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AN ELASTO-PLASTIC STRESS ANALYSIS ON A METAL MATRIX COMPOSITE BEAM OF ARBITRARY ORIENTATION SUPPORTED FROM TWO ENDS UNDER A TRANSVERSE UNIFORMLY DISTRIBUTED LOAD

Mümin Küçük*, Ümran Esendemir**
* Ege University, Bornova, İzmir, Turkey,
** Department of Mechanical Engineering, Süleyman Demirel University, Isparta, Turkey

Abstract- An analytical elastic-plastic stress analysis is carried out on a metal-matrix composite beam of arbitrary orientation supported from two ends under a transverse uniformly distributed load. The composite layer consists of stainless steel fiber and aluminum matrix. The material is assumed to be perfectly plastic in the elasto-plastic solution. The intensity of the uniform force is chosen at a small value; therefore the normal stress component of σ_y is neglected during the elasto-plastic solution. The expansion of the plastic region and the residual stress component of σ_x are determined for 0°, 30°, 45°, 60° and 90° orientation angles. Plastic yielding occurs for 0° and 90° orientation angles on the lower and upper surfaces of the beam at the same distances from the mid point. However; it starts first at the lower surface for 30°, 45° and 60° orientation angles. The intensity of the residual stress component of σ_x is found maximum at the lower and upper surfaces. However; the intensity of residual stress component τ_{xy} is maximum on or around the x axis of the beam.

Keywords- Elastic-plastic stress analysis, metal matrix composite, residual stress, simply supported beam.

1. INTRODUCTION

Metal-matrix composites offer an increased service temperature and improved specific mechanical properties over existing metal alloys. Aluminum-matrix composites, which are particularly cited for their superior performance-to-weight advantage, have many applications in the aerospace and other industries. Aluminum alloys reinforced with stainless steel offer high in-plane strength and specific stiffness. Karakuzu and Özcan [1], Canumalla et al. [2] have investigated discontinuously are viewed as candidate materials for elevated temperature applications because of their attractive high temperature strength properties and wear resistance. Jeronimidis and Parkyn [3] investigated residual stresses in carbon fiber-thermoplastic matrix laminates. Sayman et al. [4] have investigated elasto-plastic stress analysis of aluminum metal matrix composite laminated plates under in-plane loading. Karakuzu and Sayman [5] studied elasto-plastic stress analysis of fiber reinforced aluminium matal matrix rotating discs by using finite element techniques. Majumdar and Newaz [6] carried out inelastic deformation of metal matrix composites. Kang and Ku [7] investigated the infiltration limits in the fabrication of Al_2O_3 short fiber reinforced composites for various processing conditions. Cöcen et al. [8] produced SiC aluminium matal matrix composites to strengthen the aluminium matrix. Akay and Özden [9-10] have investigated the influence of residual stresses on the mechanical and thermal properties of injection moulded thermoplastics. Akay and Özden [11] measured the thermal residual stresses in injection moulded thermoplastics by removing thin layers from specimens.

In the present study, an elasto-plastic stress analysis is carried out on a fiber reinforced metal matrix composite beam supported from two ends under a transverse uniformly distributed load. Sample problems are given for various orientation angles. Elastic, elasto-plastic and residual normal and shear stresses are calculated.

2. ELASTIC ANALYSIS

The composite cantilever beam is supported from two ends subjected a transverse uniformly distributed load q, as shown in Figure 1.

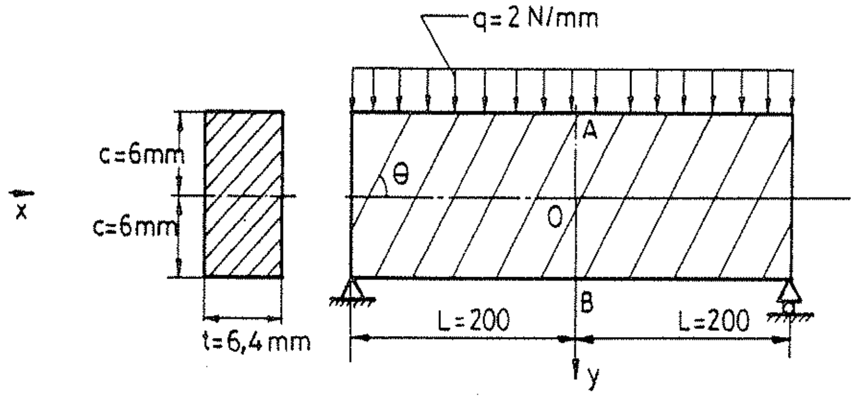

Figure 1. Composite beam supported from two ends under a transverse uniformly distributed load

The angle between the first principal axis of the composite fibers and the x axis is θ. For the plane-stress case the equation of equilibrium is given as [12],

$$\bar{a}_{22}\frac{\partial^4 F}{\partial x^4} - 2\bar{a}_{26}\frac{\partial^4 F}{\partial x^3 \partial y} + \left(2\bar{a}_{12} + \bar{a}_{66}\right)\frac{\partial^4 F}{\partial x^2 \partial y^2} - 2\bar{a}_{16}\frac{\partial^4 F}{\partial x \partial y^3} + \bar{a}_{11}\frac{\partial^4 F}{\partial y^4} = 0 \qquad (1)$$

Where F is a stress function. The constants in the Equation (2.1) are given as [13],

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{a}_{11} & \bar{a}_{12} & \bar{a}_{16} \\ \bar{a}_{12} & \bar{a}_{22} & \bar{a}_{26} \\ \bar{a}_{16} & \bar{a}_{26} & \bar{a}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \qquad (2)$$

where :

$$
\begin{aligned}
\bar{a}_{11} &= a_{11}\cos^4\theta + (2a_{12}+a_{66})\sin^2\theta\cos^2\theta + a_{22}\sin^4\theta \\
\bar{a}_{12} &= a_{12}(\sin^4\theta+\cos^4\theta) + (a_{11}+a_{22}-a_{66})\sin^2\theta\cos^2\theta \\
\bar{a}_{22} &= a_{11}\sin^4\theta + (2a_{12}+a_{66})\sin^2\theta\cos^2\theta + a_{22}\cos^4\theta \\
\bar{a}_{16} &= (2a_{11}-2a_{12}-a_{66})\sin\theta\cos^3\theta - (2a_{22}-2a_{12}-a_{66})\sin^3\theta\cos\theta \\
\bar{a}_{26} &= (2a_{11}-2a_{12}-a_{66})\sin^3\theta\cos\theta - (2a_{22}-2a_{12}-a_{66})\cos^3\theta\sin\theta \\
\bar{a}_{66} &= 2(2a_{11}+2a_{22}-4a_{12}-a_{66})\sin^2\theta\cos^2\theta + a_{66}(\sin^4\theta+\cos^4\theta)
\end{aligned}
\tag{3}
$$

and,

$$
a_{11} = \frac{1}{E_1}, a_{12} = -\frac{\upsilon_{12}}{E_1}, a_{22} = \frac{1}{E_2}, a_{66} = \frac{1}{G_{12}} \tag{4}
$$

F is chosen as a fifth order polynomial to satisfy the governing differential equation,

$$
F = \frac{x^2y^3}{6}d_5 + \frac{y^5}{20}f_5 + \frac{xy^4}{12}e_5 + \frac{x^2y}{2}b_3 + \frac{xy^2}{2}c_3 + \frac{y^3}{6}d_3 + \frac{x^2}{2}a_2 \tag{5}
$$

substituting it into the equilibrium gives

$$
e_5 = \frac{2\bar{a}_{16}}{\bar{a}_{11}}d_5 = md_5 \tag{6}
$$

$$
f_5 = \frac{-2\bar{a}_{12} - \bar{a}_{66} + 2\bar{a}_{16}m}{3\bar{a}_{11}}d_5 = nd_5 \tag{7}
$$

where: $m = \frac{2\bar{a}_{16}}{\bar{a}_{11}}$, $n = \frac{-2\bar{a}_{12} - \bar{a}_{66} + 2\bar{a}_{16}m}{3\bar{a}_{11}}$

The boundary conditions:

$$
y = c \Rightarrow \sigma_y = 0 \tag{8}
$$

$$
y = -c \Rightarrow \sigma_y = -\frac{q}{t} \tag{9}
$$

$$
y = \mp c \Rightarrow \tau_{xy} = 0 \tag{10}
$$

$$
x = L \Rightarrow \int_{-c}^{c} \tau_{xy}\, t\, dy = -qL \tag{11}
$$

$$
x = -L \Rightarrow \int_{-c}^{c} \tau_{xy}\, t\, dy = qL \tag{12}
$$

$$x = \mp L \Rightarrow \int_{-c}^{c} \sigma_x t\, dy = 0 \tag{13}$$

$$x = \mp L \Rightarrow \int_{-c}^{c} \sigma_x t\, y\, dy = 0 \tag{14}$$

The stress components are found from this F function as,

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = x^2 y d_5 + y^3 f_5 + x y^2 e_5 + x c_3 + y d_3 \tag{15}$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = \frac{y^3}{3} d_5 + y b_3 + a_2 \tag{16}$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} = -x y^2 d_5 - \frac{y^3}{3} e_5 - x b_3 - y c_3 \tag{17}$$

From the boundary conditions,

$$\begin{aligned} a_2 &= -\frac{q}{2t}, \quad d_5 = -\frac{3q}{4tc^3}, \quad b_3 = \frac{3q}{4tc}, \quad e_5 = -\frac{3}{4}\frac{qm}{tc^3} \\ f_5 &= -\frac{3qn}{4tc^3}, \quad c_3 = \frac{qm}{4tc}, \quad d_3 = \frac{3qL^2}{4tc^3} + \frac{9qn}{20tc} \end{aligned} \tag{18}$$

Hence, the elastic stress components become

$$\sigma_x = \frac{3q}{4tc^3}\left(L^2 - x^2\right) y + \frac{qm}{4tc}\left(1 - \frac{3y^2}{c^2}\right) x - \frac{3qn}{4t}\left(\frac{y^3}{c^3} - \frac{3y}{5c}\right) \tag{19}$$

$$\sigma_y = -\frac{q}{4tc^3} y^3 + \frac{3q}{4tc} y - \frac{q}{2t} \tag{20}$$

$$\tau_{xy} = \frac{3q}{4tc^3} x y^2 + \frac{qm}{4tc^3} y^3 - \frac{3q}{4tc} x - \frac{qm}{4tc} y \tag{21}$$

3. ELASTO-PLASTIC SOLUTION

The equations of equilibrium for the plane-stress case, neglecting the body force are given as,

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} &= 0 \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} &= 0 \end{aligned} \tag{22}$$

If the length of the beam is very large in comparison with the height of the beam and q is chosen at a small value, σ_y can be neglected in comparison with σ_x and τ_{xy}. Steel fiber reinforced composites have the same yield points in tension and compression; therefore the

Tsai-Hill theory is used as a yield criterion in this study. The equivalent stress in a principal material direction is given as,

$$\bar{\sigma}^2 = \sigma_1^2 - \sigma_1\sigma_2 + \frac{X^2}{Y^2}\sigma_2^2 + \frac{X^2}{S^2}\tau_{12}^2 = X^2 \tag{23}$$

where X,Y are the yield points in the 1st and 2nd principal material directions respectively and S is the shear yield strength in the 1-2 plane. The stress components in the principal material directions are written as,

$$\begin{aligned} \sigma_1 &= \sigma_x \cos^2\theta + 2\tau_{xy}\sin\theta\cos\theta \\ \sigma_2 &= \sigma_x \sin^2\theta - 2\tau_{xy}\sin\theta\cos\theta \\ \tau_{12} &= -\sigma_x \sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right) \end{aligned} \tag{24}$$

Substituting $\sigma_y = 0$ into the second equation of the equilibrium gives $\frac{\partial \tau_{xy}}{\partial x} = 0$. And putting the stress components σ_1, σ_2 and τ_{12} in Eqn. (23) and deriving it with respect to x gives

$$\frac{\partial \sigma_x}{\partial x} = 0 \tag{25}$$

integration of which gives $\sigma_x = f(y)$

Substituting σ_x in Eqn. (22), it is found that σ_x and τ_{xy} are constants. In the beam, the plastic region begins at the lower and upper edges. At these edges the shear stress is equal to zero and, for a perfectly plastic material, σ_x can be taken as a constant C at the upper and lower surfaces, where C is the yield point of the composite in the x direction. When the plastic zone is expanded for a perfectly plastic material, σ_x is again taken as C $=X_1$ and τ_{xy} is zero. Putting Eqn. (24) into the Eqn. (23) gives the yield point in the x direction as,

$$X_1 = \frac{X}{\sqrt{\cos^4\theta - \sin^2\theta\cos^2\theta + \frac{X^2\sin^4\theta}{Y^2} + \frac{X^2\sin^2\theta\cos^2\theta}{S^2}}} \tag{26}$$

Plastic region starts at the same distances from the free end for 0° and 90° orientation angles because of the symmetry of the material properties with respect to the x axis. However it starts first at the upper surface of the beam for 15° and 30° orientation angles.

The composite cantilever beam is supported from two ends to transverse uniformly distributed load q for elasto-plastic solution, as shown in Figure 2.

3.1. An elasto-plastic solution for θ=0° and θ=90° orientation angles

For satisfying both the differential equation and the boundary conditions in the elastic region of the elasto-plastic part, the stress function F is chosen as,

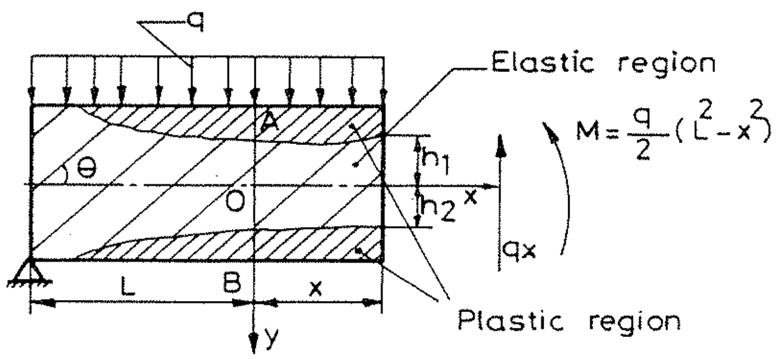

Figure 2. Composite beam supported from two ends under a transverse uniformly distributed load for the elasto-plastic solution

$$F=\frac{d_5}{6}x^2y^3+\frac{f_5}{20}y^5+\frac{g_3}{6}y^3+\frac{b_3}{2}x^2y+\frac{a_2}{2}y^2 \tag{27}$$

If we substitute the stress function in the governing differential equation we obtain,

$$\left(2\bar{a}_{12}+\bar{a}_{66}\right)2yd_5+6\bar{a}_{11}\,yf_5=0\Rightarrow f_5=-\left(\frac{2\bar{a}_{12}+\bar{a}_{66}}{3\bar{a}_{11}}\right)d_5=rd_5 \tag{28}$$

The stress components are:

$$\sigma_x=\frac{\partial^2F}{\partial y^2}=d_5x^2y+f_5y^3+g_3y+a_2 \tag{29}$$

$$\sigma_y=\frac{\partial^2F}{\partial x^2}=0 \tag{30}$$

$$\tau_{xy}=-\frac{\partial^2F}{\partial x\partial y}=-d_5xy^2-b_3x \tag{31}$$

The boundary conditions for this beam are given as,

$$y=-h_1\Rightarrow\tau_{xy}=0 \tag{32}$$

$$y=h_2\Rightarrow\tau_{xy}=0 \tag{33}$$

$$\int_{-h_1}^{h_2}t\,\tau_{xy}\,dy=-q\,x \tag{34}$$

$$y=-h_1\Rightarrow\sigma_x=-X_1 \tag{35}$$

$$y=h_2\Rightarrow\sigma_x=X_1 \tag{36}$$

The resultant of σ_x at any section is equal to zero:

$$-X_1t(c-h_1)+X_1t(c-h_2)+\int_{-h_1}^{h_2}\sigma_x t\,dy=0 \tag{37}$$

The resultant moment of σ_x at any section (x) is equal to the bending moment:

$$X_1 t(c-h_1)\frac{c+h_1}{2}+X_1 t(c-h_2)\frac{c+h_2}{2}+\int_{-h_1}^{h_2}\sigma_x t\, y\, dy=\frac{q(L^2-x^2)}{2} \tag{38}$$

From the boundary conditions, the unknown parameters are found as,

$$h_1=h_2=\sqrt{\frac{-X_1c^2+\dfrac{q(L^2-x^2)}{2t}}{-\dfrac{X_1}{3}+\dfrac{qr}{5t}}} \tag{39}$$

$$a_2=0,\; b_3=-d_5\, h_1^2,\; d_5=-\frac{q}{\dfrac{4th_1^3}{3}},\; g_3=\left[\frac{X_1}{h_1}-d_5x^2-rd_5h_1^2\right] \tag{40}$$

The stress components can be found by using these parameters.

3.2. Elasto-plastic solution for the inclined orientation angles

For the inclined orientation angles the stress function F for the elastic region in the elasto-plastic part is chosen as the following,

$$F=d_5\frac{x^2y^3}{6}+f_5\frac{y^5}{20}+e_5\frac{xy^4}{12}+b_3\frac{x^2y}{2}+c_3\frac{xy^2}{2}+d_3\frac{y^3}{6}+b_2\frac{y^2}{2} \tag{41}$$

If we substitute the derivative values in to Eqn. (1) we obtain,

$$\left[\left(2\bar{a}_{12}+\bar{a}_{66}\right)2d_5-4\bar{a}_{16}e_5+6\bar{a}_{11}f_5\right]y+\left[-4\bar{a}_{16}d_5+2\bar{a}_{11}e_5\right]x=0$$

For satisfying the equation, each term of x and y must be equal to zero. Hence,

$$e_5=\frac{2\bar{a}_{16}}{\bar{a}_{11}}d_5=md_5 \tag{42}$$

$$f_5=\frac{-2\bar{a}_{12}-\bar{a}_{66}+2\bar{a}_{16}m}{3\bar{a}_{11}}d_5=nd_5 \tag{43}$$

The stress components are:

$$\sigma_x=\frac{\partial^2F}{\partial y^2}=d_5x^2y+f_5y^3+e_5xy^2+c_3x+d_3y+b_2 \tag{44}$$

$$\tau_{xy}=-\frac{\partial^2F}{\partial x\,\partial y}=-d_5xy^2-\frac{e_5}{3}y^3-b_3x-c_3y \tag{45}$$

If we replace Eqn. (42), (43) into Eqn. (44) and (45) we obtain

$$\begin{aligned} \sigma_x &= d_5x^2y + nd_5y^3 + md_5xy^2 + c_3x + d_3y + b_2 \\ \tau_{xy} &= -d_5xy^2 - \frac{md_5}{3}y^3 - b_3x - c_3y \end{aligned} \tag{46}$$

The parameters $d_5, b_3, c_3, b_2, h_1, h_2$ are determined from the boundary conditions. From the boundary condition in Eqn. (32), (33) and (34) they are obtained as,

$$c_3 = -d_5x(h_2 - h_1) - \frac{md_5}{3}\left(h_1^2 - h_1h_2 + h_2^2\right) \tag{47}$$

$$b_3 = -d_5h_1h_2 + \frac{md_5}{3x}(h_1 - h_2)h_1h_2 \tag{48}$$

$$d_5 = \frac{-qx/t}{\frac{x}{6}(h_1 + h_2)^3 + \frac{m}{12}(h_1 + h_2)^3(h_2 - h_1)} \tag{49}$$

From the boundary conditions in Eqn. (35), (36), (37) and (38) we obtain

$$d_5 = \frac{X_1(h_1 - h_2)}{\frac{n(h_1 - h_2)(h_1 + h_2)^3}{4} - \frac{mx(h_1 + h_2)^3}{6}} \tag{50}$$

$$b_2 = \frac{-(h_1 - h_2)}{2}d_5x^2 + \frac{(h_1^3 - h_2^3)}{2}nd_5 - (h_1^2 + h_2^2)md_5x + \frac{(h_1 - h_2)}{2}d_3 + \frac{md_5x}{3}\left(h_1^2 - h_1h_2 + h_2^2\right) \tag{51}$$

$$d_3 = \frac{2X_1}{h_1 + h_2} - d_5x^2 - (h_1^2 - h_1h_2 + h_2^2)nd_5 + (h_1 - h_2)md_5x \tag{52}$$

If we equate the parameter d_5 we obtain,

$$\frac{12X_1(h_2 - h_1)}{3n(h_1 - h_2) - 2mx} = \frac{-12qx}{t(2x + m(h_2 - h_1))} \tag{53}$$

Solution of the above equation gives,

$$\begin{aligned} u &= \frac{(-2xX_1t + 3qxn) \mp \sqrt{(-2xX_1t + 3qxn)^2 + 8X_1m^2tqx^2}}{2X_1mt} \\ h_2 &= h_1 + u \end{aligned} \tag{54}$$

With the arrangement of Eqn. (38), we obtain,

$$\begin{aligned}&-\frac{nd_5}{60}(2h_1+u)\left[8(h_1+u)^4+8h_1^4+2h_1^3(h_1+u)+2h_1(h_1+u)^3-12h_1^2(h_1+u)^2\right]\\&-\frac{md_5x}{12}(2h_1+u)^3u+X_1c^2-\frac{X_1}{3}\left(h_1^2+h_1u+u^2\right)-\frac{q\left(L^2-x^2\right)}{2t}=0\end{aligned} \quad (55)$$

Finding h_1 by using the Newton-Raphson method gives the other unknown constants.

4. PRODUCTION OF THE COMPOSITE BEAM

The composite layer consists of stainless steel fiber and aluminum matrix. The production has been realized by using moulds which consist of upper and lower parts. Electrical resistance has been used to heat the moulds and material which are insulated, as illustrated in Figure 3. The hydraulic press has been used to obtain a pressure of 30 MPa to the upper mould. Manufacturing set has been heated to 600 °C. In these conditions, the yield strength of aluminum is exceeded and good bonding between matrix and fiber has been realized. The mechanical properties, and yield points are given in Table 1. It is assumed that the yield point Z (in the z direction) is equal to the yield point Y (in the y direction).

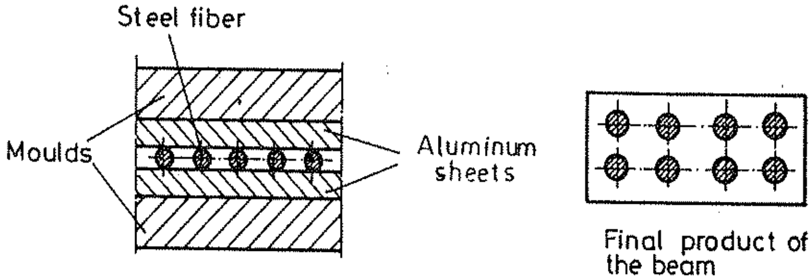

Figure 3. Production of the beam

Table 1. Mechanical properties and yield points of the composite beam

E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)	υ_{12}	Axial Strength (X) [MPa]	Transverse Strength (Y) [MPa]	Shear Strength (S) [MPa]
85000	74000	30000	0.3	230.000	24.000	48.900

5. RESULTS AND DISCUSSION

An elastic-plastic stress analysis is carried out on the composite beam supported from two ends under a transverse uniformly distributed load for 0°, 30°, 45°, 60° and 90° orientation angles. q is chosen as 2 N per mm thickness and the thickness of the beam has been taken as 6.4 mm. The height and length of the beam have taken as 12 mm, 200 mm respectively.

Plastic yielding occurs first at the lower surface for 30°, 45°, 60° orientation angles as given in Table 2. The elastic, plastic and residual stress components of σ_x and the expansion of the plastic region are given in Table 3. As seen from this table, the plastic region expands dissimilarly at the lower and upper sides of the beam oriented at 30°, 45°and 60° angles.

Table 2. The distance between the mid point and yield points

	Orientation Angles				
	0°	30°	45°	60°	90°
At the upper surface (mm)	68.41	169.92	182.41	187.65	190.58
At the lower surface (mm)	68.41	170.55	183.41	188.00	190.58

Table 3. Variations of elastic, elasto-plastic, residual normal stresses at the upper and lower edges and distances h_1 and h_2 with respect to x axis for θ=0°, 30°, 45°, 60° and 90° orientation angles.

Θ	x (mm)	h_1 (mm)	h_2 (mm)	σ_{x_e} at upper surface	σ_{x_p} at upper surface	σ_{x_r} at upper surface	σ_{x_e} at lower surface	σ_{x_p} at lower surface	σ_{x_r} at lower surface
0°	68.03	6.00	6.00	-230.00	-230.00	0.00	230.00	230.00	0.00
	58.03	5.78	5.78	-238.27	-230.00	8.27	238.27	230.00	-8.27
	48.03	5.58	5.58	-245.22	-230.00	15.22	245.22	230.00	-15.22
30°	169.92	6.00	5.88	-71.80	-71.80	0.00	72.50	71.80	-0.70
	164.92	4.95	4.83	-83.15	-71.80	11.35	84.07	71.80	-12.27
	159.92	3.70	3.58	-93.73	-71.80	21.92	94.62	71.80	-22.82
45°	182.41	6.00	5.82	-43.09	-43.09	0.00	44.50	43.09	-1.41
	177.41	3.98	3.80	-54.80	-43.09	11.71	56.17	43.09	-13.08
	174.41	2.02	1.84	-61.69	-43.09	18.60	62.94	43.09	-19.85
60°	187.65	6.00	5.84	-30.82	-30.82	0.00	31.62	30.82	-0.80
	185.65	4.95	4.79	-35.69	-30.82	4.87	36.48	30.82	-5.66
	183.65	3.61	3.45	-40.79	-30.82	9.97	41.57	30.82	-10.75
90°	190.58	6.00	6.00	-24.00	-24.00	0.00	24.00	24.00	-0.00
	188.58	4.60	4.60	-29.12	-24.00	5.12	29.12	24.00	-5.12
	186.58	2.57	2.57	-34.00	-24.00	10.00	34.00	24.00	-10.00

The intensity of the residual stress component of σ_x is maximum at the upper and lower surfaces. However; the residual stress component of σ_x at the lower surface is greater than that at the upper surface for 30°, 45°and 60° orientation angles. The residual stress component of τ_{xy} is given only on the x axis and smaller than the residual stress component of σ_x. For the same length of the beam, when the value of the orientation angle increases, the intensity of the residual stress component of σ_x increases too, as seen Table 4. The distribution of the residual stress component of σ_x along the cross sections of the beam is show in Figure 4. As seen from this Figure, it is maximum at the upper and lower surfaces. However; when the plastic zone spreads further in the beam the intensity of the residual stress component of σ_x

Table 4. The residual stress component of σ_x at the upper and lower of the simply supported beam for 45°, 60° and 90° orientation angles at x=182.41, 187.65.

Θ	x (mm)	h_1 (mm)	h_2 (mm)	σ_{x_e} at upper surface	σ_{x_p} at upper surface	σ_{x_r} at upper surface	σ_{x_e} at lower surface	σ_{x_p} at lower surface	σ_{x_r} at lower surface
45°	182.41	6.00	5.82	-43.09	-43.09	0.00	44.50	43.09	-1.41
60°	182.41	2.45	2.29	-43.44	-30.82	12.62	44.11	30.82	-13.29
60°	187.65	6.00	5.84	-30.82	-30.82	0.00	31.62	30.82	-0.80
90°	187.65	3.79	3.79	-31.10	-24.00	7.10	31.10	24.00	-7.10

at the boundary of the elastic and plastic regions becomes greater. Elastic and elastic-plastic solution of the shear stress component of τ_{xy} are given only at the last section.

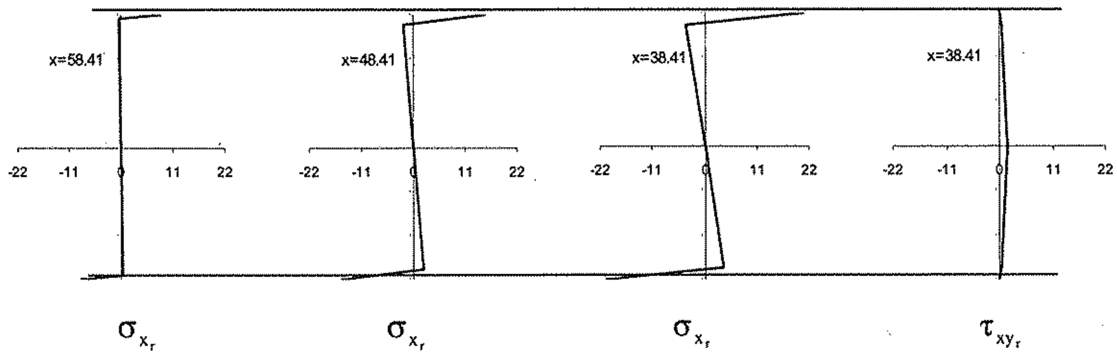

Figure 4. The distribution of the residual stress component of σ_x for 0° orientation angles along the sections of beam.

The distribution of the residual stress component of σ_x along the cross sections of the beam for 30° orientation angle is illustrated in Figure 5. The intensity of the residual stress component of σ_x at the elasto-plastic boundary becomes greater for further expanded plastic regions. It is maximum at the elasto-plastic boundary on the lower side of the beam for distant sections from the end of the beam.

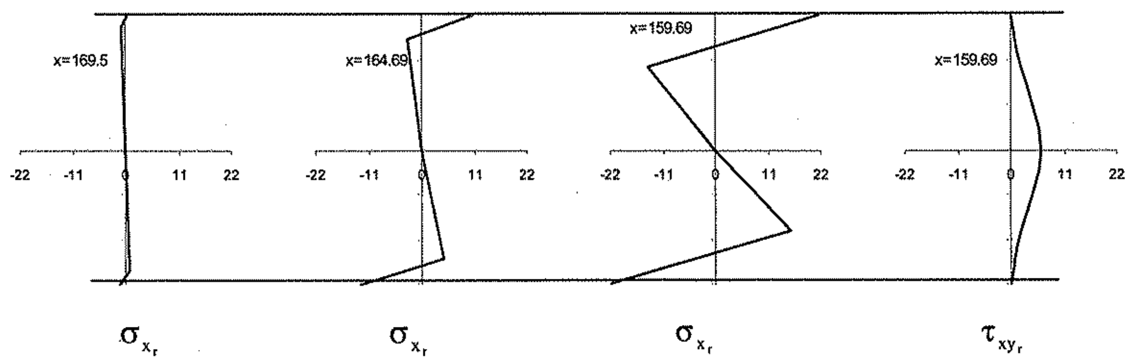

Figure 5. The distribution of the residual stress component of σ_x for 30° orientation angles along the sections of beam.

The distribution of the residual stress component of σ_x along the cross sections of the beam for 45° orientation angle in Figure 6. The intensity of the residual stress component of σ_x becomes greater at the elastic-plastic boundary for distant sections from the end of the beam.

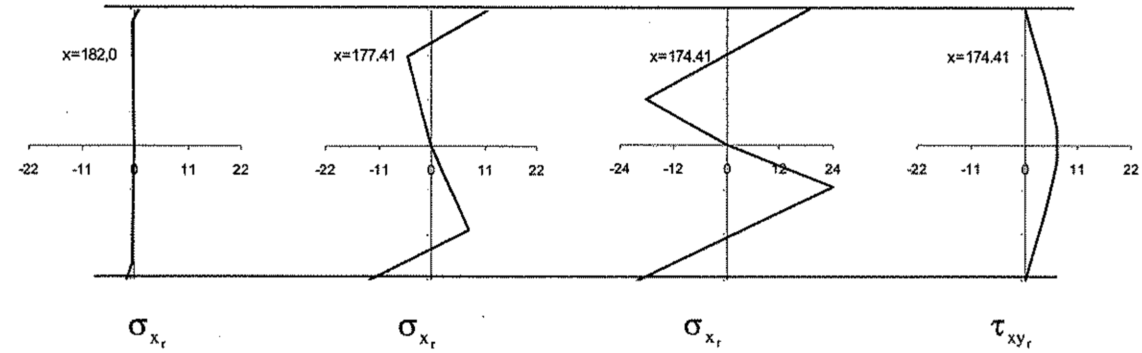

Figure 6. The distribution of the residual stress component of σ_x for 45° orientation angles along the sections of beam.

The distribution of the residual stress of σ_x and the plastic region are denoted in Figure 7, for 60° orientation angle. Yielding starts earlier at the lower surface for this orientation angle. The plastic region expands rapidly at the lower side of the beam.

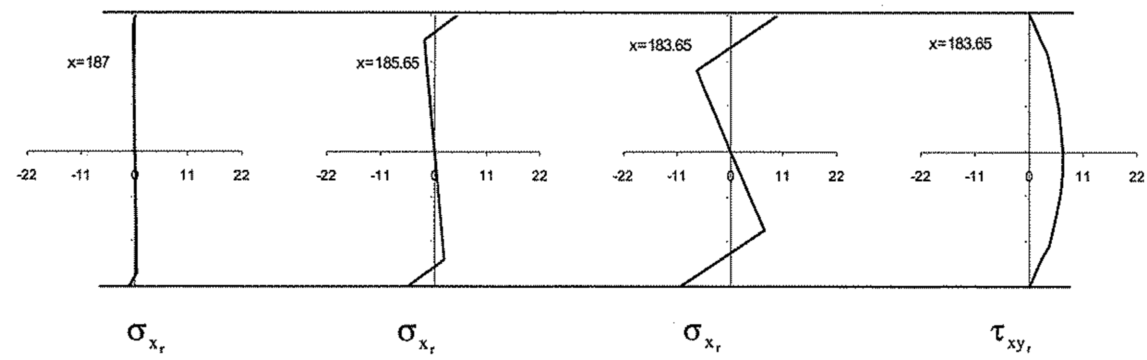

Figure 7. The distribution of the residual stress component of σ_x for 60° orientation angles along the sections of beam.

The distribution of the residual stress component of σ_x along the cross section of the beam for 90° orientation angle is shown in Figure 8. The intensity of the residual stress component of σ_x becomes greater at the boundary of elastic-plastic regions for further expanded plastic zones.

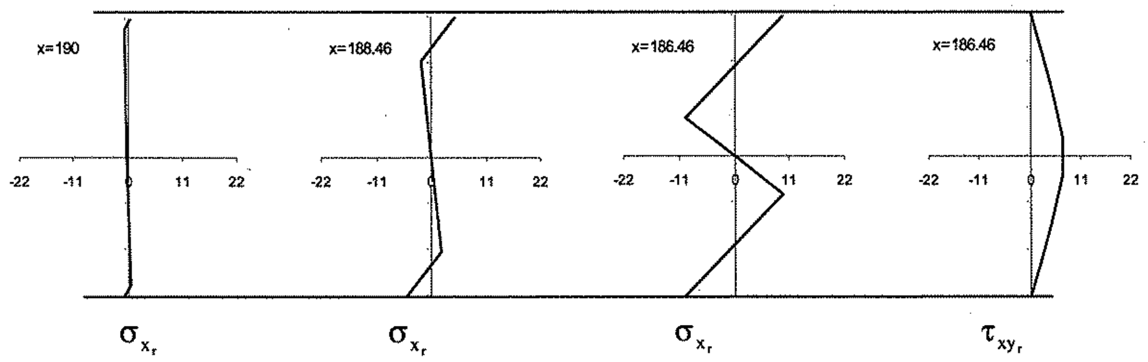

Figure 8. The distribution of the residual stress component of σ_x for 90° orientation angles along the sections of beam.

6. CONCLUSIONS

The results given below are concluded in this investigation, an elastic-plastic solution is carried out on a metal matrix composite beam supported from two ends under a transverse uniformly distributed load.

Yielding begins earlier at the lower surface for 30°, 45° and 60° orientation angles. However; it has the same distances from the middle section for 0° and 90° orientation angles due to the symmetry of the material properties with respect to the x axis.

Plastic yielding occurs first at the same distances from the middle section for 0° and 90° orientation angles. But it starts first at the lower surface for 30°, 45° and 60° orientation angles.

The intensity of the residual stress component of σ_x becomes greater at the boundary of the elastic-plastic regions for further expanded plastic zones.

The intensity of the residual stress component of σ_x is the greatest at the upper and lower surfaces The intensity of the residual stress component is maximum for 0° orientation angle.

The intensity of the residual stress component of τ_{xy} is the greatest on or around the x axis. It is much smaller than that of σ_x.

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