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A COMPARISON OF CHAOTIC CIRCUITS FROM A STATISTICAL APPROACH

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Abstract. – In this paper, we have investigated the chaotic structure of two different nonlinear circuits by a statistical approach and compared the results of analyses for these systems. We have utilized time history technique to identify the dynamic system. According to the results of the analysis using the BDS test, it has been found that each time series data belonging to circuits indicate chaotic behavior in some cases.

1.INTRODUCTION

Nowadays, electrical circuits are very important tools both to research the nonlinear behavior and to identify the complex structure. Firstly, Linsay from Massachusetts Institute of Technology carried out the first rigorous study of the chaotic behavior of an electrical circuit in 1981[Linsay,1981]. In the same year, the nonlinear resistor circuit was studied with computer simulation by Ueda and Akamatsu. Beginning from that time, many observations of chaotic behavior in electrical circuits have been reported, for instance, Van der Pol oscillator, Chua`s circuits can be mentioned [Hasler,1987]. As a consequence of those studies, it has been accepted that analyzing a time series from an electrical circuit, both experimentally and numerically, gives some advantages over any other systems. One such advantage is that the experimenter has control over many of the parameters which influence the behavior [Newell, et all..,1996].

Since the nonlinear systems can potentially explain the variations that seem to be random, carrying out a research on them is the main reason. In this case, it is very common to utilize time series of the dynamical systems [Moon, 1987]. For this process; firstly, the fluctuations of the system should be obtained and later on time series analysis should be done. In this study, we have used the BDS [Brock, Dechert and Scheinkman, 1987] test to identify the electronic system. The test was firstly used to investigate for independence in stock market data by Scheinkman and LeBaron [1989], and Hsieh [1991]. Hsieh [1989] used the BDS to find out nonlinearity and chaos in monetary exchange rates. The use of BDS statistics in order to test other time series data in finance and economics for nonlinear structure is now a fairly well established practice. Lately, the test was used to determine the chaotic structure of RL– diode circuit [Kasap and Kurt,1998].

In this paper, we have investigated the nonlinearity, especially chaotic behavior of the RL-diode circuit and the nonlinear resistor circuit using a statistical approach- the BDS test. As a result of these analyses, we have also compared the results of these studies. Section 2 covers a brief explanation of the BDS test. Section 3 gives the experimental processes and observed time series. Section 4 summarizes the results of the chaotic analysis. Finally, the main findings and comparison of the results are presented in section 5.

2. THE BDS TEST STATISTICS

The BDS approach tests the null hypothesis that the variable of interest is independently and identically distributed (IID). This test is more powerful than the alternative test of deterministic chaos or stochastic non-linear models [Brock, *et al.*, 1991]. Now, let us briefly consider the test statistic-BDS itself. It is based on the so-called correlation integral introduced by Grasberger and Procaccia [1983].

The time series to be analyzed (X_t :1,2,...,T) is used to form the so-called N-histories $X_t^N = (X_t, X_{t+1}, \ldots, X_{t+N-1})$. Each N-history can be considered to be point in an N-dimensional space, where N is called the embedding dimension. These N-histories can be used to define a correlation integral

$$C_N(e) = \frac{2}{T_N(T_N-1)} \sum_{t<s} \sum I_e(X_t^N X_s^N),$$

where $T_N = T-N+1$, and I_e is the indicator function of the event $| X_{t+i} - X_{s+i} | < e$, i=0,1,...,N-1. The correlation integral, $C_N(e)$, can be interpreted as an estimate of the probability that X_t^N and X_s^N are within a distance e. Given this interpretation, we can see that under the independence hypothesis $C_N(e) \rightarrow C_1(e)^N$, as $T \rightarrow \infty$ holds. That is, $P(| x_{t+i} - x_{s+i} | < e)$, (i=0,1,...,N-1) is, due to independence, equal to $\prod_{i=1}^{N-1} P(| X_{t+i} - X_{s+i} | < e)$, which is estimated by $C_1(e)^N$ as the variables are identically distributed [Brock, *et al.*, 1991 and Chappell, *et al.*, 1996]. Thus, the BDS statistic reduces to

$$W_N(e) = [\sqrt{T}\ (C_N(e) - C_1(e)^N)] / \hat{\sigma}_N(e),$$

where $\hat{\sigma}_N^2(e)$ is an estimate of the standard devision under the null hypothesis. The distribution of $W_N(e)$ converges to a standard normal with expectation zero and variance unity, as T approaches infinity. Thus, one can now calculate the statistic that has a standard normal asymptotic distribution under the independence hypothesis. If the absolute values of the test statistic are large, the null hypothesis of IID (randomness) is to be rejected. The critical values reported by Brock, *et al.* [1991] for significance levels of 0.05 and 0.01 are 2.22 and 3.40 respectively.

3. EXPERIMENTAL PROCESSES

Experimental studies have been realized in two circuits. This section, firstly covers the RL-diode circuit experiment and later on the nonlinear resistor circuit experiment is given.

3.1. RL- DIODE CIRCUIT

This circuit has a dynamics that indicates nonautonomous variation to time and has only one nonlinear element- silicon diffused rectifier diode. The electrical circuit studied in this subsection is shown in Fig. 1. The behavior of the circuit is dominantly impressed by diode.

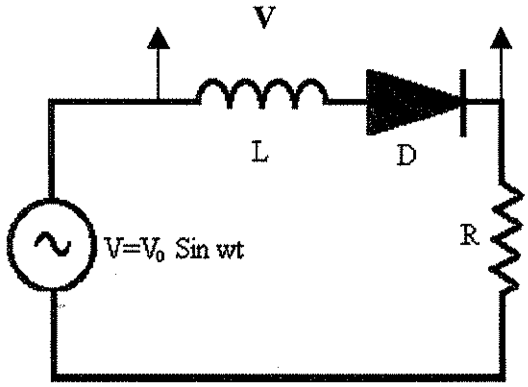


Figure 1. The diagram of circuit: R=220 Ω, L= 2 mH, D=1N4001

We can model the diode as a nonlinear capacitor in parallel with a nonlinear resistor as discussed by Matsumoto [Matsumoto, 1987]. The behavior of the circuit has been thoroughly studied in several paper during the last twenty years [see Matsumoto, *et al.*, 1984]. Therefore, we will not go into details here. It has been rigorously proven that if the circuit parameters and the external drive (input voltage) are chosen appropriately, then the system admits nonlinear oscillations.

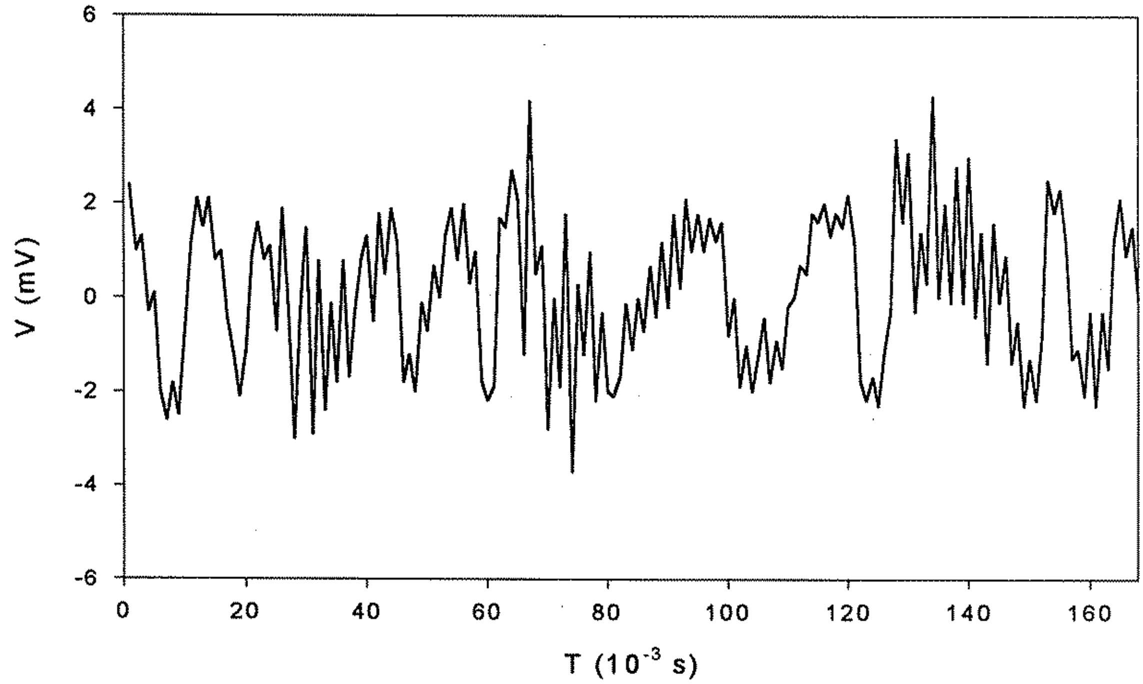
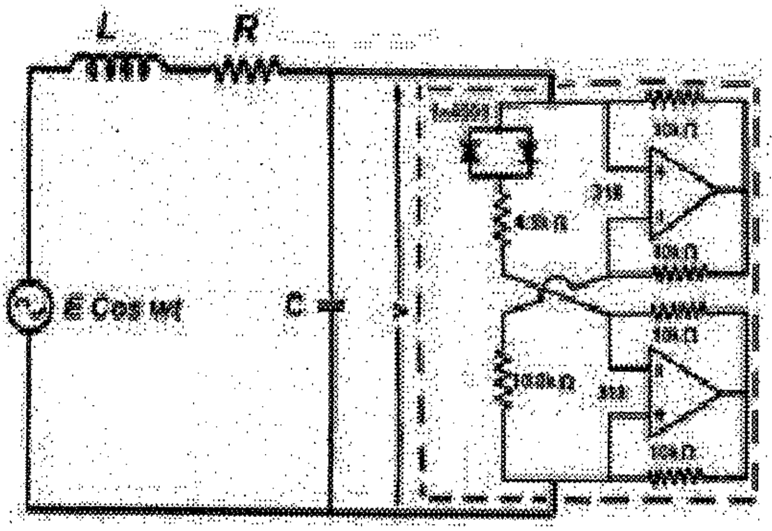


Figure 2. The nonlinear signal from RL-Diode circuit

In our experiment, we have used a 1*N*4001 diode, but any diode with a large capacitance can be used. The driven sinusoidal voltage has been obtained from a function generator; the experimental results have been obtained from an oscilloscope whose probes have been attached as in Fig. 1. The nature of the output signal depends on the value of the input voltage. We have observed period doubling for (input voltage) Vin = 160 mV and for f = 80 kHz. After a number of period doublings, a chaotic signal was observed for Vin = 120 mV and f = 6 kHz as in Fig. 2.

3.2. NONLINEAR RESISTOR CIRCUIT

This system was studied with computer simulation by Ueda and Akamatsu [Ueda and Akamatsu, 1981]. The nonlinear resistor circuit studied in this subsection is shown in Fig. 3a. A periodic excitation $V \cos \omega t$ has been injected into the circuit. We have a nonlinear resistor which has given the nonlinearity to the circuit. In that case, the nonlinear resistor has been identified as a



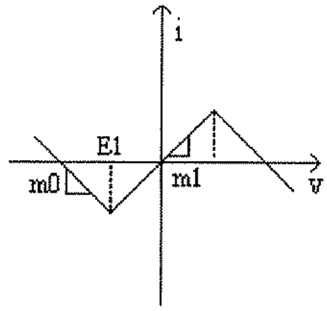


Figure 3: **a)** The diagram of the nonlinear resistor circuit: R=5.1 kΩ, L= 56 μH, C=10 μF
b) Nonlinear resistor v-i characteristic.

three-segment piecewise linear resistor [Fig. 3b]. The dynamics of this circuit was studied in several paper [see Ueda and Akamatsu, 1981]. The dynamics of the circuit is given as follows:

$$L\frac{di}{dt} + Ri + v = V\cos\omega t$$

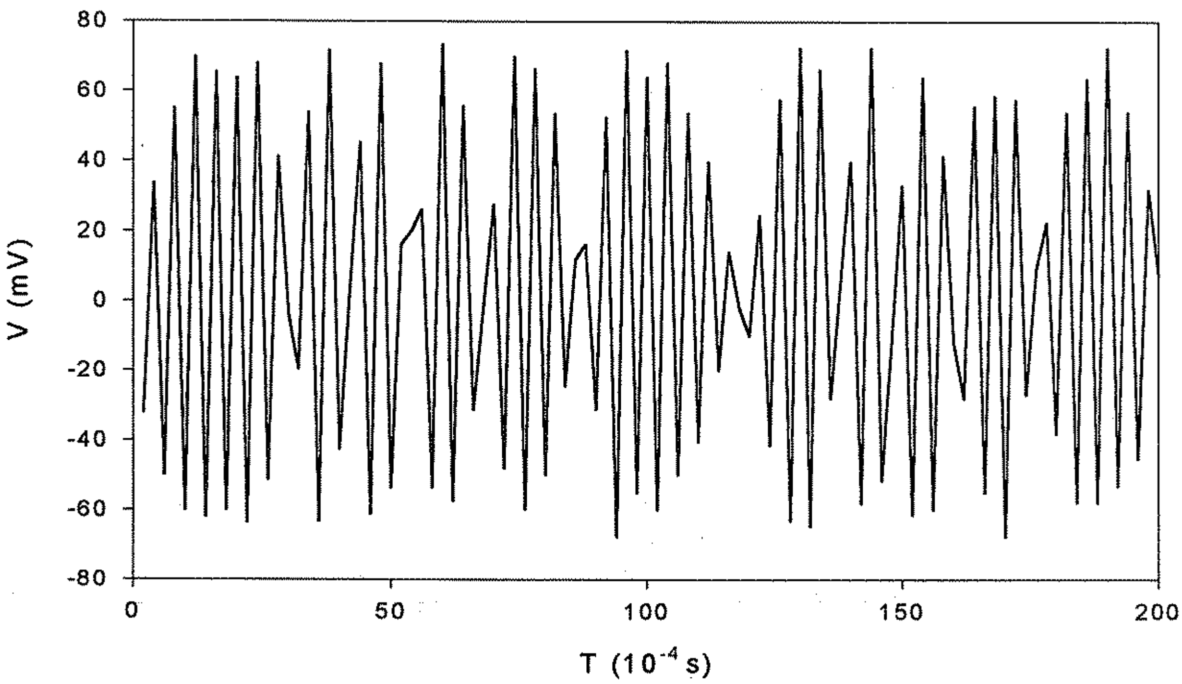


Figure 4. The nonlinear signal from nonlinear resistor circuit

The v- i characteristic of the nonlinear resistor can be described analytically by

$$f(v) = - m_0\, v + 0.5(m_0 + m_1)[\,|\, v + E_1 \,|\, - \,|\, v - E_1 \,|\,].$$

This type of resistor was also used in order to study torus breakdown, experimentally and analytically [Matsumoto, *et al.*., 1987]. In their work, they used this resistor in an autonomous way. However, we have indicated that this type of nonlinear resistor with a periodic excitation can give a nonlinear behavior. The resistor is shown with dashed line in Fig. 3a. The experimental procedure is the same as that in the previous one. The input voltage, Vin and frequency, f have been both changed till we have taken the nonlinear signal from output. In our experiment, this signal was observed for Vin = 200 mV and f = 0.23 kHz as in Fig. 4.

4. RESULTS OF THE CHAOTIC ANALYSES

The analysis process has been realized in two forms. Firstly, we have analyzed the raw data. Secondly, the first differenced data computed from the ($X_t - X_{t-1}$) has been put into practice as a stochastic approach. Because of the fact that testing the nonlinearity can be effected by autocorrelation, so it should be removed from data. This procedure was typically done by taking the first difference of the data [Hsieh, 1991]. Clearly, the BDS statistics lies within the positive tail of the standard normal distribution for these data. Hence we have rejected the null hypothesis that the data are IID. However, for a very high embedding dimension like 20, the null hypothesis cannot be rejected. In this work, the embedding dimension N has been mainly as varied from 2 to

Table 1. The results of the BDS test for the raw data taken from the RL- diode circuit.

e	*N*	*BDS/ SD*	*e*	*N*	*BDS/ SD*	*e*	*N*	*BDS/ SD*
0.1	2	6.8767	*0.5*	2	-1.877	*0.9*	2	-13.674
	3	19.415		3	0.64361		3	-18.456
	4	36.381		4	1.4302		4	-22.2
	5	52.048		5	1.942		5	-25.437
	6	76.683		6	2.3154		6	-28.357
	7	111.96		7	2.5812		7	-31.061
	8	234.55		8	2.9795		8	-33.604
	9	554.47		9	3.1066		9	-36.025
	10	1373.3		10	3.3267		10	-38.347
	20	-0.98189		20	2.8579		20	-58.87
	50	-0.089108		50	0.86307		50	-33.575

10 and e has been chosen to be 0.1, 0.5 and 0.9 for each series. In this study, our aim is not to investigate chaos for finding appropriate e values, so we have chosen these to compare process. In addition to that, we have also seen the influence of the high embedding dimension as 20 and 50.

Table 2. The results of the BDS test for the first differenced data taken from the RL- diode circuit.

e	*N*	*BDS/ SD*	*e*	*N*	*BDS/ SD*	*e*	*N*	*BDS/ SD*
0.1	2	22.375	*0.5*	2	10.07	*0.9*	2	26.71
	3	28.354		3	9.628		3	33.548
	4	48.363		4	9.538		4	44.931
	5	63.306		5	9.334		5	49.905
	6	85.462		6	9.030		6	54.972
	7	114.240		7	8.906		7	60.556
	8	146.171		8	8.774		8	65.792
	9	201.328		9	8.325		9	70.334
	10	275.680		10	8.051		10	76.659
	20	0.519		20	5.34		20	114.84
	50	-0.03303		50	2.689		50	-232.73

The results of the raw data and the first differenced data analyses for the RL- diode circuit are given in Table 1 and Table 2, respectively. As mentioned in Section 2 for the significant levels as 0.01, 0.05, 2.22 and 3.40 if $| BDS/ SD | > |2.22|$ or $| BDS/ SD | > |3.40|$ relations can be found in

Table 3. The results of the BDS test for the raw data taken from the nonlinear resistor circuit.

e	*N*	*BDS/ SD*	*e*	*N*	*BDS/ SD*	*e*	*N*	*BDS/ SD*
0.1	2	64.825	*0.5*	2	47.415	*0.9*	2	9.2514
	3	113.71		3	47.211		3	7.2188
	4	202.60		4	49.500		4	5.3877
	5	381.04		5	53.924		5	3.8282
	6	756.39		6	61.072		6	2.9027
	7	1567.3		7	69.377		7	2.5327
	8	3190.3		8	80.132		8	2.7669
	9	6166.4		9	93.907		9	2.8948
	10	12167		10	112.66		10	3.0161
	20	-0.26480		20	739.34		20	2.8825
	50	-0.0080419		50	-0.26068		50	2.0181

analysis, one can say that the data has a chaotic behavior. In the light of this information, chaotic structure has been found for *e* to be 0.1 and 0.9 for the raw data. Secondly, the analysis of the first differenced data for the RL-diode circuit given by Fig. 5 has indicated the chaotic structure for each *e* value.

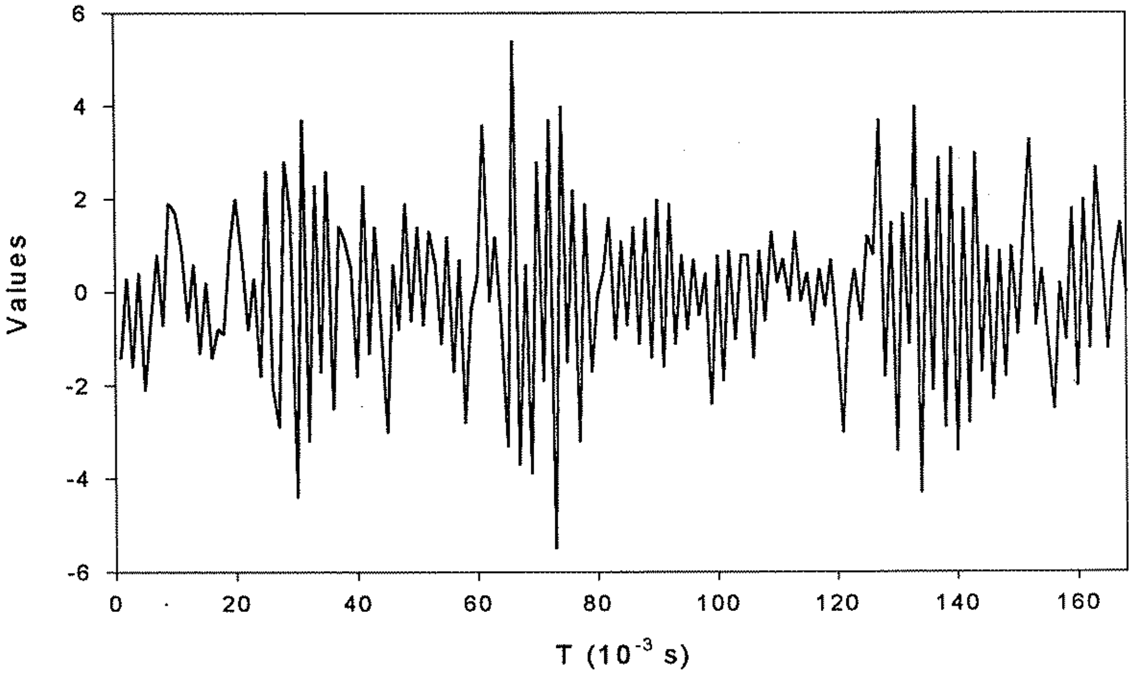


Figure 5. The first difference values of the nonlinear signal belonging to RL-diode circuit

Nonlinear resistor circuit time series data for the testing process are given by Fig. 4 and Fig. 6, respectively. The analysis results for these time series are given in Table 3 and Table 4.

Table 4. The results of the BDS test for the first differenced data taken from the nonlinear resistor circuit.

E	*N*	*BDS/ SD*	*e*	*N*	*BDS/ SD*	*e*	*N*	*BDS/ SD*
0.1	2	61.793	*0.5*	2	50.258	*0.9*	2	15.209
	3	117.84		3	51.279		3	12.686
	4	222.03		4	54.139		4	10.582
	5	468.16		5	60.333		5	8.8375
	6	1049.3		6	68.774		6	7.9218
	7	2495.1		7	79.742		7	7.7747
	8	6644.0		8	93.593		8	7.6456
	9	15382		9	113.13		9	7.6468
	10	40207		10	137.67		10	7.6668
	20	-0.23087		20	1325.4		20	6.8767
	50	-0.0061737		50	$9.8707.10^{6}$		50	5.7031

According to these results, the analysis for the raw data of this circuit indicates a chaotic behavior in this sense that the previous condition can be provided by *e* to be 0.1 and 0.5. Finally,

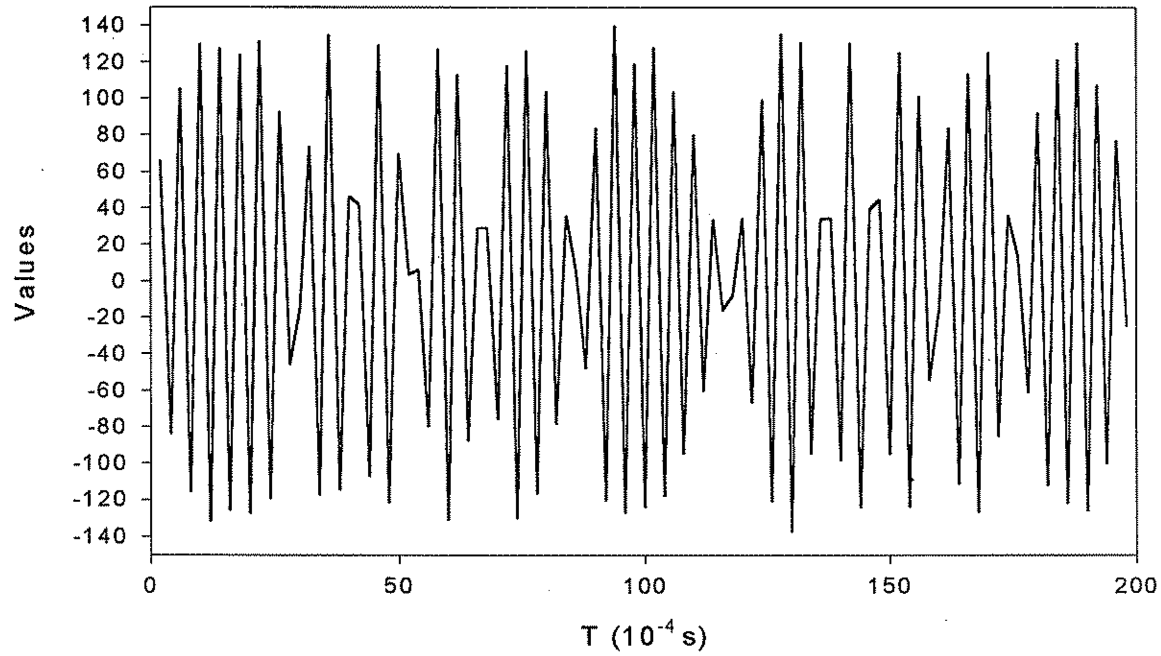


Figure 6. The first difference values of the nonlinear signal belonging to nonlinear resistor circuit

the result for the first differenced data of nonlinear resistor circuit gave the chaotic structure for each *e* values.

5. CONCLUSION

We have investigated time series data taken from the nonlinear circuits named RL- diode and nonlinear resistor. For this process, the BDS test statistics has been used both to analyze the data and compare the dynamics of systems. Firstly, the results have indicated that analyzing the raw data of each circuit doesn't give the chaotic structure of the systems in this sense that e= 0.5 for the RL-diode circuit and e= 0.9 for the nonlinear resistor circuit does not provide the necessity conditions. However, both two circuits have indicated the chaos for their first differenced data. This also provides the idea mentioned above that the nonlinearity can be affected by autocorrelation. Especially, the results for e= 0.5 for the analyses of circuits are significant since this value of e gives an averaging distance for estimating of probability in the correlation integral.

According to the results, the BDS/ SD values are upper than the limits for each circuit. This is especially clear for the nonlinear resistor circuit. Thus, it can be argued that the nonlinear resistor circuit indicates more chaotic character than the RL- diode circuit. The behavior of the higher embedding dimensions N= 20 and N= 50 are also remarkable for the first differenced time series data of circuits despite of providing the necessities of chaotic behavior.

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