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AN ELASTO-PLASTIC STRESS ANALYSIS IN A POLYMER MATRIX COMPOSITE BEAM OF ARBITRARY ORIENTATION SUBJECTED TO TRANSVERSE UNIFORMLY DISTRIBUTED LOAD

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Abstract- Polymer matrix composite beam of arbitrary orientation subjected to transverse uniformly distributed load is studied by an analytical elasto-plastic stress analysis. In the elasto-plastic solution, the material is assumed to be perfectly plastic. A composite consisting of fiber reinforced polymer matrix was produced for this work. The expansion of the plastic region and the residual stress component of σ_x are determined for 0°, 30°, 45°, 60° and 90° orientation angles. The yielding begins for 0° and 90° orientation angles at the upper and lower surfaces of the beam at the same distances from the free end. But, it starts first at the upper surface for 30°, 45° and 60° orientation angles. Sample problems are given for various orientation angles, x axis of the beam is used to obtain the location of the elasto-plastic boundary and to calculate elastic, elasto-plastic and residual normal and shear stresses. The intensity of the residual stress component of τ_{xy} is maximum on or around the x axis of the beam but the residual stress component of σ_x is maximum at the upper and lower surfaces.

1. INTRODUCTION

Composites are made up of several different things, parts or substances. There has been a rapid growth in the use of fiber reinforced materials in engineering applications in the last few years. Fiber reinforced plastics have acquired a high reputation for structural applications.

Anath and Chandra [1] have investigated fiber push-out in metallic and intermetallic matrix composites. Inelastic deformation of metal matrix composites has been carried out by Majumdar and Newaz [2]. They have studied on plasticity and damage machanisms. Karakuzu and Sayman [3] have studied elasto plastic finite element analysis of fiber reinforced aliminium metal matrix rotating discs by using finite element tecniques. Canumalla et al. [4] have investigated the mechanical behavior of mullite fiber reinforced aluminum alloy composites. Jeronimidis and Parkyn [5] have investigated residual stresses in APC-2 cross-ply laminates. They have compared classical laminate theory with measured levels of residual stress obtained from a number of experimental techniques. Karakuzu and Özcan [6] have studied an analytical elasto-plastic stress analysis in an aluminium metal matrix composite cantilever beam subjected to a single transverse force applied to the free end of the beam and a uniformly distributed load. Sayman [7] has carried out an elasto-plastic stress analysis in stainless steel fiber reinforced aluminum metal matrix laminated plates loaded transversely. Arnold et al. [8] have investigated the use of the compliant-layer concept in reducing residual stresses resulting from processing. Experimental investigations on the forming of thermoplastic composites can be found in References [9-13]. Yeh and Krempl [14] have introduced the vanishing fiber diameter model together with the thermoviscoplasticity theory based on overstress.

In this study an analytical elasto-plastic stress analysis is carried out for a fiber- reinforced polymer matrix composite beam subjected to a uniformly distributed load. Sample problems are given for different orientation angles.

2. ELASTIC ANALYSIS

The composite cantilever beam is loaded by the uniformly distributed load q, as shown in Figure 1. The angle between the principal axis of the composite fibers and the x axis is θ.

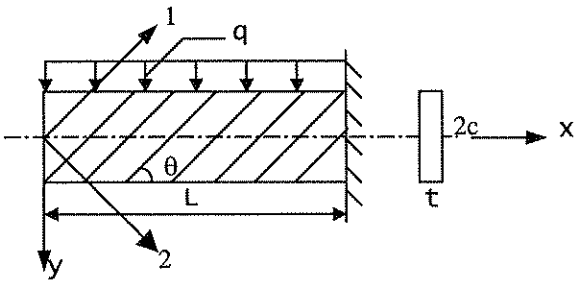

Figure 1. Composite cantilever beam subjected to a uniformly distributed load

For the plane-stress case the equation of equilibrium is given by Lekhnitskii [15] as,

$$\bar{a}_{22}\frac{\partial^4 F}{\partial x^4}-2\bar{a}_{26}\frac{\partial^4 F}{\partial x^3\partial y}+\left(2\bar{a}_{12}+\bar{a}_{66}\right)\frac{\partial^4 F}{\partial x^2\partial y^2}-2\bar{a}_{16}\frac{\partial^4 F}{\partial x\partial y^3}+\bar{a}_{11}\frac{\partial^4 F}{\partial y^4}=0 \tag{1}$$

where F is a stress function. Solutions of some anisotropic beams are given by Lekhnitskii [16]. The constants in equation (1) are given by Jones [17] as,

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{Bmatrix}=\begin{bmatrix}\bar{a}_{11} & \bar{a}_{12} & \bar{a}_{16}\\ \bar{a}_{12} & \bar{a}_{22} & \bar{a}_{26}\\ \bar{a}_{16} & \bar{a}_{26} & \bar{a}_{66}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} \tag{2}$$

where:

$$\begin{aligned}
\bar{a}_{11}&=a_{11}\cos^4\theta+(2a_{12}+a_{66})\cos^2\theta\sin^2\theta+a_{22}\sin^4\theta\\
\bar{a}_{12}&=a_{12}(\sin^4\theta+\cos^4\theta)+(a_{11}+a_{22}-a_{66})\sin^2\theta\cos^2\theta\\
\bar{a}_{22}&=a_{11}\sin^4\theta+(2a_{12}+a_{66})\sin^2\theta\cos^2\theta+a_{22}\cos^4\theta\\
\bar{a}_{16}&=(2a_{11}-2a_{12}-a_{66})\sin\theta\cos^3\theta-(2a_{22}-2a_{12}-a_{66})\sin^3\theta\cos\theta\\
\bar{a}_{26}&=(2a_{11}-2a_{12}-a_{66})\sin^3\theta\cos\theta-(2a_{22}-2a_{12}-a_{66})\cos^3\theta\sin\theta\\
\bar{a}_{66}&=2(2a_{11}+2a_{22}-4a_{12}-a_{66})\sin^2\theta\cos^2\theta+a_{66}(\sin^4\theta+\cos^4\theta)
\end{aligned} \tag{3}$$

$$a_{11}=\frac{1}{E_1},\quad a_{12}=-\frac{\upsilon_{12}}{E_1},\quad a_{22}=\frac{1}{E_2},\quad a_{66}=\frac{1}{G_{12}} \tag{4}$$

F is chosen as a fifth order polynomial to satisfy the differential equation,

$$F_5 = d_5 \frac{x^2 y^3}{6} + f_5 \frac{y^5}{20} + e_5 \frac{xy^4}{12} \tag{5}$$

Substituting it into the equilibrium gives

$$\left(2\bar{a}_{12} + \bar{a}_{66}\right) d_5 2y - 2\bar{a}_{16} d_5 x 2 + \bar{a}_{11} f_5 6y - 2\bar{a}_{16} e_5 2y + \bar{a}_{11} e_5 2x = 0 \tag{6}$$

$$x(-4a_{16}d_5 + 2a_{11}e_5) + y(4a_{12}d_5 + 2a_{66}d_5 + 6a_{11}f_5 - 4a_{11}e_5) = 0 \tag{7}$$

For satisfying equation (7) each term of x and y must be equal to zero. Hence,

$$e_5 = md_5 \, , \quad m = \frac{2\bar{a}_{16}}{\bar{a}_{11}} \text{ and,} \tag{8}$$

$$f_5 = nd_5, \quad n = \frac{-2a_{12} - a_{66} + 2a_{16}m}{3a_{11}} \tag{9}$$

The boundary conditions are:

$$\begin{aligned} y = -c &\Rightarrow \sigma_y = -q, \tau_{xy} = 0 \\ y = +c &\Rightarrow \sigma_y = 0, \tau_{xy} = 0 \end{aligned} \tag{10}$$

For satisfying the differential equation with boundary conditions, further, the polynomials are chosen in second and third orders.

$$\begin{aligned} F_2 &= a_2 \frac{x^2}{2} \\ F_3 &= b_3 \frac{x^2 y}{2} + c_3 \frac{xy^2}{2} + d_3 \frac{y^3}{6} \end{aligned} \tag{11}$$

The stress components are found from this F function as,

$$F = F_5 + F_3 + F_2 \tag{12}$$

$$\begin{aligned} \sigma_x &= \frac{\partial^2 F}{\partial y^2} = d_5 x^2 y + f_5 y^3 + e_5 xy^2 + c_3 x + d_3 y \\ \sigma_y &= \frac{\partial^2 F}{\partial x^2} = \frac{d_5}{3} y^3 + b_3 y + a_2 \\ \tau_{xy} &= -\frac{\partial^2 F}{\partial x \partial y} = -d_5 xy^2 - \frac{e_5}{3} y^3 - b_3 x - c_3 y \end{aligned} \tag{13}$$

From the boundary conditions

$$a_2 = -\frac{q}{2t}, \quad b_3 = \frac{qc^2}{2I}, \quad d_5 = -\frac{q}{2I} \tag{14}$$

are obtained. Where I is the moment of inertia of the cross-section of the beam, and

$$I = \frac{(2c)^3 t}{12} = \frac{2c^3 t}{3} \tag{15}$$

Using $\tau_{xy} = 0$ at $y = \mp c$

$$b_3 = -d_5 c^2, \quad c_3 = -\frac{e_5}{3} c^2 \tag{16}$$

are obtained. At the free end, according to Saint Venant's principle, the resultant and bending moment of σ_x must be equal to zero. Thus,

for x=0

$$\int_{-c}^{c}\sigma_x t dy=0,\quad \int_{-c}^{c}\sigma_x t y dy=0 \Rightarrow \int_{-c}^{c}\left(f_5 y^3+d_3 y\right)t dy=0,\quad \int_{-c}^{c}\left(f_5 y^4+d_3 y^2\right)t dy=0 \tag{17}$$

From this condition, it is found that,

$$d_3=-\frac{3f_5c^2}{5} \tag{18}$$

The elastic stress components become,

$$\begin{aligned}\sigma_x&=-\frac{q}{2I}\left(x^2y+ny^3+mxy^2-\frac{1}{3}mc^2x-\frac{3}{5}nc^2y\right)\\ \sigma_y&=-\frac{q}{2I}\left(\frac{y^3}{3}-c^2y\right)-\frac{q}{2t}\\ \tau_{xy}&=-\frac{q}{2I}\left(-xy^2-\frac{1}{3}my^3+c^2x+\frac{1}{3}mc^2y\right)\end{aligned} \tag{19}$$

3. ELASTO-PLASTIC SOLUTION

The equations of equilibrium for the plane-stress case are

$$\begin{aligned}&\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0\\ &\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}=0\end{aligned} \tag{20}$$

If the length of the beam is very large in comparasion with its height, σ_y can be neglected in comparasion with σ_x and τ_{xy}. For calculation of stresses in the two-dimensional case, the equivalent stress for an orthotropic material is usually obtained according to the Tsai-Hill theory. The equivalent stress in the plane stress case is

$$\bar{\sigma}=\sqrt{\sigma_1^2-\sigma_1\sigma_2+\frac{X^2}{Y^2}\sigma_2^2+\frac{X^2}{S^2}\tau_{12}^2}=X \tag{21}$$

where X,Y are the yield strengths in the 1 and 2 principal material directions, respectively and S is the shear yield strength in the 1-2 plane. It is assumed that X=Y=Z, because of the same alignment of the fibers in the second and third principal material directions. σ_1,σ_2 and τ_{12} are the stress components in the principal material directions, given by

$$\begin{aligned}\sigma_1&=\sigma_x\cos^2\theta+2\tau_{xy}\sin\theta\cos\theta\\ \sigma_2&=\sigma_x\sin^2\theta-2\tau_{xy}\sin\theta\cos\theta\\ \tau_{12}&=-\sigma_x\sin\theta\cos\theta+\tau_{xy}\left(\cos^2\theta-\sin^2\theta\right)\end{aligned} \tag{22}$$

Writing $\sigma_y=0$ in the second differential equations of equilibrium gives that τ_{xy} is a function of y in the ordinary form or $\tau_{xy}=f(y)$. Deriving Equation (21) with respect to x and using

$\frac{\partial \tau_{xy}}{\partial x} = 0$, gives $\frac{\partial \sigma_x}{\partial x} = 0$. Substituting $\frac{\partial \sigma_x}{\partial x}$ in the first differential equations of equilibrium gives τ_{xy} as a constant. Plastic region begins at the upper or lower surfaces of the beam when the stress component σ_x reaches,

$$X_1 = \frac{X}{\sqrt{\cos^4\theta - \sin^2\theta\cos^2\theta + \frac{X^2\sin^4\theta}{Y^2} + \frac{X^2\sin^2\theta\cos^2\theta}{S^2}}} \tag{23}$$

where θ is the orientation angle of the fibers. When the plastic region starts on these surfaces σ_x becomes X_1 and τ_{xy} equals 0. From this condition $\sigma_x = X_1$ is found as a constant in the plastic region and the shear stress is equal to zero. The composite cantilever beam is loaded by the uniformly distributed load q for elasto-plastic solution, as shown in Figure 2.

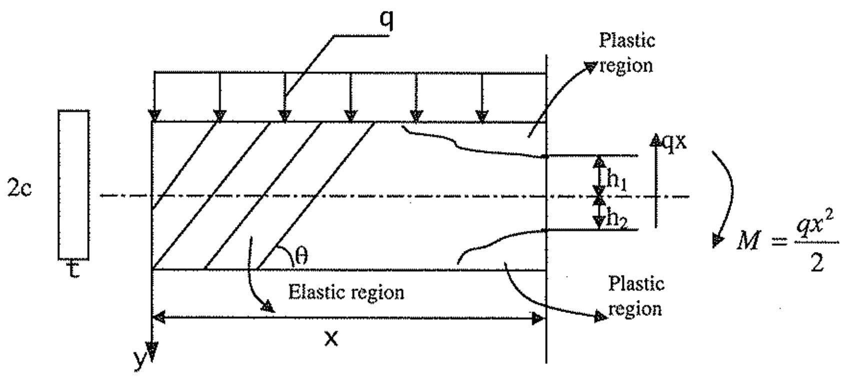

Figure 2. Composite cantilever beam subjected to a uniformly distributed load for the ealsto plastic solution

3.1. Elasto-plastic solution for θ=0° and 90° orientation angle

In order to satisfy both the differential equation and the boundary conditions, the stress function F is chosen as

$$F = \frac{d_5}{6}x^2y^3 + \frac{f_5}{20}y^5 + \frac{g_3}{6}y^3 + \frac{b_3}{2}x^2y + \frac{a_2}{2}y^2 \tag{24}$$

If we substitude the stress function in the governing differential equation, we obtain

$$\left(2\bar{a}_{12} + \bar{a}_{66}\right)2yd_5 + 6\bar{a}_{11}\, y f_5 = 0 \Rightarrow f_5 = -\frac{2\bar{a}_{12} + \bar{a}_{66}}{3\bar{a}_{11}} d_5$$

$$n = -\frac{2\bar{a}_{12} + \bar{a}_{66}}{3\bar{a}_{11}} \Rightarrow f_5 = nd_5 \tag{25}$$

The stress components are:

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} = d_5x^2y + f_5y^3 + g_3y + a_2$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2} = 0 \tag{26}$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} = -d_5 xy^2 - b_3 x$$

The boundary conditions for this beam are given as,

$$y = -h_1 \Rightarrow \tau_{xy} = 0 \tag{27}$$

$$y = h_2 \Rightarrow \tau_{xy} = 0 \tag{28}$$

$$\int_{-h_1}^{h_2} t\,\tau_{xy}\, dy = -q\, x \tag{29}$$

$$y = -h_1 \Rightarrow \sigma_x = X_1 \tag{30}$$

$$y = h_2 \Rightarrow \sigma_x = -X_1 \tag{31}$$

The resultant of σ_x at any section is equal to zero:

$$X_1 t(c - h_1) - X_1 t(c - h_2) + \int_{-h_1}^{h_2} \sigma_x t\, dy = 0 \tag{32}$$

The resultant of σ_x at any section (x) is equal to the bending moment:

$$X_1 t(c - h_1)\frac{c + h_1}{2} + X_1 t(c - h_2)\frac{c + h_2}{2} - \int_{-h_1}^{h_2} \sigma_x t\; y\, dy = \frac{qx^2}{2} \tag{33}$$

where positive σ_x produces an opposite moment of $\frac{qx^2}{2}$, therefore it takes a negative sign. From the boundary conditions, the unknown parameters are found as,

$$h_1 = h_2 = \sqrt{\frac{X_1 c^2 - \frac{qx^2}{2t}}{\frac{nq}{5t} + \frac{X_1}{3}}} \tag{34}$$

$$a_2 = 0$$

$$b_3 = d\, h_1^2 \tag{35}$$

$$d_5 = -\frac{q}{\frac{4th_1^3}{3}} \tag{36}$$

$$g_3 = -\left[\frac{X_1}{h_1} + d_5 x^2 + n d_5 h_1^2\right] \tag{37}$$

The stress components can be found by using these parameters.

3.2. Elasto-plastic solution for the inclined orientation angles

For the inclined orientation angles the stress function F is chosen as follows

$$F = d_5 \frac{x^2 y^3}{6} + f_5 \frac{y^5}{20} + e_5 \frac{xy^4}{12} + b_3 \frac{x^2 y}{2} + c_3 \frac{xy^2}{2} + d_3 \frac{y^3}{6} + \frac{a_2}{2} x^2 + b_2 \frac{y^2}{2} \tag{38}$$

Substituting F into Eqn. (1) one obtains,

$$\left(2\bar{a}_{12}+\bar{a}_{66}\right)d_5 2y-4\bar{a}_{16}\,d_5x+6\bar{a}_{11}\,f_5y-4\bar{a}_{16}\,e_5+2\bar{a}_{11}\,e_5x=0 \tag{39}$$

$$\left[\left(2\bar{a}_{12}+\bar{a}_{66}\right)2d_5-4\bar{a}_{16}\,e_5+6\bar{a}_{11}\,f_5\right]y+\left[-4\bar{a}_{16}\,d_5+2\bar{a}_{11}\,e_5\right]x=0 \tag{40}$$

For satisfying the equation, each term of x and y must be equal to zero. Hence,

$$e_5=\frac{2\bar{a}_{16}}{\bar{a}_{11}}d_5=md_5,\quad m=\frac{2\bar{a}_{16}}{\bar{a}_{11}} \tag{41}$$

$$f_5=nd_5,\quad n=\frac{-2\bar{a}_{12}-\bar{a}_{66}+2\bar{a}_{16}\,m}{3\bar{a}_{11}} \tag{42}$$

The stress components are determined as,

$$\begin{aligned}
\sigma_x&=\frac{\partial^2F}{\partial y^2}=d_5x^2y+f_5y^3+e_5xy^2+c_3x+d_3y+b_2\\
\sigma_y&=\frac{\partial^2F}{\partial x^2}=\frac{d_5}{3}y^3+b_3y+a_2\\
\tau_{xy}&=-\frac{\partial^2F}{\partial x\,\partial y}=-d_5xy^2-\frac{e_5}{3}y^3-b_3x-c_3y=0
\end{aligned} \tag{43}$$

In the above calculations , one neglects σ_y in comparasion with σ_x and τ_{xy}. If Eqn. (41), (42) are replaced to Eqn. (43) one has

$$\begin{aligned}
\sigma_x&=d_5x^2y+nd_5y^3+md_5xy^2+c_3x+d_3y+b_2\\
\tau_{xy}&=-d_5xy^2-\frac{md_5}{3}y^3-b_3x-c_3y
\end{aligned} \tag{44}$$

Parameters $d_5, b_3, c_3, b_2, h_1, h_2$ are found from the boundary conditions. From the boundary condition in Eqn. (27), (28) and (29) one obtains

$$c_3=-d_5x\left(h_2-h_1\right)-\frac{md_5}{3}\left(h_1^2-h_1h_2+h_2^2\right) \tag{45}$$

$$b_3x=-d_5xh_1h_2+\frac{md_5}{3}\left(h_1-h_2\right)h_1h_2 \tag{46}$$

$$d_5=\frac{-qx/t}{\frac{x}{6}\left(h_1+h_2\right)^3+\frac{m}{12}\left(h_1+h_2\right)^3\left(h_2-h_1\right)} \tag{47}$$

From the boundary conditions in Eqn. (30), (31), (32) and (33) one has

$$d_5=\frac{X_1\left(h_1-h_2\right)}{\frac{n\left(h_1-h_2\right)\left(h_1+h_2\right)^3}{4}-\frac{mx\left(h_1+h_2\right)^3}{6}} \tag{48}$$

$$c_3x + b_2 = \frac{(h_1 - h_2)}{2} d_5 x^2 + \frac{(h_1^3 - h_2^3)}{2} n d_5 - \frac{(h_1^2 + h_2^2)}{2} m d_5 x + \frac{(h_1 - h_2)}{2} d_3 \tag{49}$$

$$d_3 = -\frac{2X_1}{h_1 + h_2} - d_5 x^2 - (h_1^2 - h_1 h_2 + h_2^2) n d_5 + (h_1 - h_2) m d_5 x \tag{50}$$

If one equates parameter d_5 by Eqs. (47) and (48) one has

$$\frac{12X_1(h_1 - h_2)}{3n(h_1 - h_2) - 2mx} = \frac{-12qx}{t(2x + m(h_2 - h_1))} \tag{51}$$

Solution of the above equation gives

$$h_1 - h_2 = u = \frac{\left(\frac{2x}{m} + \frac{3qxn}{X_1 mt}\right) \pm \sqrt{\left(\frac{2x}{m} + \frac{3qxn}{X_1 mt}\right)^2 - \frac{8qx^2}{X_1 t}}}{2} \tag{52}$$

Arranging Eqn. (52), one obtains

$$u = \frac{(2xX_1 t + 3qxn) \pm \sqrt{(2xX_1 t + 3qxn)^2 - 8X_1 m^2 tqx^2}}{2X_1 mt} \tag{53}$$

Whereas from Eqn. (33) one retrieves , it is obtained as

$$\begin{aligned} &\frac{nd_5}{60}(h_1 + h_2)\left[8(h_1^4 + h_2^4) + 2h_1h_2(h_1^2 + h_2^2) - 12h_1^2h_2^2\right] - \frac{md_5 x}{12}(h_1 + h_2)^3(h_1 - h_2) \\ &+ X_1c^2 - \frac{X_1}{3}(h_1^2 + h_2^2 - h_1h_2) = \frac{qx^2}{2t} \end{aligned} \tag{54}$$

If one replaces $h_1 - h_2 = u \Rightarrow h_1 = h_2 + u$ into Eqn. (54), one has

$$\begin{aligned} f = &\frac{nd_5}{60}(2h_2 + u)\left[8(h_2 + u)^4 + 8h_2^4 + 2h_2(h_2 + u)^3 + 2h_2^3(h_2 + u) - 12h_2^2(h_2 + u)^2\right] \\ &- \frac{md_5 x}{12}(2h_2 + u)^3 u - \frac{qx^2}{2t} + X_1c^2 - \frac{X_1}{3}\left[(h_2 + u)^2 + h_2^2 - h_2(h_2 + u)\right] \end{aligned} \tag{55}$$

Solving Eqn. (55) by the Newton-Raphson method gives h_2 and then the other constants can be determined.

4.PRODUCTION OF COMPOSITE BEAM

In this study, low density polyethylene (LDPEF2.12, Petkim company) has been used as a thermoplastic matrix. Polyethylene granules have been placed on the rectangular mould . The amount of granules is enough to cover the mould surface to prevent holes. After they were melted at a temperature about 160 °C, they were held for 5 minutes under 2.5 MPa pressure. Then by raising the pressure to 15 MPa and decreasing the temperature to 30 °C, melted granules were held for 3 minutes more. Thus a polyethylene layer was produced. Second polyethylene layer was produced in the same way. Putting steel fiber between two polyethylene layers, a composite layer was produced by using the above process. The thickness of the composite layer was 2 mm. A beam is constructed by using the four layer under the same manufacturing process. This was placed between the first and the second polyethylene matrices on the second mould as shown in Figure 3.

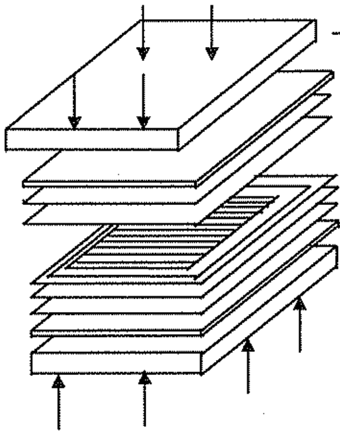

Figure 3. Press Operation

At the same time acetate papers were placed to the bottom and top surfaces of this sandwich type polyethylene plates. The mechanical properties and yield points of the composite material are given in Table 1 and 2.

Table 1. Mechanical properties of the composite beam

E_1(GPa)	E_2(GPa)	G_{12}(Gpa)	υ_{12}
4.3	0.966	0.58	0.4

Table 2. Yield points of the composite beam

Axial yield point, X (MPa)	23.05
Transverse yield point , Y (MPa)	6.26
Shear yield point , S (MPa)	6.24

5.SAMPLE PROBLEM

q is chosen as 0.1 N per mm thickness. For uniformly distributed load, elasto-plastic solutions were carried out for orientation angles of 0°, 30°, 45°, 60°, 90° and the thickness of the beam has been taken as 6 mm. The height of the beams (2c) has been taken as 15 mm.

For the uniformly distributed loads the bending moments and shear forces at any section are $qx^2/2$ and qx respectively . This bending moment must be equal to the sum of bending moments of the elastic and plastic stresses and the resultant of the stress component σ_x must be equal to zero at any elasto-plastic section. The subtraction of the elastic stresses from the elasto-plastic stresses gives the residual stresses in the beam. Elastic, elasto-plastic and residual stresses at any section are shown schematically in Fig 4.

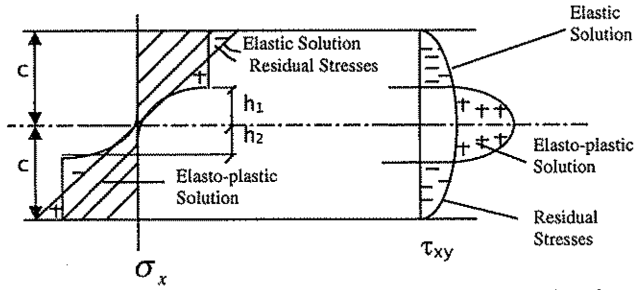

Figure 4. Schematic representation of elastic, elasto-plastic and residual stresses at any section

h_2 is calculated from Eqn. (55) by Newton Raphson method. Where h_1 and h_2 are the distances from the upper and lower elasto-plastic boundaries with respect to the x axis at any section, respectively. The yielding begins for 0°and 90° at the upper and lower surfaces of the beam at the same distances from the free end because of the symetry of the material properties with respect to x axis. However; it starts first at the upper surface for 30°, 45° and 60° orientation angles as seen in Table 3. Expansion of the plastic region and residual stress component of σ_x for 30°, 45° and 60° orientation angles in the region between upper and lower yield points as seen in Table 4. For the same length of the beam when the value of the orientation angle increases the intensiy of the residual stresses component of σ_x increases as seen in Table 5. Elastic, elasto-plastic and residual normal and shear stresses are shown in Tables 6-10 for each case.

Table 3. The distance between the free end and yield points

	Orientation angles				
	0°	30°	45°	60°	90°
At the upper surface (mm)	322.15	226.81	196.61	179.24	167.88
At the lower surface (mm)	322.15	235.21	202.33	182.50	167.88

Table 4. Expansion of the plastic region and the residual stress component of σ_x for 30°, 45° and 60° orientation angles in the region between upper and lower yield points.

Orientation Angle	Distance between the free end and yield point	h_1 (mm)	$(\sigma_x)_{residual}$ at the upper surface [MPa]	$(\sigma_x)_{residual}$ at the lower surface [MPa]
30°	226.81	7.50	0.000	-0.846
	230.00	7.28	-0.328	-0.530
	235.12	6.96	-0.877	0.000
45°	196.61	7.50	0.000	-0.499
	200.00	7.23	-0.302	-0.205
	202.33	7.07	-0.514	0.000
60°	179.24	7.50	0.000	-0.260
	181.00	7.35	-0.142	-0.120
	182.50	7.23	-0.264	0.000

Table 5. The residual stress component of σ_x at the upper and lower surfaces of the cantilever for 30°, 45°, 60° and 90° orientation angles at x=235.21, 202.33, 187.50 mm

Orientation Angles	x (mm)	h_1 (mm)	h_2 (mm)	σ_{x_e} at upper s.	σ_{x_p} at upper s.	σ_{x_r} at upper s.	σ_{x_e} at lower s.	σ_{x_p} at lower s.	σ_{x_r} at lower s.
30°	235.21	6.96	7.50	12.732	11.855	-0.877	-11.855	-11.855	0.000
45°	235.21	3.16	3.69	12.591	8.839	-3.752	-11.993	-8.839	3.154
45°	202.33	7.07	7.50	9.353	8.839	-0.514	-8.839	-8.839	0.000
60°	202.33	5.14	5.41	9.241	7.267	-1.974	-8.947	-7.267	1.680
60°	187.50	6.77	7.04	7.946	7.267	-0.679	-7.674	-7.267	0.407
90°	187.50	5.32	5.32	7.809	6.260	-1.549	-7.809	-6.260	1.549

Table 6. Variations of elastic, elasto-plastic, residual normal stresses and distances h_1 and h_2 with respect to the x axis for θ=0° at the upper and lower edges.

x (mm)	h_1 (mm)	h_2 (mm)	σ_{x_e} at upper s.	σ_{x_p} at upper s.	σ_{x_r} at upper s.	σ_{x_e} at lower s.	σ_{x_p} at lower s.	σ_{x_r} at lower s.
322.15	7.50	7.50	23.050	23.050	0.000	-23.050	-23.050	0.000
332.15	7.00	7.00	24.500	23.050	-1.450	-24.500	-23.050	1.450
342.15	6.46	6.46	26.000	23.050	-2.950	-26.000	-23.050	2.950
352.15	5.85	5.85	27.540	23.050	-4.490	-27.540	-23.050	4.490
362.15	5.15	5.15	29.130	23.050	-6.080	-29.130	-23.050	6.080

Table 7. Variations of elastic, elasto-plastic, residual normal stresses and distances h_1 and h_2 with respect to the x axis for θ=30° at the upper and lower edges.

x (mm)	h_1 (mm)	h_2 (mm)	σ_{x_e} at upper s.	σ_{x_p} at upper s.	σ_{x_r} at upper s.	σ_{x_e} at lower s.	σ_{x_p} at lower s.	σ_{x_r} at lower s.
235.21	6.96	7.50	12.732	11.855	-0.877	-11.855	-11.855	0.000
240.21	6.57	7.11	13.269	11.855	-1.414	-12.373	-11.855	0.518
245.21	6.18	6.72	13.818	11.855	-1.963	-12.903	-11.855	1.048
250.21	5.77	6.31	14.377	11.855	-2.522	-13.444	-11.855	1.589
255.21	5.31	5.85	14.948	11.855	-3.093	-13.996	-11.855	2.141

Table 8. Variations of elastic, elasto-plastic, residual normal stresses and distances h_1 and h_2 with respect to the x axis for θ=45° at the upper and lower edges.

x (mm)	h_1 (mm)	h_2 (mm)	σ_{x_e} at upper s.	σ_{x_p} at upper s.	σ_{x_r} at upper s.	σ_{x_e} at lower s.	σ_{x_p} at lower s.	σ_{x_r} at lower s.
202.33	7.07	7.50	9.353	8.839	-0.514	-8.839	-8.839	0.000
207.33	6.64	7.07	9.814	8.839	-0.975	-9.287	-8.839	0.448
212.33	6.19	6.62	10.287	8.839	-1.448	-9.747	-8.839	0.908
217.33	5.70	6.13	10.770	8.839	-1.931	-10.218	-8.839	1.379
222.33	5.14	5.57	11.265	8.839	-2.426	-10.700	-8.839	1.861

Table 9. Variations of elastic, elasto-plastic, residual normal stresses and distances h_1 and h_2 with respect to the x axis for θ=60° at the upper and lower edges.

x (mm)	h_1 (mm)	h_2 (mm)	σ_{x_e} at upper s.	σ_{x_p} at upper s.	σ_{x_r} at upper s.	σ_{x_e} at lower s.	σ_{x_p} at lower s.	σ_{x_r} at lower s.
182.50	7.23	7.50	7.531	7.267	-0.264	-7.267	-7.267	0.000
187.50	6.77	7.04	7.946	7.267	-0.679	-7.674	-7.267	0.407
192.50	6.28	6.55	8.372	7.267	-1.105	-8.092	-7.267	0.825
197.50	5.73	6.00	8.808	7.267	-1.541	-8.522	-7.267	1.255
202.50	5.12	5.39	9.256	7.267	-1.989	-8.963	-7.267	1.696

Table 10. Variations of elastic, elasto-plastic, residual normal stresses and distances h_1 and h_2 with respect to the x axis for θ=90° at the upper and lower edges.

x (mm)	h_1 (mm)	h_2 (mm)	σ_{x_e} at upper s.	σ_{x_p} at upper s.	σ_{x_r} at upper s.	σ_{x_e} at lower s.	σ_{x_p} at lower s.	σ_{x_r} at lower s.
167.88	7.50	7.50	6.260	6.260	0.000	-6.260	-6.260	0.000
172.88	7.03	7.03	6.638	6.260	-0.378	-6.638	-6.260	0.378
177.88	6.51	6.51	7.030	6.260	-0.770	-7.030	-6.260	0.770
182.88	5.93	5.93	7.429	6.260	-1.169	-7.429	-6.260	1.169
187.88	5.27	5.27	7.840	6.260	-1.580	-7.840	-6.260	1.580

6. RESULTS AND CONCLUSIONS

An analytical elasto-plastic solution is given for a fiber reinforced thermoplastic composite cantilever. The analytical elasto-plastic stress analysis is carried out for the composite cantilever beam subjected to a linearly distributed load. The yield strength of the composite in the x direction is X_1=23.05, 11.855, 8.839, 7.267 and 6.26 MPa for θ =0°, 30°, 45°, 60° and 90° respectively. The intensity of the residual stress component of σ_x is maximum at the upper and lower surfaces. The expansion of the plastic zone is the same at the upper and lower sides for 0° and 90° orientation angles because of the symmetry of the material properties with respect to the x axis. The plastic region begins first at the upper surfaces for the inclined orientation angles such as 30°, 45° and 60°. When the value of the orientation angle decreases, the length of the beam increases. For the same lengths of the beam, when the value of the orientation angle increases, the intensity of the residual stress component of σ_x increases too.The intensity of the residual stress component of τ_{xy} is maximum around the x axis. The intensity of the residual stress component of τ_{xy} is smaller than that of σ_x.

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