Mathematical & Computational Applications, Vol. 6, No. 3, pp. 177-188, 2001

PROPAGATION OF CYLINDRICAL MAGNETO-GASDYNAMIC SHOCK WAVE WITH VARYING ENERGY IN A ROTATING, GRAVITATING, NON-UNIFORM ATMOSPHERE

Ashok Ganguly and Shilpa Shinde
Department of Applied Mathematics
Shri G.S. Institute of Technology and Science
Indore (M.P.) 452 003 India

Abstract- A self similar, theoretical model of propagation of cylindrical magneto-gasdynamic shock wave with varying energy in a non-uniform atmosphere, in the interplanetary region, in the presence of rotation and gravitation, has been developed. Subsequently, its comparison with the gas dynamic case has been shown with the help of variations of flow variables with distance graphically. It is seen that the presence of magnetic field generates very high energy at the instant of explosion and its absence in gasdynamic model makes the variation of energy docile.
Keywords- Magneto-gasdynamic shock wave, self similar motion, thermodynamic equilibrium.

1. INTRODUCTION

The problem of propagation of magneto-gasdynamic shock waves in a rotating interplanetary atmosphere assumes special significance for research workers in the study of astrophysical phenomena. It has been observed that the outer atmosphere of the planets rotates due to the rotation of the planets. Macroscopic motion with supersonic speed occurs in interplanetary atmosphere with rotation and shock waves are generated. Further, the interplanetary magnetic field is connected with the rotation of the sun which implies that a large scale of magnetic field might appear in the rapidly rotating stars. The processes which take place in the outer layer of nova and supernova, cepheids, solar atmosphere and corona are significantly affected by their rotation. It has been observed that steller outbursts consist of the non-steady motion of large masses of gas with violent increase in emitted energy.

Following technique of Sedov [3] of dimensional similarity, in this paper we study the propagation of magneto-gasdynamic cylindrical shock waves with varying energy in a rotating non-uniform atmosphere, where we have taken the effects of magnetic field as well as gravitation into consideration. The rotating atmosphere has been taken in cylindrical symmetry. The total energy content within the inner expanding surface and the shock front is assumed to be increasing with time. The flow is analysed under the assumption that the plasma is non-viscous ideal gas of infinite electrical conductivity. The gas is assumed to be grey and opaque and shock to be transparent and isothermal. The total energy content within the inner expanding surface and the shock front is assumed to be increasing with time. Further, under the equilibrium condition, we have neglected the effect of rotation of gravitation. The variations of flow variables behind the shock waves have been obtained numerically and graphically. The above problem in the absence of magnetism for gasdynamic case was studied by sedov [3].

2. FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

The fundamental equations governing the motions of an inviscid perfect gas in which effect of magnetic field, gravitation, and rotation may be significant are given by

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{\partial u}{r}=0 \qquad \text{................(1)}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{h}{\rho r}\frac{\partial}{\partial r}(hr)+\frac{Gm}{r}-\frac{v^2}{r}=0 \qquad \text{................(2)}$$

$$\frac{\partial h}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(ruh)=0 \qquad \text{................(3)}$$

$$\frac{d}{dt}(v.r)=0 \qquad \text{................(4)}$$

$$\frac{\partial e}{\partial t}+u\frac{\partial e}{\partial r}-\frac{p}{\rho^2}\frac{\partial \rho}{\partial t}-\frac{pu}{\rho^2}\frac{\partial \rho}{\partial r}=0 \qquad \text{................(5)}$$

$$\frac{\partial m}{\partial r}=2\pi\rho r \qquad \text{................(6)}$$

Where u, v, ρ, m, r, t, γ, G, h are the radial component of velocity, azimuthal component of velocity, the density, the mass per unit volume, the raidal distance from the centre, time, the ratio of specific heats, gravitational constant, magnetic field.

The field variable just ahead of shock are denoted by the suffix 0 and are

$$u_0=0\,; \qquad v_0=0\,; \qquad \rho_0=bR^{\alpha}\ (-3\le\alpha\le 0)\,;$$

$$p_0=CR^{\beta}\ (\beta\le 0)\,; \quad E=st^{\delta}\ (\delta\ge 0)$$

$$V=\frac{dR}{dt}\,; \qquad e=\frac{p}{\rho(\gamma-1)} \qquad \text{................(7)}$$

Where γ is the adiabatic exponent and α, β, δ are constants, V is the speed of the shock wave, R is the radius, E is the total energy inside the shock of radius R and s is a constant.

The boundary conditions at the shock are given by (zel'dovich and Raizer [4]; and Ojha et al. [2])

$$u_1=\frac{2V}{(\gamma+1)}, \qquad v_1=\frac{2V}{(\gamma+1)},$$

$$\rho_1=\left\{\frac{(\gamma+1)}{(\gamma-1)}\right\}\rho_0, \qquad p_1=\frac{2\rho_0 V^2}{(\gamma+1)}$$

$$e_1 = \frac{2V^2}{(\gamma+1)^2}, \qquad h_1 = \left\{\frac{(\gamma+1)}{(\gamma-1)}\right\} h_o$$

$$m_1 = m_0 = \frac{2\pi b r^{(2+\alpha)}}{(2+\alpha)}, \qquad \text{Where } V = \frac{dR}{dt} \qquad \text{................(8)}$$

is the speed of the shock wave and ρ_0, h_0 are the density and the magnetic field in the undisturbed media. The suffix 1 denotes the quantities just behind the shock. The Alf'ven Mach number and the usual Mach number are defined as

$$M_A^2 = \frac{\rho_o V^2}{h_0^2} \quad \text{and} \quad M^2 = \frac{\rho_o V^2}{\gamma p_o} \qquad \text{................(9)}$$

Under the equilibrium conditions, we have

$$G = - \left[\frac{1}{(\gamma M^2)} + \frac{1}{(2M_A^2)}\right] \frac{(2+\alpha)(1+\alpha)V^2}{\pi b r^{1+\alpha}} \qquad \text{................(10)}$$

let us assume the solutions of the fundamental equations in the form (cf. Nath and Sinha[1])

$$u = \frac{r}{t} U(\eta) \qquad \text{................(11)}$$

$$v = \frac{r}{t} K(\eta) \qquad \text{................(12)}$$

$$\rho = r^k t^\lambda F(\eta) \qquad \text{................(13)}$$

$$p = r^{k+2} t^{\lambda-2} P(\eta) \qquad \text{................(14)}$$

$$m = r^{k+2} t^\lambda M(\eta) \qquad \text{................(15)}$$

$$h = r^{(k+2)/2} t^{(\lambda-2)/2} H(\eta) \qquad \text{................(16)}$$

$$e = r^{k+2} t^{\lambda-2} Q(\eta) \qquad \text{................(17)}$$

$$G = r^{-k+1} t^{-\lambda-2} T(\eta) \qquad \text{................(18)}$$

Where $\eta = r^a t^b$ and k, λ, a, b are constants. ....(19)

Let us choose η_0 = constant, the value of η at the shock front. This choice fixed the velocity of the shock surface as $V = -\frac{b}{a} . \frac{R}{t}$ ....(20)

The total energy E inside the shock wave of radius R is given by

$$E = \int_{r^*}^{R} [\{(\rho u^2/2) + p/(\gamma-1) + (h^2/2) - (Gm\rho/r)\} r^2] dr = s t^{-\delta} \text{......(21)}$$

Where **r*** the co-ordinate of the inner expanding surface. In terms of the variable η the above integral can be written as

$$E = (2\pi/a) \int_{\eta^*}^{\eta_0} [\{F(\eta)U^2(\eta)/2 + P(\eta)/(\gamma-1) + H^2(\eta)/2 - T(\eta)M(\eta)F(\eta)\}$$

$$\eta^{\{((k+4)/a)-1\}} t^{\{\lambda-2-(b/a)(k+4)\}}] d\eta, \qquad \text{..............(22)}$$

Where η_0, η^* are the values of η at the shock front and the inner expanding surface, respectively.
Equation (22) yields.

$k = 0, \qquad \lambda = -2, \qquad a = -4, \qquad b = \delta + 4$ and

$$\lambda - 2 - (b/a)(k+4) = \delta \qquad \text{...............(23)}$$

With the help of (22) and (19) Equation, we get

$$R = \eta_0^{-1/4}\, t^{(4+\delta)/4} \qquad \text{...............(24)}$$

$$V = [(4+\delta)/4]\frac{R}{t} \qquad \text{...............(25)}$$

3. SOLUTION OF THE PROBLEM

Using equations (11) - (20) and (23), in the fundamental equations (1) - (6), we get the set of differential equations

$$\frac{\partial \rho}{\partial t} = -\frac{8}{(4+\delta)}\rho\frac{V}{R} - V\frac{r}{R}\frac{\partial \rho}{\partial r} \qquad \text{...............(26)}$$

$$\frac{\partial p}{\partial t} = \frac{(2\delta-8)}{(4+\delta)}p\frac{V}{R} - V\frac{r}{R}\frac{\partial p}{\partial r} \qquad \text{...............(27)}$$

$$\frac{\partial h}{\partial t} = \frac{(\delta-4)}{(4+\delta)}h\frac{V}{R} - V\frac{r}{R}\frac{\partial h}{\partial r} \qquad \text{...............(28)}$$

$$\frac{\partial e}{\partial t} = \frac{(2\delta-8)}{(4+\delta)}e\frac{V}{R} - V\frac{r}{R}\frac{\partial e}{\partial r} \qquad \text{...............(29)}$$

$$\frac{\partial u}{\partial t} = \frac{\delta}{(4+\delta)}u\frac{V}{R} - V\frac{r}{R}\frac{\partial u}{\partial r} \qquad \text{...............(30)}$$

$$\frac{\partial v}{\partial t} = \frac{\delta}{(4+\delta)}v\frac{V}{R} - V\frac{r}{R}\frac{\partial v}{\partial r} \qquad \text{...............(31)}$$

Substituting these results in the fundamental equations (1)-(6), we get a set of ordinary differential equations as follows :

$$\frac{dU}{dx} = \left[U(x-U)\frac{\delta}{(4+\delta)} + \frac{2\gamma(\gamma-1)}{(\gamma+1)^2}.\frac{P}{Y}\left[\frac{2U}{x} + \frac{(2\delta-16)}{\gamma(4+\delta)}\right]\right.$$
$$\left. + \frac{(\gamma+1)}{(\gamma-1)}.\frac{1}{M_A^2}.\frac{\xi^2}{Y}.\frac{2\delta}{(4+\delta)} - (x-U)\left[\frac{1}{(\gamma M^2)} + \frac{1}{(2M_A^2)}\right]\frac{2(1+\alpha)M}{x} - \frac{(x-U)K^2}{x}\right]$$
$$\div\left[(x-U)^2 - \frac{2\gamma(\gamma-1)}{(\gamma+1)^2}.\frac{P}{Y} - \frac{(\gamma+1)}{(\gamma-1)}.\frac{1}{M_A^2}.\frac{\xi^2}{Y}\right] \qquad \text{...............(32)}$$

$$\frac{dY}{dx} = \frac{Y}{(x-u)}\left[\frac{dU}{dx} + \frac{U}{x} - \frac{8}{(4+\delta)}\right] \qquad \text{...............(33)}$$

$$\frac{d\xi}{dx} = \frac{\xi}{(x-U)}\left[\frac{U}{x} + \frac{dU}{dx} + \frac{(\delta-4)}{(4+\delta)}\right] \qquad \text{................(34)}$$

$$\frac{dK}{dx} = \frac{K}{(x-U)}\left[\frac{\delta}{(4+\delta)} + \frac{U}{x}\right] \qquad \text{...............(35)}$$

$$\frac{dP}{dx} = \frac{\gamma P}{(x-U)}\left[\frac{(2\delta-16)}{\gamma(4+\delta)} + \frac{dU}{dx} + \frac{2U}{x}\right] \qquad \text{...............(36)}$$

$$\frac{dE}{dx} = \frac{E(\gamma-1)}{(x-U)}\left[\frac{(2\delta-8)}{(\gamma-1)(4+\delta)} + \frac{dU}{dx} + \frac{2U}{x}\right] \qquad \text{...............(37)}$$

$$\frac{dM}{dx} = \frac{(\gamma+1)(2+\alpha)}{(\gamma-1)}\cdot\frac{Y}{x} - (2+\alpha)M \qquad \text{...............(38)}$$

Where $u/V=U$, $v/V = K$, $h/h_1 = \xi$, $\rho/\rho_1 = Y$, $e/e_1 = E$,
$p/p_1 = P$, $m/m_1 = M$ and $r/R = x$ ....(39)

Boundary conditions :
$U = 2/(\gamma+1)$, $K = 2/(\gamma+1)$, $Y = 1$, $\xi = 1$,
$P = 1$, $E = 1$, $M = 1$ for $x = 1$.
...............(40)

4. RESULTS AND DISCUSSIONS

The set of differential equations (32)-(38) have been integrated numerically by using Runga-Kutta method under the boundary conditions equations (40). The numberical values of the constants are taken as $\gamma = 4/3, 7/5$; $\alpha = -2, -2.5$; $M^2 = 5$; $M_A^2 = 10$. The variations of various physical parameters (flow variables) with distances for different values of γ and α in both magneto-gasdynamic and gasdynamic (when magnetic field absent) cases are shown in Tables (1)-(8) and Fig. (1) to (7). A comparison has been drawn between the two models. We observe that radial velocity, rotational velocity, magnetic field decrease as we move towards the centre of the shock while density, pressure and energy increase.

In case of mass, the mass transfer through the shock wave is more of uniform nature in both the models. In fact, mass increases as we go towards the centre of the shock, reflecting heavy mass transfer in course of shock propagation.

However, for $\gamma = 4/3, 7/5$, $\alpha = -2$, it is seen that in the presence of magnetic field, the variation in mass remains uniform as we go towards the centre of the shock, which means that mass transfer is not significant.

In case of Radial velocity distribution pattern there is no appreciable change in both the cases in both the model or in the cases of different values of γ and α.

There is slight influence of lower values of γ and $\alpha = -2.5$ towards the fall of rotational velocity as one moves towards the center. However, the decrease in rotational velocity to slower in gasdynamic model. This reveals strong justification for consideration of rotational effect in the interplanetary atmosphere.

In case of gasdynamical model (GM) increase in density, as one moves towards the center is slower than the magneto-gasdynamical model (M.GM). The same behaviour is observed in case of pressure and energy.

It is to be remark the variation of energy is more signification when magnetic field is present. Peculiarity in case of gasdynamical model is that the energy distribution pattern is confined to small variation. But the magnetic field leads to a generation of very high energy at the instant of explosion, its absence in gasdynamic model makes the variation of energy very docile.

All Present

1. **Table**

γ is : 4/3
δ is : 0.5
M^2 is : 5

α is : -2
M_A^2 : 10

x	U	Y	ξ	k	E	P	M
1	0.857143	1	1	0.857143	1	1	1
0.999	0.857132	1.006174	0.999369	0.851337	1.007183	1.014	1
0.998	0.857118	1.012403	0.998712	0.845524	1.014431	1.021381	1
0.997	0.857102	1.018685	0.998029	0.839706	1.021743	1.028783	1
0.996	0.857084	1.025023	0.997318	0.833882	1.02912	1.036251	1
0.995	0.857063	1.031417	0.99658	0.828053	1.036563	1.043787	1
0.994	0.857041	1.037866	0.995814	0.822218	1.044073	1.05139	1
0.993	0.857015	1.044373	0.99502	0.816378	1.05165	1.059062	1
0.992	0.856987	1.050937	0.994197	0.810533	1.059294	1.066803	1

Magnetic Field Absent

2. **Table**

γ is : 4/3
δ is : 0.5
M^2 is : 5

α is : -2
M_A^2 : 10

x	U	Y	ξ	k	E	P	M
1	0.857143	1	0	0.857143	1	1	1
0.999	0.85661	1.002651	0	0.851337	1.002371	1.007987	1
0.998	0.85609	1.005399	0	0.845549	1.004866	1.010573	1
0.997	0.855582	1.008245	0	0.839779	1.007486	1.013255	1
0.996	0.855086	1.011191	0	0.834026	1.010233	1.016064	1
0.995	0.854603	1.014238	0	0.82829	1.013108	1.019004	1
0.994	0.854132	1.017388	0	0.822571	1.016122	1.022074	1
0.993	0.853674	1.020642	0	0.816867	1.019248	1.025279	1
0.992	0.853228	1.024002	0	0.811178	1.022517	1.028618	1

All Present

3. **Table**

γ is : 4/3
δ is : 0.5
M^2 is : 5

α is : -2.5
M_A^2 : 10

x	U	Y	ξ	k	E	P	M
1	0.857143	1	1	0.857143	1	1	1
0.999	0.857107	1.006005	0.999195	0.851337	1.006952	1.013712	1.002998
0.998	0.857068	1.012058	0.998361	0.845525	1.01396	1.020849	1.006021
0.997	0.857026	1.01816	0.997497	0.83971	1.021025	1.027998	1.009071
0.996	0.856982	1.02431	0.996604	0.83389	1.028145	1.035204	1.012147
0.995	0.856935	1.03051	0.99568	0.828065	1.035322	1.042468	1.01525
0.994	0.856885	1.036759	0.994725	0.822236	1.042555	1.049791	1.01838
0.993	0.856833	1.043057	0.993739	0.816404	1.049846	1.057171	1.021537
0.992	0.856778	1.049407	0.992722	0.810567	1.057193	1.06461	1.024721

Magnetic Field Absent

4. **Table**

γ is : 4/3
δ is : 0.5
M^2 is : 5

α is : -2.5
M_A^2 : 10

x	U	Y	ξ	k	E	P	M
1	0.857143	1	0	0.857143	1	1	1
0.999	0.856461	1.001631	0	0.851337	1.000979	1.006248	1.002998
0.998	0.855788	1.003333	0	0.845556	1.002047	1.007382	1.006006
0.997	0.855125	1.005105	0	0.8398	1.003204	1.008579	1.009025
0.996	0.854471	1.006947	0	0.834068	1.004452	1.009867	1.012055
0.995	0.853827	1.008862	0	0.828361	1.00579	1.011248	1.015097
0.994	0.853193	1.010849	0	0.822677	1.00722	1.01272	1.018151
0.993	0.852568	1.012908	0	0.817016	1.008742	1.014285	1.021217
0.992	0.851952	1.015042	0	0.811377	1.010356	1.015944	1.024295

All Present

5. Table

γ is : 7/5
δ is : 0.5
M^2 is : 5

α is : -2
M_A^2 : 10

x	U	Y	ξ	k	E	P	M
1	0.833333	1	1	0.833333	1	1	1
0.999	0.833291	1.005235	0.999417	0.828614	1.005619	1.010823	1
0.998	0.833247	1.010505	0.998811	0.82389	1.011268	1.016549	1
0.997	0.8332	1.015809	0.998183	0.819162	1.016947	1.022279	1
0.996	0.83315	1.02115	0.997531	0.81443	1.022656	1.028039	1
0.995	0.833098	1.026525	0.996857	0.809694	1.028395	1.03383	1
0.994	0.833044	1.031936	0.996158	0.804954	1.034164	1.039651	1
0.993	0.832987	1.037383	0.995436	0.80021	1.039963	1.045502	1
0.992	0.832928	1.042867	0.994689	0.795462	1.045791	1.051384	1

Magnetic Field Absent

6. Table

γ is : 7/5
δ is : 0.5
M^2 is : 5

α is : -2
M_A^2 : 10

x	U	Y	ξ	k	E	P	M
1	0.833333	1	0	0.833333	1	1	1
0.999	0.832972	1.003397	0	0.828614	1.002971	1.00742	1
0.998	0.832621	1.006877	0	0.823901	1.006064	1.010568	1
0.997	0.832281	1.010441	0	0.819194	1.009225	1.013803	1
0.996	0.831952	1.014091	0	0.814493	1.012511	1.017146	1
0.995	0.831634	1.017828	0	0.809797	1.015904	1.020597	1
0.994	0.831324	1.021653	0	0.805106	1.019406	1.024159	1
0.993	0.831029	1.025568	0	0.800419	1.02302	1.027834	1
0.992	0.830742	1.029575	0	0.795737	1.026747	1.031623	1

All Present

7. **Table**

γ is : 7/5
δ is : 0.5
M^2 is : 5

α is : -2.5
M_A^2 : 10

x	U	Y	ξ	k	E	P	M
1	0.833333	1	1	0.833333	1	1	1
0.999	0.833263	1.00507	0.999246	0.828614	1.005381	1.010517	1.002498
0.998	0.833189	1.010169	0.998467	0.823891	1.010785	1.015994	1.005015
0.997	0.833113	1.015299	0.997663	0.819165	1.016212	1.021468	1.007552
0.996	0.833034	1.020458	0.996834	0.814436	1.021661	1.026965	1.010109
0.995	0.832952	1.025648	0.995978	0.809704	1.027133	1.032485	1.012685
0.994	0.832867	1.030868	0.995097	0.804968	1.032627	1.038027	1.015282
0.993	0.83278	1.036118	0.99419	0.80023	1.038143	1.043591	1.017898
0.992	0.832689	1.041398	0.993255	0.795489	1.04368	1.049177	1.020535

Mag. Field Absent

8. **Table**

γ is : 7/5
δ is : 0.5
M^2 is : 5

α is : -2.5
M_A^2 : 10

x	U	Y	ξ	k	E	P	M
1	0.833333	1	0	0.833333	1	1	1
0.999	0.832829	1.002568	0	0.828614	1.001778	1.005886	1.002498
0.998	0.832334	1.005199	0	0.823906	1.003631	1.007795	1.005008
0.997	0.831847	1.007891	0	0.819208	1.005561	1.009765	1.00753
0.996	0.831369	1.010647	0	0.814521	1.007568	1.011812	1.010064
0.995	0.830899	1.013467	0	0.809845	1.009653	1.013939	1.012611
0.994	0.830438	1.016351	0	0.805178	1.011817	1.016145	1.015171
0.993	0.829986	1.019302	0	0.800521	1.01406	1.018431	1.017744
0.992	0.829542	1.022319	0	0.795874	1.016385	1.0208	1.02033

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1. $\gamma = 4/3$, $\alpha = -2$, All Present
2. $\gamma = 4/3$, $\alpha = -2.5$, All Present
3. $\gamma = 4/3$, $\alpha = -2.5$, Mag. Field Absent
4. $\gamma = 7/5$, $\alpha = -2$, All Present
5. $\gamma = 7/5$, $\alpha = -2.5$, All Present

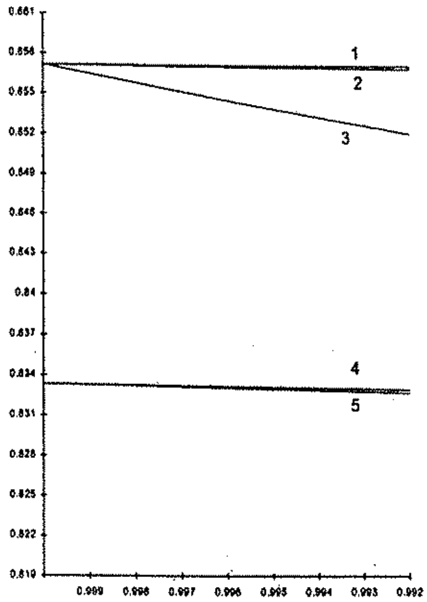

Fig 1 : Variation of radial velocity with distance

1. $\gamma = 4/3$, $\alpha = -2.5,-2$ All Present
2. $\gamma = 4/3$, $\alpha = -2.5$, Mag. Field Absent
3. $\gamma = 7/5$, $\alpha = -2.5,-2$ All Present

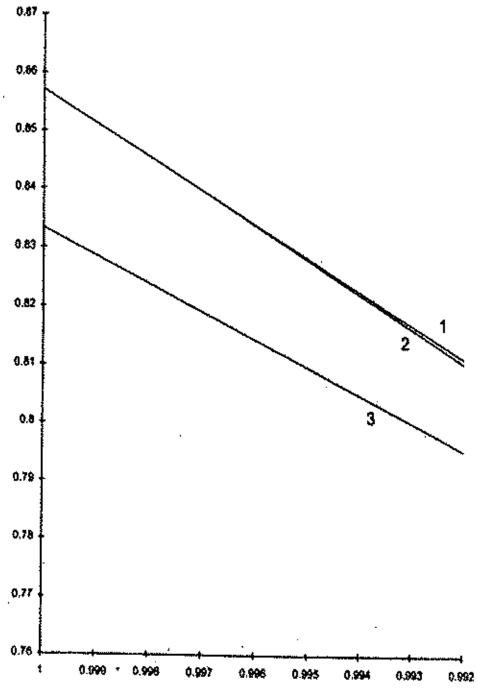

Fig 2 : Variation of rotational velocity with distance

1. $\gamma = 4/3$, $\alpha = -2$, All Present
2. $\gamma = 4/3$, $\alpha = -2.5$, All Present
3. $\gamma = 7/5$, $\alpha = -2$, All Present
4. $\gamma = 7/5$, $\alpha = -2.5$, All Present
5. $\gamma = 4/3$, $\alpha = -2.5$, Mag. Field Absent

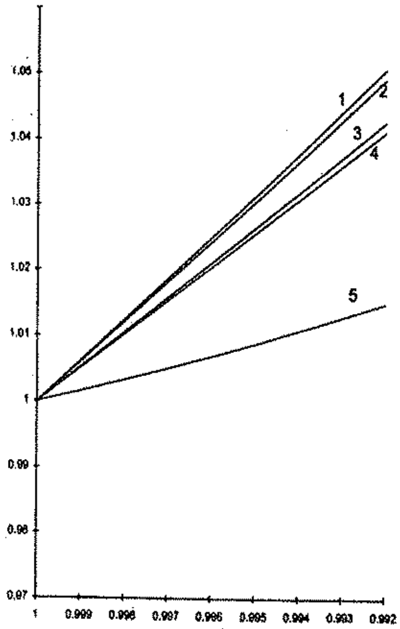

Fig 3 : Variation of density with distance

1. $\gamma = 4/3$, $\alpha = -2$, All Present
2. $\gamma = 4/3$, $\alpha = -2.5$, All Present
3. $\gamma = 7/5$, $\alpha = -2$, All Present
4. $\gamma = 7/5$, $\alpha = -2.5$, All Present
5. $\gamma = 4/3$, $\alpha = -2.5$, Mag. Field Absent

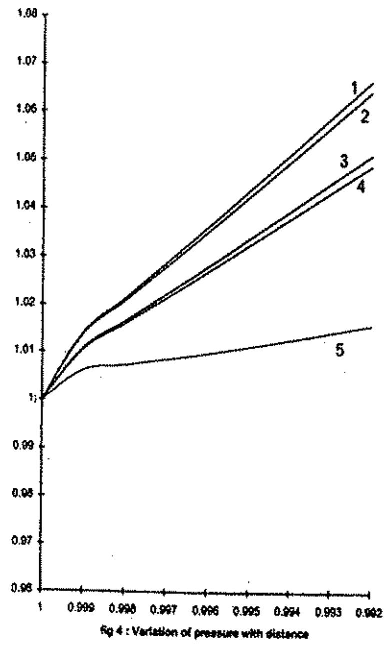

fig 4 : Variation of pressure with distance

1. $\gamma = 4/3$, $\alpha = -2$, All Present
2. $\gamma = 4/3$, $\alpha = -2.5$, All Present
3. $\gamma = 7/5$, $\alpha = -2$, All Present
4. $\gamma = 7/5$, $\alpha = -2.5$, All Present
5. $\gamma = 4/3$, $\alpha = -2.5$, Mag. Field Absent

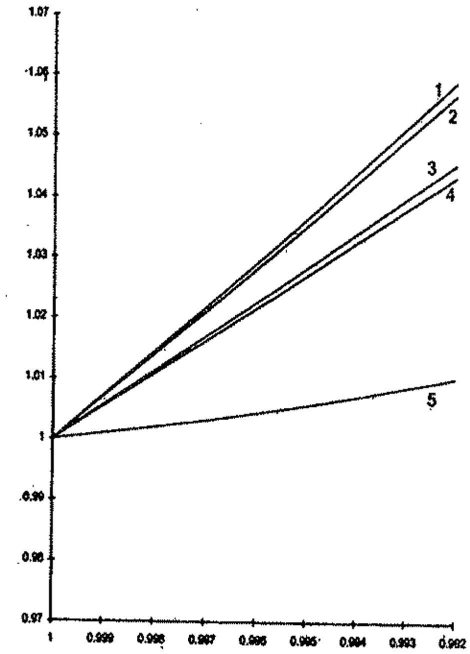

fig 5 : Variation of energy with distance

1. $\gamma = 4/3$, $\alpha = -2.5$, All Present
2. $\gamma = 4/3$, $\alpha = -2.5$, Mag Field Absent
3. $\gamma = 7/5$, $\alpha = -2.5$, All Present
4. $\gamma = 4/3, 7/5$, $\alpha = -2$, All Present

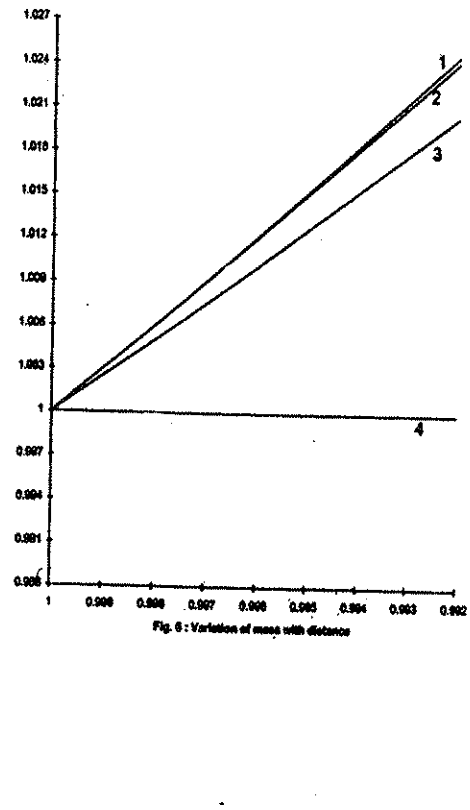

Fig. 6 : Variation of mass with distance

1. $\gamma = 7/5$, $\alpha = -2$, All Present
2. $\gamma = 4/3$, $\alpha = -2$, All Present
3. $\gamma = 7/5$, $\alpha = -2.5$, All Present
4. $\gamma = 4/3$, $\alpha = -2.5$, All Present

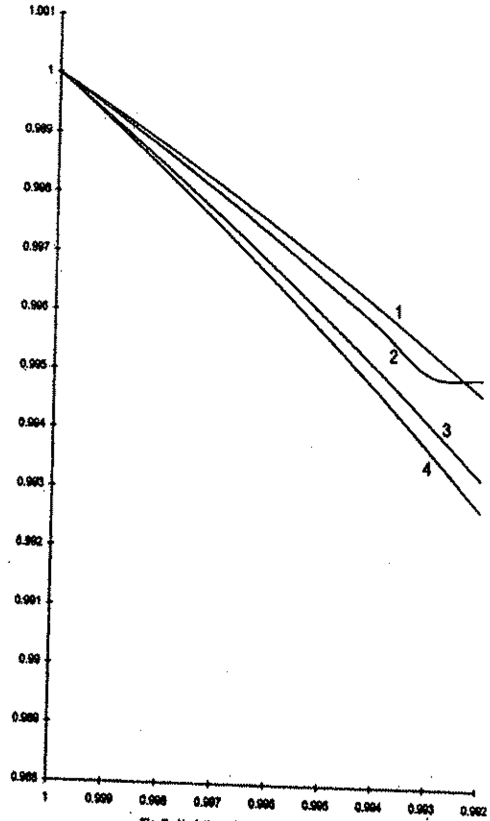

Fig. 7 : Variation of magnetic field with distance