Supplementary material

Variable energy fluxes and exact relations in Magnetohydrodynamics turbulence

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Computational work load in the simulation

We employ a pseudo-spectral parallel code named TARANG [1, 2] to solve the following equations numerically in a three-dimensional (3D) cubic periodic box of size $(2\pi)^3$:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\boldsymbol{\nabla})\,\mathbf{u} = -\boldsymbol{\nabla} p + \mathbf{F}_u(\mathbf{b},\mathbf{b}) + \nu\nabla^4\mathbf{u} + \mathbf{F}_{ext}, \tag{1}$$

$$\frac{\partial \mathbf{b}}{\partial t} + (\mathbf{u}\cdot\boldsymbol{\nabla})\,\mathbf{b} = \mathbf{F}_b(\mathbf{b},\mathbf{u}) + \eta\nabla^4\mathbf{b}, \tag{2}$$

$$\boldsymbol{\nabla}\cdot\mathbf{u} = 0, \tag{3}$$

$$\boldsymbol{\nabla}\cdot\mathbf{b} = 0, \tag{4}$$

where $\mathbf{u}$ and $\mathbf{b}$ are velocity and magnetic fields (normalised in velocity unit) respectively, p is the total pressure (a sum of kinetic and magnetic pressures); ν and η are respectively the hyperviscosity and hyperdiffusivity. In addition, the random large-scale force term is $\mathbf{F}_{ext}$, and $\mathbf{F}_u$, $\mathbf{F}_b$ respectively, represent the Lorentz force and the stretching of the magnetic field by the velocity field. We nondimensionalized the above equations using characteristic velocity (U_0), length (2π), and the eddy turn over time ($2\pi/U_0$). We use total 27 double precision arrays in TARANG to simulate these nondimensionalised equations on Cray XC40 (Shaheen II) of KAUST supercomputing laboratory, Saudi Arabia.

We performed simulation on a 512^3 grid with hyperviscous and hyperdiffusion parameters $\nu = \eta = 3\times10^{-7}$. We set random initial conditions for both velocity and magnetic fields (details about the random initial conditions are given in the main text). We use fourth-order Runge-Kutta scheme for time marching and optimize time steps using Courant-Friedrich-Lewis (CFL) condition. We performed two-dimensional decomposition (pencil) of our 3D data among the processors to execute time marching in parallel (as shown in Fig. 1). We took a total of 4096 processors on Cray XC40 for our simulation. Our data is divided such that the no. of divisions along X and Y direction are 64. Our simulation took a total 108 GB of RAM on these nodes. We performed simulation till 15 eddy turn over time and the total simulation time is 96 hours on the cluster. We summarize these details in Table 1. Note that TARANG uses FFTK to compute the nonlinear terms using FFT. FFTK shows very good computation and communication scaling on dragonfly topology of Cray XC40 [2]. In addition to these details, the time interval for saving output fields at a different time step is set to 1 eddy turn over time in our simulation. Our simulation took 6.4 GB and 4.2 GB space respectively in the hard disk to save these real and Fourier space velocity and magnetic field arrays.

Table 1: **Workload of the simulation: the grid-resolution N, the total number of processor p , data division along x direction p_{col}, data division along y direction p_{row}, total RAM used in GB M_{RAM}, simulation time t (eddy turnover time) and simulation ran time in hours T.**

N	p	p_{col}	p_{row}	M_{RAM}	t	T
512^3	4096	64	64	108	15	96

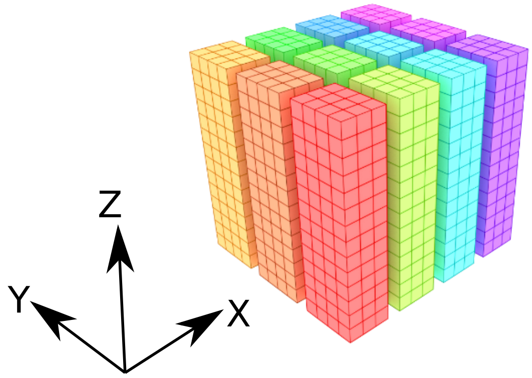

Fig 1: Schematic diagram for the pencil decomposition of a 3D array. From Chatterjee et al. [2]. Reprinted with permission from Elsevier.

References

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