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LINEAR FORCED IN-PLANE AND OUT-OF-PLANE VIBRATIONS OF FRAMES HAVING A CURVED MEMBER

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Abstract- The forced, in-plane and out-of-plane vibrations of frames comprised of straight and curved members are investigated using Finite Element Methods. The straight and curved beams are assumed as Euler-Bernoulli type and they have circular cross-sections. The frame lies in a single plane. In the analysis, elongation, bending and rotary inertia effects are included. Four degrees of freedom for in-plane vibrations and three degrees of freedom for out-of-plane vibrations are assumed. The in-plane and out-of-plane point and transfer receptances are obtained in order to determine the sensitive and non-sensitive frequency interval of the frame system.

Keywords- vibration of frames, curved beams, forced vibrations

1. INTRODUCTION

The vibrations of straight and curved beams have been studied by many scientists. They can be used in structures such as gears, electrical machines, pumps and turbines, ships, to model the behaviour of horizontally curved multispan continuous bridges or in the design of ribs, edge stiffeners in bridge deck slabs and stiffened shell characteristics of turbomachines and rockets, etc. In addition to them, there are many general and special areas. The governing equations for these problems were presented together with their solutions in the book by Love [1]. The dynamic responses of curved beams were investigated widely. Ojalvo *et al.* [2] studied the elastic stability of ring segments with a thrust or a pull directed along the chord neglecting warping effect. Irie *et al.* [3] analyzed the steady state response of a Timoshenko curved beam with internal damping. Silva and Urgueira [4] used an analytical model for out-of-plane vibrations. Ibrahimbegović [5] discussed the beam elements whose reference axes were arbitrary space-curved lines including forcing effect. Khdeir and Reddy [6] presented a generalized modal approach to solve the dynamic response of cross-ply laminated arches with arbitrary boundary conditions and for arbitrary loadings. Khan and Pise [7] presented an analytical model and associated computer program which was developed to investigate the dynamic behavior of curved piles embedded in a homogeneous elastic half-space and subjected to forced harmonic vertical vibrations, wherein the movement of piles in the axial and lateral directions were considered. Kang and Bert [8] applied DQM for computation of the eigenvalues of flexural-torsional buckling of arches including a warping contribution and bending moments and radial loads. Bozhevolnaya and Kildegaard [9] investigated a sandwich curved beam subjected to a uniform loading experimentally. Load-deflection and thrust-deflection dependencies are shown to be nonlinear, while load-deformation dependencies for sandwich faces are found to be linear.

Dynamic stiffness method was used to analyze the transient response of the curved beams [10]. Walsh and White [11] investigated coupled extensional-flexural wave propagation by considering the mobility of a semi-infinite beam with a constant radius of curvature. Both theoretical and experimental results were discussed and formulae for the point and cross mobilities of the structure were presented. Huang [12] presented a systematic method for analyzing the out-of-plane dynamic behaviours of non-circular curved beams by taking into account the effects of shear deformation, rotary inertia, and viscous damping.

The studies about vibrations of frames having both curved and straight members are as follows. Cortinez *et al.* [13] calculated the inextensional natural frequencies of a fixed-free straight-curved beam system having a concentrated mass at the end of curved member for in-plane vibrations by excluding rotary inertia. The authors used Rayleigh-Schmidt technique, and compared the results with the results of Dunkerley's approach and FEM. Wang *et al.* [14] presented theoretical and numerical analyses for the combined system of a spatial curved rod and straight rods (spiral rods) in free vibration using transfer matrix method. Kashimoto *et al.* [15] presented the dynamic stress concentration problem of an inhomogeneous rod of infinite length, consisting of two infinite straight portions and one finite portion of arbitrary curvature by using transfer matrix method. The authors gave natural frequency values for only curved part. Wang [16,17] set up the displacements, which are three displacement components; two bending slopes and one twist angle, for a curved frame to derive the governing equations of a T-type curved frame via the same beam theory. An analytical method for both the in-plane motion and out-of-plane motion of a curved hollow shaft was presented for two types of shaft structures, which were a curved hollow shaft and a fixed-fixed straight-curved-straight-hollow shaft by considering torsion and bending. The author found that the first in-plane modal frequency of a structure was greater than the first out-of-plane modal frequency of the same structure using transfer matrix method. Petrolito and Legge [18] developed a general nonlinear analysis method for structural frames with curved members to calculate the complete load-deflection response.

In this study, the linear, forced in-plane and out-of-plane vibrations of the frames comprised of a straight and a curved member are investigated by using Finite Elements Method, FEM. The curvature of the curved member is on a single plane. The bending, torsion effects for the straight part and bending, torsion and rotatory inertia effects for the curved part are included. The system is modelled using energy equations and analysis is made for different cases. The energy equations of the straight beam are written in cartesian coordinates and those of the curved beam are written in radial-tangential coordinates. The in-plane and out-of-plane point and transfer receptances are plotted and the effect of loss factor is studied.

2. FINITE ELEMENT FORMULATION

The finite element method [19] will be used to obtain the response of vibrating frame. The frame is comprised of one straight and one curved beam as shown in Figure 1. The beams have rigid connection to each other, and L_T , S are the lengths of straight and curved members respectively. R is the radius of curvature, and γ is arch angle of the curved beam. E is modulus of elasticity, I is mass moment of inertia, J is polar moment of inertia, A is cross-sectional area.

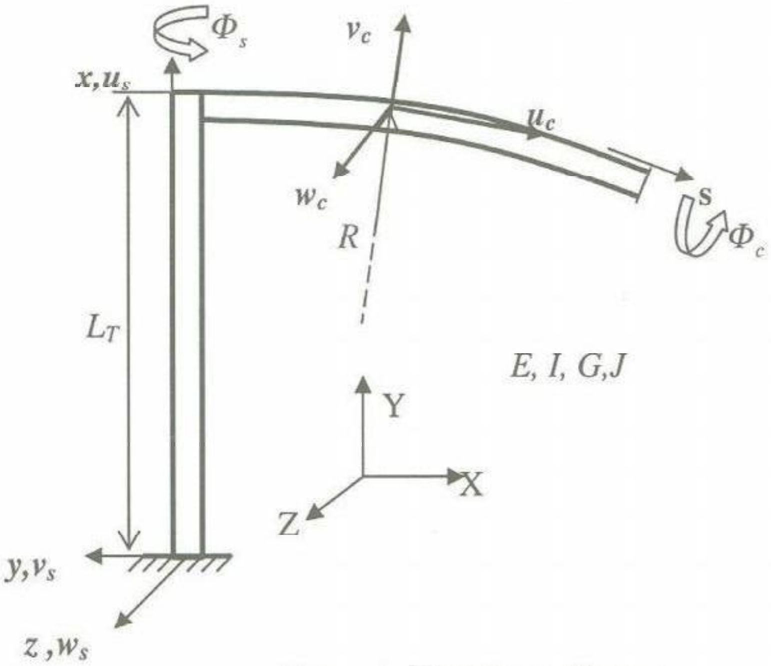


Figure 1. The Frame System

Sub-indices s and c denote straight and curved members respectively. The cubic interpolation functions for tangential, radial displacements in in-plane vibrations and linear torsional and cubic transverse displacements in out-of-plane vibrations are assumed.

The elastic and kinetic energies of the frame can be expressed as follows

$$U_{in} = \frac{1}{2}E\int_s \left[A\varepsilon_{c\,in}^2 + I\kappa_{c\,in}^2\right]ds + \frac{1}{2}E\int_x \left[A\varepsilon_{s\,in}^2 + I\kappa_{s\,in}^2\right]dx \tag{1}$$

$$T_{in} = \frac{1}{2}\rho\int_s \left[A(\dot{u}_{c\,in}^2 + \dot{v}_{c\,in}^2) + I\dot{\beta}_{c\,in}^2\right]ds + \frac{1}{2}\rho\int_x \left[A\left(\dot{u}_{s\,in}^2 + \dot{v}_{s\,in}^2\right) + I\dot{\beta}_{s\,in}^2\right]dx \tag{2}$$

$$U_{out}=\frac{1}{2}EI\int_L \kappa_{s\,out}^2\,dx+\frac{1}{2}GJ\int_L \varphi_{s\,out}^2\,dx+\frac{1}{2}EI\int_S \kappa_{c\,out}^2\,ds+\frac{1}{2}GJ\int_S \varphi_{c\,out}^2\,ds \tag{3}$$

$$T_{out}=\frac{1}{2}\rho A\int_L \dot{w}_{s\,out}^2\,dx+\frac{1}{2}\rho J\int_L \dot{\Phi}_{s\,out}^2\,dx+\frac{1}{2}\rho A\int_S \dot{w}_{c\,out}^2\,ds+\frac{1}{2}\rho I\int_S \dot{\Psi}_{c\,out}^2\,ds+\frac{1}{2}\rho J\int_S \dot{\Phi}_{c\,out}^2\,ds \tag{4}$$

In these equations $(\dot{\ })$ denotes differentiation with respect to time t. In-plane strain, net cross-sectional rotation and curvature change of the curved and straight member in equations (1) and (2), and out-of-plane curvature change and torsion in equations (3) and (4) are as follows

$$\varepsilon_{cin}=\frac{\partial u_{cin}}{\partial s}+\frac{v_{cin}}{R} \qquad \varepsilon_{sin}=\frac{\partial u_{sin}}{\partial x} \tag{5}$$

$$\beta_{cin}=\frac{\partial v_{cin}}{\partial s}-\frac{u_{cin}}{R} \qquad \beta_{sin}=\frac{\partial v_{sin}}{\partial x} \tag{6}$$

$$\kappa_{cin}=\frac{\partial \beta_{cin}}{\partial s}=\frac{\partial^2 v_{cin}}{\partial s^2}-\frac{1}{R}\frac{\partial u_{cin}}{\partial s} \qquad \kappa_{sin}=\frac{\partial \beta_{sin}}{\partial x}=\frac{\partial^2 v_{sin}}{\partial x^2} \tag{7}$$

$$\kappa_{cout}=\frac{\Phi_{cout}}{R}-\frac{\partial^2 w_{cout}}{\partial s^2} \qquad \kappa_{sout}=\frac{\partial^2 w_{sout}}{\partial x^2} \tag{8}$$

$$\varphi_{cout}=\frac{\partial \Phi_{cout}}{\partial s}+\frac{1}{R}\frac{\partial w_{cout}}{\partial s} \qquad \varphi_{sout}=\frac{\partial \Phi_{sout}}{\partial s} \tag{9}$$

$$\Psi_{cout}=\partial w_{cout}/\partial s \tag{10}$$

3. FORCED VIBRATIONS

The characteristics of forced vibrations have importance due to the need of controlling of vibration amplitudes. That's why, the point and transfer receptances of the frame by forcing at the free end are calculated by including the structural damping. The general equation of motion for a harmonically forced system is as follows [19]:

$$M\ddot{u}+C\dot{u}+Ku=f\exp(i\omega t) \tag{11}$$

where u is the nodal displacement vector, C is viscous damping matrix and f is the vector formed by forces on the nodes. There are some mathematical models for the damping expressing energy losses in mechanical systems [20,21]. The structural damping can be considered by replacing $K(1+i\eta)$ with the viscous damping. In this case, equation (11) becomes

$$u = \left[K - \omega^2 M + i\eta K\right]^{-1} f \exp(i\omega t) \tag{12}$$

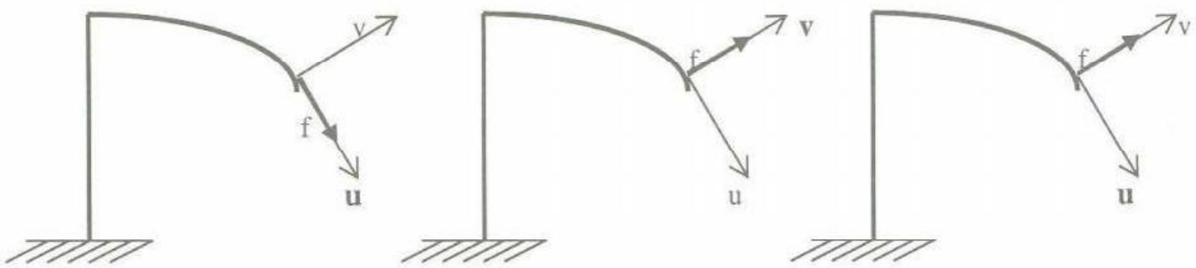
where $i = \sqrt{-1}$ and η is the loss factor. Equation (12) can be written as

$$u = \left[A_R + iA_I\right]^{-1} f \exp(i\omega t) \tag{13}$$

where $K - \omega^2 M = A_R$ and $\eta K = A_I$. In the analysis the material properties are assumed as follows: E=2*10^{11} Pa, G=0.84*10^{11} Pa, ρ=7800 kg/m^3, S and L_T=1 m, γ is 30°, cross-sectional radii are 0.02 m,

3.1. In-Plane Vibrations

In this section, an external force is assumed at free end which has effects in radial and tangential directions of curved part. The point receptance models due to the force acting in tangential, radial directions are shown in Figures 2a-b. The transfer receptance model for the displacement of tangential direction due to the force acting in radial direction is shown in Figure 2c.



(a) Point receptance (u / f) (b) Point receptance (v / f) (c) Transfer receptance (u / f)
Figure 2. Receptances for in-plane vibrations

In Figure 3 and 4, the point receptance amplitudes of tangential and radial directions at the free end are shown respectively as functions of the forcing frequencies. 0.01, 0.05 and 0.2 are taken as loss factor. The large amplitudes obviously show the natural frequencies, and as it's seen, the increase of loss factor limits the amplitudes. In Figure 5, the transfer receptance amplitudes of frame is shown corresponding to the mode 1 seen in Figure 2c.

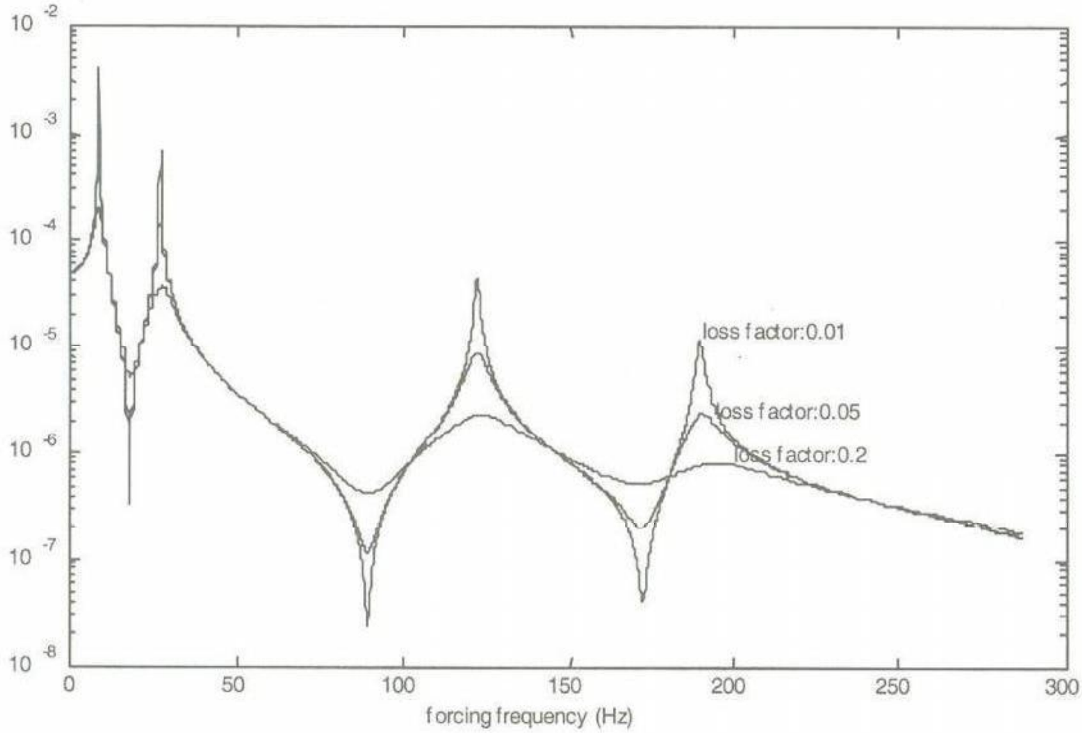


Figure 3. The in-plane point receptance amplitudes of tangential displacement (u / f)

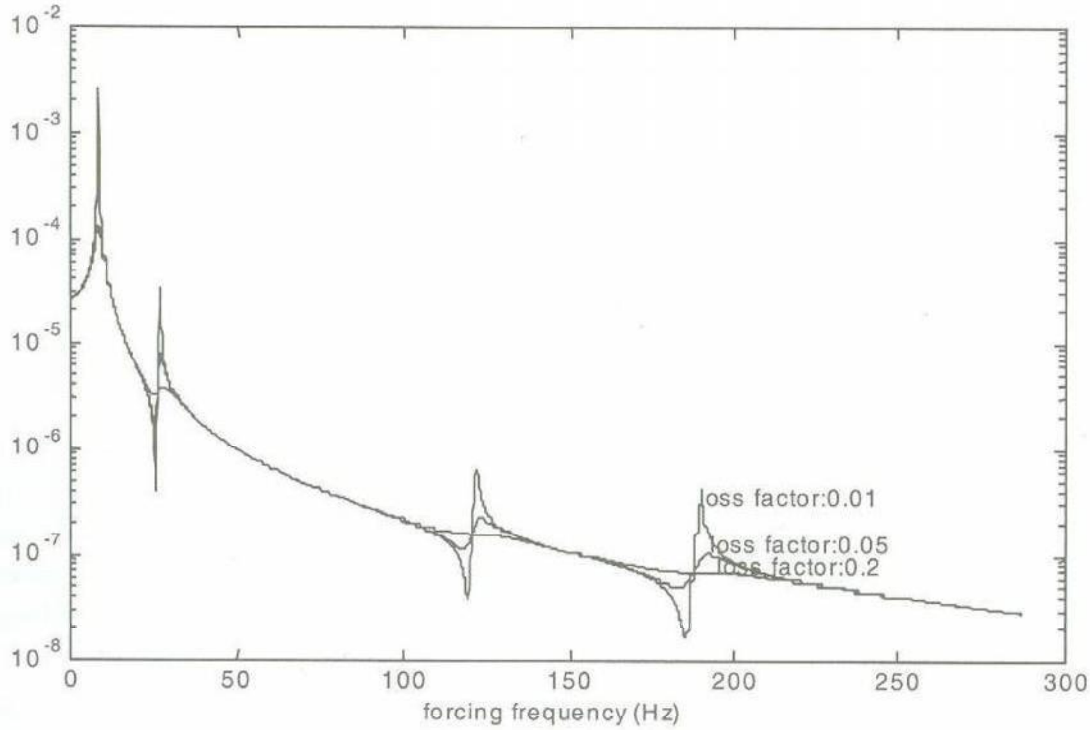


Figure 4. The in-plane point receptance amplitudes of radial displacement (v / f)

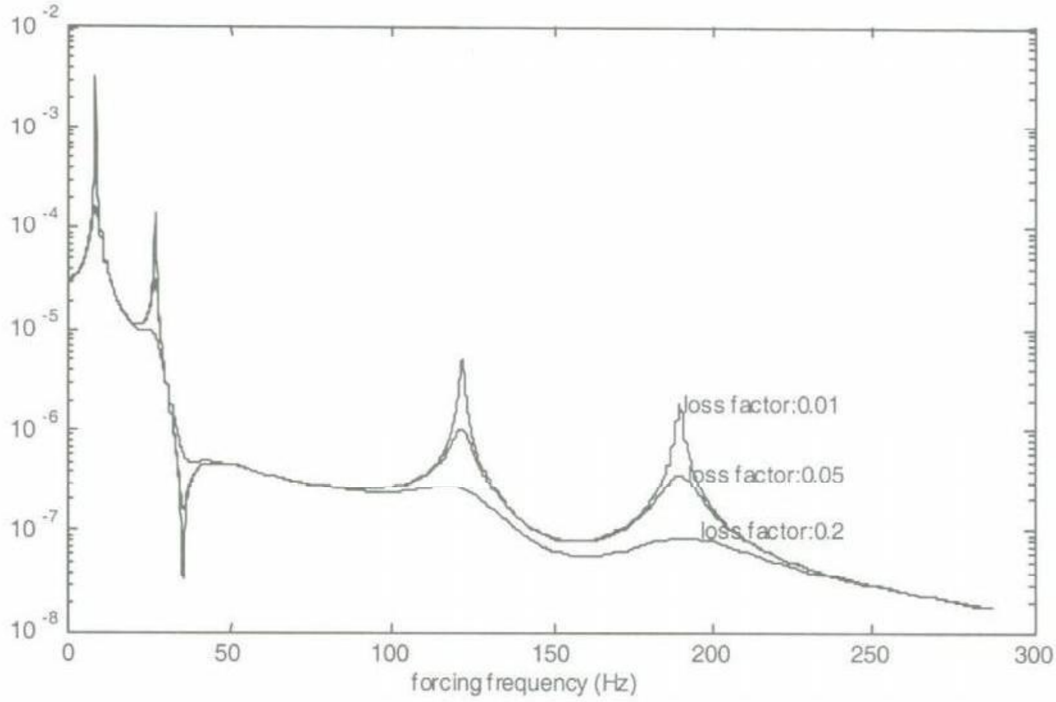


Figure 5. The in-plane transfer receptance amplitudes of tangential displacement due to the force acting on the direction of radial displacement (u / f)

3.2. Out-of-Plane Vibrations

The external force is also applied in the direction of out-of-plane displacement w. In Figure 6, the point receptance (a) and the transfer receptance (b) models for free end are shown. In Figures 7 and 8, the changes of point and transfer receptances due to the forcing frequencies are shown respectively. The point receptance of the displacement w is shown in Figure 7, and the torsional transfer receptance due to the force acting on the direction of w in Figure 8. As seen, the characteristics of the receptances are naturally similar to those of the previous section. The increase of the loss factor again reduces the amplitudes.

(a) Point receptance (w / f) (b) Transfer receptance (ϕ / f)

Figure 6. Receptances for out-of-plane vibrations

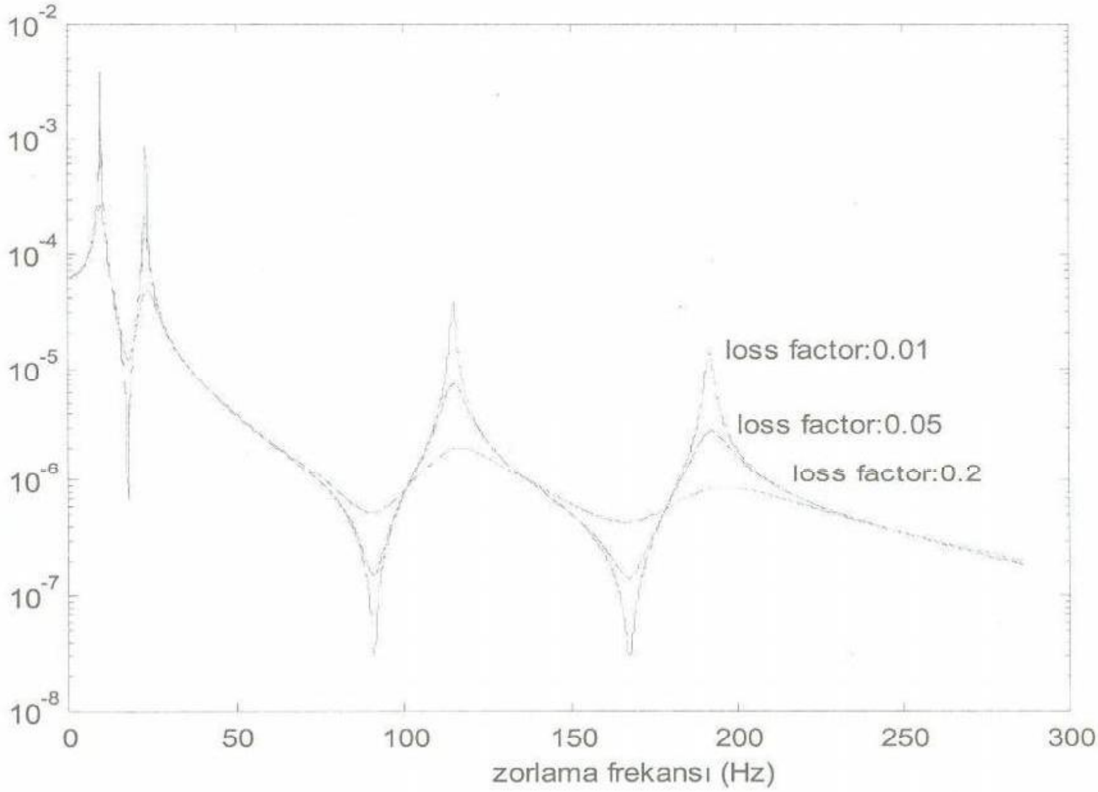


Figure 7. The out of-plane point receptance amplitudes of out-plane displacement (w / f)

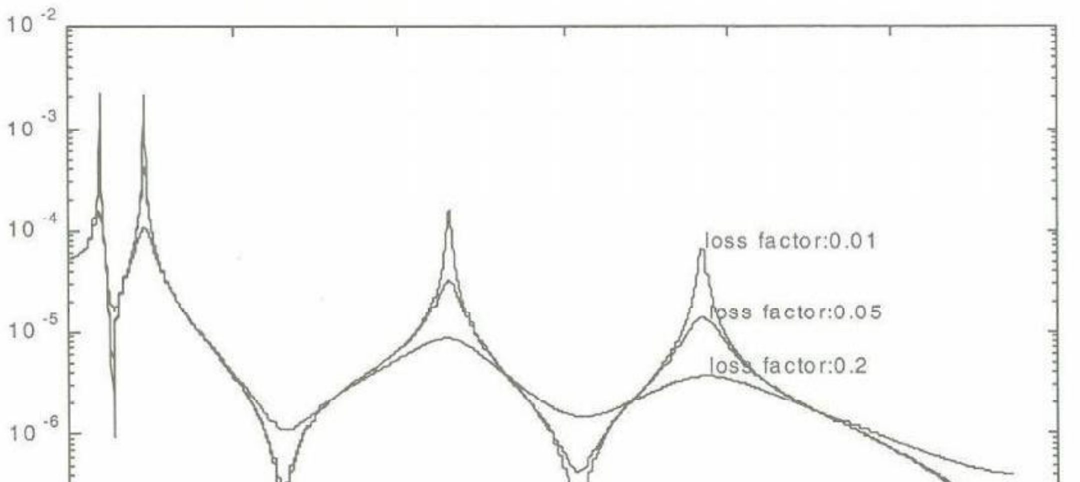


Figure 8. The out of-plane transfer receptance amplitudes of twist angle (ϕ) due to the force acting on the direction of displacement w (ϕ / f)

4. CONCLUSIONS

In this study, the forced vibrations of the frame comprised of a straight and a curved member are investigated. The in-plane and out-of-plane receptances are obtained due to the external force acting to the free end of the frame. The changes of the point and transfer receptance amplitudes are plotted and the effect of loss factor is observed. The sensitive and non-sensitive frequency intervals are determined for frame system. It should also be noted that, the in-plane and out-of-plane natural frequencies of the frame can be obtained from the related figures which show the change of receptances.

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