The correlation between Goos–Hänchen Shift and complex refractive index

According to Maxwell's equations, the refractive index can be expressed as

$$\varepsilon_r = n^2 \tag{1}$$

where n is the refractive index of the medium and ε_{r} is the dielectric constant.

Maxwell's electromagnetic theory can explain many important properties of light propagation in materials, such as interference and diffraction. However, this theory is based on the assumption of continuous material structures and does not consider the electrical structure of the atoms and molecules that constitute the material. Therefore, Maxwell's electromagnetic theory struggles to explain the phenomenon of light dispersion and cannot provide expressions for the absorption coefficient and refractive index of materials [1]. To gain a deeper understanding of the refractive index of materials, it is necessary to analyze the electric dipole model, known as the Lorentz model. According to Lorentz's electron theory, under the influence of light, the forced oscillations of these electrons can be described by the following equations:

$$\frac{d^2r}{dt^2} + \gamma\frac{dr}{dt} + \omega_0^2 r = -\frac{eE}{m} \tag{2}$$

$$\gamma = \frac{e^2\omega_0^2}{6\pi\varepsilon_0 c^3 m} \tag{3}$$

Among them, γ is the damping coefficient, ω_0 is the angular frequency of the inherent vibration of the electron, e is the charge of the electron, and r is the displacement of the electron from the equilibrium position when subjected to an external electric field. E is the electric field intensity of the incident light, m is the electron mass, and c is the speed of light in vacuum. According to the above equation, the displacement of electrons in the medium under the action of a light field can be obtained as

$$r = \frac{-\frac{e}{m}}{\left(\omega_0^2 - \omega^2\right) - i\gamma\omega} E \tag{4}$$

where ω is the angular frequency of incident light. The vibration of the electron turns the atom into an oscillating electric dipole with a dipole moment of qr, where q is the charge. The response of a medium to light is the polarization of electric dipoles under the action of light. For thin gases or low-concentration solutions, assuming there are atoms in unit volume, the induced electric polarization intensity is

$$P = Ner = \frac{-\frac{Ne^2}{m}}{\left(\omega_0^2-\omega^2\right)-i\gamma\omega}E \tag{5}$$

From Maxwell's electromagnetic field theory, the expression for the electric displacement vector is given by

$$D = \varepsilon_0 E + P = \varepsilon_0 \varepsilon_r P \tag{6}$$

The induced electric polarization of the material can then be expressed as

$$P = \varepsilon_0 \chi E \tag{7}$$

Therefore, the material's electric susceptibility can be defined as

$$\chi = \chi' + i\chi'' \tag{8}$$

$$\chi' = \frac{Ne^2}{\varepsilon_0 m}\frac{\omega_0^2-\omega^2}{\left(\omega_0^2-\omega^2\right)+\gamma^2\omega^2} \tag{9}$$

$$\chi'' = \frac{Ne^2}{\varepsilon_0 m}\frac{\gamma\omega}{\left(\omega_0^2-\omega^2\right)+\gamma^2\omega^2} \tag{10}$$

It can be seen that the polarization rate of the medium is complex, therefore, the refractive index of the material can be expressed as a complex number:

$$\hat{n} \approx \sqrt{\varepsilon_r} = \sqrt{1+\chi} = n + i\eta \tag{11}$$

In the formula, n is the refractive index of the medium and η is the extinction coefficient of the medium. Furthermore, the complex refractive index can be related to the complex dielectric function:

$$\hat{\varepsilon}_r = 1+\chi = n^2-\eta^2+2n\eta i \tag{12}$$

$$\begin{aligned}\sqrt{1+\chi} = 1 &+ \frac{Ne^2}{2\varepsilon_0 m}\frac{\omega_0^2-\omega^2}{\left(\omega_0^2-\omega^2\right)+\gamma^2\omega^2} + i\frac{Ne^2}{2\varepsilon_0 m}\frac{\gamma\omega}{\left(\omega_0^2-\omega^2\right)+\gamma^2\omega^2} - \\ &\frac{N^2e^4}{8\varepsilon_0^2 m^2}\frac{\left(\omega_0^2-\omega^2\right)^2-\gamma^2\omega^2}{\left[\left(\omega_0^2-\omega^2\right)+\gamma^2\omega^2\right]^2} - i\frac{N^2e^4}{4\varepsilon_0^2 m^2}\frac{2\left(\omega_0^2-\omega^2\right)\gamma\omega}{\left[\left(\omega_0^2-\omega^2\right)+\gamma^2\omega^2\right]^2} + \ldots\ldots\end{aligned} \tag{13}$$

Due to the high frequency of light, the above equation can be abbreviated as

$$\eta \approx \frac{Ne^2}{2\varepsilon_0 m}\frac{\gamma\omega}{\left(\omega_0^2-\omega^2\right)+\gamma^2\omega^2} \tag{14}$$

The relationship between the refractive index and the extinction coefficient of the medium can be obtained from the above two equations as follows:

$$n \approx 1 + \frac{\omega_0^2-\omega^2}{\gamma\omega}\eta - \frac{1}{2}\frac{\left(\omega_0^2-\omega^2\right)^2-\gamma^2\omega^2}{\gamma^2\omega^2}\eta^2 \tag{15}$$

When light propagates through a medium, the phenomenon where the light intensity decreases with increasing depth into the medium is known as absorption. Absorption can be further divided into true absorption, where light is converted into thermal energy within the medium, and scattering, where light is scattered in all directions by the medium [2]. Therefore, the introduction of absorption and scattering characteristic parameters is used to describe the interaction between light and matter. The absorption coefficient μ_a and scattering coefficient μ_s represent the probability of a photon being absorbed or scattered per unit length in the medium, respectively. The absorption coefficient, mentioned earlier, has a physical meaning consistent with the absorption coefficient in Lambert's law and can also be described as the relative change in light intensity after passing through a unit distance [3].

When light, represented by a monochromatic plane wave, penetrates a medium of thickness dx vertically along the X-direction and experiences a decrease in intensity from I_0 to $I - dI$, the absorption coefficient can be defined as

$$I = I_0 e^{-\mu_a x} \tag{16}$$

Here, I_0 represents the incident light intensity, and I is the intensity of light entering the medium at a distance x from the entry point. The extinction coefficient is related to the absorption and scattering of the medium and can be expressed as

$$\eta = \frac{\mu\lambda}{4\pi} = \frac{(\mu_a + \mu_s)\lambda}{4\pi} \tag{17}$$

Therefore, the complex refractive index of the medium can be defined as

$$\hat{n} = n + i\eta = n + i\frac{\mu\lambda}{4\pi} = n + i\frac{(\mu_a + \mu_s)\lambda}{4\pi} \tag{18}$$

Among them, $\mu = \mu_a + \mu_s$ indicates the total attenuation coefficient, μ_a is the absorption coefficient, and μ_s is the scattering coefficient.

When light transitions from non-absorbing, uniform medium 1 into medium 2, taking into account the interaction of light with medium 2 (including absorption and scattering), and applying the complex refractive index into the Fresnel equations, we obtain

$$r_s = \frac{n_1 \cos\theta_1 - \hat{n}_2 \cos\theta_2}{n_1 \cos\theta_1 + \hat{n}_2 \cos\theta_2} \tag{19}$$

$$r_p = \frac{\hat{n}_2 \cos\theta_1 - n_1 \cos\theta_2}{\hat{n}_2 \cos\theta_1 + n_1 \cos\theta_2} \tag{20}$$

Further, based on the law of refraction, we have

$$r_s = \frac{n_1 \cos\theta_1 - \sqrt{\left(n_2 + i\frac{(\mu_a + \mu_s)\lambda}{4\pi}\right)^2 - n_1^2 \sin^2\theta_1}}{n_1 \cos\theta_1 + \sqrt{\left(n_2 + i\frac{(\mu_a + \mu_s)\lambda}{4\pi}\right)^2 - n_1^2 \sin^2\theta_1}} \tag{21}$$

$$r_p = \frac{\left(n_2 + i\frac{(\mu_a+\mu_s)\lambda}{4\pi}\right)^2 \cos\theta_1 - \sqrt{n_1^2\left(n_2 + i\frac{(\mu_a+\mu_s)\lambda}{4\pi}\right)^2 - n_1^2 \sin^2\theta_1)}}{\left(n_2 + i\frac{(\mu_a+\mu_s)\lambda}{4\pi}\right)^2 \cos\theta_1 + \sqrt{n_1^2(\left(n_2 + i\frac{(\mu_a+\mu_s)\lambda}{4\pi}\right)^2 - n_1^2 \sin^2\theta_1)}} \quad (22)$$

From these equations, we can calculate the phase of the reflection coefficients as

$$\phi_s = \mathrm{Im}\left\{\ln\left[\frac{n_1\cos\theta_1 - \sqrt{\left(n_2 + i\frac{(\mu_a+\mu_s)\lambda}{4\pi}\right)^2 - n_1^2\sin^2\theta_1}}{n_1\cos\theta_1 + \sqrt{\left(n_2 + i\frac{(\mu_a+\mu_s)\lambda}{4\pi}\right)^2 - n_1^2\sin^2\theta_1}}\right]\right\} \quad (23)$$

$$\phi_p = \mathrm{Im}\left\{\ln\left[\frac{\left(n_2 + i\frac{(\mu_a+\mu_s)\lambda}{4\pi}\right)^2 \cos\theta_1 - \sqrt{n_1^2\left(n_2 + i\frac{(\mu_a+\mu_s)\lambda}{4\pi}\right)^2 - n_1^2 \sin^2\theta_1)}}{\left(n_2 + i\frac{(\mu_a+\mu_s)\lambda}{4\pi}\right)^2 \cos\theta_1 + \sqrt{n_1^2(\left(n_2 + i\frac{(\mu_a+\mu_s)\lambda}{4\pi}\right)^2 - n_1^2 \sin^2\theta_1)}}\right]\right\} \quad (24)$$

With these phase values, GHS can be expressed as

$$D_i = -\frac{\lambda_0}{2\pi n_1}\frac{d\phi_i}{d\theta_1} = -\frac{\lambda_0}{2\pi n_1}\frac{d\,\mathrm{Im}[\ln(\phi_i)]}{d\theta_1} \quad (25)$$

It can be observed that the magnitude of the GHS is influenced by several factors, including the wavelength of light, polarization state, angle of incidence, and refractive index of the medium, as well as the absorption and scattering coefficients. However, when the wavelength of light, polarization state, and angle of incidence are held constant, the magnitude of the GHS is determined by the refractive index, absorption coefficient, and scattering coefficient, which reflect the electromagnetic properties of the material itself. In essence, the GHS can serve as a characterization of the intrinsic electromagnetic properties of the material.

References

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