Mathematical & Computational Applications, Vol. 6, No. 2, pp. 103-111, 2001

THE EFFECT OF THE GALILEO INVARIANCE PAIRING ON THE 1^- STATES IN SPHERICAL NUCLEI

A. A. Kuliev
Department of Physics, Faculty of Arts and Sciences, Sakarya University, Adapazarı-TURKEY

C. Selam, A. Küçükbursa
Department of Physics, Faculty of Arts and Sciences, Dumlupınar University, Kütahya-TURKEY

Abstract-Experimentally investigation low lying 1^- excitations in heavy nuclei plays important role in modern nuclear structure physics. In particular the measurements of the E1 strength in the region $E_x \leq 3$ MeV may shed some light on the mechanism by means of which isospin symmetry or Galilean invariance of pairing is broken in nuclei. In our search we investigate the characteristics of 1^- states for Te, Ba, Ce isotopes in translational invariance GDR model including Galilean invariance pairing. Our calculations show that integrated cross sections is sensitive to restoration broken Galilean invariance of pairing. This restoration does not influence the positions of the low lying 1^- states, but changes its E1 strengths. In spite of restoration Galilean invariance of pairing it is impossible to carry out to the experimental results with spherical structure of nuclei which have been examined.

1. INTRODUCTION

The information methods about the structure of the atomic nuclei by studying the low-lying states have recently been attracting scientists interest. Some scientists believe that the low-lying states play an important role in the explanation of the symmetry breaking in nuclei/1/.

It has been known by some scientists for a long time that there are the low-lying states which make the E1 transition with a greater probability in non-spherical or deformed nuclei /2-11/. In the experiments performed with the (e, e`) and (γ, γ`) reactions, it has been shown that the E1 transition probability from the ground state to these states was around (10-20)e^2fm^2 /12-13/. Scientists have tried to explain this large value of the E1 transitions probability with the different mechanisms. For example, F. Iachello has explained this with the a-claster structure and the static octupole deformation of the nuclei /1/. Some scientists also believe that the contribution of the Giant Dipole Resonances to the low-lying states during the breaking isotopic spin symmetry is responsible for the reason of the large value of the E1 transition probability /14/.

These states have sufficiently investigated within the microscopic model of nuclei /15-23/. Although the theoretical results are in agreement with the experimental value for energy, this agreement is not valid for the dipole transition probability.

It has been declared in the later experimental study /24-33/ that the existence of the low-lying states in the spherical nuclei is also possible. But the calculations of the microscopic model states that the energy for the lowest state in the spherical nuclei is Ex = 6-7 MeV, and this value is Ex = 2-2.5 MeV in the experimental results. However, the energy of the state that as produced by the interactions of the low-lying 2^+ and 3^- states is in agreement with the energy value of the one of few states found in the experiments. But one can't say this about the dipole transition probability.

In order to compensate this disagreement in the theoretical calculations the translational invariance was shown important for the dipole transitions /21/. It is a fact that the pairing interaction potential which is considered between the nucleons in heavy nuclei is also not a Galilean invariance.

The restoration of this invariance, of course, can affect the concerned transition probability.

In this paper, the properties of the 1^- states for nuclei in the spherical deformation limit have been studied by using the Pyatov-Salamov method and considering the Galileo invariance of the pairing forces in the Giant Dipole Resonances Model.

2. TRANSLATIONAL INVARIANCE HAMILTONIEN WITH GALILEAN INVARIANCE PAIRING

If the processes done in/21/ have been performed in the quasi-particle space by using the Pyatov-Salamov method then the Hamilton operator is:

$$H = H_{SQP} + h_0 + h_1 + h_\Delta \tag{1}$$

The Hamilton operator H_{SQP} of the independent quasi-particles system is expressed:

$$H_{SQP} = \sum_j E_j B_{jj} \tag{2}$$

where E_j is the single quasi-particle energy, and B_{jj} is defined as:

$$B_{jj} = \sum \alpha^+_{jm} \alpha_{jm} \tag{3}$$

$\alpha^+_{jm}(\alpha_{jm})$- are the quasi-particle creation(annihilation) operators. The h_0 is the kinematics correlation term and it conserves the momentum in the average quasi-particle field. This term is given by:

$$h_0 = -\frac{1}{8\aleph_0} \sum b_{12} b_{34} \left[C_{12}(\mu) + (-1)^{\mu+1} C^+_{12}(-\mu)\right] \cdot \left[C^+_{34}(\mu) + (-1)^{\mu+1} C_{34}(-\mu)\right] \tag{4}$$

The term h_1 represents the isovector dipole forces which have the translational invariance and it is expressed as:

$$h_1 = \frac{\aleph_1}{6} \sum \frac{a^\tau_{12} a^\tau_{34}}{N_\tau N_{\tau'}} \left[C_{12}(\mu) + (-1)^{\mu+1} C^+_{12}(-\mu)\right] \cdot \left[C^+_{34}(\mu) + (-1)^{\mu+1} C_{34}(-\mu)\right] \tag{5}$$

The last term h_Δ is given by:

$$h_\Delta = -\frac{1}{12\aleph_\Delta} \sum d_{12} d_{34} \left[C_{12}(\mu) - (-1)^{\mu+1} C^+_{12}(-\mu)\right] \cdot \left[C^+_{34}(\mu) - (-1)^{\mu+1} C_{34}(-\mu)\right] \tag{6}$$

and it is a term which makes the pairing interaction potential a Galilean invariance. Here the operator $C^+_{12}(\mu)$ is the two quasi-particle creation operator and it is defined as:

$$C^+_{12}(-\mu) = \sum_{m_1 m_2} \sqrt{\frac{3}{2j_1 + 1}} (j_2 m_2 1\mu / j_1 m_1)(-1)^{j_2 - m_2} \alpha^+_{j_1 m_1} \alpha^+_{j_2 - m_2} \tag{7}$$

The commutation relation between the two quasi-particle creation and annihilation operators is given by:

$$\left[C_{12}(\mu), C_{34}^{+}(\mu')\right] = \left(\delta_{j_1 j_3}\delta_{j_2 j_4} + (-1)^{j_1 - j_2 + 1}\delta_{j_1 j_4}\delta_{j_2 j_3}\right)\delta_{\mu\mu'}. \tag{8}$$

The constants $\aleph_0$, $\aleph_1$, $\aleph_\Delta$ can be obtained from the following relations:

$$\aleph_0 = \sum_{3,4,\tau} E_{34} b_{34}^2, \qquad \aleph_\Delta = \sum_{3,4,\tau} \Delta_\tau a_{34} d_{34}, \qquad \aleph_1 = 300A^{-5/3}$$

were:

$$a_{34} = (j_3\|rY\|j_4)(u_3 v_4 + u_4 v_3), \; \Delta_\tau = 12/\sqrt{A}$$

$$b_{34} = (j_3\|rY\|j_4)(u_3 v_4 + u_4 v_3)E_{34} - \Delta_\tau (u_1 u_2 - v_1 v_2), N_\tau = \begin{cases} N & \text{for neutrons} \\ -Z & \text{for protons} \end{cases}$$

$$d_{34} = (j_3\|rY\|j_4)(u_3 u_4 + v_4 v_3)\Delta_\tau \qquad\qquad E_{34} = E_{j_3} + E_{j_4}$$

3. THE SOLUTIONS OF THE EQUATION OF MOTION WITHIN OF THE RPA METHOD

The eigenfunctions and eigenvalues of the Hamilton operator in eq. (1) can be obtained from the following equation of motion:

$$\left[H, Q_i^{+}(\mu)\right]|0\rangle = \omega_i Q_i^{+}(\mu)|0\rangle \tag{9}$$

where ω_i's are the energies of the 1⁻ states. The phonon operator which produces the 1⁻ states has been chosen as:

$$Q_i^{+}(\mu)|0\rangle = \left\{\sqrt{\frac{1}{2}}\sum\left[\Psi_{12}^{i} C_{12}^{+}(\mu) - \varphi_{12}^{i} C_{12}(\mu)\right]\right\}|0\rangle \tag{10}$$

Here $|0\rangle$ represents the phonon vacuum, i.e., $Q_i\,|0\rangle = 0$. As a result of some derivations, the dispersion equation is given by:

$$\omega^2 \cdot \begin{vmatrix} F_{11} & \omega_i \aleph_1 F_{12} & F_{13}/\aleph_\Delta \\ \omega_i F_{12} & \dfrac{3}{4}\left(\dfrac{A}{NZ}\right)^2 + \aleph_1 F_{22} & \omega_i F_{23}/\aleph_\Delta \\ F_{13} & \aleph_1 F_{23} & -\left(\dfrac{3}{2} - F_{33}/\aleph_\Delta\right) \end{vmatrix} = 0 \tag{11}$$

As shown, one of the roots of this equation ($\omega_i = 0$) corresponds to the collective motion of the system. The other ones ($\omega_i \neq 0$) are the energies of the excited 1⁻ states. The functions F_{ij} are defined as:

$$F_{11} = \sum \frac{E_{34} b_{34}^2}{E_{34}^2 - \omega_i^2}, \; F_{12} = \sum \frac{b_{34} a_{34}}{N_\tau\,(E_{34}^2 - \omega_i^2)}, \; F_{13} = \sum \frac{E_{34} b_{34} d_{34}}{E_{34}^2 - \omega_i^2},$$

$$F_{22} = \sum \frac{E_{34} a_{34}^2}{N_\tau^2 \, (E_{34}^2 - \omega_i^2)}, \quad F_{23} = \sum \frac{d_{34} a_{34}}{N_\tau \, (E_{34}^2 - \omega_i^2)}, \quad F_{33} = \sum \frac{E_{34} d_{34}^2}{E_{34}^2 - \omega_i^2}$$

The amplitudes ψ_{ij} and φ_{ij} which are contained in the wave-functions is given by the relations:

$$\begin{aligned} \Psi_{34}^\tau &= \frac{1}{E_{34} - \omega_i} \left[\frac{b_{34} E_{34}}{\aleph_0} - \frac{4\aleph_1}{3} \left(\frac{NZ}{A} \right)^2 \frac{a_{34}}{N_\tau} L_1 + \frac{2 d_{34}}{3\aleph_\Delta} L_2 \right] \frac{X}{2} \\ \varphi_{34}^\tau &= \frac{1}{E_{34} + \omega_i} \left[\frac{b_{34} E_{34}}{\aleph_0} - \frac{4\aleph_1}{3} \left(\frac{NZ}{A} \right)^2 \frac{a_{34}}{N_\tau} L_1 - \frac{2 d_{34}}{3\aleph_\Delta} L_2 \right] \frac{X}{2} \end{aligned} \tag{12}$$

Here the constant X can be found from the normalization condition:

$$\sum_{3,4} \left(\left| \Psi_{34}^i \right|^2 - \left| \varphi_{34}^i \right|^2 \right) = 1$$

The expressions for L_1 and L_2 in terms F_{ij} can be written as:

$$L_1(\omega_i) = \frac{\left[\frac{2}{3\aleph_\Delta} F_{13} F_{23} + F_{12} \left(1 - \frac{2F_{33}}{3\aleph_\Delta} \right) \right] \frac{\omega_i^2}{\aleph_0}}{\left[1 + \frac{4\aleph_1}{3} \left(\frac{NZ}{A} \right)^2 F_{22} \right] \cdot \left[1 - \frac{2}{3\aleph_\Delta} F_{33} \right]}$$

$$L_2(\omega_i) = \frac{\left[-\frac{4\aleph_1 \omega_i}{3} \left(\frac{NZ}{A} \right)^2 F_{12} F_{23} + F_{13} \left(1 + \frac{4\aleph_1}{3} \left(\frac{NZ}{A} \right)^2 F_{22} \right) \right] \frac{\omega_i}{\aleph_0}}{\left[1 + \frac{4\aleph_1}{3} \left(\frac{NZ}{A} \right)^2 F_{22} \right] \cdot \left[1 - \frac{2}{3\aleph_\Delta} F_{33} \right]}$$

4. THE ELECTRIC DIPOLE TRANSITION PROBABILITIES AND THE INTEGRAL CROSS SECTIONS

The transition probability from the ground state (0^+) to the excited 1^- states for the even-even nuclei is given by:

$$B(E1, 0^+ \to 1_i^-) = \left| \langle 0 | [Q_i^+, D_{1\mu}] | 0 \rangle \right|^2 \tag{13}$$

Here $D_{1\mu} = e \sum_i r_i Y_{1\mu}(\vartheta_i, \varphi_i)$ is the electric dipole operator. The following relation for the the transition probability is obtained by using the equations (10) and (13):

$$B(E1, 0^+ \to 1_i^-) = e^2 \left| \sum_{1,2} a_{12} \left[\Psi_{12}^i + (-1)^{\mu+1} \varphi_{12}^i \right] \right|_p^2$$

The symbol p in the relation shows that the summation is only over the proton system.

Since the breaking of the Galilean invariance of pairing affects the transition probabilities, it is useful to calculate the cross sections which are the integral characteristics of the 1^- states. These cross sections are calculated as/10/:

$$\sigma_n = \int E^n \sigma(E) dE = \frac{16\pi^3}{9\hbar c}\sum_i \omega_i^{n+1} B(E1, 0^+ \to 1_i^-)$$

5. RESULTS AND DISCUSSIONS

In this paper the pairing interaction forces considered here have been provided to be Galileo invariance while the characteristics of the 1^- states in the spherical nuclei were being calculated. The Woods-Saxon potential has been chosen as the average potential. The eigenfunctions and the eigenvalues of Schrödinger equation which is solved by the parameters of Chepurnov have been used as a base states in the calculations. The condition of being a necessary complete set is satisfied by confirming the sum rule/39/ for the electric dipole matrix elements. The pairing interaction constants – correlation functions – for the proton and neutron systems has been accepted as $\Delta_n = \Delta_p$. The calculations for the Sn, Te, Ba, Ce, Nd, Sm isotopes have been done for both the Galileo Invariance (GI) and Non-Galileo Invariance (NGI) cases.

The dependence of the excited 1^- states for ^{140}Ce isotope on the pairing interaction parameter has been shown in Figure 1 – 3. As shown from figures, the dependence of the cross-sections on the pairing interaction is not strong and the amount of the cross-sections increases more rapidly as the pairing interaction forces increase in the non-Galileo invariant case. In other words, as the pair interaction increases, the degree of the Galileo invariant breaking also increases.

The E1 transition probability distribution of the 1^- states for the different correlation functions has been shown in Figure 4. As observed, the pairing interaction has separated the Giant Dipole Resonance. One can note that the restoration of the Galileo invariance has a less effect on this distribution.

The cross-sections and the energy and the E1 transition probability of the lowest two states for ^{128}Te have been shown in Table. As shown from the table, the Galileo invariance of the pairing forces does not affect the energy of the low-lying 1^- states. But the transition probability changes depending on the pairing interaction constant. The calculations show that these changes can both be in the positive and negative direction with respect to the nucleus considered.

In Figure 5 the dependence of the cross-section σ_{-2} on the parameter $\aleph_1$ has been shown. The calculations have approved that this parameter could affect necessarily both the distributions of the transition probabilities and the cross sections.

We can conclude the following statements as a result of the calculations:

a) The Galileo invariance of the pairing interaction has a great importance when the integral cross-sections for the 1^- states are calculated;

b) The restoration of the Galileo invariance does not affect the energies of the low-lying 1^- states but changes the value of the E1 transition probability;

c) Obtaining the experimental result with the spherical structure of the nuclei is not possible;

d) The restoration of the Galileo invariance can affect more the low-lying states in the deformed nuclei. In other words, the study of the low – lying 1^- states in the deformed nuclei can give important clues about the Galileo invariance of the pairing interaction.

REFERENCES

1. F.Iachello, Phys. Lett., B160, 1, 1985
2. M.R.Schemark, Nucl.Data Sheets, B4, 561, 1970
3. I.A.Ellis, Nucl. Data Sheets, B4, 134, 581, 1970
4. B.M.Gorbachov, Y.S.Zamyatin, A.Lobov, Atomizdat, M.,1970
5. M.Gai, J.F.Ennis, M.Ruscev, E.C.Schloemer, B.Shivakumar, S.M.Sterbenz, N.Tsoupas and B.P.Browley, Phys.Rev.Lett., 51, 646, 1983
6. W.Bonin, M.Dahlinger, S.Glienke, E.Kahkeleit, M.Krömer, D.Habs, B.Schwartz and H.Backe, A310, 249, 1983
7. W.Bonin, H.Backe, M.Dahlinger, S.Glienke, D.Habs, E.Hanelt, E.Kahkeleit, and B.Schwartz, Z.Phys., A322, 59, 1985
8. P.Zeyen, B.Ackermann, U.Dammrich, K. Euler, V.Grafen, C.Günther, P.Herzog, M.M.Tölle, B.Prillwitz, R.Tölle, Ch.Lauterbach, H.J.Maier, Z.Phys., A328, 399, 1987
9. M.Gai, J.F.Ennis, D.A.Bromly, H.Emling, F.Azgui, E.Grosse, H.J.Wollersheim, C.Mittag and F.Riess, Phys.Lett., B215, 242, 1988
10. D.Bohle, G.Kilgus, A.Richter, C.W. de Jager, H. de Vries, Phys.Lett., B195, 326, 1987
11. R.D.Heil, H.H.Pitz, *et al.*, Nucl.Phys., A476, 39, 1988
12. B.Buck and A.A.Pilt, Nucl.Phys., 280, 133, 1977
13. F.Iachello, Phys.Rev., C23, 2778, 1981
14. A.Zilges, P.von Brentano and A.Richter, Z.Phys., A341, 489, 1992
15. V.G.Solovyov, NAUKA,M.,1971
16. S.Yoshida, Nucl.Phys., 38, 380, 1962
17. V.G.Solovyov, *et al.*, Izv.AN SSSR, ser.phys., 28, 599, 1964
18. A.Faessler, A.Plastino, Nucl.Phys., A94, 580, 1967
19. K.Neergard, P.Vogel, Nucl.Phys., A145, 33, 1970
20. S.P.Ivanova, *et al.*, JINR, P4-8459, 1974
21. N.I.Pyatov, D.I.Salamov, Nukleonika, 22, 126, 1977
22. N.I.Pyatov, JINR, P4-8208, P4-8380, 1974
23. M.I.Baznat, N.I.Pyatov, D.I.Salamov, JINR, P4-9805, 1976
24. M.Gai, *et al.*, Phys.Rev.Lett., 51, 646, 1983
25. T.Ghur, K.D.Hummel, G.Kilgus, D.Bohle and A.Richter, Nucl.Phys., A501, 95, 1989
26. H.H.Pitz, *et al.*, Nucl.Phys., A509, 587, 1990
27. F.R.Metzger, Phys.Rev., C14, 543, 1976
28. H.H.Pitz, *et al.*, Nucl.Phys., A492, 411, 1989
29. A.Zilges, *et al.*, Nucl,Phys., A507, 399, 1990
30. W.Ziegler, *et al.*, Nucl.Phys., A564, 366, 1993
31. U.Knissel, *et al.*, Prog.Part.Nucl.Phys.34, 285, 1995
32. R.Shwengner, *et al.*, Nucl.Phys., A620, 277, 1997
33. C.Fransen, *et al.*, Phys.Rev., C57, 129, 1998
34. R.Schwengner, G.Winter, *et al.*, Nucl.Phys., A620, 277, 1997
35. P.A.Butler,W.Nazarewich, Nucl.Phys., A533, 249, 1991
36. A.Zilges, P.von Brentano, A.Richter, Z.Phys., A341, 489, 1992
37. A.Zilges, *et al.*, Phys.Rev.Lett., 70, 288, 1993
38. V.A.Chepurnov, Sov.J.Nucl.Phys., 6, 955, 1967
39. A.B.Migdal, JETPh, 15, 81, 1945

Table -The integrated photonuclear cross sections and two low lying 1^- states in ^{128}Te

$\Delta_n=\Delta_p$, $A^{1/2}$		0	2	4	6	8
σ_{-2}	GI	12.55	12.86	13.57	18.79	15.08
	NGI		12.48	11.98	11.91	12.40
σ_{-1}	GI	167.85	169.26	173.76	237.52	198.91
	NGI		165.96	158.98	159.32	169.26
σ_0	GI	2.30	2.30	2.32	3.13	2.73
	NGI		2.27	2.17	2.20	2.38
σ_{+2}	GI	456.35	456.86	455.59	607.47	570.31
	NGI		453.33	434.90	452.77	515.17
ω_1	GI	6.99	7.02	7.11	7.24	7.39
	NGI		7.02	7.11	7.24	7.39
B(E1, ω_1)	GI	3.5	2.9	2.2	1.5	0.9
	NGI		3.2	2.5	1.7	1.0
ω_2	GI / NGI	7.41	7.42	7.49	7.58	7.69
B(E1, ω_2)	GI	17.0	14.9	12.1	6.8	4.7
	NGI		17.3	15.4	9.6	7.4

Δ_n and Δ_p (MeV), σ_{-2} (mb/MeV), σ_{-1}(mb), σ_0(MeV·b), σ_{+2}(MeV^3·b)
ω(MeV), B(E1, ω)($10^{-3}e^2fm^2$) unites. $\aleph_1 = 300A^{-5/3}$ MeV fm^{-2}
GI – Galilean Invariance , NGI – Non-Galilean Invariance

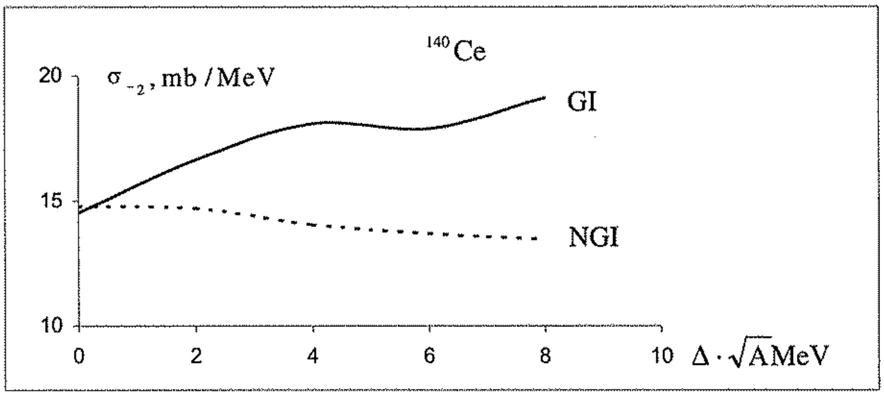

Fig 1. The dependence integrated photonuclear cross sections σ_{-2} on pairing correlation parameter ($\aleph_1 = 300A^{-5/3}$ MeV fm^{-2})

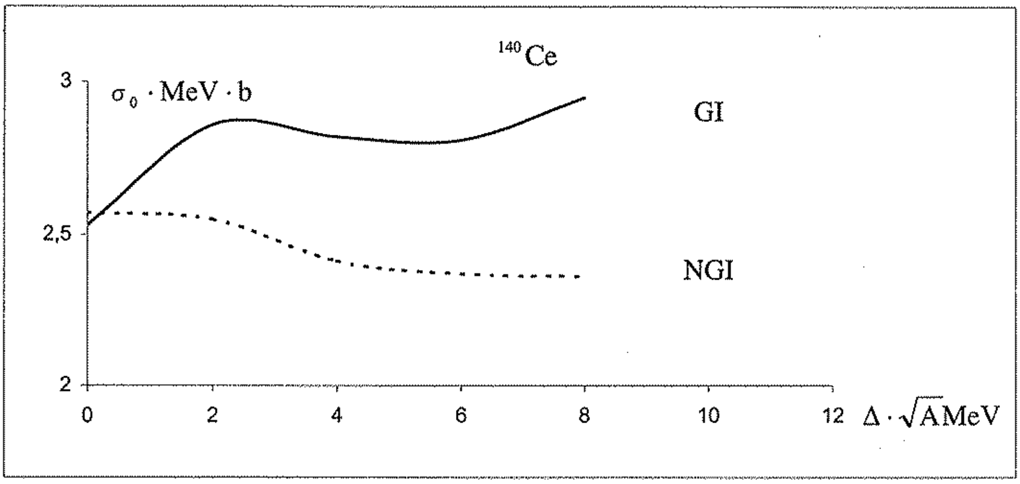

Fig 2. The dependence integrated photonuclear cross section σ_0
On pairing correlation parameter
($\aleph_1 = 300A^{-5/3}$ MeV fm^{-2})

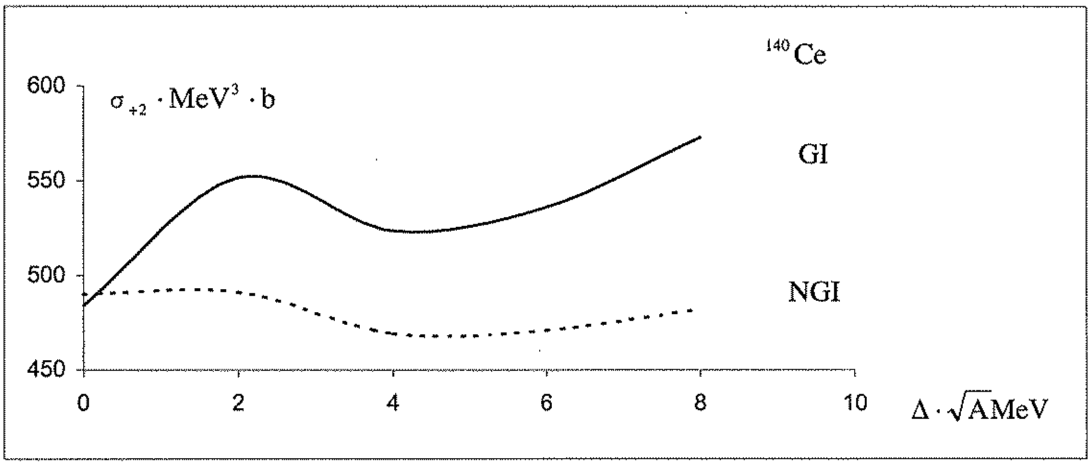

Fig 3. The dependence integrated photonuclear cross sections σ_{+2}
on pairing correlation parameter
($\aleph_1 = 300A^{-5/3}$ MeV fm^{-2})

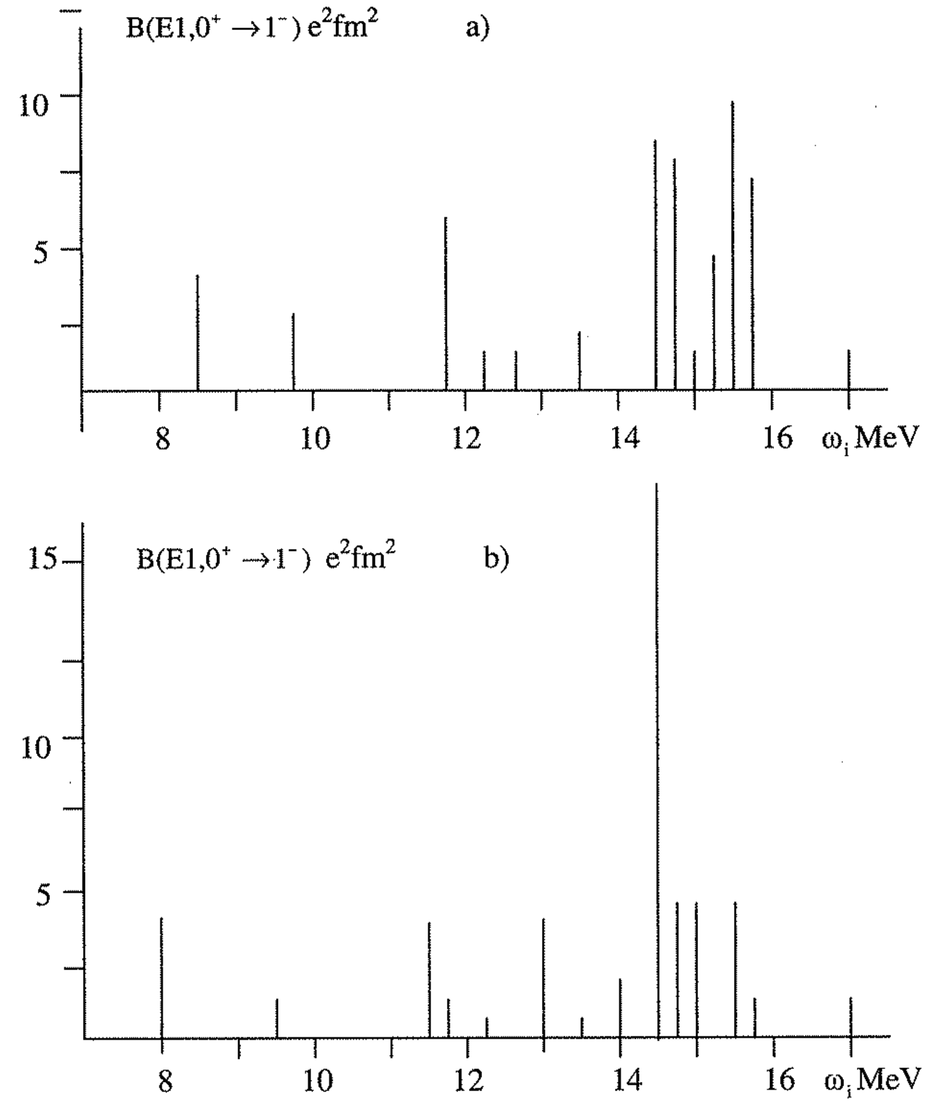

Fig. 4. Energy diagram of B(E1) for ^{138}Ce
a) $\Delta_n = \Delta_p = 12/A^{1/2}$, b) $\Delta_n = \Delta_p = 6/A^{1/2}$, MeV

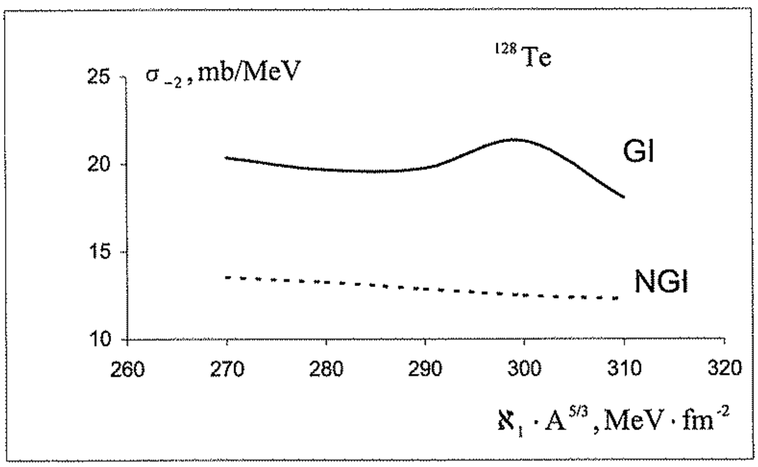

Fig 5. The dependence integrated photonuclear cross sections σ_{-2} on $\aleph_1$
$\Delta_n = \Delta_p = 6/A^{1/2}$,MeV