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A RELATION BETWEEN PRIME NUMBERS AND TWIN PRIME NUMBERS

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Abstract-Every mathematician has been concerned with prime numbers, and has metwith mysterious surprises about them.Besides intuition, using empirical methods has an important role to findrelations between prime numbers. A relation between any prime numberand any twin prime number has been obtained.

1.INTRODUCTION

Integers are classified as unity, prime numbers and composite numbers. The sum, the difference and the product of two integers is an integer again. But the quotient of two integers is not always an integer. Any integer, greater than 1, that has no divisor except itself and 1 is said to be a prime number. An integer is called to be a composite number if it is not 1 and it is not prime. If p and p+2 are two prime numbers, then (p,p+2) is called a pair of twin primes.

The sieve of Eratosthenes is a fairly practical method to locate prime numbers. However, it is not the only method . Now we give some important theorems about prime numbers.

Theorem 1.
Every composite number can be written uniquely as a product of prime numbers.

Theorem 2. (Euclid)
There exist infinitely many prime numbers.

There are many proofs of this theorem. But the easiest and shortest one is Euclid's proof.

Proposition 3.
The odd numbers, bigger than 3 and not the multiple of 3, can be expressed in the form of 6n - 1 or 6n +1, where $n \in \mathbb{N}$.

Theorem 4.(Dirichlet)
If a and b are relatively prime numbers, then there are infinitely many prime numbers in the form of $a\text{n}+b$, where $n \in \mathbb{N}$.

From Theorem 4 and Proposition 3 above we can deduce the following result.

Corollary 5.

Every prime number, greater than 3, can be expressed in the form of 6n-1 and 6n+1.

2.THE MAIN THEOREM

A new relation between prime and twin prime numbers are given in the following theorem.

Theorem 6.

If (p , p+2) is a pair of any twin primes and A is any prime number , then

$$3 \mid A - p \qquad \text{or} \qquad 3 \mid A - (p + 2) ,$$

where p and A are greater than 3.

Proof.

Since (p , p+2) is a pair of twin primes and A is a prime number, we can write

$p = 6n - 1$, $p + 2 = 6n + 1$ and $A = 6m - 1$ or $A = 6m + 1$, for some $n , m \in$ IN. Hence we have two cases.

i) First case:

If $A = 6m - 1$, we choose the first of twin prime $p = 6n - 1$. Then We have

$$\begin{aligned} A - p &= (6m-1) - (6n-1) \\ &= 6(m-n) , \end{aligned}$$

and so

$$3 \mid A - p.$$

ii) Second case:

If $A = 6m+1$, we choose the second of twin prime $p+2 = 6n + 1$. Then we have

$$\begin{aligned} A - (p+2) &= (6m+1) - (6n+1) \\ &= 6(m+n) \end{aligned}$$

and so

$$3 \mid A - (p+2) .$$

REFERENCES

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