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# ROBUST CONTROL OF A SPATIAL ROBOT USING SLIDING MODES

Nurkan Yagiz

Istanbul University,
Department of Mechanical Engineering,
Avcilar, Istanbul, Turkey

**Abstract-**In this work the control of a spatial robot by sliding mode control is studied. The robot model has three degrees of freedom. These robots are usually used in material handling in remote dangerous environments and production line. First, the mathematical model of the system is formulated. The equations of motion are driven by employing Lagrangian formulation based on the energy equations. Then the sliding mode control theory is applied. Special care is given to the chattering problem. The chattering may have serious damaging effect on gear systems as well as motor drive systems. Hence, the chattering character of classical sliding mode control is overcome and a new version of the control method is applied on robot. The simulation results are presented in graphical form and the robust character of the selected control method is shown to be capable of controlling the robots successfully against the disturbances as well as having the robot follow the desired trajectory.

**Key Words-**Spatial robot, sliding mode control, chattering problem, robust character.

## 1.INTRODUCTION

Sliding mode control is one of the most important approaches in robotics with large uncertainties, nonlinearities and external disturbances. Generally, a MIMO linear sliding mode is first designed to describe the desired system error dynamics, a robust controller drives the swithching plane variables to reach the sliding mode and the convergence of error dynamics can be obtained on the linear sliding mode. The application of non-chattering and robust sliding mode control [1] is the goal of this study. Sliding mode control has been proposed by Emelyanov and friends in Soviet Union in 1950 [2], [3]. The reasons of preferring the sliding mode control theory in this study was the applicability on linear and non-linear systems, multi inport-outport systems and the improvement of the discrete time techniques besides its non-chattering and robust character. The most important character of this method is its robustness. In other words, its insensitiveness to system parameter changes and disturbances from outside [4], [5]. Nowadays, sliding mode control has been applied in the design of robot control [6]-[8], flight control, motor control and power systems. The studies on sliding mode control can be classified under two main subject: First, the new application areas and second, the chattering reduction.

## 2.ROBOT MODEL

The physical model of the Spatial Robot has been shown in Figure 1. The robot model has three degrees of freedom. In this model, joint frictions have been neglected.

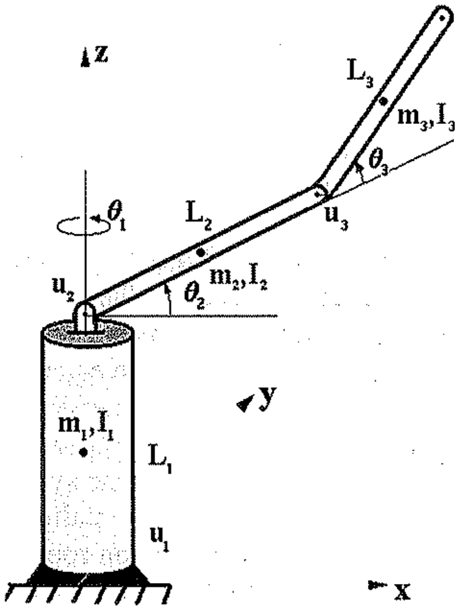


Figure 1. Three Degrees of Freedom Spatial Robot Model.

Considering the model, $m_1$, $I_1$, $L_1$ represent the mass, angular inertia and length of the first arm; $m_2$, $I_2$, $L_2$ represent the mass, angular inertia, length of the second arm; finally, $m_3$, $I_3$, $L_3$ represent the mass, angular inertia, length of the third arm. The position of the center of mass of the arms are at half length. The system parameters are given at the Appendix. The angular positions are represented by $\theta_1$, $\theta_2$ and $\theta_3$. The $u_1$, $u_2$ and $u_3$ are the control moment inputs to the robot to have it follow the desired trajectory with or without the existance of the any disturbance. Let ( $p_x$ , $p_y$ , $p_z$ ) show the end point cartesian coordinates of the spatial robot following the desired trajectory. Making necessary kinematic arrangements, the angular motions to follow the desired robot trajectory can be obtained as follows:

$$\theta_1 = \arctan\left(\frac{p_y}{p_x}\right) \tag{1}$$

$$\theta_2 = \arctan\left(\frac{P_z - L_1}{\dfrac{P_x}{\cos\theta_1}}\right) - \arctan\left(\frac{L_3 \sin\theta_3}{(L_2 + L_3\cos\theta_3)\cos\theta_1}\right) \tag{2}$$

$$\theta_3 = \arccos\left(\frac{\left(\frac{Py}{\sin\theta_1}\right)^2 + (Pz - L_1)^2 - L_2^2 - L_3^2}{2L_2L_3}\right) \tag{3}$$

Since the cartesian coordinates of the mass centers of the arms are:

$$\left.\begin{aligned} x_2 &= \frac{L_2}{2}\cos\theta_2\cos\theta_1 \\ y_2 &= \frac{L_2}{2}\cos\theta_2\sin\theta_1 \\ z_2 &= L_1 + \frac{L_2}{2}\sin\theta_2 \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} x_3 &= (L_2\cos\theta_2 + \frac{L_3}{2}\cos\theta_3)\cos\theta_1 \\ y_3 &= (L_2\cos\theta_2 + \frac{L_3}{2}\cos\theta_3)\sin\theta_1 \\ z_3 &= L_1 + L_2\sin\theta_2 + \frac{L_3}{2}\sin\theta_3 \end{aligned}\right\} \tag{5}$$

Using the Lagrange Formulae, the equations of the motion of the three degrees of freedom spatial robot are obtained:

$$\begin{aligned} &\left[I_1 + m_3\left(L_2C_2 + \frac{L_3}{2}C_{23}\right)^2\right]\ddot{\theta}_1 + \left(2m_3(L_2^2C_2S_2 + \frac{L_3}{2}C_{23}L_2S_2) - m_2\frac{L_2^2}{2}C_2S_2\right) \\ &\dot{\theta}_1\dot{\theta}_2 + \left[m_2C_2^2\frac{L_2^2}{4}\right]\ddot{\theta}_2 + \left[2m_3\left(L_2\frac{L_3}{2}C_2 + \frac{L_3^2}{4}C_{23}S_{23}\right)\right]\dot{\theta}_1(\dot{\theta}_2 + \dot{\theta}_3) = u_1 \end{aligned} \tag{6}$$

$$\begin{aligned} &\left[I_2 + m_2\frac{L_2^2}{4} + \frac{m_3}{2}(2L_2^2 + \frac{L_3^2}{2} + 2L_2L_3C_{23})\right]\ddot{\theta}_2 + \left[\frac{m_3}{2}(L_3^2 + L_2L_3C_2)\right]\ddot{\theta}_3 \\ &+ \frac{m_3}{2}L_2L_3(2\dot{\theta}_2\dot{\theta}_3 - \dot{\theta}_3^2S_3) + C_2(\frac{L_2}{2}m_2g + m_3gL_2) + m_3g\frac{L_3}{2}C_{23} = u_2 - u_3 \end{aligned} \tag{7}$$

$$I_3\ddot{\theta}_3 + m_3\left[L_2\frac{L_3}{2}\left(\ddot{\theta}_2C_3 - \dot{\theta}_2\dot{\theta}_3S_3\right) + \frac{L_3^2}{4}(\ddot{\theta}_2 + \ddot{\theta}_3)\right] + m_3g\frac{L_3}{2}C_{23} = u_3 \tag{8}$$

In these equations $C_i$, $S_i$, $C_{ij}$ and $S_{ij}$ stand for $\cos\theta_i$, $\sin\theta_i$, $\cos(\theta_i\text{-}\theta_j)$ and $\sin(\theta_i\text{-}\theta_j)$ respectively.

## 3.CHATTERING FREE SLIDING MODE CONTROLLER DESIGN

Sliding mode controller provides an effective control in the presence of parameter uncertainties and unmodeled dynamics. The method is based on transforming an $n^{th}$ order tracking problem into first order stability problem. In theory, successful performance is obtained but the uncertainties in the model structure makes a compromise between tracking performance in a given band width and parametric uncertainty. In practise, it is equal to replace a control method in which the direction of the control changes too fast with a smoother one. A controlled non-linear dynamic system is described by Equation (9) as

$$\dot{\underline{x}} = \underline{f}(x) + [B]\underline{u} \tag{9}$$

The aim of the controller is to control the variable $\underline{x}$ under the system uncertainties and hold the system on a sliding surface S as shown in Figure 2. This surface is described as:

$$S = \{\, \underline{x} : \underline{\sigma}\,(x,t) = 0 \} \tag{10}$$

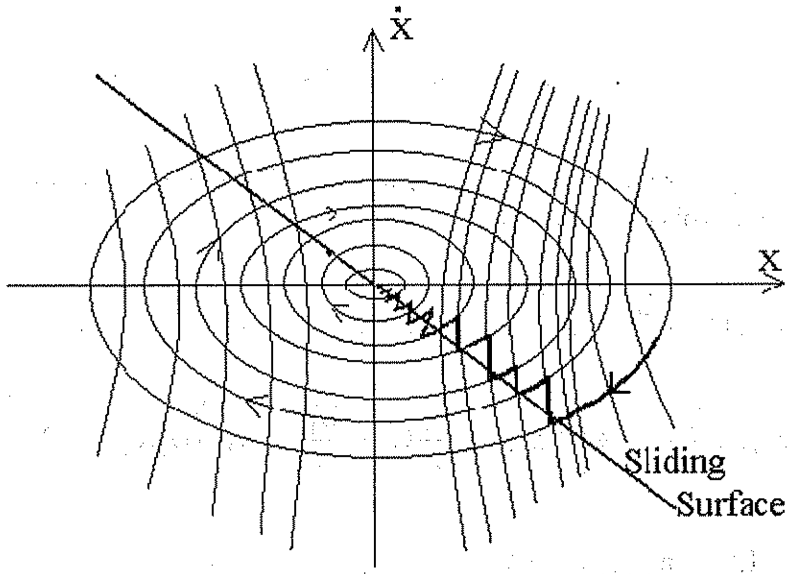


Figure 2. The Trajectory of the Controlled System on the Phase Plane

The sliding surface equation for a system can be selected as follows:

$$\underline{\sigma} = [G]\Delta\underline{x} \tag{11}$$

$\Delta\underline{x} = x_r - x$ is the difference between the reference value and the system response. [G] is the matrix which represents the sliding surface slope. For stability, the following

Lyapunov function candidate has to be positive definite and its derivative has to be negative semi-definite:

$$v(\underline{\sigma}) = \frac{\underline{\sigma}^T \underline{\sigma}}{2} > 0 \tag{12}$$

$$\frac{d\,v(\underline{\sigma})}{d\,t} = \frac{\underline{\dot{\sigma}}^T \underline{\sigma}}{2} + \frac{\underline{\sigma}^T \underline{\dot{\sigma}}}{2} \leq 0 \tag{13}$$

The equation (11) is seperated as follows:

$$\underline{\sigma} = \underline{\phi}(t) - \underline{\sigma}_a(x) \tag{14}$$

where:

$$\underline{\phi}(t) = [G]\, \underline{x}_r \tag{15}$$

$$\underline{\sigma}_a\,(x) = [G]\, \underline{x} \tag{16}$$

If the limit condition is applied to (13), then:

$$\frac{d\underline{\sigma}}{d\,t} = \frac{d\,\underline{\phi}(t)}{d\,t} - \frac{\partial\,\underline{\sigma}_a(x)}{\partial\,x}\frac{d\,\underline{x}}{d\,t} = 0 \tag{17}$$

From (9) and (11),

$$\frac{d\,\underline{\phi}}{d\,t} = [G]\,\underline{f}(x) + [G][B]\underline{u}_{eq} \tag{18}$$

$\underline{u}_{eq}$ is the controller force of the limit case and from the equation (18),

$$\underline{u}_{eq} = [G\,B]^{-1}\left(\frac{d\,\underline{\phi}(t)}{d\,t} - [G]\underline{f}(x)\right) \tag{19}$$

with the condition that $[GB]^{-1}$ must exist. For mechanical systems, $[GB]^{-1}$ is always pseudo-inverse and equals to mass matrix. Since the equivalent control in Equation (18) is only valid on the sliding surface an additional term must be found in order to induce the system to follow the constraints by holding it on the surface. The derivative of another Lyapunov function candidate may be chosen as follows:

$$\underline{\dot{v}} = -\underline{\sigma}^T\,[\Gamma]\underline{\sigma} < 0 \tag{20}$$

The equations (13) and (20) have to be equal. By substituting the necessary terms, the following expression is obtained:

$$\underline{u} = \underline{u}_{eq} + [K]\,\underline{\sigma} \tag{21}$$

Here,

$$[K] = [GB]^{-1}[\Gamma] \tag{22}$$

$[\Gamma]$ is assumed to be constant matrix. The value of the terms are determined at the design stage by trial. Higher values give better results but restricted by actuator limits and other conflicting criteria. If $\underline{f}(x)$ and [B] matrices are not well known, then the equivalent control inputs will be too far from the actual equivalent control inputs. In the literature a number of approaches are proposed for the estimation of $\underline{u}_{eq}$ rather than calculating it. In this study, it is suggested that the equivalent control is the average of the total control. The design of an averaging filter for the calculation of the equivalent control can be as below:

$$\hat{\underline{u}}_{eq} = \frac{1}{\tau s + 1}\underline{u} \tag{23}$$

This means that the control input enters a low-pass filter. The value of $1/\tau$ gives the cut-off frequency. Low frequencies determine the characteristics of the signal and high frequencies come from unmodeled dynamics. So the filter smoothens the control input. The resultant controller input is:

$$\underline{u} = \hat{\underline{u}}_{eq} + [K]\underline{\sigma} \tag{24}$$

## 4.CONTROL OF THE SPATIAL ROBOT AND ROBUSTNESS VERIFICATION

The simulation has been resulted using the sliding mode control technique proposed. In classical sliding mode control, the control parameter value has been found by adding a constant number K times the sign of sliding function to the equivalent control value [1], [2], [3]. Since the sign of sliding function has been used, a high frequency chattering having the value of $\pm$ K has been added to the equivalent control. If such motions are expected from robot arm, the gear mechanism fails. Therefore soft torque transition output must be expected from motor drives. That means chattering must be prevented. Since the controller proposed in this study is selected as the equivalent control value plus a constant K, a non-chattering motion has been obtained. Besides, equivalent control has been estimated and the need for system knowledge has been minimized.

To observe the performance of the controller, a trajectory on the working space limits of the robot is defined as shown in Figure 3. where:

$$\left.\begin{aligned} p_x &= x_f + (x_i - x_f)e^{-50t^3} \\ p_y &= y_f + (y_i - y_f)e^{-50t^3} \\ p_z &= z_f + (z_i - z_f)e^{-50t^3} \end{aligned}\right\} \tag{25}$$

$(x_i,y_i,z_i)$ and $(x_f,y_f,z_f)$ are the cartesian coordinates of the start and end points of the trajectory where robot arm end point is to follow and given at the Appendix.

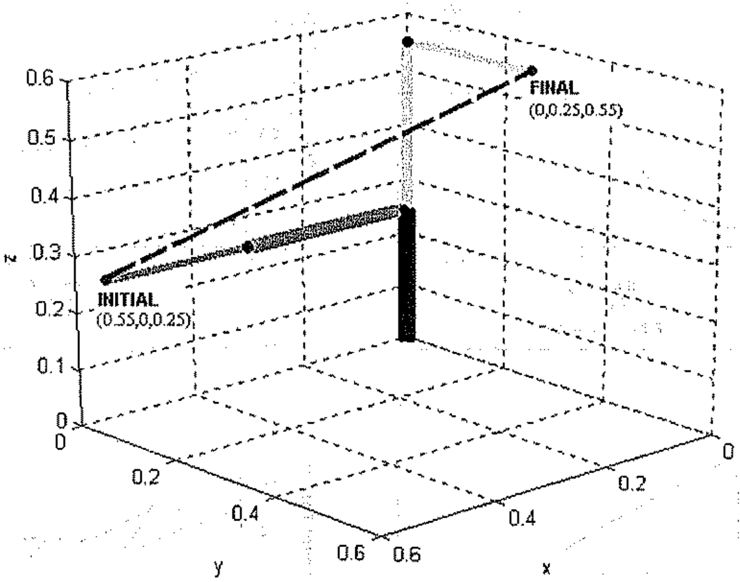


Figure 3. Desired Robot Arm Trajectory

The block diagram of the system is presented in Figure 4. Using the trajectory desired and performing inverse kinematic manipulations, necessary robot joint motions are calculated and used as referance motions. The differance between these referance values and values of the spatial robot angular motions become the error values used in sliding mode controller which produces joint torques necessary to maintain the trajectory.

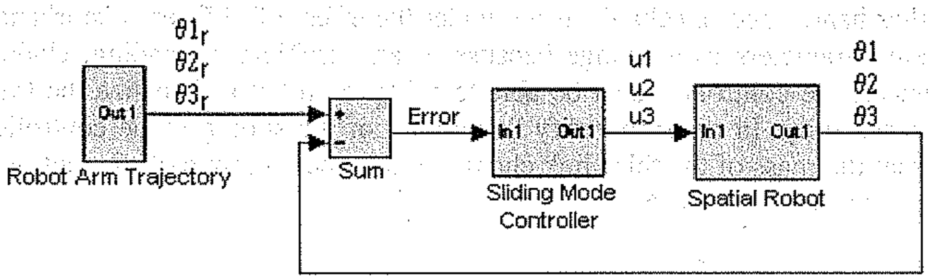


Figure 4. Block Diagram of the System.

In Figure 5, the simulation results are presented. Instead of using dynamic relations in producing necessary joint torques, sliding mode controller is used to produce them. Necessary joint angular motions as a result of control process are shown in Figure 5.a. The error of angular motions are presented in Figure 5.b. Figure 5.c gives the phase plane plot of the error of the angular displacement of the second joint. The necessary control inputs to follow the desired trajectory are plotted in Figure 5.d.

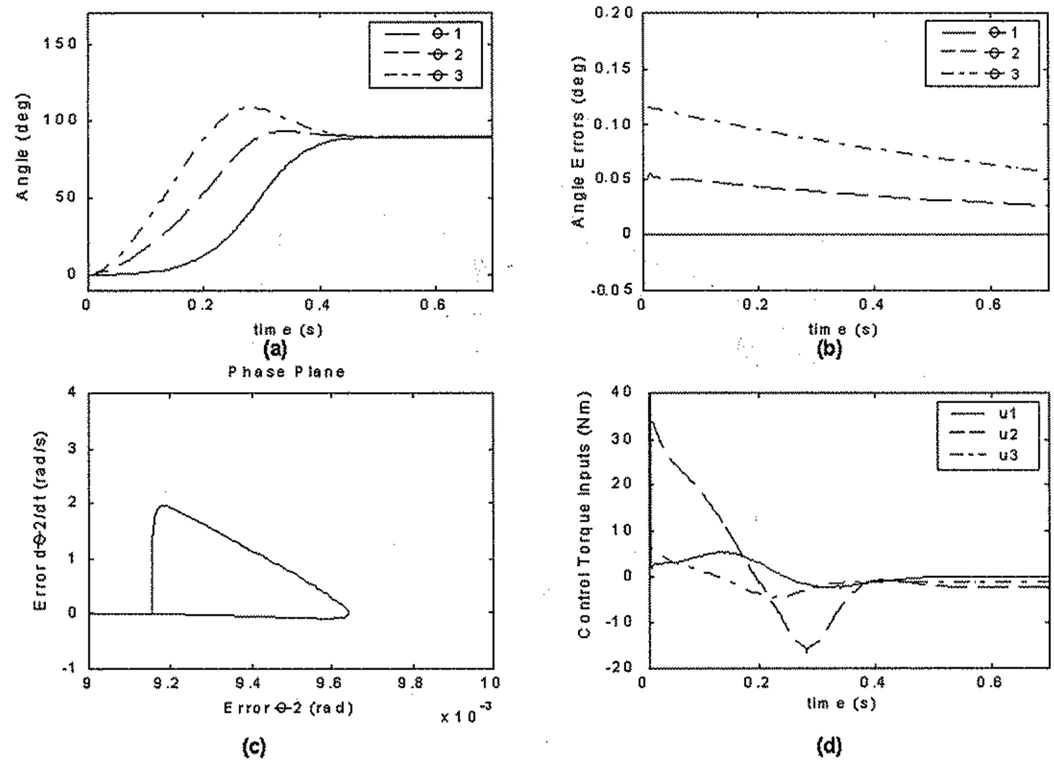


Figure 5. Simulation Results, (a) The output trackings of joint 1, joint 2 and joint 3, (b) The output tracking errors of joint 1, joint 2 and joint 3, (c) Phase plane plot of tracking error of joint 2, (d) The control inputs of joint 1, joint 2 and joint 3.

On the other hand, robot mechanisms are under the effect of different disturbances and the system parameters may change because of any problem or loading. Under these circumstances, the controller must be able to drive the robot and realize the trajectory motion successfully. In order to verify this ability of the sliding mode controller, it is assumed that the mass of the third link changes after a certain time of the motion on the trajectory as shown in Figure 6.

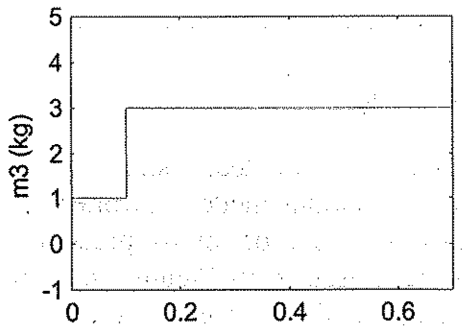


Figure 6. Change at the Mass of the Third Link

When the mass of the third link changes at 0.1st second, the simulation is performed and results are presented in Figure 7. In this Figure, it is observed that the necessary joint angular motions are produced successfully verifying the robust character of the sliding mode controller. Only the control torque inputs change as expected when compared with the ones in Figure 5 as shown in Figure 7.d.

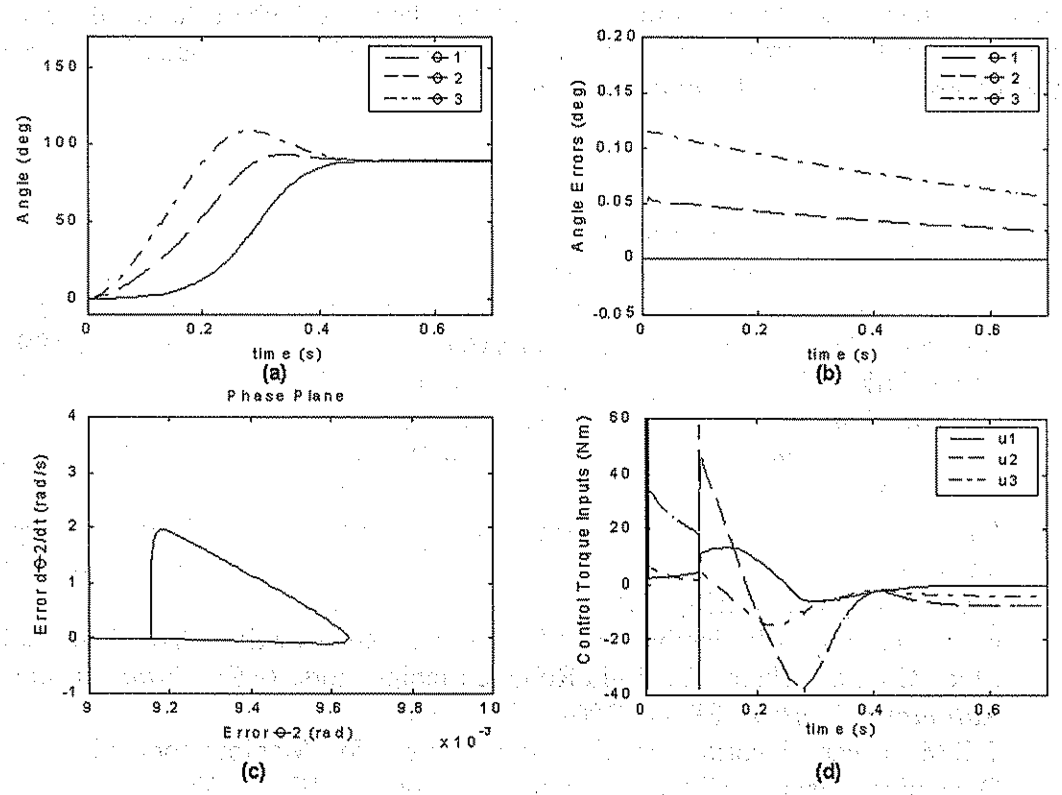


Figure 7 When Mass of the Third Link Changes Suddenly, (a) The output trackings of joint 1, joint 2 and joint 3, (b) The output tracking errors of joint 1, joint 2 and joint 3, (c) Phase plane plot of tracking error of joint 2, (d) The control inputs of joint 1, joint 2 and joint 3.

When the control torque inputs both in Figure 5.d and Figure 7.d are checked, the non-chattering character of the proposed sliding mode controller is witnessed. This is very important since chattering torque profile can harm the joint motors and system components.

## 5.CONCLUSIONS

In this study, a robust non-chattering sliding mode controller has been designed for a spatial robot and the simulation results have been presented. The main reason in proposing sliding mode control for the robot systems was its robustness and non-chattering character. Since the robot dynamics might change a lot depending on position and load, the approach in control strategy necessitates robust character. Causing a sudden change at the mass of the third link of the robot, the robust character of the controller has been checked. It is observed that the sliding mode controller is insensitive to the parameter changes of the dynamic system and continiues to control the robot with high performance and success. Additionally, if sudden changes are expected from robot arm motors as is the case in classical sliding mode theory, the gear mechanism fails. Therefore soft torque transition output must be expected from motor drives. That means chattering must be prevented. In this study, a chattering free sliding mode control technique is introduced to overcome this problem. The smooth character of the control torque inputs of the control action in this study also verified the non-chattering character of the proposed sliding mode control approach.

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## APPENDIX

Spatial robot parameters:

$m_1 = 3$ kg.
$m_2 = 2$ kg.
$m_3 = 1$ kg.
$L_1 = 0.25$ m.
$L_2 = 0.30$ m.
$L_3 = 0.25$ m.
$I_i = 1/12\ m_i L_i^2$.
$(x_i,y_i,z_i) = (0.55,0.00,0.25)$
$(x_f,y_f,z_f) = (0.00,0.25,0.55)$