308	0.1137
L_m	0.53	0.4281	19.2259	0.5247	1.0108
R_r	0.85	1.2154	42.3944	0.8506	0.0744
L_r	0.0012	0.0015	28.4394	0.0012	0.9381
a_r	1.3030	1.5407	18.2424	1.3152	1.7956
α	0.45	0.442	1.1778	0.451	0.222

The results shown in Examples 1–3 show that the ordinary least squares method has more inaccurate estimates than the proposed modification of GTLS. Examples 1 and 3 illustrate how the presence of noise affects the appearance of a false fault. In example 1, the true resistance of the rotor and its estimates are $R_r = a_0/b_0 = 3.347$, $\hat{R}_{r_{ls}} = 11.007$, and $\hat{R}_{r_{gtls}} = 3.221$. In example 3, the resistance of the rotor and its estimates are $R_r = 0.85$, $\hat{R}_{r_{ls}} = 1.215$, and $\hat{R}_{r_{gtls}} = 0.8506$. The OLS ratings are highly inflated, which can be misinterpreted as broken rotor bars.

4. Discussion

The simulation results show that model (1) allows the identification of parameters at the lowest signal-to-noise ratio. The disadvantage of this model is poor accuracy at high frequencies. Model (2) is the most accurate of those considered in this study. However, it is very sensitive to noise. It should be noted that due to the presence of the first-order derivative and the derivative of the order $0.5 + \alpha \approx 1$ in the regression vector, the problem is ill-conditioned. Model (3) occupies an intermediate position between model (1) and model (2); it is less resistant to noise than (1), but it has an accuracy comparable to (2). For construction, the experimental model (3) is the most preferable.

5. Conclusions

This article is the first to consider the identification of three fractional-order models of an induction motor in the presence of errors in variables. Fractional-order models allow one to simulate the skin effect. Further development of identification methods can include the construction of algorithms based on regularized total least squares [34,35] and the use of implicit models of transfer functions [36,37].

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Data Availability Statement: The data was generated in Matlab. Simulation conditions are described in detail in the corresponding section.

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