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Supplementary Information

Enhancement and Tunability of Near-Field Radiative Heat Transfer Mediated by Surface Plasmon Polaritons in Thin Plasmonic Films. *Photonics* 2015, *2*, 659-683

Svetlana V. Boriskina *, Jonathan K. Tong, Yi Huang, Jiawei Zhou, Vazrik Chiloyan and Gang Chen *

Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA; E-Mails: jtong@mit.edu (J.K.T.); yi_huang@mit.edu (Y.H.); zhoujw20@mit.edu (J.Z.); vazrik@mit.edu (V.C.)

* Authors to whom correspondence should be addressed; E-Mails: sborisk@mit.edu (S.V.B.); gchen2@mit.edu (G.C.).

1. Transmission Coefficients for Various Absorber/Emitter Configurations

The transmission coefficients for radiative ($k \le k_0$) and evanescent ($k \ge k_0$) waves contributing to the total heat transfer are defined as in Equation (2) of main text, which is included as Equation (1) below:

$$\mathcal{T}_j = \begin{cases} \left(1-\left|r_{01}^j\right|^2\right)\cdot\left(1-\left|r_{02}^j\right|^2\right) \Big/ \left|1-r_{01}^j r_{02}^j e^{2ik_{\perp 0}d}\right|^2, & k \le k_0 \\ 4e^{-2\,\mathrm{Im}(k_{\perp 0})d}\cdot \mathrm{Im}\left(r_{01}^j\right)\cdot \mathrm{Im}\left(r_{02}^j\right) \Big/ \left|1-r_{01}^j r_{02}^j e^{-2\,\mathrm{Im}(k_{\perp 0})d}\right|^2, & k > k_0 \end{cases} \tag{1}$$

where j denotes the light polarization (s or p), and r_{0m}^s, r_m^p are the reflection coefficients between the vacuum in the gap and either absorber or emitter medium (which may be composite). It was mentioned in the main text that for the simplest case of bulk emitter and absorber the reflection coefficients take the familiar form of the Fresnel coefficients for the two interfaces between the vacuum and bulk materials:

$$r_{0m}^s = \left(k_{\perp 0} - k_{\perp m}\right)/\left(k_{\perp 0} + k_{\perp m}\right),\ r_{0m}^p = \left(\varepsilon_m k_{\perp 0} - \varepsilon_0 k_{\perp m}\right)/\left(\varepsilon_m k_{\perp 0} + \varepsilon_0 k_{\perp m}\right) \tag{2}$$

For the symmetrical absorber-emitter configurations considered in this paper, Equation 1 can be further simplified as follows:

$$\mathcal{T}_j = \begin{cases} \left(1-\left|R^j\right|^2\right)^2 \Big/ \left|1-\left(R^j\right)^2 e^{2ik_{\perp 0}d}\right|^2, & k \le k_0 \\ 4e^{-2\,\mathrm{Im}(k_{\perp 0})d}\cdot \left(\mathrm{Im}\left(R^j\right)\right)^2 \Big/ \left|1-\left(R^j\right)^2 e^{-2\,\mathrm{Im}(k_{\perp 0})d}\right|^2, & k > k_0 \end{cases} \tag{3}$$

where $R^j = r_{01}^j = r_{02}^j$. The reflection coefficient R^j has a more complex form if the emitter is composite (*i.e.*, multi-layered) rather than homogeneous.

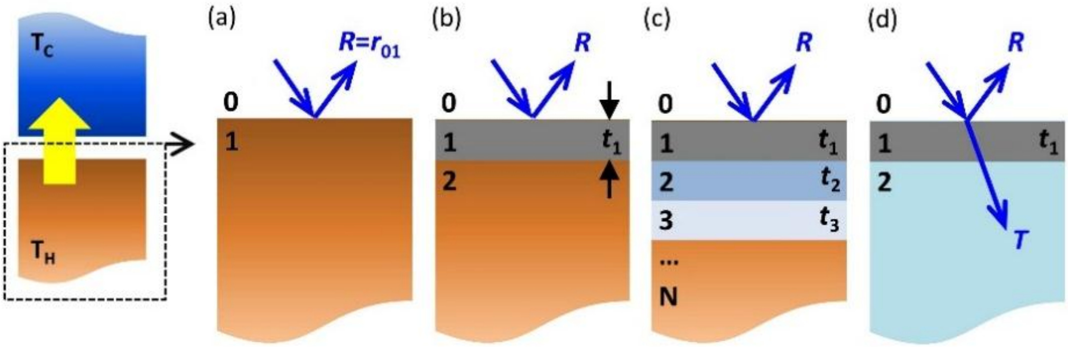

Figure S1. Schematics of various configurations of composite multi-layered emitters/absorbers.

Figure S1 summarizes different types of composite emitters/absorbers considered in this paper, including (a) a homogeneous bulk absorber; (b) a film-on-substrate absorber; (c) a multi-layered absorber; and (d) a suspended thin-film absorber. In the situation when all the radiated photons are either absorbed on the cold side or reflected back to the hot emitter (as in Figure S1a–c), Equation 3 can be used to find the total heat flux from the emitter to the absorber provided that the reflection coefficients R^j account for the reflection from the composite absorber as a whole. In particular, for the case of a thin-film on a substrate (Figure S1b) the reflection coefficients take the following form:

$$R^j = \frac{r_{01}^j + r_{12}^j \cdot \exp(2ik_{\perp 1}t_1)}{1 + r_{01}^j \cdot r_{12}^j \cdot \exp(2ik_{\perp 1}t_1)} \tag{4}$$

where r_{01}^j, r_{12}^j are the Fresnel coefficients of individual interfaces in Figure S1b, which can be found by using Equation (2). In the more complex case of multiple stacked films on a substrate (or on a back mirror) shown in Figure S1c, the stack reflection coefficient can be calculated by the repetitive application of Equation (4), starting at layer N-1 adjacent to the substrate [1]:

$$R_n{}^j = \frac{r_{n-1,n}^j + R_{n+1}^j \cdot \exp(2ik_{\perp n}t_n)}{1 + r_{n-1,n}^j \cdot R_{n+1}^j \cdot \exp(2ik_{\perp n}t_n)}, \quad n = N-1,\ldots 1 \tag{5}$$

where coefficients R_{N-1}^j are the standard Fresnel coefficients on the interface between the substrate and the adjacent layer (N−1), while R_1^j is an effective Fresnel reflection coefficient of the whole multi-layer stack shown in Figure S1c. It is easy to see that Equation (5) reduces to Equation (4) if $N = 2$, and to Equation (2) if $N = 1$ (bulk absorber). Furthermore, in the case of non-symmetrical cabsorber-emitter configuration, Equation (5) can be used to find the effective reflecting coefficients of both the absorber and the emitter stacks, which can then be substituted into Equation S1 to find the heat transfer coefficients. Equation (5) can be derived by using the transfer matrix method (TMM).

In the case of near field radiative heat transfer between suspended thin films (Figure S1d), Equations (1) and (3) must be modified to account for transmission losses through the films. Using a combination of the transfer matrix and scattering matrix methods, analytical expressions can then be derived, which provide the radiative heat flux entering and exiting the film [2–4]. By taking the difference between these heat fluxes, the net heat flux absorbed by the film can be found, and the corresponding heat transmission coefficients take the following forms:

$$\mathcal{T}_s = \begin{cases} \dfrac{\left(1-\left|R^s\right|^2 - \mathrm{Re}\left(\dfrac{k_{\perp 2}^* \cdot \mathrm{Re}(k_{\perp 0})}{\left|k_{\perp 0}\right|^2}\right)\cdot\left|T^s\right|^2\right)^2}{\left|1-\left(R^s\right)^2 e^{2ik_{\perp 0}d}\right|^2}, & k \le k_0 \\ 4e^{-2\,\mathrm{Im}(k_{\perp 0})d} \cdot \dfrac{\left(\mathrm{Im}\left(R^s\right) - \dfrac{1}{2}\mathrm{Re}\left(\dfrac{k_{\perp 2}^* \cdot \mathrm{Im}(k_{\perp 0})}{\left|k_{\perp 0}\right|^2}\right)\cdot\left|T^s\right|^2\right)^2}{\left|1-\left(R^s\right)^2 e^{-2\,\mathrm{Im}(k_{\perp 0})d}\right|^2}, & k > k_0 \end{cases} \tag{6}$$

$$\mathcal{T}_p = \begin{cases} \dfrac{\left(1-\left|R^p\right|^2 - \mathrm{Re}\left(\dfrac{k_2^* \cdot k_{\perp 2} \cdot \mathrm{Re}(k_{\perp 0})}{k_2 \cdot \left|k_{\perp 0}\right|^2}\right)\cdot\left|T^p\right|^2\right)^2}{\left|1-\left(R^p\right)^2 e^{2ik_{\perp 0}d}\right|^2}, & k \le k_0 \\ 4e^{-2\,\mathrm{Im}(k_{\perp 0})d} \cdot \dfrac{\left(\mathrm{Im}\left(R^p\right) - \dfrac{1}{2}\mathrm{Re}\left(\dfrac{k_2^* \cdot k_{\perp 2} \cdot \mathrm{Im}(k_{\perp 0})}{k_2 \cdot \left|k_{\perp 0}\right|^2}\right)\cdot\left|T^p\right|^2\right)^2}{\left|1-\left(R^p\right)^2 e^{-2\,\mathrm{Im}(k_{\perp 0})d}\right|^2}, & k > k_0 \end{cases} \tag{7}$$

Here, T^j is the transmission coefficient of the film:

$$T^j = \frac{\tau_{01}^j \cdot \tau_{12}^j \cdot \exp\left(2ik_{\perp 1}t_1\right)}{1 + r_{01}^j \cdot r_{12}^j \cdot \exp\left(2ik_{\perp 1}t_1\right)} \tag{8}$$

where τ_{01}^j, τ_{12}^j are the standard Fresnel transmission coefficients for each interface in Figure S1d. The corresponding formulas for a general case for non-symmetrical emitters and absorbers can be found in supplementary materials of [2].

It should also be noted that our calculations indicate that the transmission losses only become important to consider at larger vacuum gap separations, where near-field coupling is weak between the suspended films. For the gap separations of 20 nm and below, transmission losses are negligible and do not need to be included into the analysis.

2. Power Laws in the Heat Flux Scaling as a Function of Gap Width

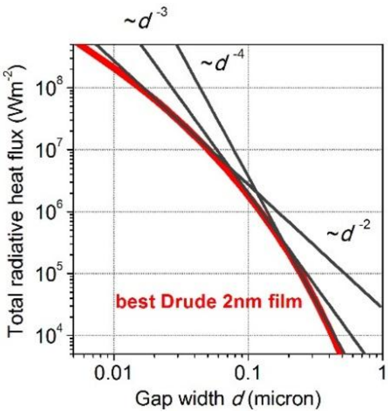

Figure S2. Radiative heat flux between thin plasmonic films with optimum Dude model parameters as a function of the vacuum gap width d. Regimes of d^{-2} d^{-3}, and d^{-4} power scaling laws are identified. The hot and cold side temperatures are fixed at 1000 K and 300 K, respectively.

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