A Swing of Beauty:

Pendulums, Fluids, Forces, and Computers!

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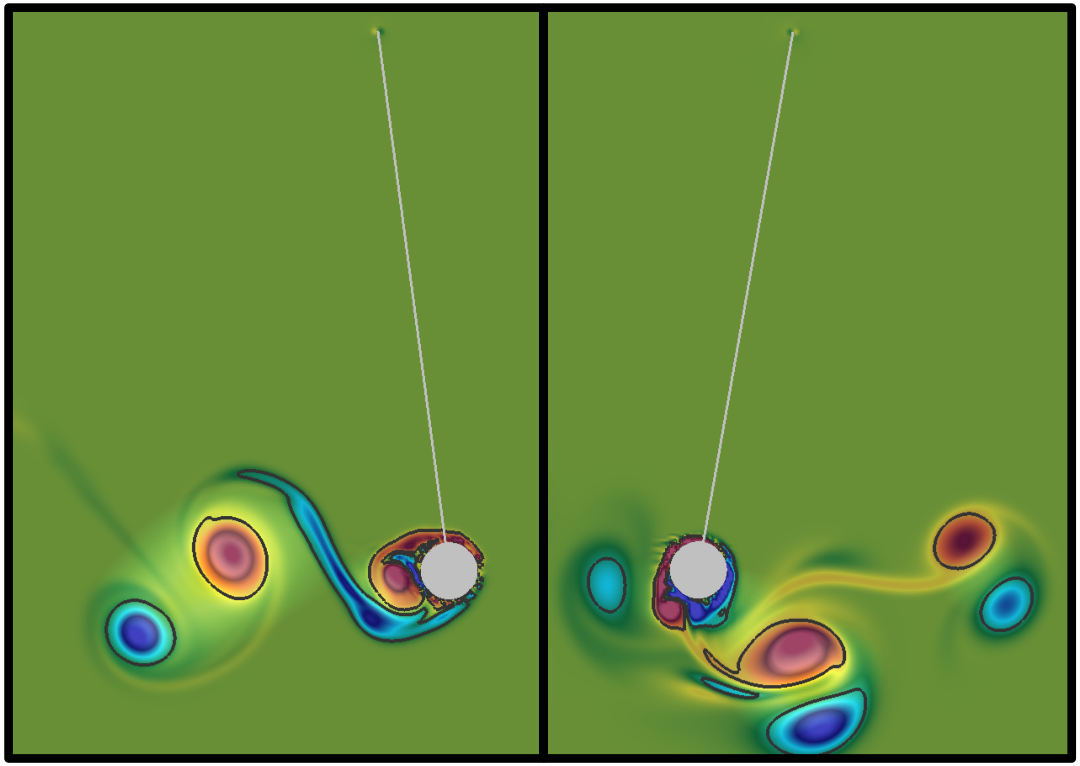
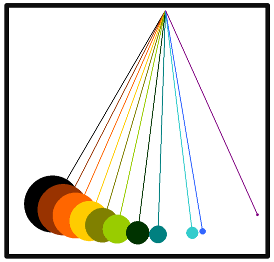
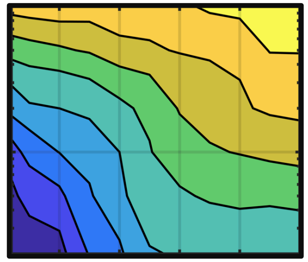
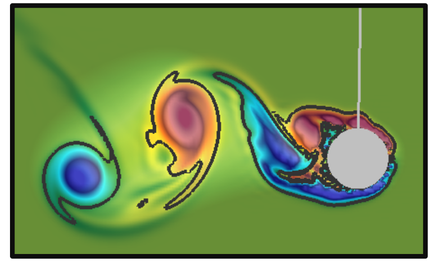
School of Science
The College of New Jersey

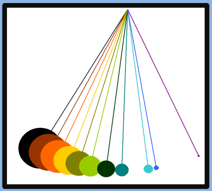
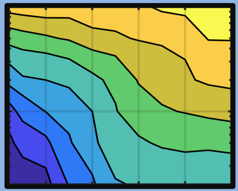
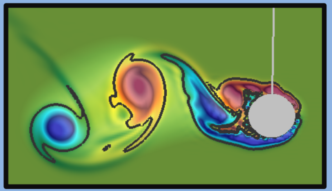


Outline:

- **A pendulum tale…**
  - **History & Reduced-Order Model**
  - Fluid-Structure Interaction (FSI) Model
  - Comparison and Validation

A Brief History…

Pendulums have been studied for millennia…

nature

News | Published: 12 April 1888

Pendulum Seismometers

JOHN MILNE

Nature **37**, 570–571(1888) | Cite this article

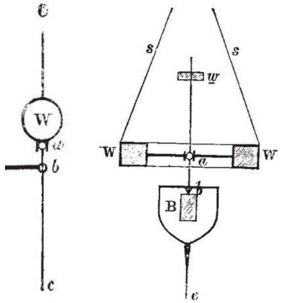


FIG. 1. FIG. 2.

Ideas dating back thousands of years!

Antiquity

Volume 92, Issue 361 February 2018 , pp. 217-232

Experimental stone-cutting with the Mycenaean pendulum saw

Nicholas G. Blackwell (a1)

How ancient civilizations may have built palaces thousands of years ago!

J Extra Corpor Technol. 2003 Sep;35(3):172-83.

History of extracorporeal circulation: the conceptional and developmental period.

Boettcher W[1], Merkle F, Weitkemper HH.

Pendulum Clocks dating back to the 1660s and continual improvements thereafter!

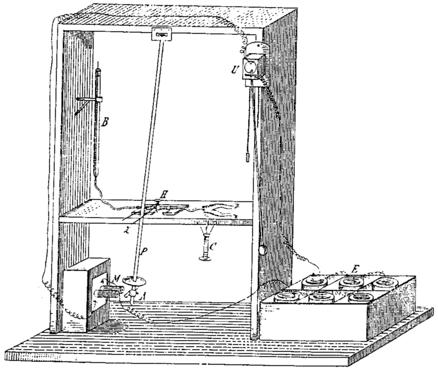
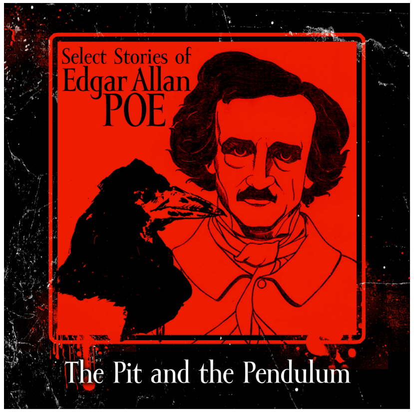


Figure 8. The "electrical pendular cock" developed by Hamel (1889). A pendulum (P), activated by a magnet (M), opens and closes a stop-cock (H), resulting in intermittent perfusion pressure.

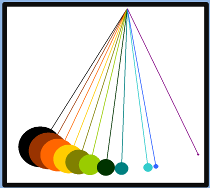
The Pit and the Pendulum Audiobook

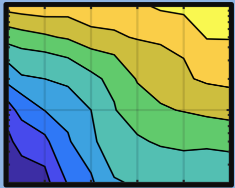
Author: Edgar Allan Poe
Narrator: Chris Lutkin
Publisher: Dreamscape

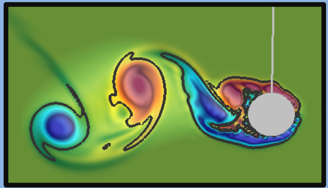
…and even in pop culture!



Even recently…







New complex dynamics of pendulums are continually being discovered…

Open . Published Online: 01 October 2007 Accepted: June 2007

Enhanced upswing in immersed collisions of tethered spheres

Physics of Fluids **19**, 101701 (2007); https://doi.org/10.1063/1.2771657

Hui-Chi Hsu *and* Hervé Capart

European Journal of Physics

PAPER

Dynamics of damped oscillations: physical pendulum

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Published 23 October 2017 • © 2017 European Physical Society

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A pendulum experiment on added mass and the principle of equivalence

American Journal of Physics **75**, 226 (2007); https://doi.org/10.1119/1.2360993

Douglas Neill Dean Livelybrooks *and* Russell J. Donnelly

ELSEVIER

Journal of Fluids and Structures

Volume 56, July 2015, Pages 124-133

Galloping instability and control of a rigid pendulum in a flowing soap film

Alessandro Orchini[a,1], Hamid Kellay[b], Andrea Mazzino[c,d]

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Phys. Rev. E **81**, 046317 – Published 26 April 2010

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Volume 862 10 March 2019 , pp. 348-363

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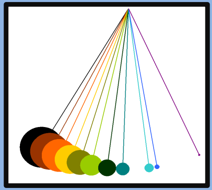

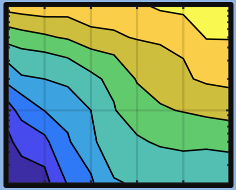

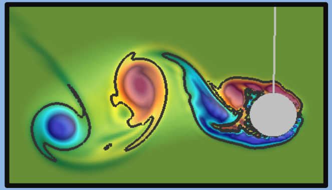

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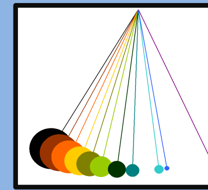

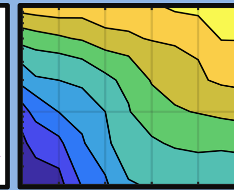

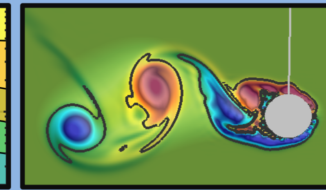

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A pendulum … principle of e…

American Journal of Physic…

Douglas Neill Dean Livelybroo…

They all study the *interactions* between the pendulum and the **fluid** environment it is immersed within!

Alessandro Orchini, Hamid Kellay, Andrea Mazzino

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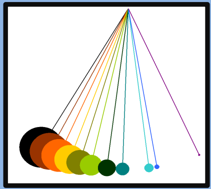

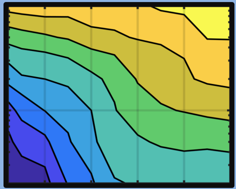

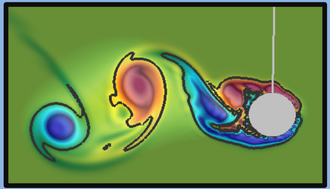

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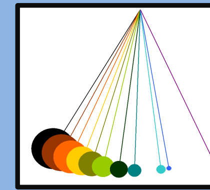
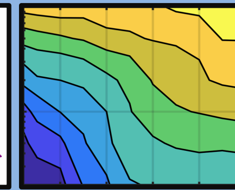
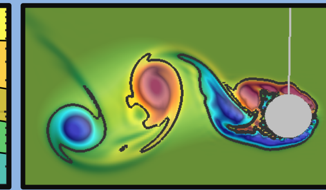
M. M. Bandi[1, a)], A. Concha[1, b)], R. Wood[1], *and* L. Mahadevan[1,2]

Let’s begin by recalling the ordinary differential equation model of a pendulum in a vacuum.

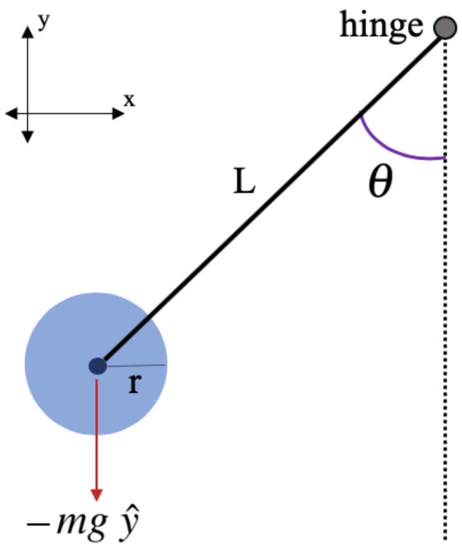
*Not a complete list.







Simple Gravity Pendulum (not in a fluid)

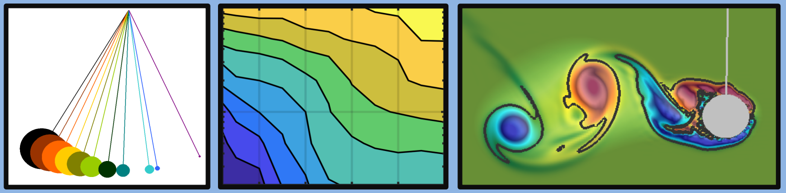


Initial angular displacement of a circular bob of radius, *r*, and mass, *m,* held at a pendulum of length, *L*

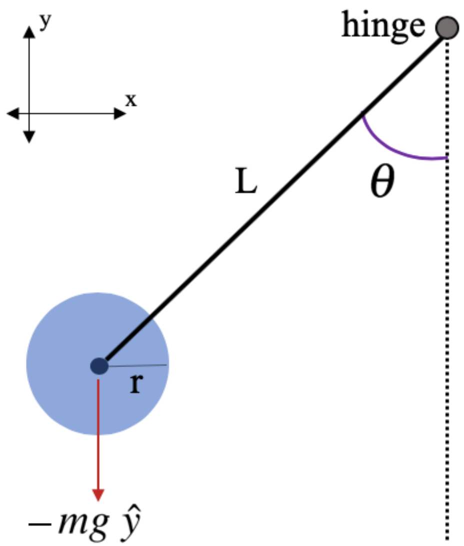
Ordinary Differential Equation Model of a **Simple Gravity Pendulum**

$$I\frac{d^2\theta}{dt^2} + mgL\sin\theta = 0$$

Simple Gravity Pendulum



Simple Gravity Pendulum (not in a fluid)



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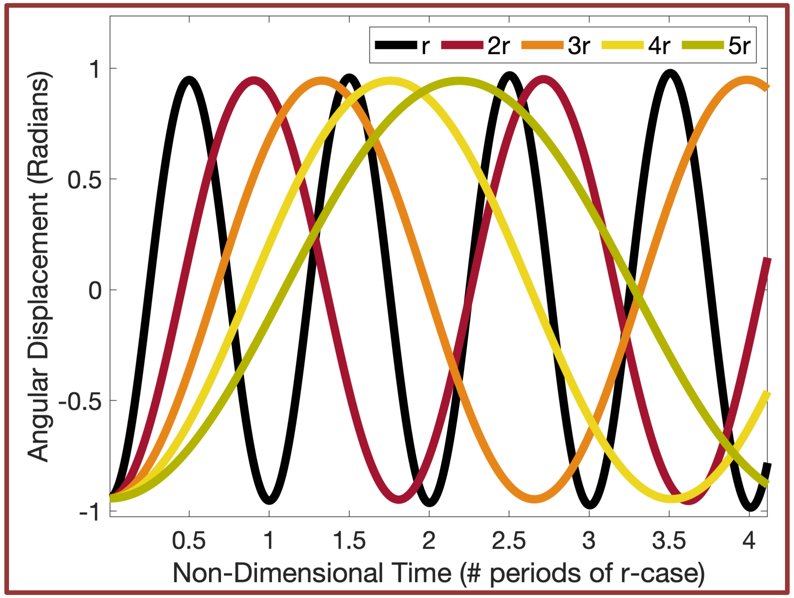
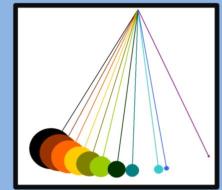
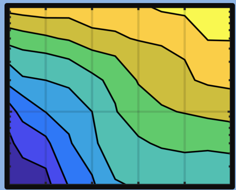
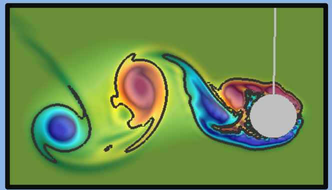
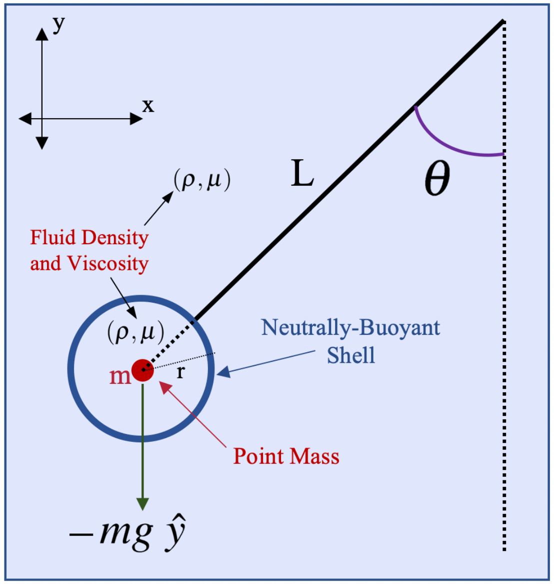


No loss of amplitude over time. Nothing to stop the pendulum from swinging.

Putting the Pendulum into a Fluid

There will be *drag*, fluid drag



Pendulum now immersed in a fluid!

Ordinary Differential Equation Model of a
Damped Pendulum

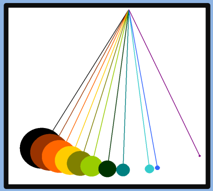
$$I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + mgL\sin\theta = 0$$

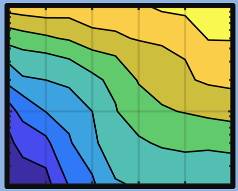
New
damping term

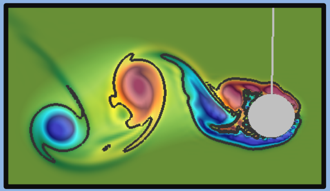
The **damping term**:

1. Slows down the pendulum’s motion
2. Has a “bulk” parameter, *b*, hiding all details about the fluid!

Putting the Pendulum into a Fluid







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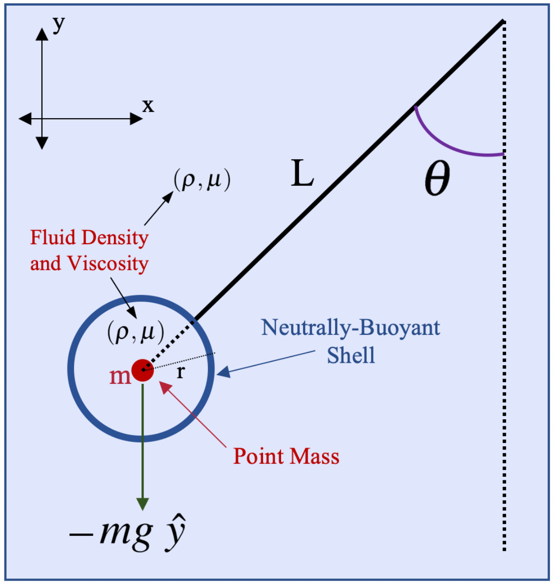

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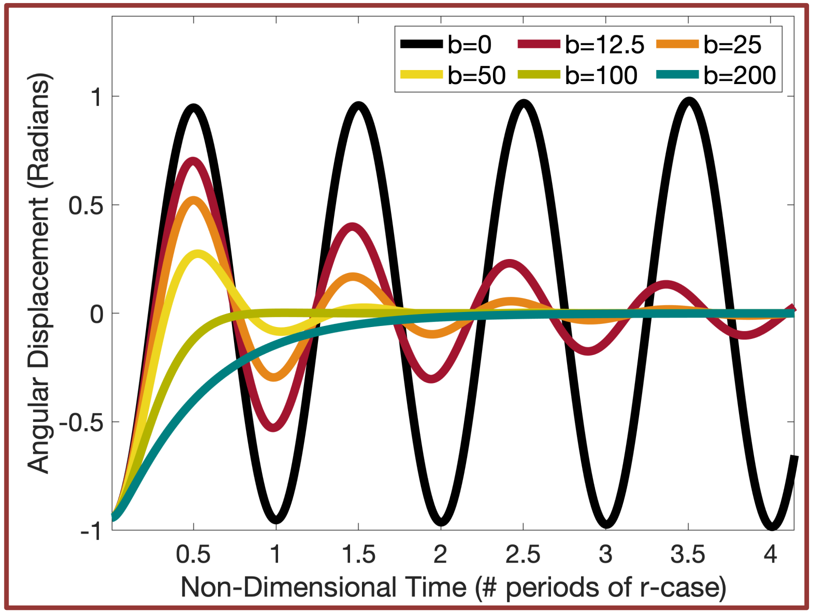
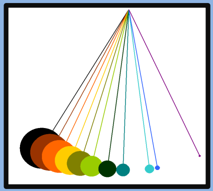
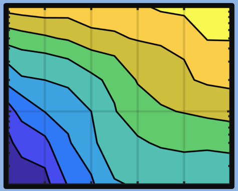
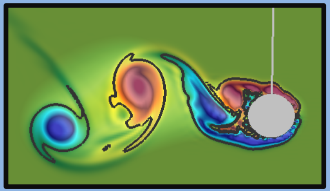


As damping increases (*'b' increases*), the oscillatory amplitude decreases

Putting the Pendulum into a Fluid

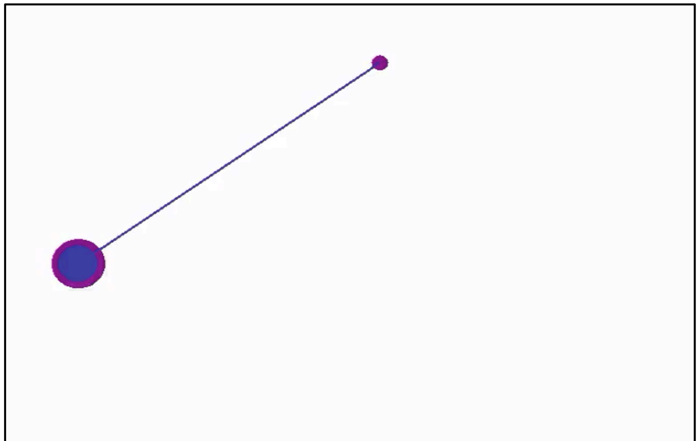




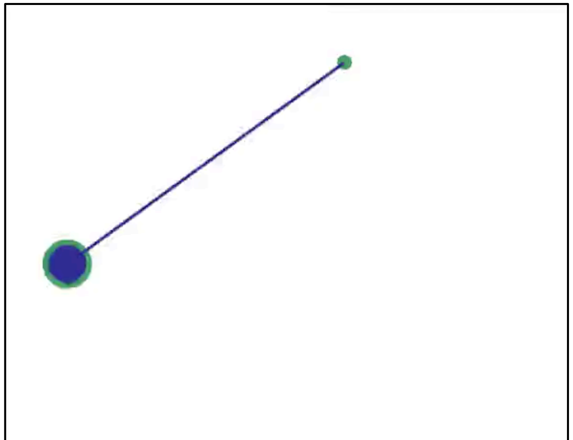


Damping can lead to 3 different behaviors:

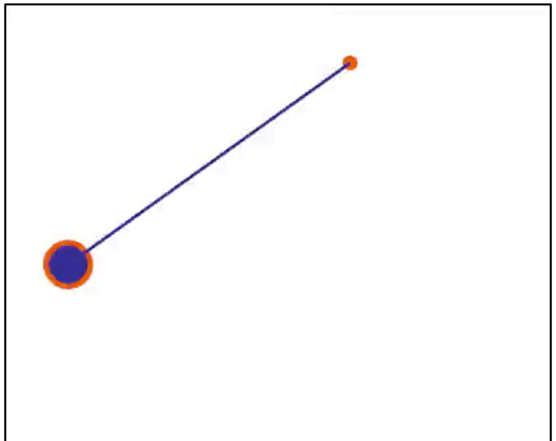
$$I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + mgL\sin\theta = 0$$



“UNDER-damped"

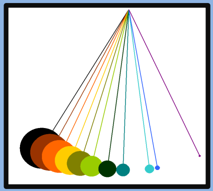


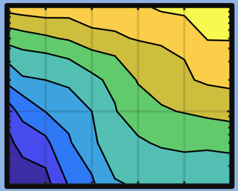
“CRITICALLY-damped"

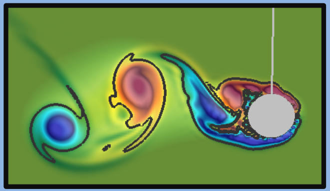


“OVER-damped"

Putting the Pendulum into a Fluid





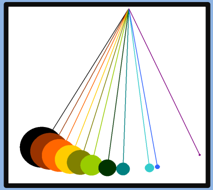


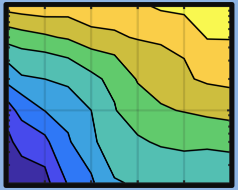
Why does damping this form?

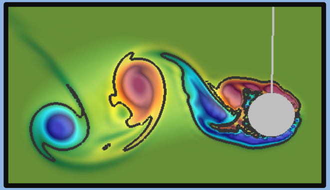
(e.g., why does the fluid's effect on the bob take this form?)

$$I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + mgL\sin\theta = 0$$

The fluid slows down the pendulum bob proportional to its angular velocity?







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The fluid slows down the pendulum bob proportional to its angular velocity?

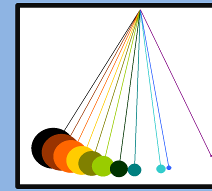
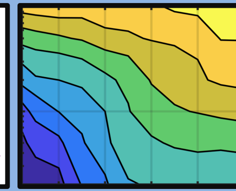
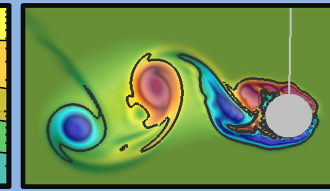
This goes back to:

Sir George Gabriel Stokes

Was kind of a big deal in fluid dynamics (…and mathematics and physics!)

https://commons.wikimedia.org/wiki/File:Ggstokes.jpg

Stokes and Pendulums

Stokes Drag Law first introduced in 1850:

X. *On the Effect of the Internal Friction of Fluids on the Motion of Pendulums.* *By* G. G. STOKES, M.A., *Fellow of Pembroke College, and Lucasian Professor of Mathematics in the University of Cambridge.*

[Read *December* 9, 1850.]

The drag force on a sphere

(at low Reynolds Numbers, Re)

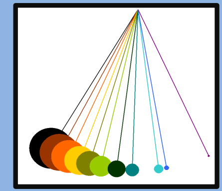
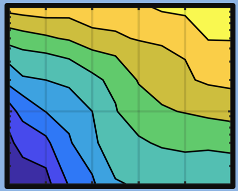
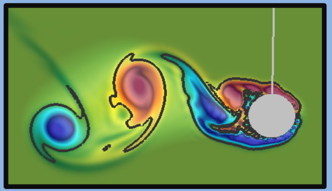
$$F_D = 6\pi\mu r v$$

(Drag force is proportional to velocity!)

Force Law Parameters:

- μ - Fluid's viscosity ('stickiness')
- r - Radius of sphere
- v - Speed of moving sphere

Wait, what is a Reynolds Number?

Reynolds Number:

Ratio of inertial forces to viscous forces in a fluid system

$$Re = \frac{\rho L U}{\mu}$$

can help categorize fluid-scale of the system allowing one to get an intuitive sense of the dynamics a priori

Fluid Parameters:

ρ - Fluid Density

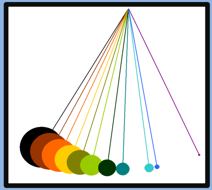
μ - Fluid (dynamic) Viscosity

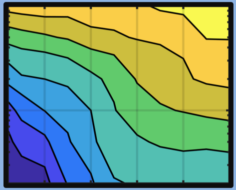
System Parameters:

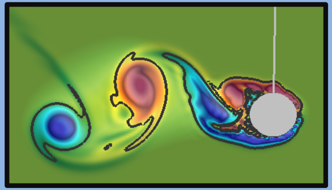
L - *Characteristic* Length-scale of system

U - *Characteristic* Speed-scale of system

Wait, what is a Reynolds Number?







Reynolds Number:

Ratio of inertial forces to viscous forces in a fluid system

$$Re = \frac{\rho L U}{\mu}$$

can help categorize fluid-scale of the system allowing one to get an intuitive sense of the dynamics a priori

Low Re (Re<<1)

Can occur when:

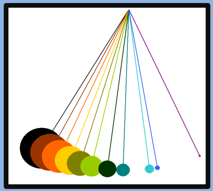
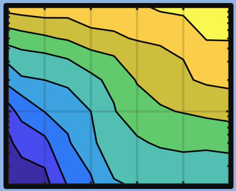
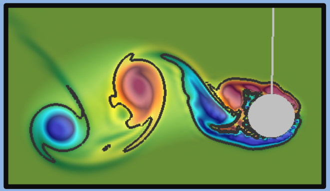
- Small length scales (like a cell)
- Very low speeds

High Re (Re>>1)

Can occur when:

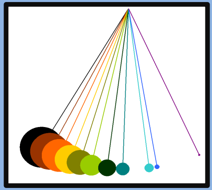
- Large length scales (like a fish)
- Very high speeds

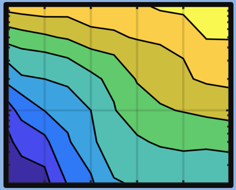
Is Stokes Drag valid for a Pendulum?

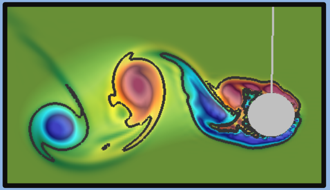




What if pendulum is <u>not</u> in a Low Reynolds Number setting?

Is Stokes Drag valid for a Pendulum?



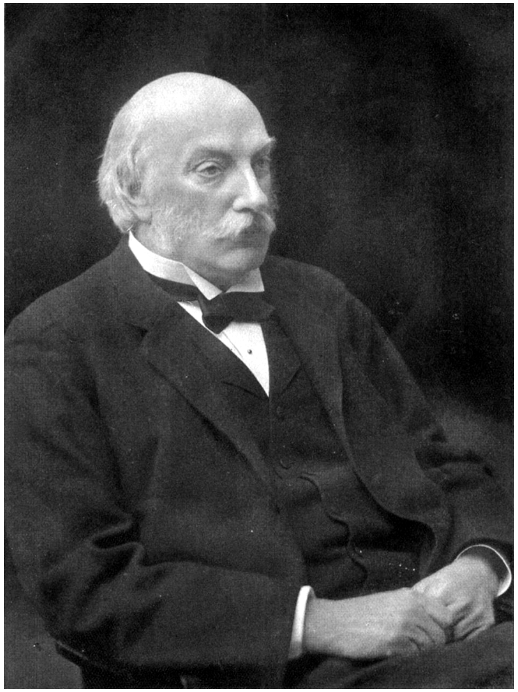




There's another drag law:

Lord Rayleigh's Drag Law

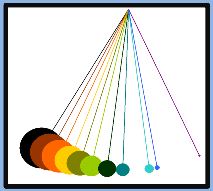
...and in comes former student on Sir G.G. Stokes!

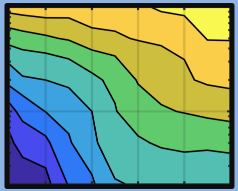


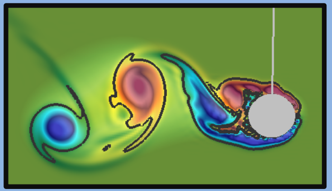
Lord Rayleigh (aka John William Strutt)

https://commons.wikimedia.org/wiki/File:John_William_Strutt.jpg

Is Stokes Drag valid for a Pendulum?







There's another drag law:

Lord Rayleigh's Drag Law

$$F_D = \frac{1}{2}\rho A C_D v^2$$

(Drag force is proportional to velocity-squared!)

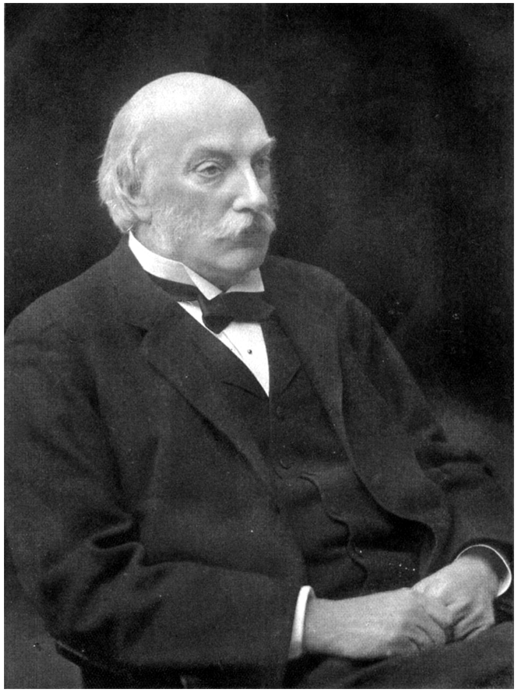
Force Law Parameters:

ρ - Fluid Density

A - Cross-sectional area of object in flow

v - Speed

C_D - Drag coefficient



Lord Rayleigh

(aka John William Strutt)

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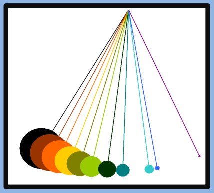
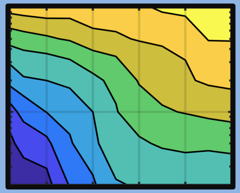
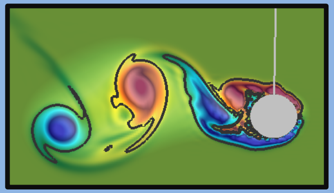
Is Stokes Drag valid for a Pendulum?

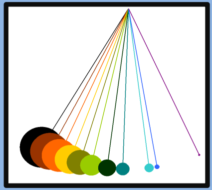
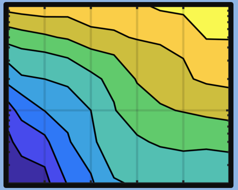
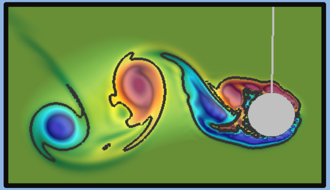
What about pendulums?

$$I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + mgL\sin\theta = 0$$

Questions:

- The damping term takes the form of Stokes Drag Law – is this valid? Why or why not Lord Rayleigh's?
- Maybe both are valid for pendulums in different situations?
- Would the mass and size of a pendulum bob affect these assumptions?

Turning to numerical simulation…

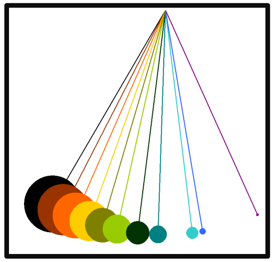
How to test this?

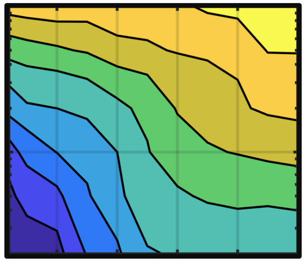
$$I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + mgL\sin\theta = 0$$

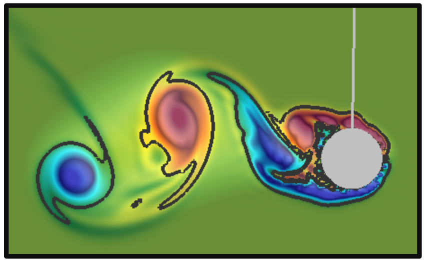
Note:

- Need experimental data to validate either drag force law in the above equation
- Could construct physical models of pendulums in air (or water)…
- But, unless we use sophisticated flow visualization techniques (like particle image velociometry for example), we wouldn't obtain details about how the fluid reacts to the pendulum bob's swinging motion (e.g., what the underlying fluid dynamics are)
- We can turn to computers, numerical simulation experiments, and fluid-structure interaction models to give us the details!

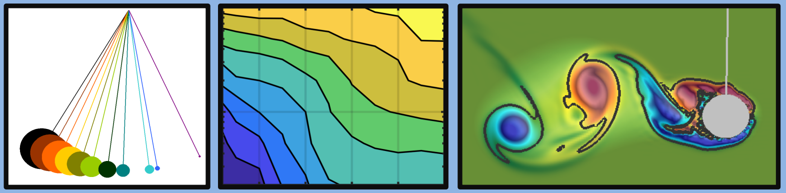
Outline:





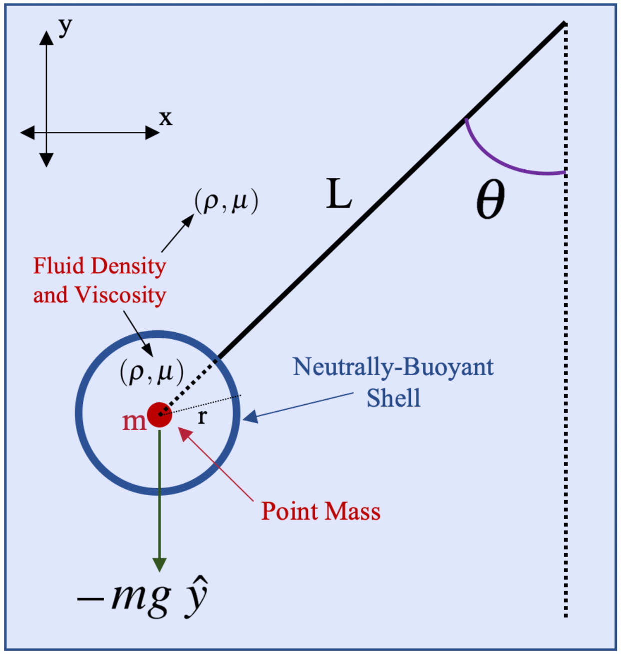


- **A pendulum tale…**
  - History & Reduced-Order Model
  - **Fluid-Structure Interaction (FSI) Model**
  - Comparison and Validation



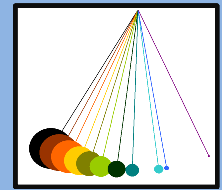
Computational Setup

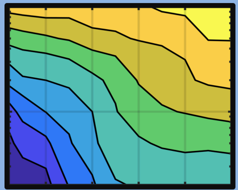
Idea:

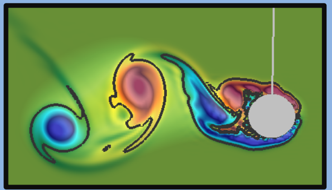


- Construct computational fluid dynamics (CFD) model using fluid-structure interaction (FSI) techniques using the *immersed boundary method*
- Create circular pendulum bobs, w/ mass concentrated at center of bob, of radius, r, and pendulum length, L
- Allow bob to freely swing under gravity thereby interacting with the fluid it is immersed within
- Beyond gravity's downward direction, do not prescribe any other force acting on the bob; fluid drag will happen naturally in the FSI framework

Fluid-Structure Interaction Model

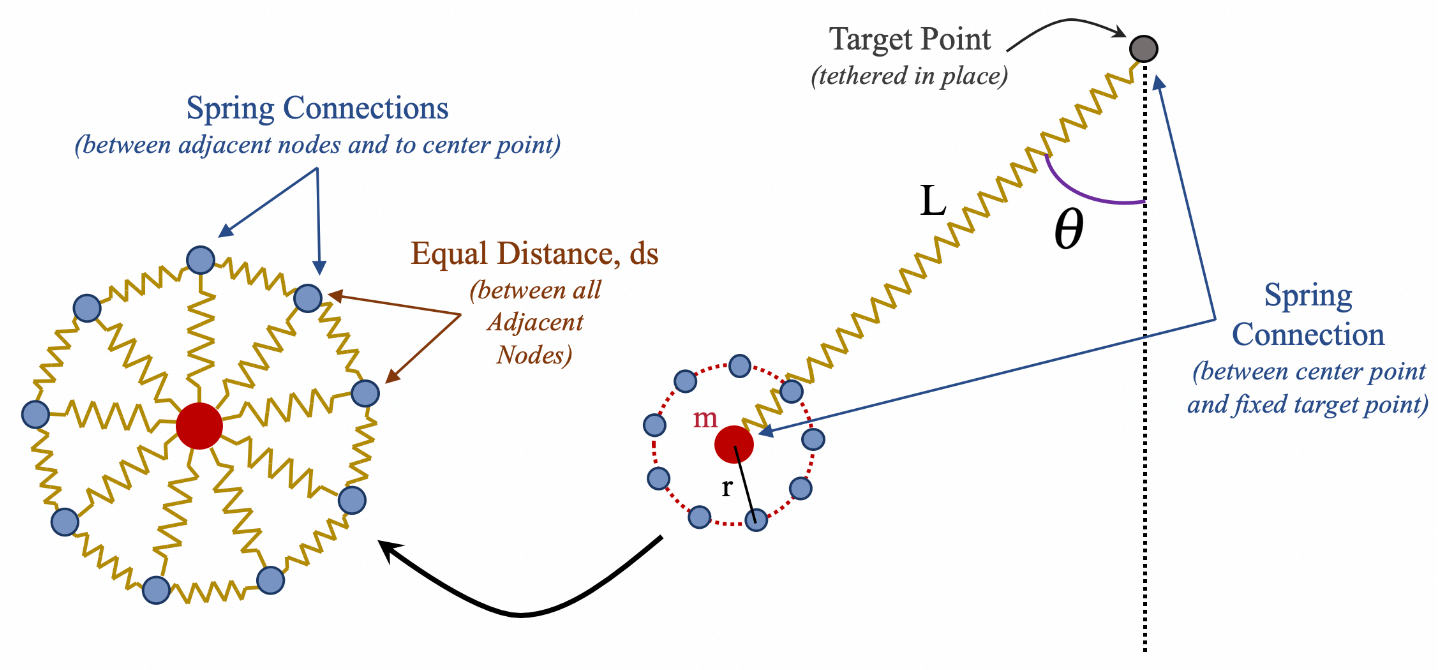






Computational Geometry

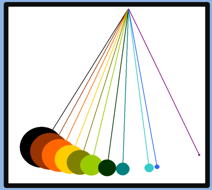
(how the discretized geometry and model is initiated)

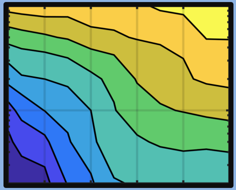


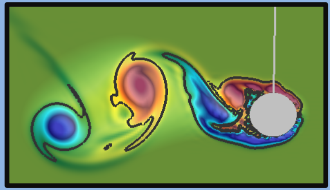
Key Ideas:

- Computers need discretized boundaries - they don't know real numbers!
- Can form boundaries in *immersed boundary* using springs and other ideas from foundational physics courses

FSI Pendulum Results

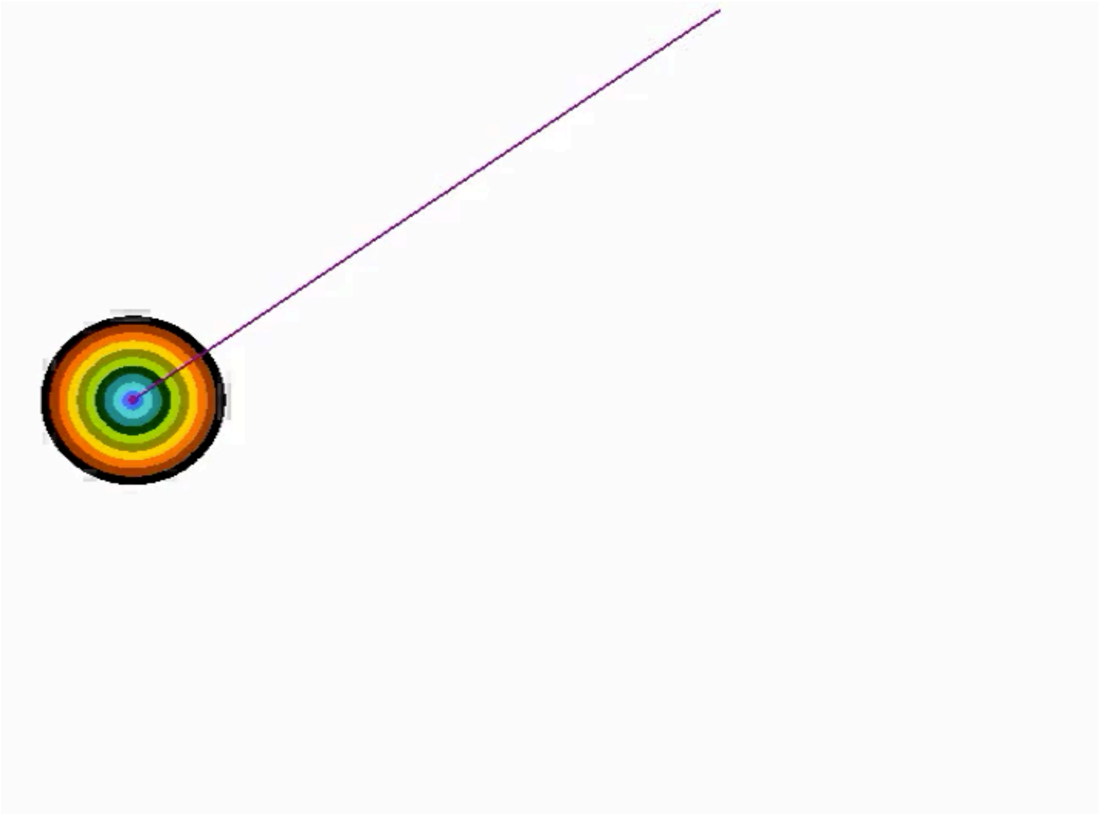






Lagrangian Data

(position and forces on the pendulum over time)



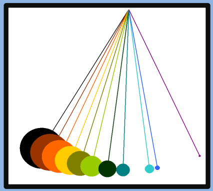
Each Pendulum Bob has:

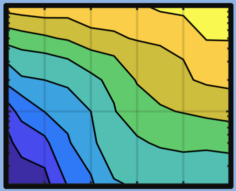
- Same point mass value at center
- Different radii

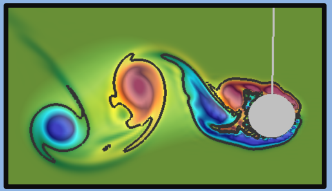
Note:

- Larger bobs are slower
- Larger bobs have less angular displacement
- **Does this mean more drag?**

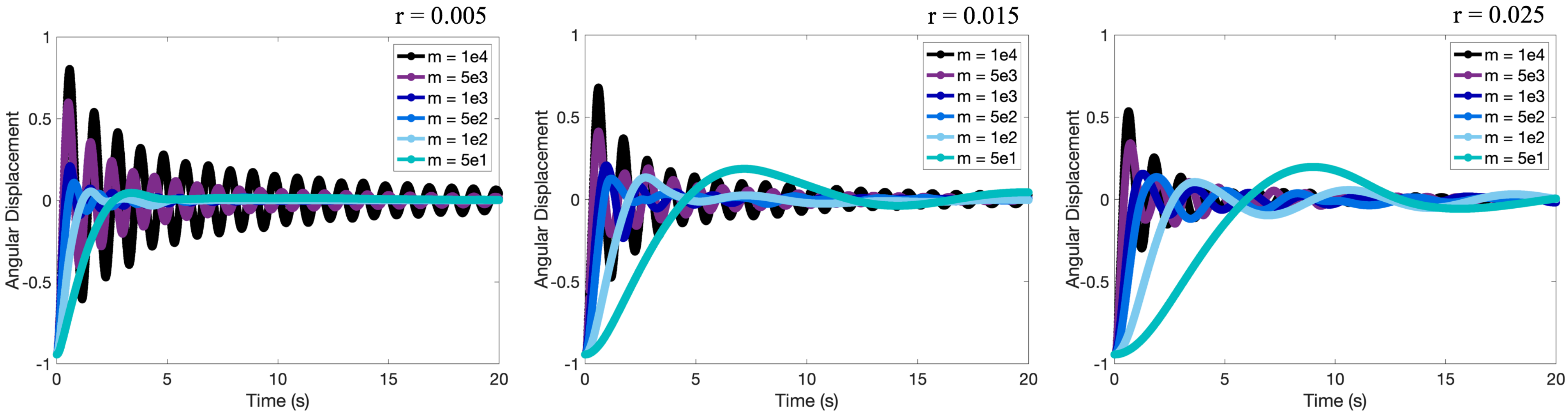
FSI Pendulum Results: Angular Displacement







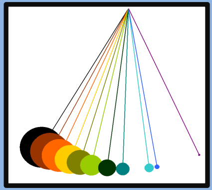
Angular Displacement (radians) vs. Time (s)

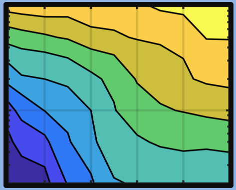


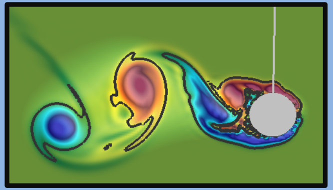
Note:

- Both the mass and radii of the bob affect its angular displacement over time
- Same radii bob, could display different damping behavior depending on its mass.

FSI Pendulum Results: Angular Displacement

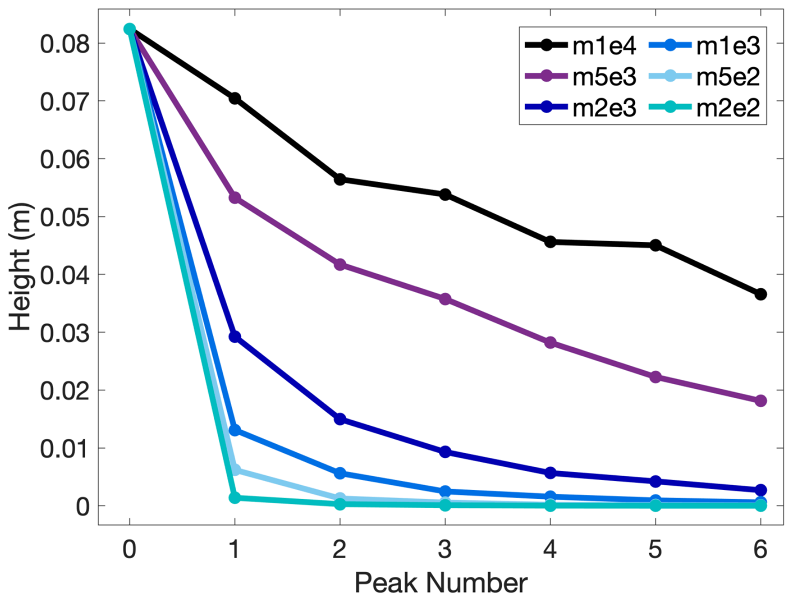


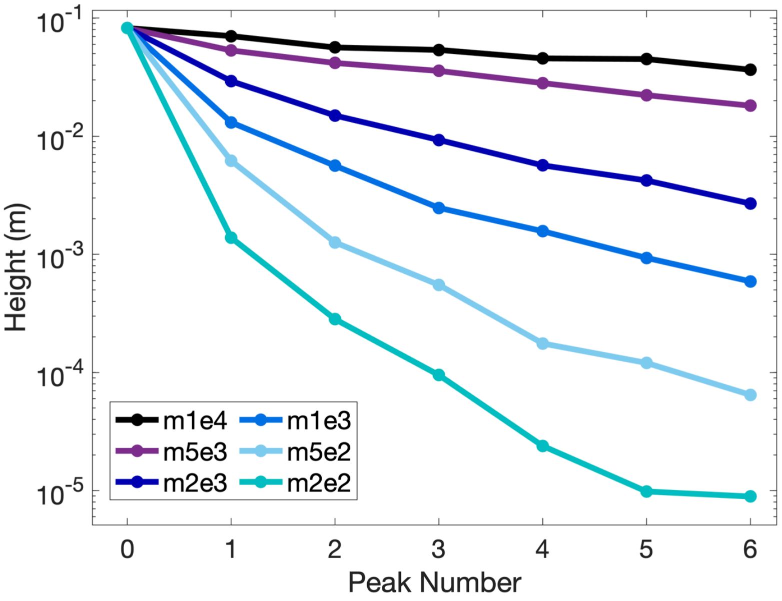




Height at peak displacement (m) vs. Peak Number

(radius of bob = 0.005m)

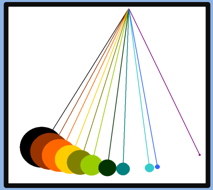


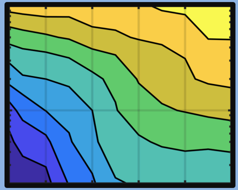


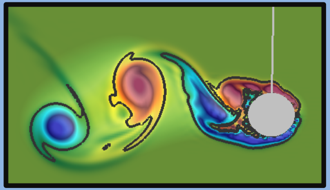
Note:

- Higher masses maintain higher amplitude oscillations and hence heights for a given radii
- Semi-logarithmic plot illustrates linear relationship between log(height) and peak number -> **exponential relationship** after a few swings!

FSI Pendulum Results: Angular Displacement

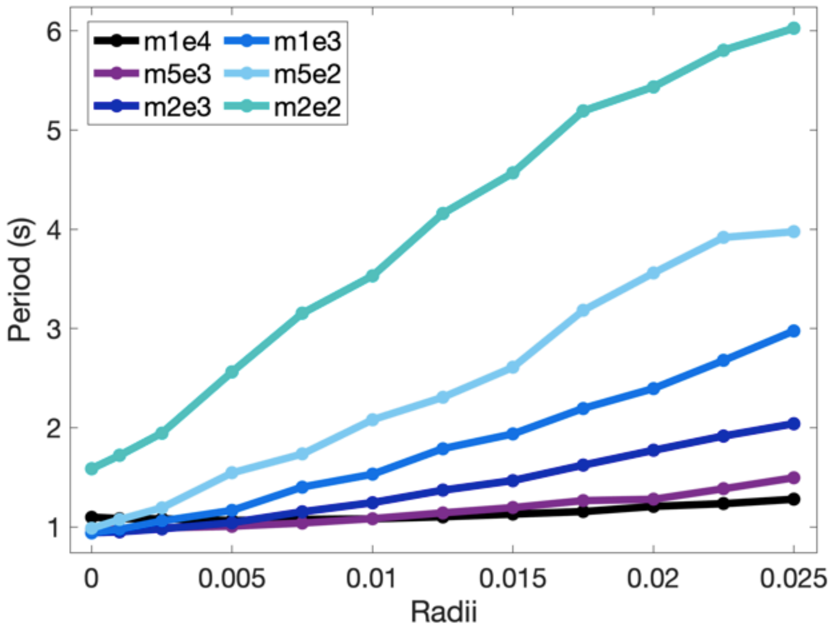


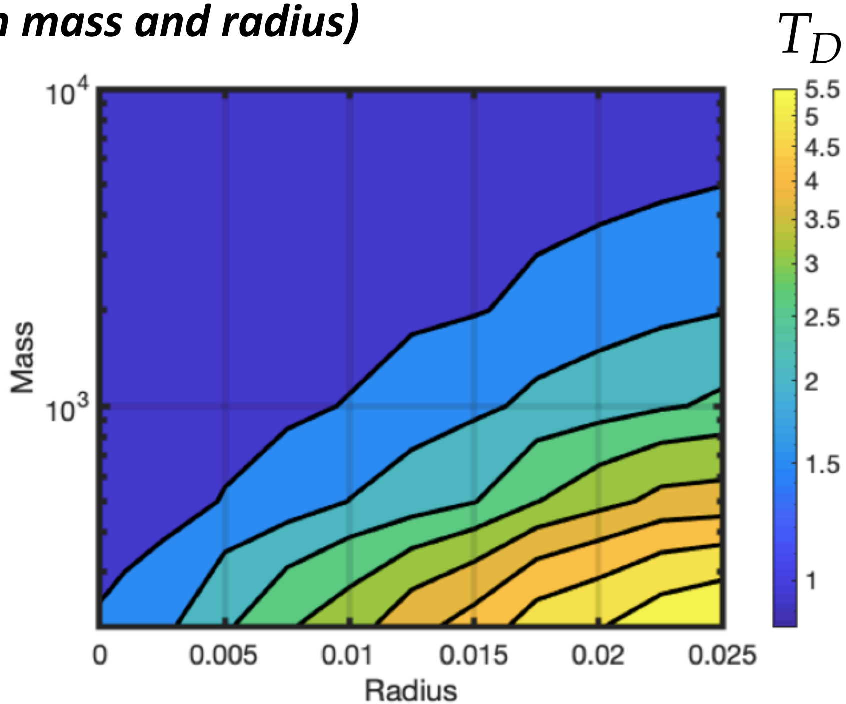




Damped Period of Oscillation (s)

(as function of both mass and radius)

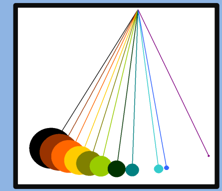


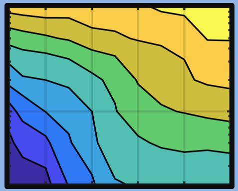


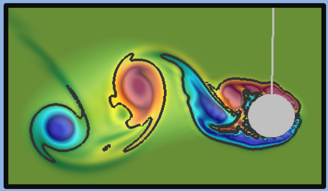
Note:

- **Higher mass cases generally have lower periods**
- **As radius increases, the period also increases**

FSI Pendulum Results: Speed

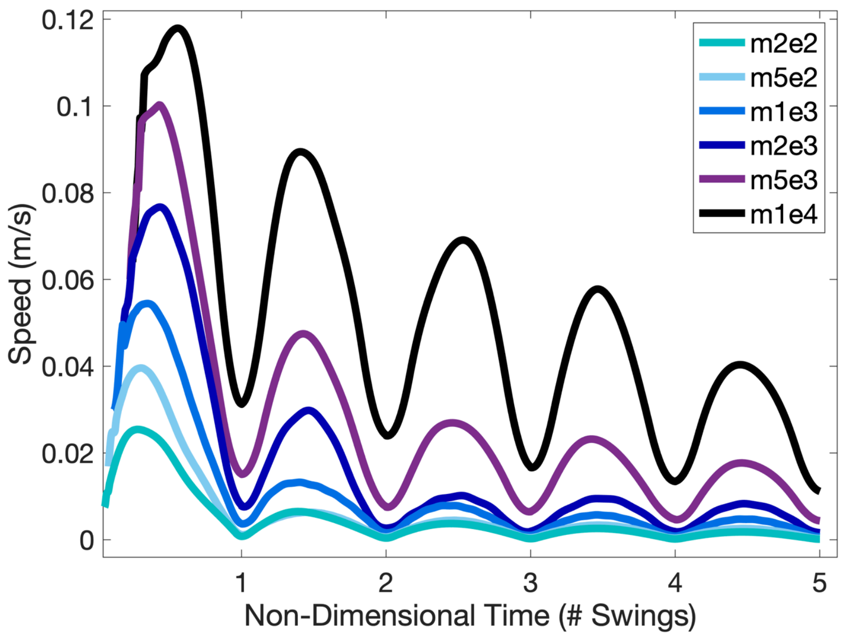


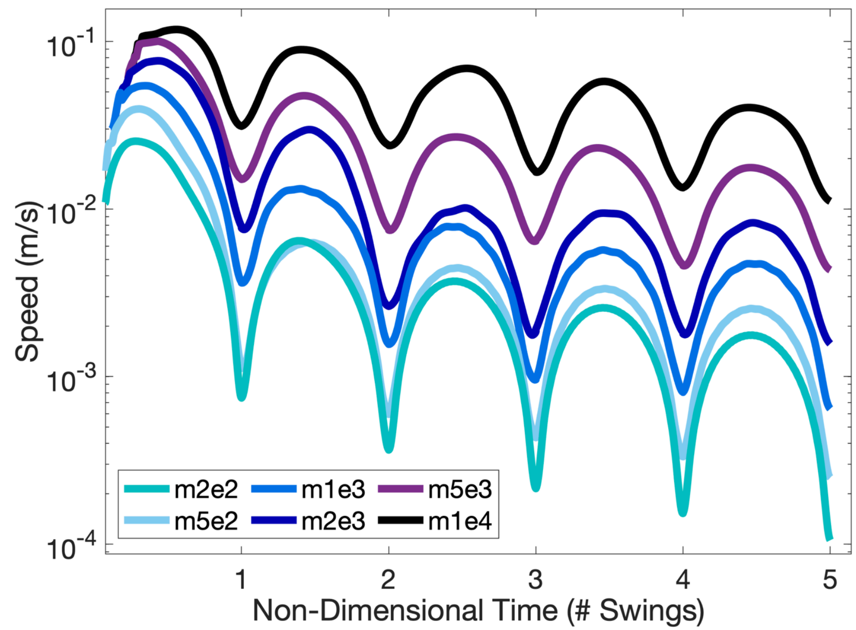




Linear Speed (m/s) of Pendulum Bob

(radius = 0.015m)

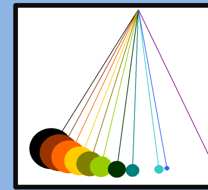


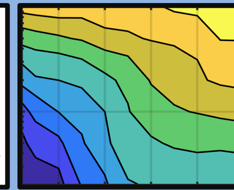


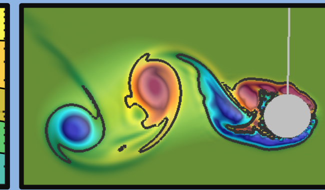
Note:

- **Higher mass cases are faster than lower mass cases for a given radii**
- **Semi-logarithmic plot shows some linear relationships in peak speed…although looks like there are multiple regimes…**

FSI Pendulum Results: Speed





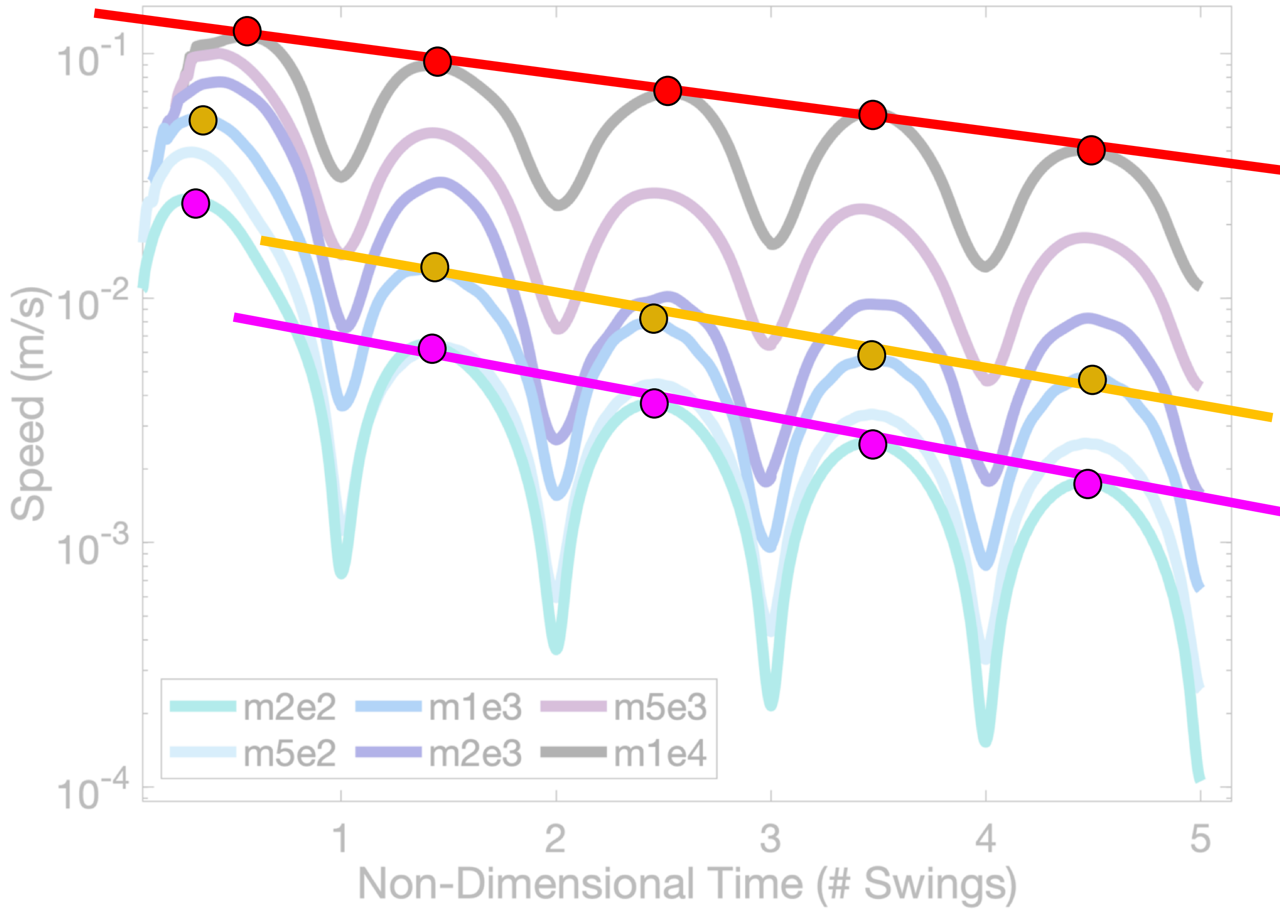


Linear Speed (m/s) of Pendulum Bob

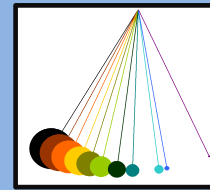
(radius = 0.015m)

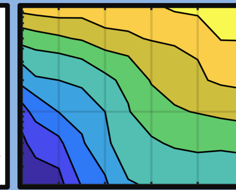
Note:

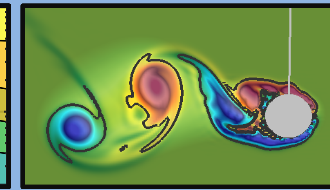
- **Peak Speed drops off significantly from 1st to 2nd peak** (for m<1e4)
- **There looks like the damping mechanism is enhanced during the first swing!**



FSI Pendulum Results: Speed

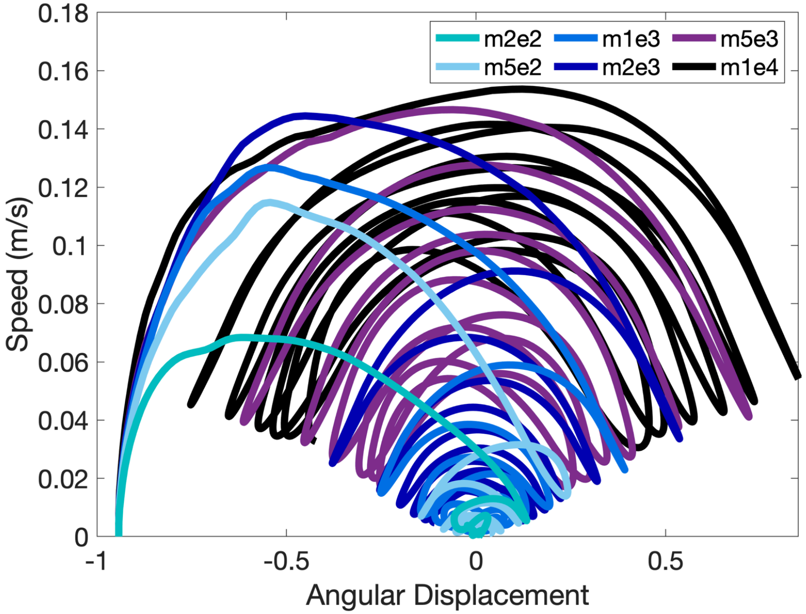


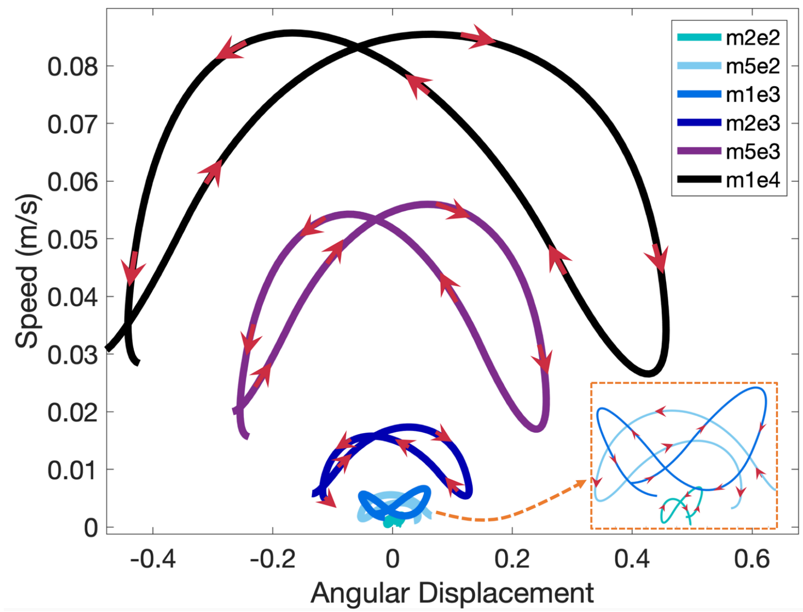




Phase Space: Speed (m/s) vs. Angular Displacement (radians)

(radius = 0.001m)

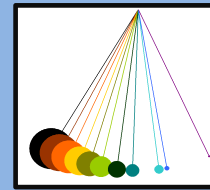


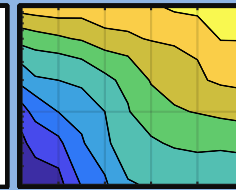


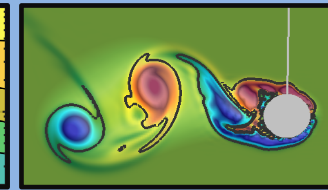
Note:

- **All data seems to collapse onto parabolically-capped cone**
- **Speeds increase when reaching zero displacement from vertical**

FSI Pendulum Results: Forces

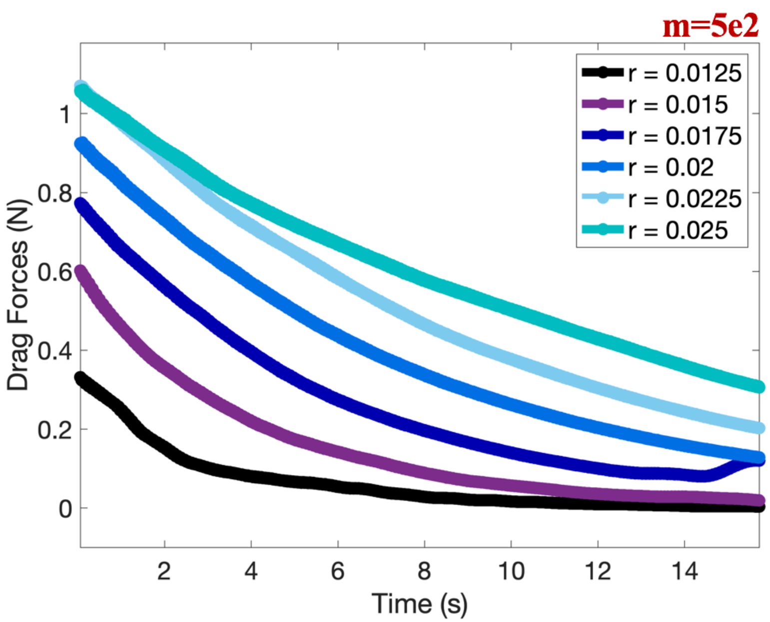


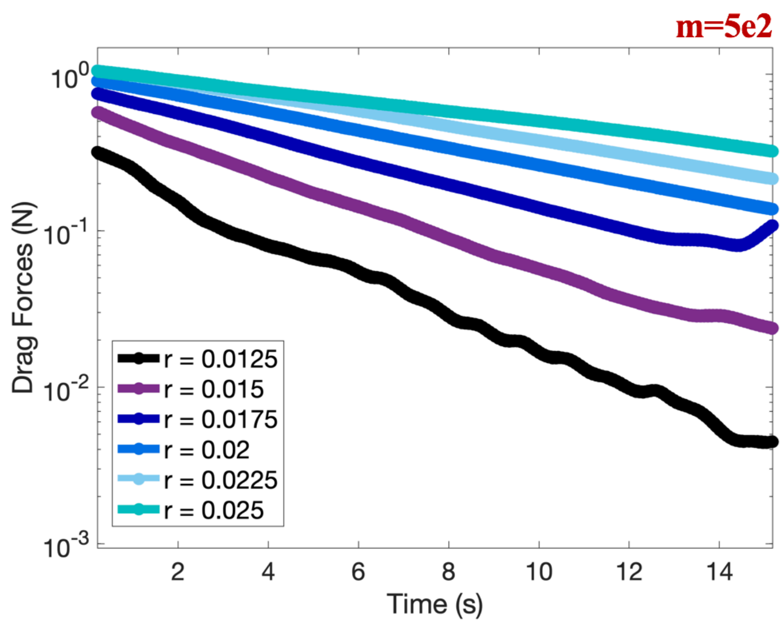




Drag Forces (N) vs. Time (s)

(radius = 0.001m)

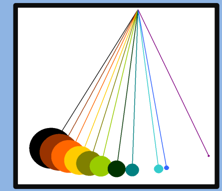


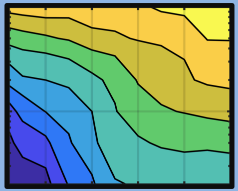


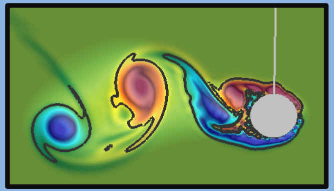
Note:

- **Drag acts in opposite direction to bob's motion**
- **Larger radii undergo more fluid drag**
- **Linear relationship between log(Drag) and Time -> Exponential Relationship!**

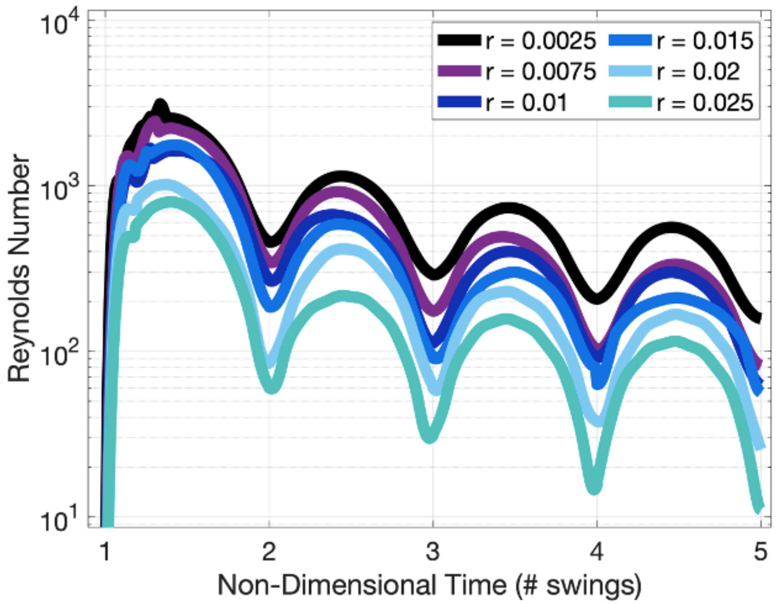
FSI Pendulum Results
Forces





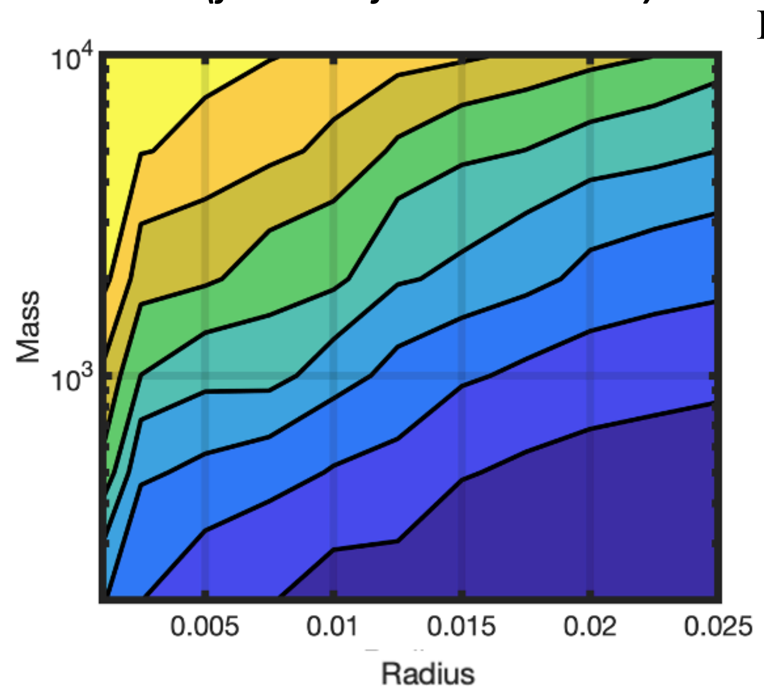


Time-dependent Reynolds Number vs. Time (s)



Avg. Reynolds Number During 1st Swing

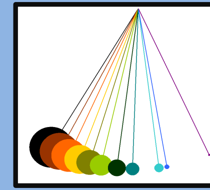
(function of mass and radius)

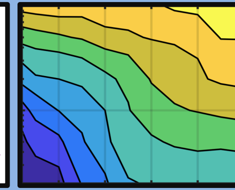


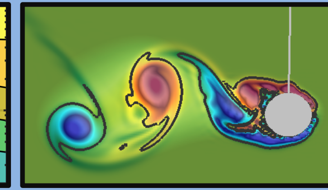
Note:

- **The average Re >> 1 during most of the first swing**
- **Re minimizes when speed of bob nears zero during each swing**
- **Since each swing's peak speed will continually decrease as the pendulum oscillates, Re will get smaller and smaller**
- **Lower radii and higher masses give rise to larger Re**

FSI Pendulum Results
Forces

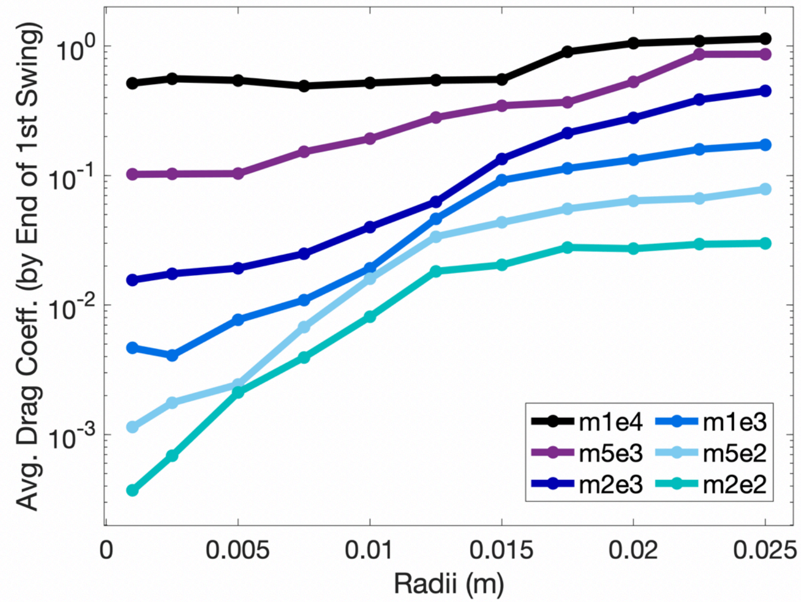


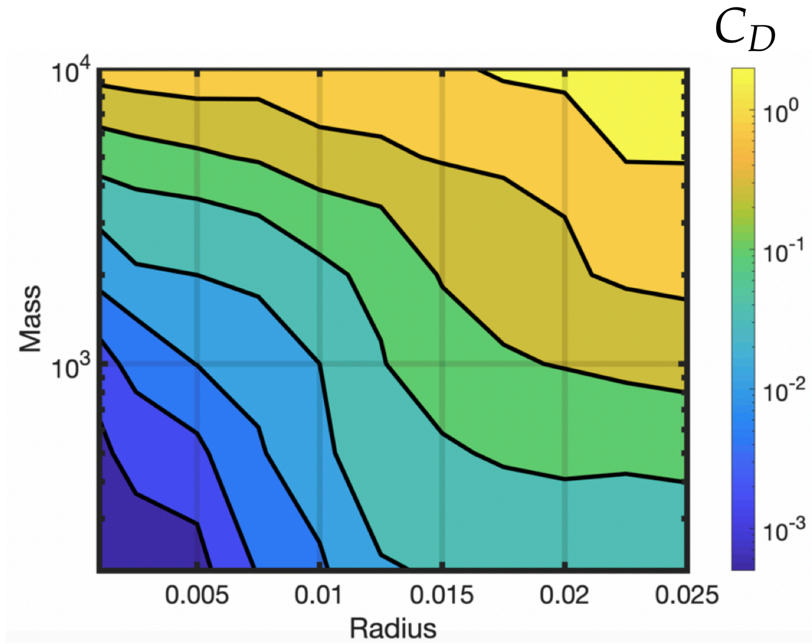




Avg. Drag Coefficient During the 1st Swing

(as function of mass and radii)

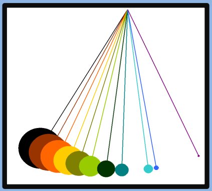
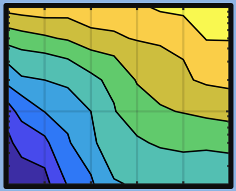
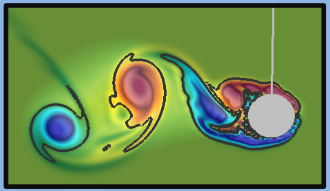




Note:

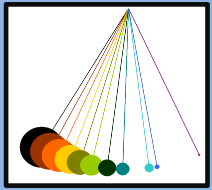
- **Drag Coefficient increases as radii increases**
- **Drag Coefficient increases as mass increases as well**
- **Non-Linear relationship between drag coefficient, mass, and radii.**

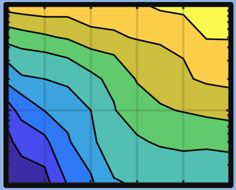
FSI Pendulum Results: Fluid Dynamics

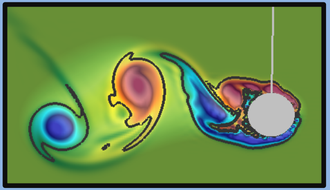




Well, what is the fluid doing in response to the pendulum swinging through it?

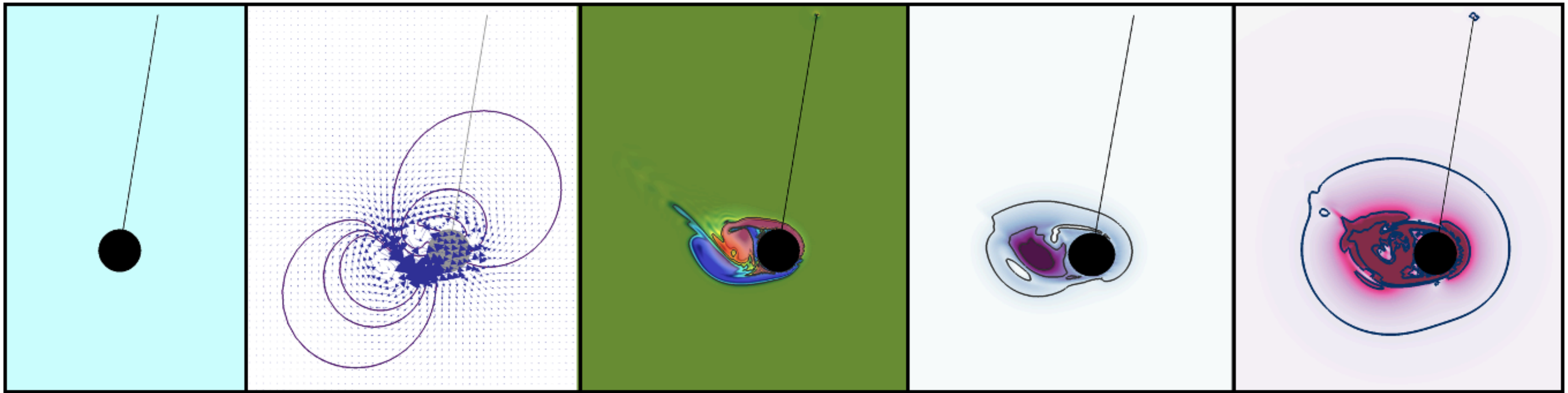
FSI Pendulum Results: Fluid Dynamics







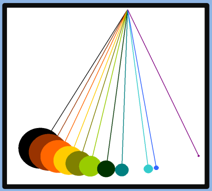
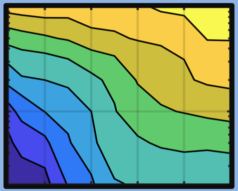
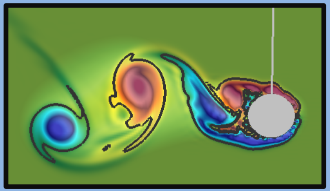
What the fluid is doing!



Lagrangian Points	Fluid Velocity Field *(w/ streamlines)*	Vorticity *(w/ vorticity contours)*	Magnitude of Velocity *(w/ mag. velocity contours)*	Finite-Time Lyapunov Exponent *(w/ FTLE contours)*

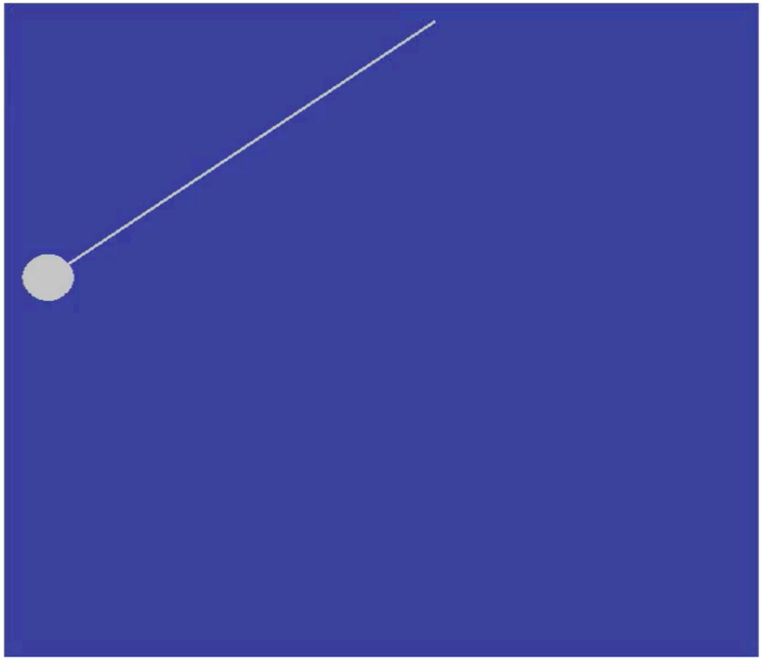
Through CFD simulations are able to gain information about the fluid dynamics as well as the pendulum bob!

FSI Pendulum Results: Fluid Dynamics

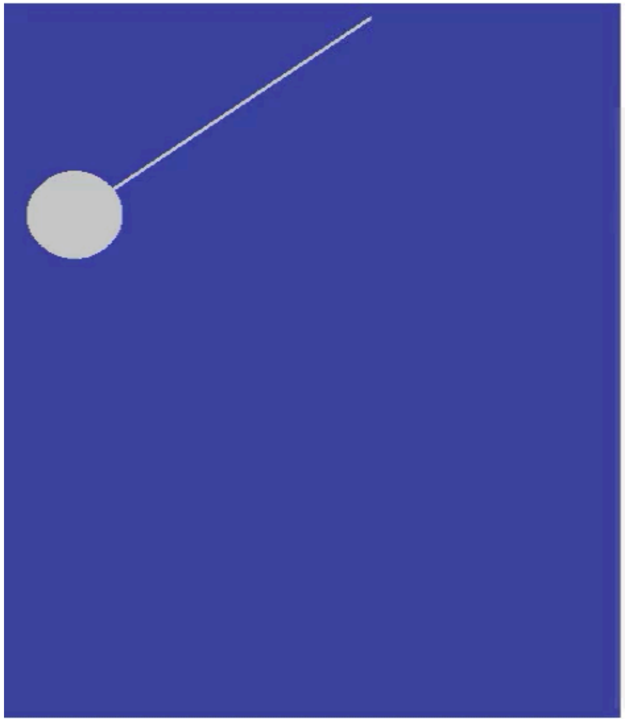




What the fluid is doing!

Colormap: Magnitude of Velocity



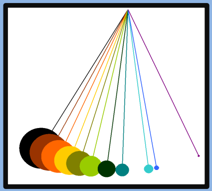
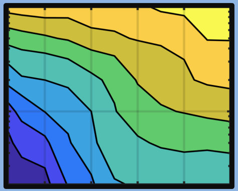
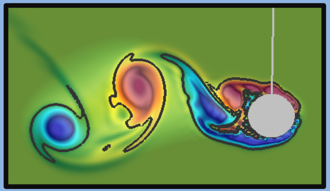
(m=5e2, r=0.01m)



(m=5e2, r=0.025m)

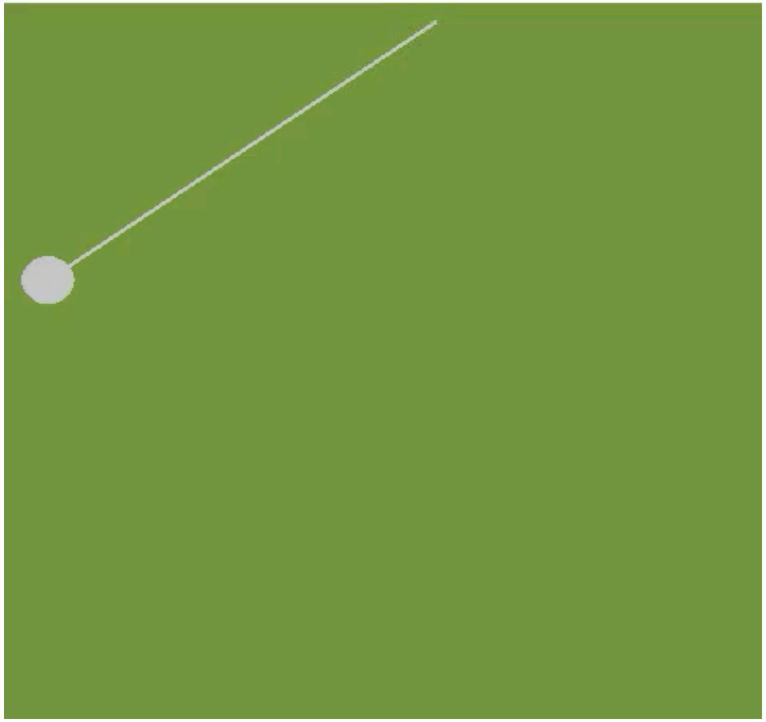
Fastest moving fluid is directly behind the pendulum bob!

FSI Pendulum Results: Fluid Dynamics

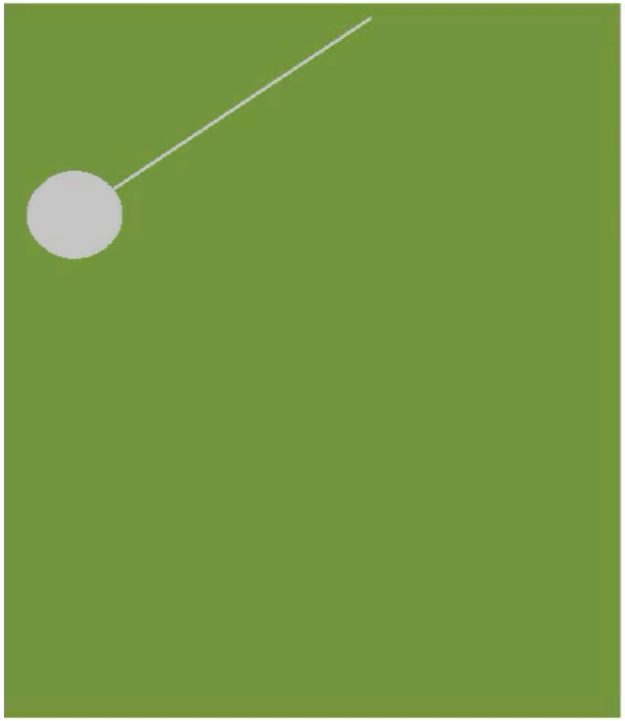




What the fluid is doing!

Colormap: Vorticity



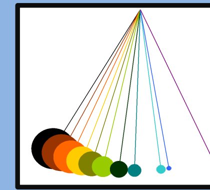
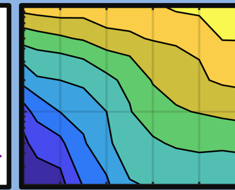
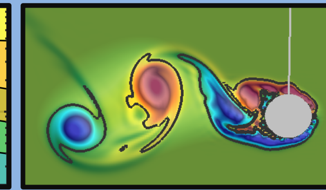
(m=5e2, r=0.01m)



(m=5e2, r=0.025m)

Vortices are being shed off of the pendulum bob!

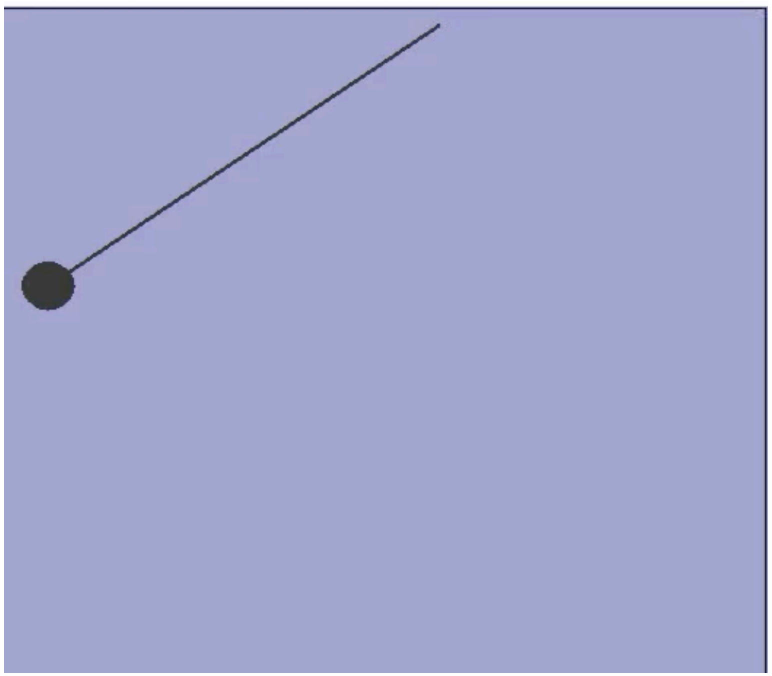
FSI Pendulum Results: Fluid Dynamics

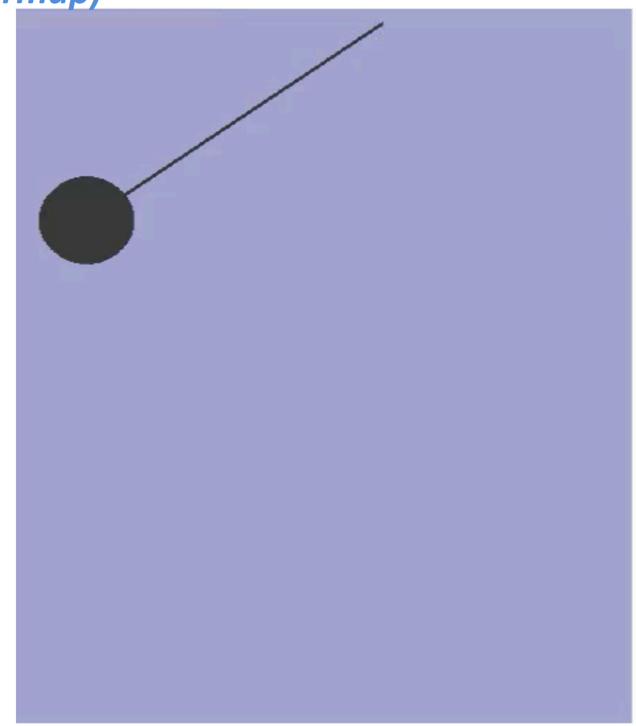
What the fluid is doing!

Vector Field: Velocity

(w/ Mag. Of Velocity colormap)



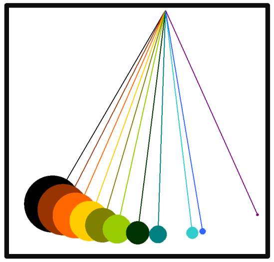
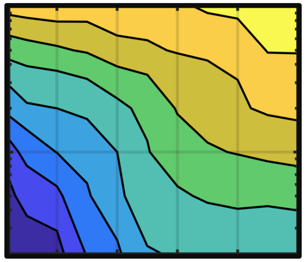
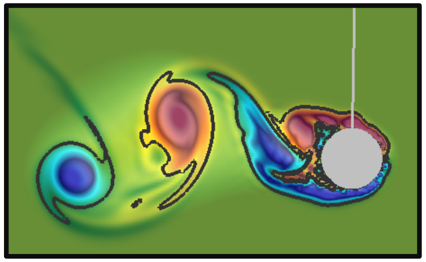
(m=5e2, r=0.01m)



(m=5e2, r=0.025m)

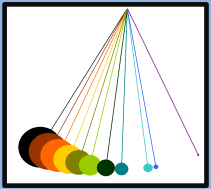
Velocity Field Shows that due to vortex formation, fluid is accelerated towards the fluid bob -> *Drafting*!

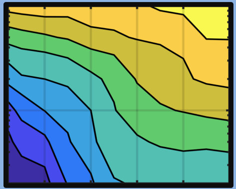
Outline:

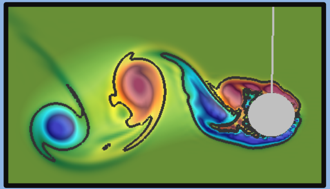




- **A pendulum tale…**
  - History & Reduced-Order Model
  - Fluid-Structure Interaction (FSI) Model
  - **Comparison and Validation**

Comparing:
FSI vs. ODE Model







Damping Pendulum ODE Model

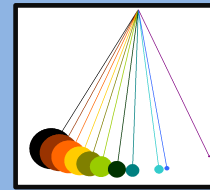
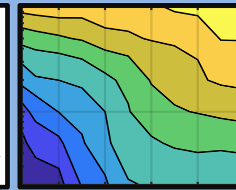
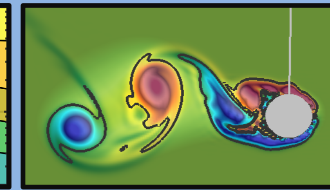
$$\frac{d^2\theta}{dt^2} + \frac{b}{I}\frac{d\theta}{dt} + \frac{mgL}{I}\sin\theta = 0$$

$\frac{b}{I}$: Coefficient on the damping term

$\frac{mgL}{I}$: ω_N^2 (Natural, undamped angular frequency)

Comparing:

FSI vs. ODE Model

Damping Pendulum ODE Model

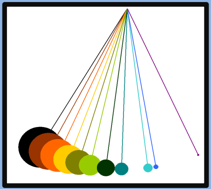
$$\frac{d^2\theta}{dt^2} + \frac{b}{I}\frac{d\theta}{dt} + \frac{mgL}{I}\sin\theta = 0$$

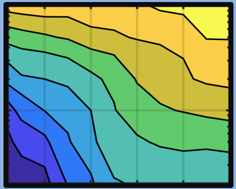
$\frac{b}{I}$: Coefficient on the damping term

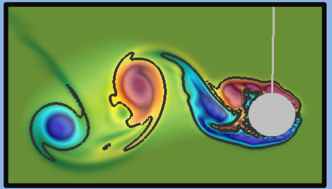
$\frac{mgL}{I}$: ω_N^2 (Natural, undamped angular frequency)

Caution: Not the true, natural, undamped angular frequency b/c we did not invoke the small angle approximation to say $\sin\theta \approx \theta$ for small θ

Comparing:
FSI vs. ODE Model





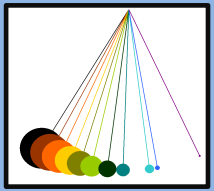
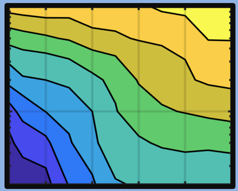
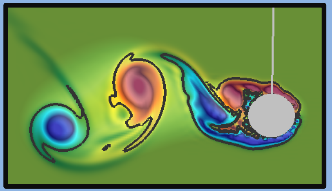


Damping Pendulum ODE Model

$$\frac{d^2\theta}{dt^2} + \frac{b}{I}\frac{d\theta}{dt} + \frac{mgL}{I}\sin\theta = 0$$

Need to match these parameter values to FSI Model to perform a comparison.

Comparing:
FSI vs. ODE Model

1. Finding the damping coefficient: $b\,/\,I$

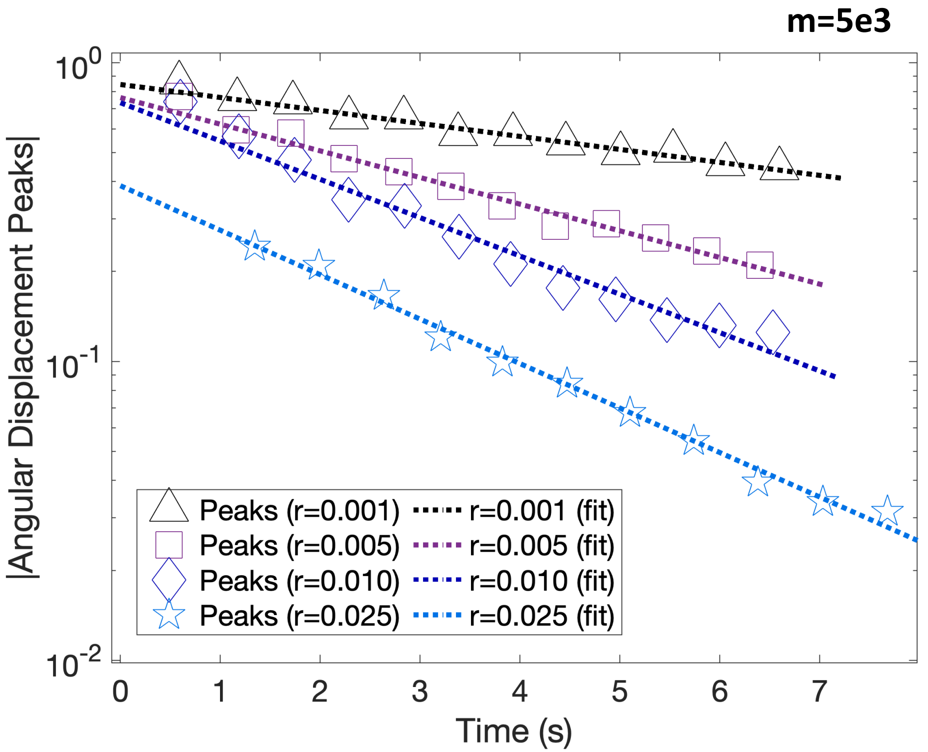
Plan:

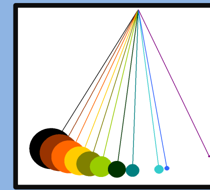
1. **Find exponential decay rate:**
   1. Fit linear regression through logarithm of peak angular displacement vs. time:

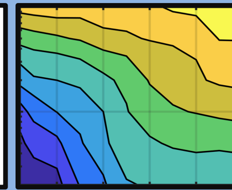
$$\gamma = -\frac{b}{2I}$$

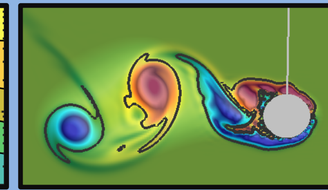
   2. Multiply by -2

$$b\,/\,I = -2\gamma$$









2. Finding natural angular frequency: ω_N^2

Plan:

1. **Compute damped angular frequency from damped period:**

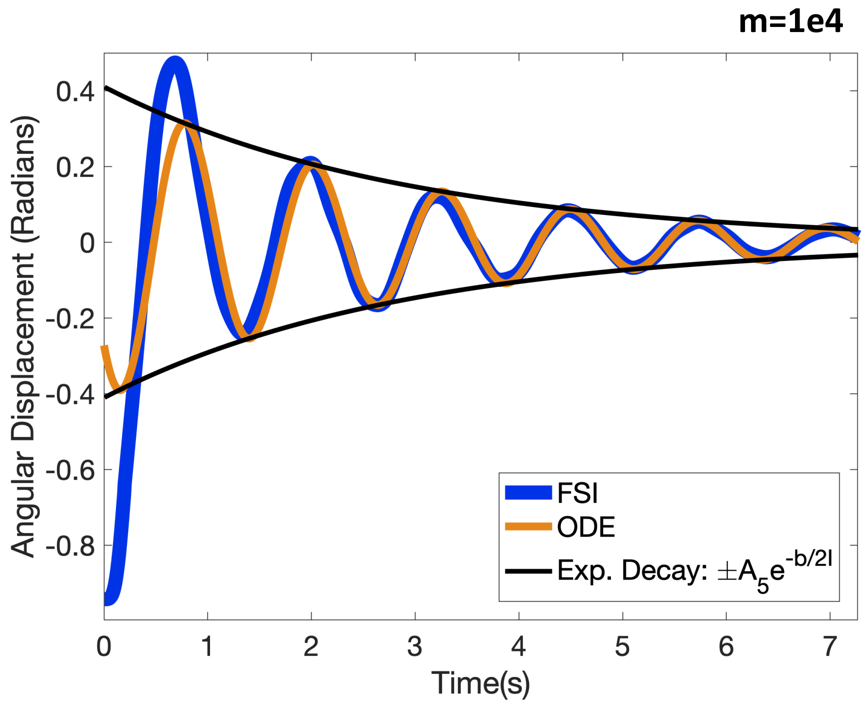
$$\omega_D = \frac{2\pi}{T_D}$$

2. **Recall relationship between damped and natural angular frequency and damping,**

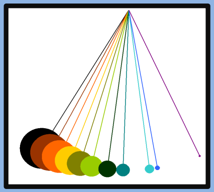
$$\omega_D^2 = \omega_N^2 - \gamma^2$$

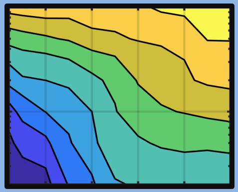
3. **Compute natural (undamped frequency):**

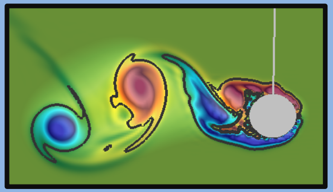
$$\omega_N^2 = \frac{mgL}{I} = \frac{4\pi^2}{T_D^2} + \gamma^2$$



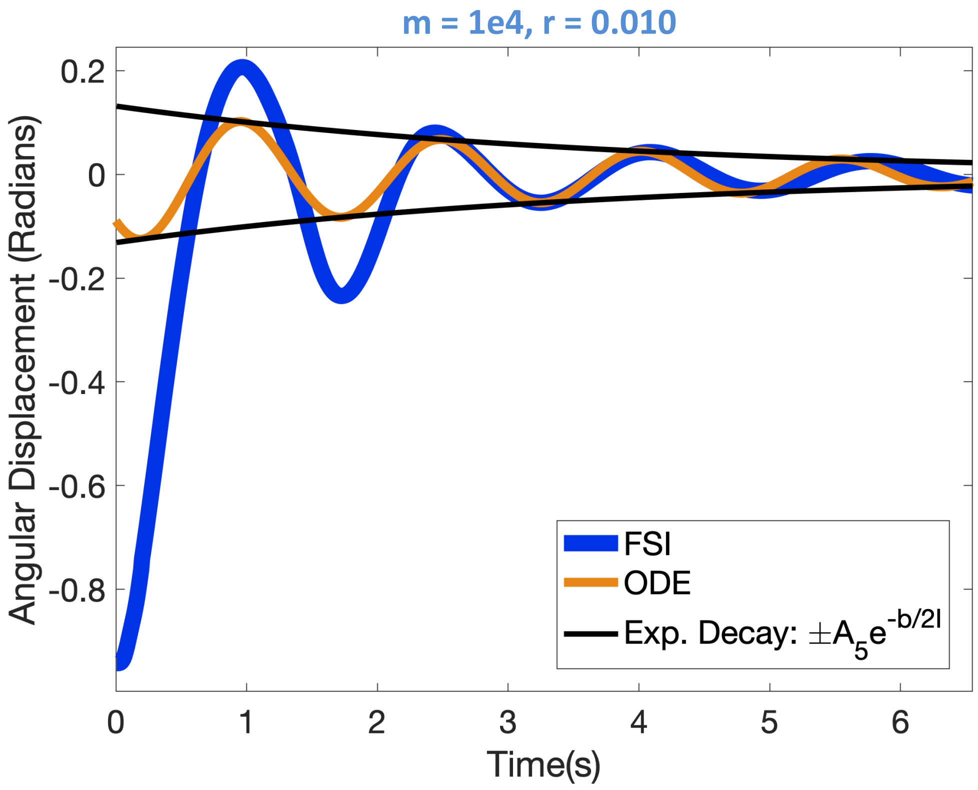
Note: Moment of Inertia not straight-forward to calculate here due principle of added mass







Comparisons!

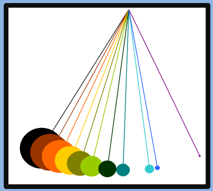


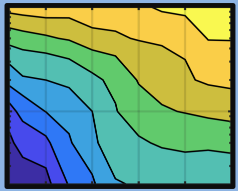
Note:

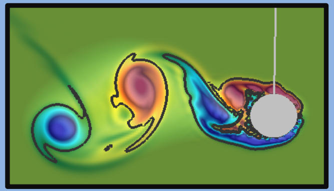
- ODE model solution found by starting at **5^{th} peak** and propagating *forward **<u>and</u>** backward* in time
- Exponential decay plotted in similar manner
- Both exponential decay and ODE model agree well with FSI model after a few peaks

Comparing:

FSI vs. ODE Model

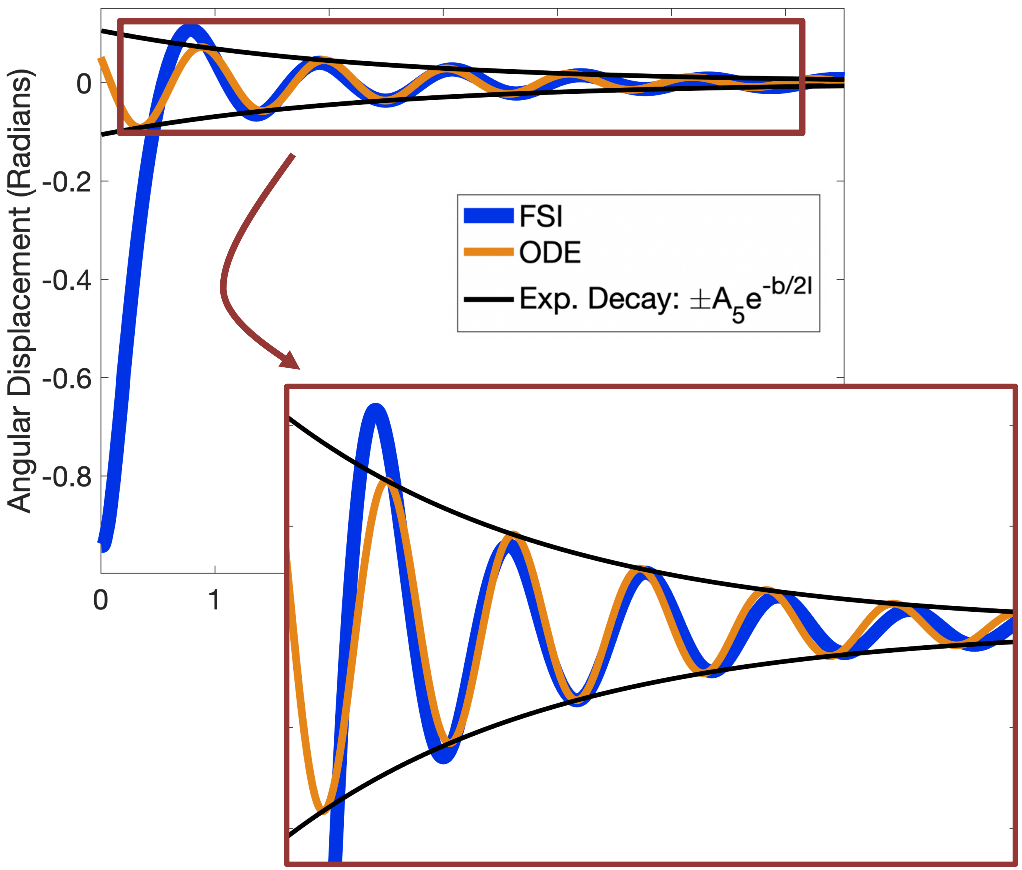






Comparisons!

m = 5e2, r = 0.005

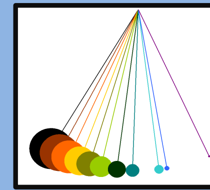


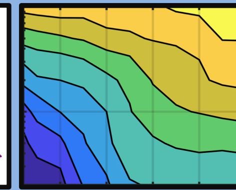
Note:

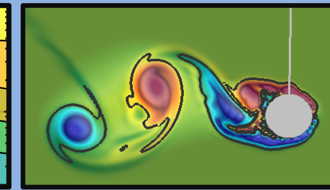
- ODE model solution found by starting at **5th peak** and propagating *forward* ***and*** *backward* in time
- Exponential decay plotted in similar manner
- Both exponential decay and ODE model agree well with FSI model after a few peaks

Comparing:

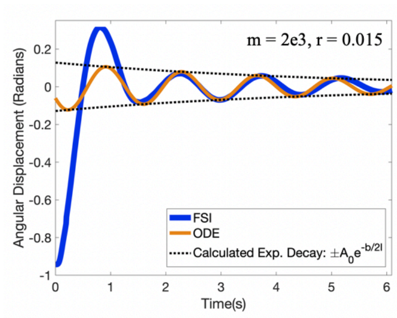
FSI vs. ODE Model

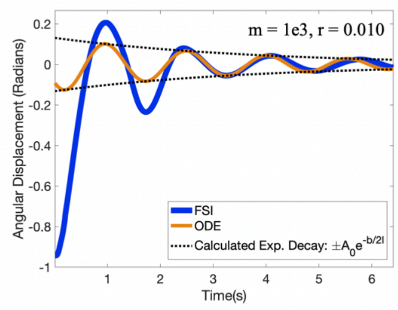


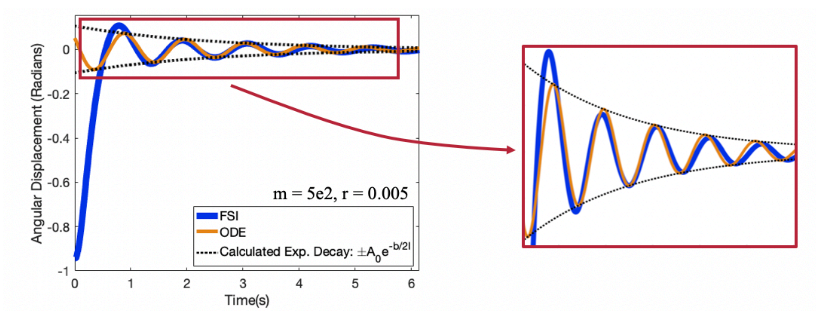


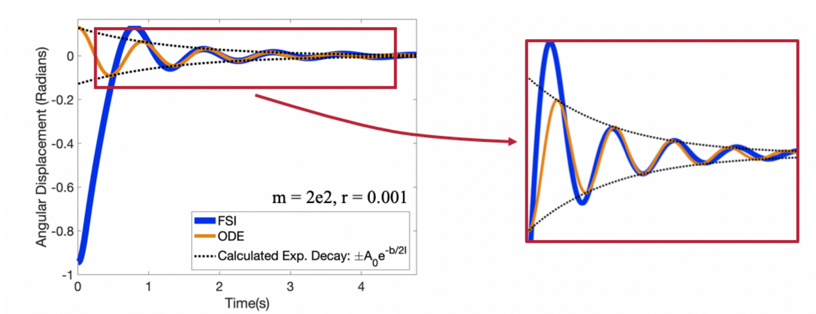


Validation is good…*after* a couple swings!





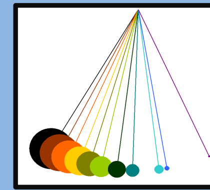
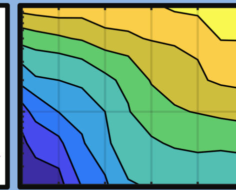
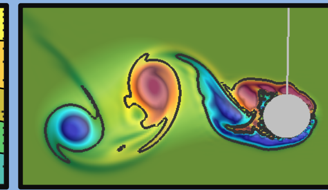




Note:

- Reduced-order ODE model shows qualitative agreement after a few swings
- The first few swings there is not agreement due to *limitations of the damping term in the ODE model*!
- The damping term is unable to capture any enhanced drag due to the fluid
- So, ***Stokes Drag Law works well in the regime in which it's supposed to!***

Comparing:
FSI vs. ODE Model

Finding damping parameter
(as function of mass and radius)

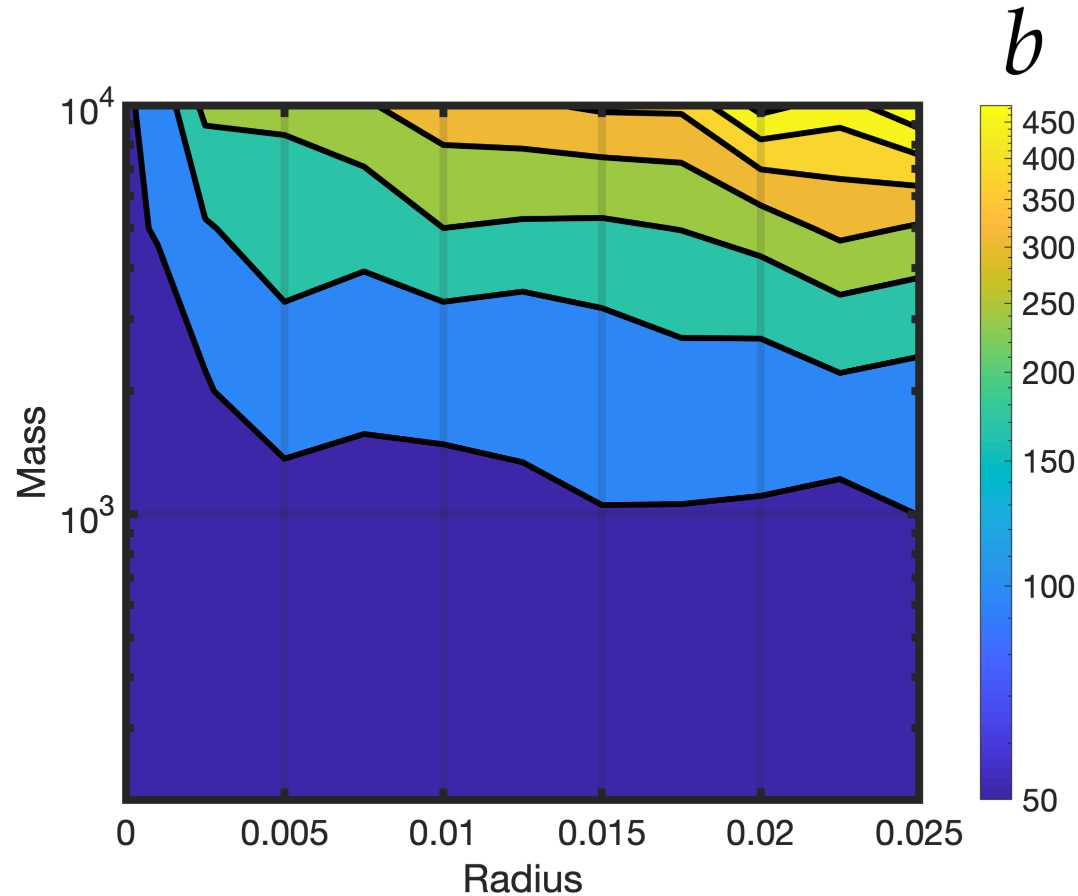
Plan:

- Find the *effective moment of inertia*:

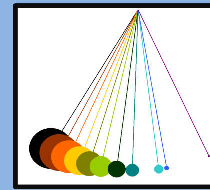
$$I = \frac{mgL}{\frac{4\pi^2}{T_D^2} + \gamma^2}$$

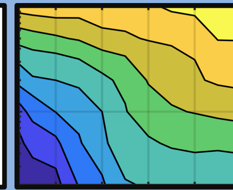
- Using *exponential decay* from fits, compute

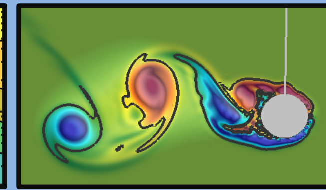
$$b = -2\gamma I$$



Comparing:
FSI vs. ODE Model

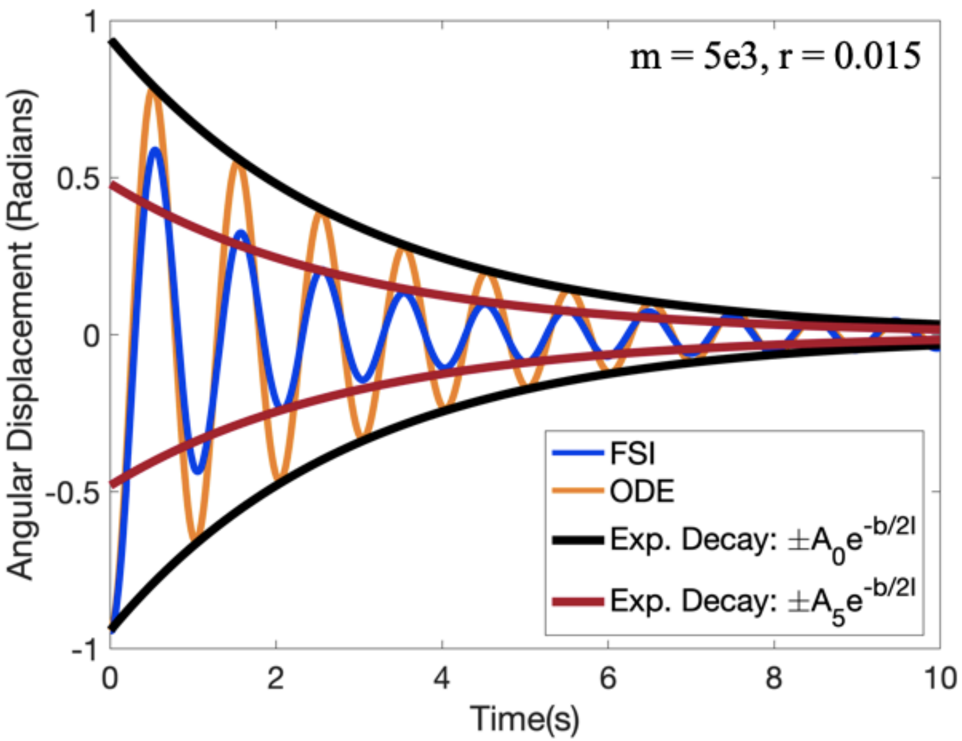






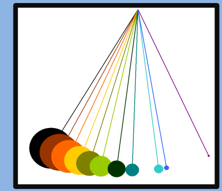
Danger!

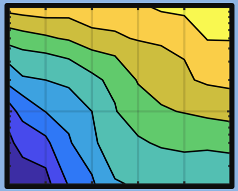
What if you don't start the ODE Model at the amplitude of the FSI model after a few swings…

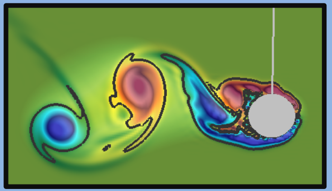


Note:

- ODE model solution found by propagating solution *starting at time t=0* with *same initial displacement of FSI model*
- Exponential decay plotted in two ways: same as before starting around 5th peak amplitude, and starting at t=0.
- **Does not show agreement with FSI model!**
- FSI model illustrates enhanced drag during the first few swings - the ODE model cannot reconcile FSI dynamics even with exponential decay fit!

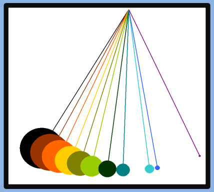


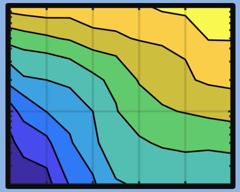


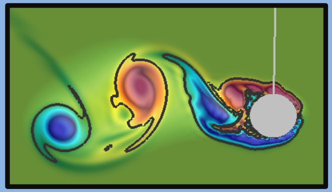


Pendulum Summary…

Pendulums







Some Takeaways:

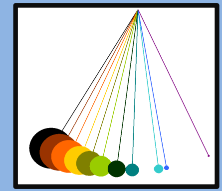
- The fluid-structure interaction (FSI) and reduced-order ODE model agree after a few swings of the pendulum

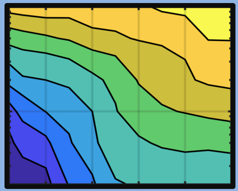
- ***ODE model cannot*** *provide* ***detail****s about* ***the underlying fluid dynamics***, but the ***FSI model certainly can***!

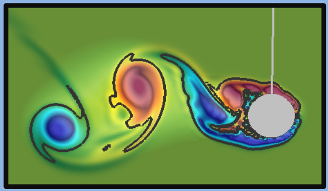
- The ***FSI model unveiled*** some of the ***hidden complex dynamics*** within the fluid system, e.g.,
    - Differences in ***vortex dynamics*** (formation and shedding)
    - "***Drafting***" off the backside of the pendulum bob as it swings

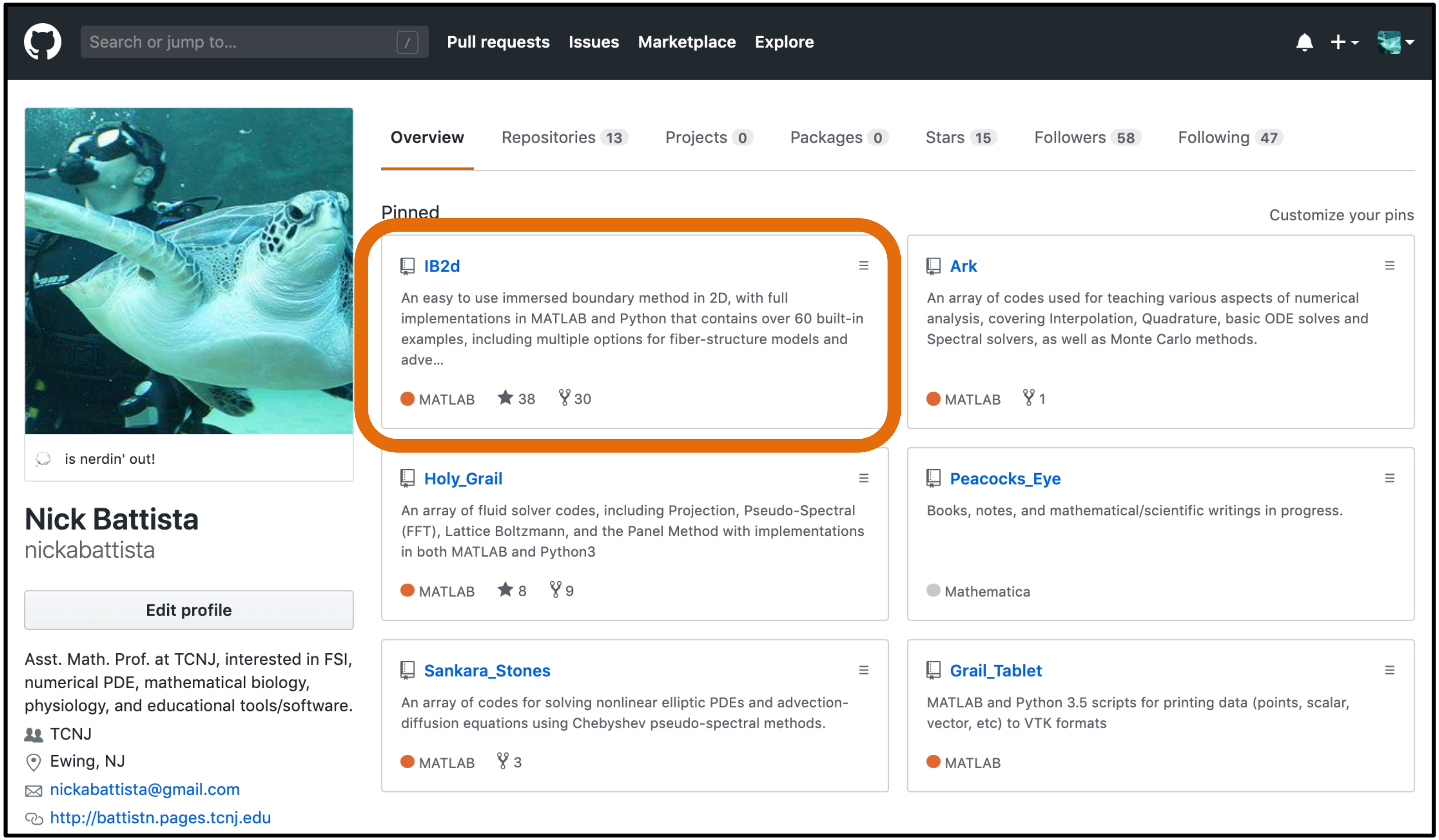
- Although an old and historic device, there is ***much more to uncover about pendulums*** and ***CFD Simulations may hold the key***!

Pendulums









Open Source Codes Available: github.com/nickabattista