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FRICTIONLESS CONTACT BETWEEN A RIGID STAMP AND AN ELASTIC LAYERED COMPOSITE RESTING ON SIMPLE SUPPORTS

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Abstract - The frictionless contact between a rounded or flat-ended rigid stamp and an elastic layered composite is considered according to theory of Elasticity. The elastic layered composite consisted of two materials with different elastic constants and heights is rested on simple supports. It is assumed that the layered composite is subjected to a concentrated load with a magnitude of 2P by means of a rigid stamp on its top surface and the effect of gravity is neglected. Stresses and displacements are expressed depending on the contact pressure using Fourier transforms technique and the problem is formulated in terms of a singular integral equation for the contact pressure. The singular integral equation is solved numerically by using appropriate Gauss- Chebyshev integration. Numerical results obtained for various dimensionless quantities for the contact pressure are presented in graphical form.

1. INTRODUCTION

The contact problems have different applications in structural engineering. For examples; contact between machine fractions, engineering problems which appear due to applications of concentrated load to beams and layers resting on an elastic half space by means of elastic or rigid stamps. In contact problems, the layer usually rests on a continuous foundation which may be either elastic $[1-8]$ or rigid $[9-12]$. There are also studies in which the layer rests on rigid supports $[13-14]$. In some works the layer is supported by two elastic quarter planes [15]. In most of the previous works the effect of gravity is neglected and the layer is subjected to a uniform or a concentrated load either directly or by means of rigid stamps. The effect of gravity is taken into account in some previous studies $[6-12]$.

In this study, the frictionless contact between a rounded or flat-ended rigid stamp and an elastic layered composite is considered according to theory of Elasticity. The elastic layered composite consisted of two materials with different elastic constants and heights is rested on simple supports (see Figure 1). The layered composite is subjected to a concentrated load with a magnitude of 2P by means of a rigid stamp on its top surface and the body forces are neglected. In the problem, assuming that all surfaces are frictionless, contact pressure distribution under the stamp is investigated. In the case of rigid rounded stamp, contact area is also unknown as well as the contact pressure distribution. The contact area and the contact pressure distribution is depended on the elastic characteristics of the layers, the applied load, and support widths.

2. FORMULATION OF THE PROBLEM

The layered composite consisted of two elastic layers with different heights and elastic constants resting on simple supports considered is shown in Fig. 1. The composite is subjected to a concentrated load on its top surface by means of a rounded or flat-ended rigid stamp.

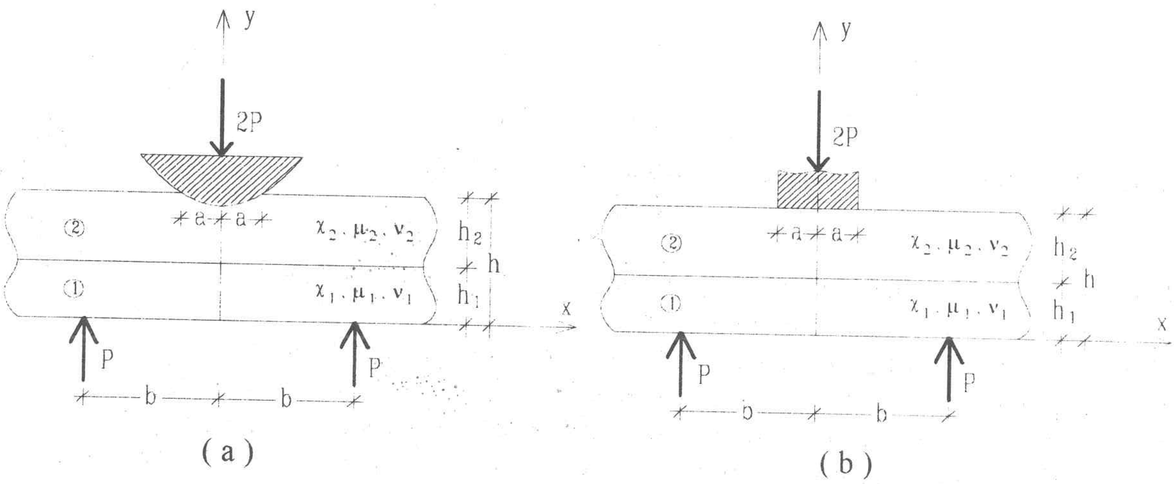

Figure 1. Geometry of the problem and loading condition : (a) the layered composite with a rigid rounded stamp, (b) the layered composite with a flat-ended rigid stamp

In the absence of body forces , the two dimensional Navier equations may be written in the following form

$$\mu_i \nabla^2 u_i + (\lambda_i + \mu_i) \frac{\partial}{\partial x}\left(\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y}\right) = 0 , \tag{1}$$

$$\mu_i \nabla^2 v_i + (\lambda_i + \mu_i) \frac{\partial}{\partial y}\left(\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y}\right) = 0 , \qquad (i=1,2) \tag{2}$$

where λ_i and μ_i (i =1,2) represent the Láme constant and shear modules of the layers ,respectively. u_i and v_i are the x- and y- components of the displacement vectors.

Observing that x=0 is a plane symmetry, it is sufficient to consider the problem in the region $0 \le x < \infty$ only. Using the symmetry consideration, the following expressions may be written.

$$u_i(x,y) = -u_i(-x,y) , \tag{3}$$

$$v_i(x,y) = v_i(-x,y) , \tag{4}$$

If it is assumed that u_i and v_i are written as;

$$u_i(x,y) = \frac{2}{\pi}\int_0^\infty \Phi_i(\alpha,y)\sin(\alpha x)d\alpha , \tag{5}$$

$$v_i(x,y) = \frac{2}{\pi}\int_0^{\infty} \Psi_i(\alpha, y)\cos(\alpha x)d\alpha\,, \tag{6}$$

where the unknown functions $\Phi_i(\alpha, y)$ and $\Psi_i(\alpha, y)$ are inverse Fourier transforms of $u_i(x,y)$ and $v_i(x,y)$, respectively. If necessary derivatives of equations (5) and (6) are substituted into equations (1) and (2) , then resulting ordinary differential equations are solved , one may be obtained expressions of displacements u_i and v_i in the form,

$$u_i(x,y) = \frac{2}{\pi}\int_0^{\infty} \left[(A_i + B_i y)e^{-\alpha y} + (C_i + D_i y)e^{\alpha y}\right]\sin(\alpha x)d\alpha\,, \tag{7}$$

$$v_i(x,y) = \frac{2}{\pi}\int_0^{\infty} \left\{\left[A_i + \left(\frac{\chi_i}{\alpha} + y\right)B_i\right]e^{-\alpha y} + \left[-C_i + \left(\frac{\chi_i}{\alpha} - y\right)D_i\right]e^{\alpha y}\right\}\cos(\alpha x)d\alpha\,; \tag{8}$$

After obtaining u_i and v_i, stresses may be evaluated using Hook's Law. The components of the stresses at the interface and boundaries may be expressed as

$$\frac{1}{2\mu_i}\sigma_{ix}(x,y) = \frac{2}{\pi}\int_0^{\infty}\left\{\left[\alpha(A_i + B_i y) - \frac{3-\chi_i}{2}B_i\right]e^{-\alpha y} + \left[\alpha(C_i + D_i y) + \frac{3-\chi_i}{2}D_i\right]e^{\alpha y}\right\}\cos(\alpha x)d\alpha\,, \tag{9}$$

$$\frac{1}{2\mu_i}\sigma_{iy}(x,y) = \frac{2}{\pi}\int_0^{\infty}\left\{-\left[\alpha(A_i + B_i y) + \frac{1+\chi_i}{2}B_i\right]e^{-\alpha y} + \left[\alpha(C_i + D_i y) - \frac{1+\chi_i}{2}D_i\right]e^{\alpha y}\right\}\cos(\alpha x)d\alpha\,, \tag{10}$$

$$\frac{1}{2\mu_i}\tau_{ixy}(x,y) = \frac{2}{\pi}\int_0^{\infty}\left\{-\left[\alpha(A_i + B_i y) + \frac{\chi_i - 1}{2}B_i\right]e^{-\alpha y} + \left[\alpha(C_i + D_i y) - \frac{\chi_i - 1}{2}D_i\right]e^{\alpha y}\right\}\sin(\alpha x)d\alpha\,, \tag{11}$$

where A_i, B_i, C_i, and D_i (i =1,2) are unknown functions which will be determined from boundary conditions of the problem. χ_i is a constant being $\chi_i = (3-\nu_i)/(1+\nu_i)$ in plane stress and $\chi_i = 3-4\nu_i$ in plane strain. ν_i is the Poisson ratio.

3. BOUNDARY CONDITION AND INTEGRAL EQUATION

The boundary conditions of the problem can be written as ;

$${}_{2xy}(x,h) = 0, \qquad (0 \le x < \infty), \tag{12}$$

$${}_{2xy}(x,h_1) = 0, \qquad (0 \le x < \infty), \tag{13}$$

$${}_{1xy}(x,h_1) = 0, \qquad (0 \le x < \infty), \tag{14}$$

$$\sigma_{2y}(x, h_1) = \sigma_{1y}(x, h_1), \qquad (0 \le x < \infty), \tag{15}$$

$$\tau_{1xy}(x,0) = 0, \qquad (0 \le x < \infty), \tag{16}$$

$$\sigma_{1y}(x,0) = -P\delta(x-b), \tag{17}$$

$$\frac{\partial}{\partial x}\left[v_2(x,h_1) - v_1(x,h_1)\right] = 0, \qquad (0 \le x < \infty), \tag{18}$$

$$\sigma_{2y}(x,h) = \begin{cases} -p(x), 0 \le x < a \\ 0, a < x < \infty \end{cases}, \tag{19}$$

$$v_2(x,h) = F(x), \qquad (0 \le x < a), \tag{20}$$

where a is the half-width of the contact area and b is the support width. p(x) represents the unknown contact pressure, F(x) is a function which defines the profile of the stamp and (x) is Dirac delta function. h_1 and h_2 are the heights of the layers. h is the total height of the layered composite.

One may note the boundary condition (19) and (20) are of mixed type. In order to have the same type condition, we may replace condition (20) by

$$\frac{\partial}{\partial x}v_2(x,h) = \frac{d}{dx}F(x) = f(x). \qquad (0 \le x < a), \tag{21}$$

Substituting equations (7-11) into boundary conditions (12-19), one may obtain A_i, B_i, C_i, and D_i (i =1,2) in terms of the unknown function p(x). Thus, the stresses and the displacements can be expressed depending on contact pressure p(x).

If A_2, B_2, C_2 and D_2 which belong to the layer 2 substituted into equation (8) and if the resulting expressions substituted into condition (21) , after some routine manipulations, one may obtain the following singular integral equation :

$$\int_0^a \left[\frac{1}{t+x} - \frac{1}{t-x} + N(x,t)\right] p(t)\,dt = -M(x), \qquad (0 \le x < a), \tag{22}$$

for p(x), where

$$N(x,t) = \frac{1}{h}\int_0^\infty \left\{ \frac{1}{\Delta^*}\left\{\left(e^{-4\omega r} + e^{-4\omega} - 2e^{-2\omega}e^{-2\omega r}\right)\left[1 + e^{2\omega r}\left(-2 - 4\omega^2 r^2 + e^{2\omega r}\right)\right]\right.\right.$$
$$\left.\left. + \beta\left[e^{-4\omega} - e^{-4\omega r} - 2e^{-2\omega}e^{-2\omega r}\left(2\omega - 2\omega r\right)\right]\left[1 - e^{2\omega r}\left(4\omega r + e^{2\omega r}\right)\right] \right\} - 1 \right\}$$
$$\cdot\left[\sin(t+x)\frac{\omega}{h} - \sin(t-x)\frac{\omega}{h}\right] d\omega, \tag{23}$$

$$M(x) = -4\frac{P}{h}\int_0^\infty \frac{1}{\Delta^*}\beta\, e^{-\omega}\left\{\left(1 - e^{2\omega r}\right)\left[(1+\omega)e^{-2\omega r} + (-1+\omega)e^{-2\omega}\right] + \omega r\left[\left(1 + e^{2\omega r}\right)\left(e^{-2\omega r} + e^{-2\omega}\right)\right.\right.$$
$$\left.\left.\cdot\left(-\omega + \omega r\right) + 2\left(e^{-2\omega}e^{2\omega r} - e^{-2\omega r}\right)\ \right]\right\}\left[\sin(b-x)\frac{\omega}{h} - \sin(b+x)\frac{\omega}{h}\right] d\omega - \frac{4\mu_2\pi}{1+\chi_2} f(x), \tag{24}$$

$$\Delta^* = \left[-1+e^{2\omega r}\left(2+4\omega^2 r^2 - e^{2\omega r}\right)\right]\left[-e^{-4\omega r}+e^{-4\omega}-2e^{-2\omega}e^{-2\omega r}\left(2\omega-2\omega r\right)\right]$$

$$+\beta\left[-1+e^{2\omega r}\left(4\omega r+e^{2\omega r}\right)\right]\left[e^{-4\omega r}+e^{-4\omega}-e^{-2\omega}e^{-2\omega r}\left(2+4\omega^2+4\omega^2 r^2-8\omega^2 r\right)\right],$$

$$\beta = \frac{1+\chi_1}{1+\chi_2}\frac{\mu_2}{\mu_1}, \qquad \omega = \alpha h, \qquad r = h_1 / h.$$

Using the symmetry conditions, if the definition of p(x) is extended into$(-a,0)$ in such a way that

$$p(x) = p(-x), \tag{25}$$

the integral equation (22) may be expressed as

$$\int_{-a}^{a}\left[\frac{1}{t-x}+k(x,t)\right]p(t)\,dt = M(x), \qquad (-a<x<a), \tag{26}$$

where

$$k(x,t) = \frac{1}{h}\int_0^\infty \left\{\frac{1}{\Delta^*}\left\{\left(e^{-4\omega r}+e^{-4\omega}-2e^{-2\omega}e^{-2\omega r}\right)\left[1+e^{2\omega r}\left(-2-4\omega^2 r^2+e^{2\omega r}\right)\right]\right.\right.$$

$$\left.\left.+\beta\left[e^{-4\omega}-e^{-4\omega r}-2e^{-2\omega}e^{-2\omega r}\left(2\omega-2\omega r\right)\right]\left[1-e^{2\omega r}\left(4\omega r+e^{2\omega r}\right)\right]\right\}-1\right\}\sin(t-x)\frac{\omega}{h}d\omega \tag{27}$$

is bounded in the closed interval $-a \le x \le a$.

The equilibrium condition for the layered composite may be written as

$$\int_{-a}^{a} p(t)\,dt = 2P. \tag{28}$$

In order to simplify the numerical analysis, we introduce the following dimensionless quantities :

$$x = a\,\xi, \qquad t = a\,\eta, \qquad m(\xi) = \frac{M(a\,\xi)}{P/h}, \qquad g(\eta) = \frac{p(a\,\eta)}{P/h}. \tag{29}$$

Then, equations (26) and (28) may be rewritten in the form of

$$\int_{-1}^{1}\left[\frac{1}{\eta-\xi}+a\,k(\xi,\eta)\right]g(\eta)\,d\eta = m(\xi), \qquad (-1<\xi<1), \tag{30}$$

$$\frac{a}{h}\int_{-1}^{1} g(\eta)\,d\eta = 2. \tag{31}$$

4. NUMERICAL SOLUTIONS OF THE INTEGRAL EQUATION

4.1. The rigid rounded stamp case :

The contact pressure $p(x)$ (or $g(\eta)$) and the half-width of the contact area a are unknown. Since end points of the contact area are smooth (smooth contact), $g(\pm 1) = 0$ and the index of the singular integral equation (30) is − [16]. Therefore, the consistency condition of the integral equation must also be satisfied [16].

Writing the solution of equation (30) in the form [7, 13, 16]

$$g(\eta) = G(\eta)\,(1-\eta^2)^{1/2}, \qquad (-1<\eta<1), \tag{32}$$

where $G(\eta)$ is bounded in $(-1 \le \eta \le 1)$ and using the appropriate Gauss-Chebyshev integration [17,18], equations (30) and (31) may be reduced to

$$\sum_{i=1}^{n}\left(1-\eta_i^2\right)\left[\frac{1}{\eta_i-\xi_j}+a\,k\left(\xi_j,\eta_i\right)\right]G\left(\eta_i\right)=\frac{n+1}{\pi}m\left(\xi_j\right), \qquad (j=1,\ldots,n+1),$$

$$\frac{a}{h}\sum_{i=1}^{n}\left(1-\eta_i^2\right)G\left(\eta_i\right)=\frac{2(n+1)}{\pi} \tag{33}$$

where

$$\eta_i=\cos\left(\frac{i\,\pi}{n+1}\right), \qquad (i=1,\ldots,n),$$

$$\xi_j=\cos\left(\frac{\pi}{2}\,\frac{2j-1}{n+1}\right), \qquad (j=1,\ldots,n+1). \tag{34}$$

In [17 − 18] it has been shown that the extra equation in (33) corresponds to the consistency condition of the original integral equation (30). It may also be shown that the $(n/2+1)^{th}$ equation in (33) is automatically satisfied. Thus the equations given by (33) constitute a system of n+1 equations for the n+1 unknowns $G(\eta_i)$, $(i=1,\ldots,n)$, and a. Note that the system is highly nonlinear in a and an interpolation scheme is required to determine those unknowns.

4.2. Flat-ended rigid stamp case :

Since vertical displacements under the rigid stamp are constant, in Equation (26) $f(x)=0$. Hence, equation (30) is also valid for $f(x)=0$ (or $f(a\xi)=0$). Flat-ended rigid stamp have 90^0 sharp corners. Therefore, the contact stress σ_y, and consequently the contact pressure p(x), will be singular at the corners and referring to references [4, 5, 12, 16] the solution of the integral equation will be in the form of

$$g(\eta) = G(\eta)\,(1-\eta^2)^{-1/2}, \qquad (-1<\eta<1), \tag{35}$$

where $G(\eta)$ is bounded in $(-1 \le \eta \le 1)$. Then, using the appropriate Gauss-Chebyshev integration [17,18], equations (30) and (31) are replaced by

$$\sum_{i=1}^{n} W_i\left[\frac{1}{\eta_i-\xi_j}+a\,k\left(\xi_j,\eta_i\right)\right]G\left(\eta_i\right)=\frac{1}{\pi}m\left(\xi_j\right), \qquad (j=1,\ldots,n-1),$$

$$\frac{a}{h}\sum_{i=1}^{n} W_i\, G(\eta_i) = \frac{2}{\pi} \quad , \tag{36}$$

where

$$W_1 = W_n = \frac{1}{2(\ n-1)}\ , \qquad W_i = \frac{1}{n-1}\ , \qquad (\ i = 2, \ldots, n-1\ ),$$

$$\eta_i = \cos\left(\frac{i-1}{n-1}\pi\right), \qquad (\ i = 1, \ldots, n\ ),$$

$$\xi_j = \cos\left(\frac{\pi}{2}\frac{2j-1}{n-1}\right), \qquad (\ j = 1, \ldots, n-1\ ). \tag{37}$$

The system in equations (36) contain n linear algebraic equations for n unknowns, $G(\eta_i)$, $(i = 1, \ldots, n)$. After $G(\eta_i)$ is obtained, using equation (35), the contact pressure under the flat-ended rigid stamp, $p(x)$, is also obtained.

5. RESULTS

It may be realized from equations (22-30) that it is necessary to specify a/h, h_1/h, β, R/h and $\dfrac{1+\chi_2}{\mu_2}\dfrac{P}{h}$ for numerical analysis. Dimensionless contact pressure distribution $p(x)/(P/h)$ under the stamp and the contact area a/h, for only rounded stamp, are investigated for different numerical values of the material constant (β), the load factor ($\dfrac{1+\chi_2}{\mu_2}\dfrac{P}{h}$), the radius of the stamp (R/h), the thickness of the layer I (h_1/h) and the support width (b/h).

5.1. Rigid Rounded Stamp Case :

Since the formulation of the problem is rather general, any stamp profile may be treated. In this case, a circular stamp with a radius of R is considered. Hence,

$$F(x) = (x^2 - R^2)^{1/2} + R \tag{38}$$

and

$$f(x) = x(R^2 - x^2)^{-1/2} \ . \tag{39}$$

Using equations (33) and (34), the contact pressure and the contact area are investigated for this stamp profile.

Figure 2 shows the variation of the size of the contact area (a/h) with the support width (b/h) for $\beta = 0.10$, $h_1/h = 0.20$, $\dfrac{1+\chi_2}{\mu_2}\dfrac{P}{h} = 0.008$ and different R/h. As expected, the contact area increases with increasing both support width and the radius of the stamp.

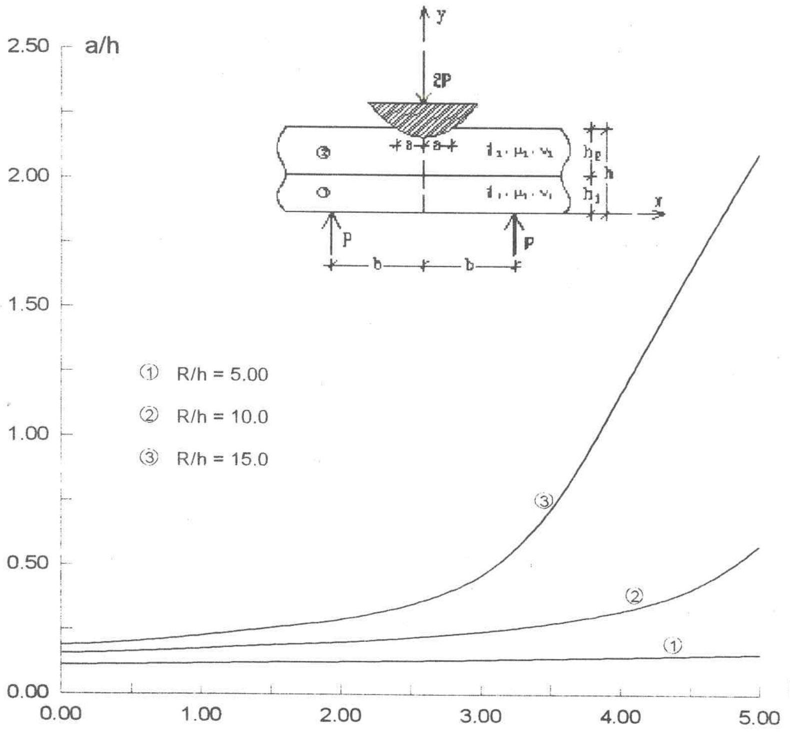

Figure 2. Variation of size of the contact area with support width for a rigid rounded stamp ($\beta = 0.10$, $h_1 / h = 0.20$ and $\frac{1+\chi_2}{\mu_2} P / h = 0.008$)

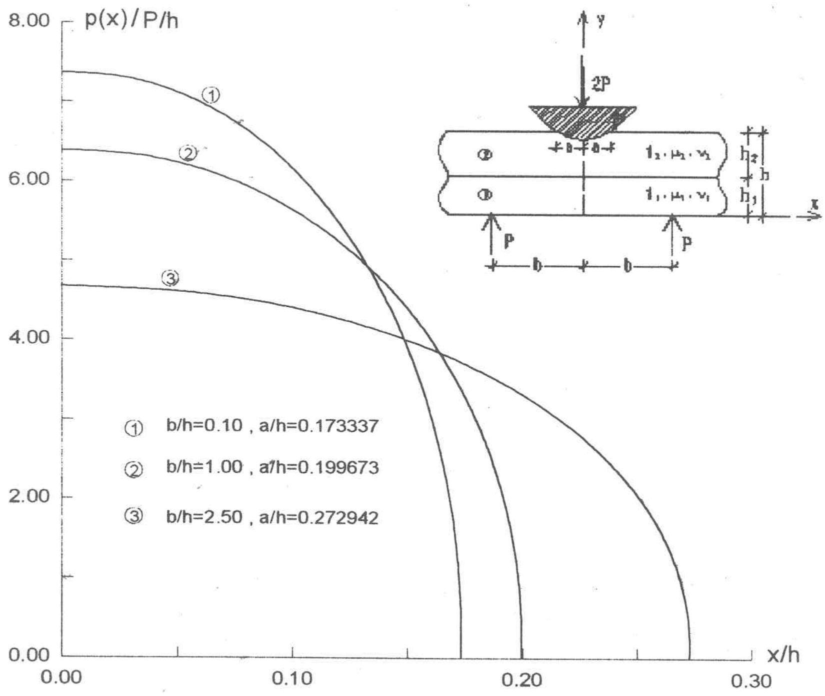

Figure 3. Contact pressure distribution for a rigid rounded stamp with circular profile ($\beta = 0.10$, $h_1 / h = 0.20$, $R / h = 10.0$ and $\frac{1+\chi_2}{\mu_2} P / h = 0.01$)

Figure 3 and 4 show $p(x)/(P/h)$, the dimensionless contact pressure distribution for $R/h = 10$ and $\frac{1+\chi_2}{\mu_2}\frac{P}{h} = 0.01$. The contact pressure is maximum at $x = 0$. In Figure 3, $\beta = 0.10$ and $h_1/h = 0.20$ are fixed. Note that as support width increases, the size of contact area increases. Consequently, the contact pressure $p(x)/(P/h)$ decreases. If b/h is chosen larger, the contact pressure is evenly distributed. Contrary, if b/h is chosen smaller, it is accumulated around $x = 0$ with a high peak.

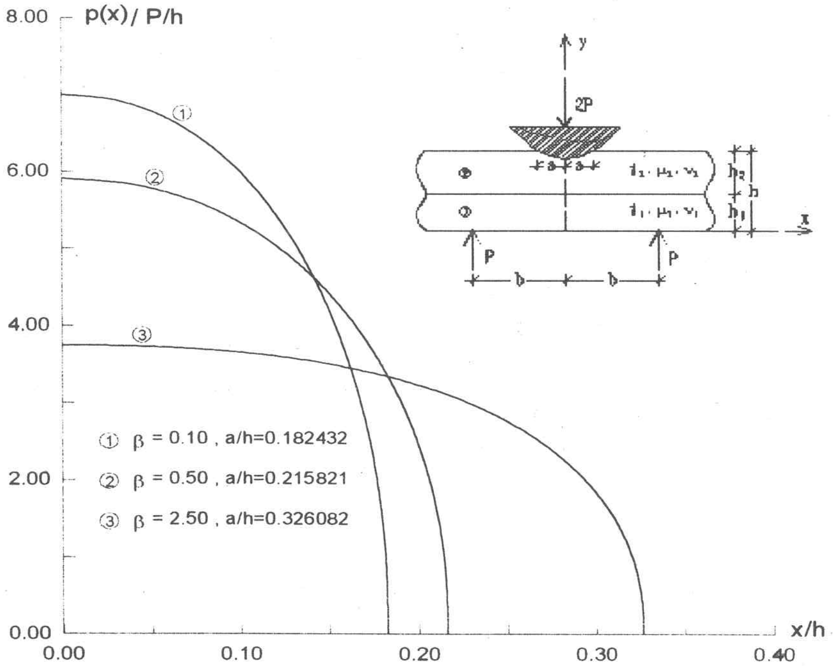

Figure 4. Contact pressure distribution for a rigid rounded stamp with circular profile ($b/h = 1.0$, $h_1/h = 0.50$, $R/h = 10.0$ and $\frac{1+\chi_2}{\mu_2}P/h = 0.01$)

In Figure 4, $b/h = 1$, $h_1/h = 0.50$ are fixed and $p(x)/(P/h)$ is given for β = 0.10, 0.50 and 2.50. The contact pressure decreases and, because of increasing a/h, exhibits smoothness with increasing the ratio of the material constants (β). In case, increasing of β means that the lower layer is softer than the upper layer.

Figure 5 shows the dimensionless stress $\sigma_x(0,y)/(P/h)$ along the symmetry plane $x = 0$ for $R/h = 10$, $\beta = 0.10$, $h_1/h = 0.20$ and $\frac{1+\chi_2}{\mu_2}\frac{P}{h} = 0.01$. If the layered composite is considered as a beam, σ_x represents the axial bending stress. For $b/h = 0.1$ the axial stress is compressive in outer portions of the upper layer and it is tensile in the interior portion as a consequence of the applied compressive traction by means of the stamp at $y = h$ and the support reactions at $y = 0$. On the other hand, the axial stress is tensile in the upper portion and compressive in the lower portion of the lower layer. Note that the resultant axial force must be zero. For larger support widths, b/h, compressive stresses in the upper portion and tensile stresses in the lower portion of both layer appear. Also as b/h increases, σ_x increases.

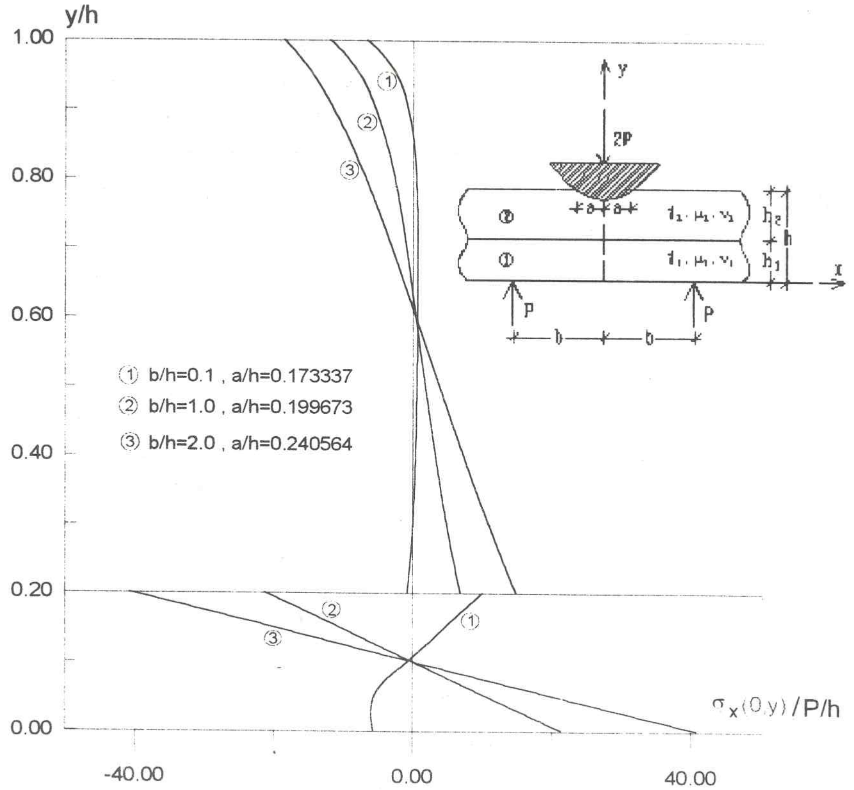

Figure 5. Axial stresses $\sigma_x(0,y)/(P/h)$ for a rigid rounded stamp with circular profile

$$(\beta = 0.10 \ , \ h_1/h = 0.20 \ , \ R/h = 10.0 \ \text{and} \ \frac{1+\chi_2}{\mu_2}P/h = 0.01)$$

As a result, the size of the contact area increases depending on increasing of the support width and the radius of the stamp. Consequently, the contact pressure decreases. The contact pressure stresses are reached their maximum values at symmetry plane $x = 0$. These results agree with the results given in [13].

5.2. Flat-ended Rigid Stamp Case :

Using equations (36) and (37) the contact pressure under the flat-ended rigid stamp is investigated. In this case , $b/h = 1$ and $h_1/h = 0.5$ are fixed and the contact pressure distribution , $p(x)/(P/h)$, is treated in Figures 6 and 7.

Figure 6 shows that the contact pressure decreases with increasing a/h for $b/h = 0.0$. If a/h is larger a separation occurs between the stamp and the layered composite. As expected, the contact pressure stress goes to infinity at the corners of the stamp because of singularities at the end points and gets smaller as it closes to the symmetry plane $x = 0$. It should also be mentioned that the pressure ratio $p(x)/(P/h)$ was found to be practically independent of the magnitude of P. Figure 7 shows that, for $a/h = 0.25$, a separation occurs between the stamp and the composite layer while the lower layer is softer than the upper layer. If the lower layer is more rigid than the upper layer, $\mu_1 > \mu_2$ or a/h is much smaller , the separation is not occurring for $b/h = 1$. These results agree with references [12] and [14].

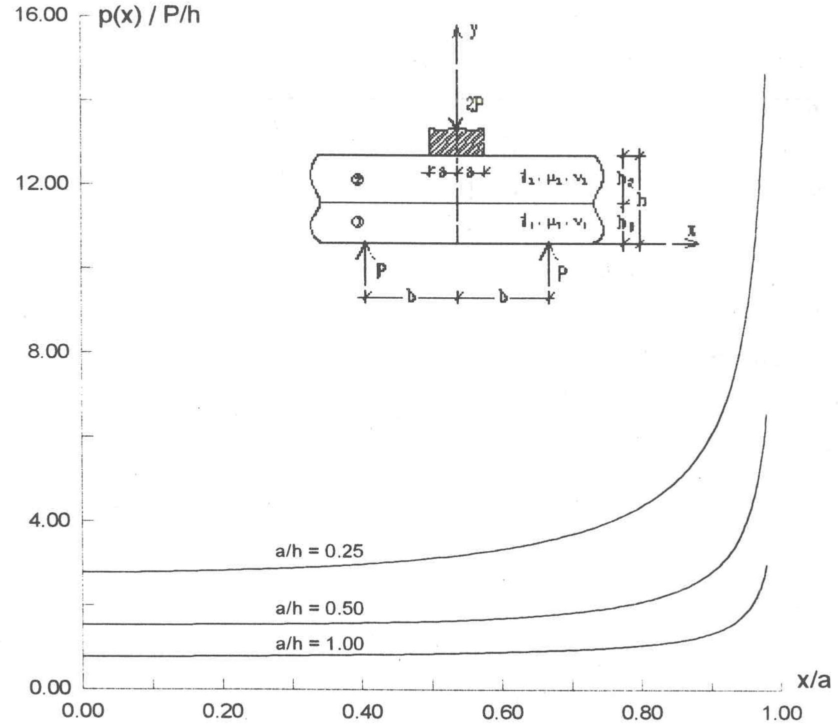

Figure 6. Contact pressure distribution for a flat-ended rigid stamp

($\beta = 0.01$, $h_1 / h = 0.50$ and $b / h = 1.0$)

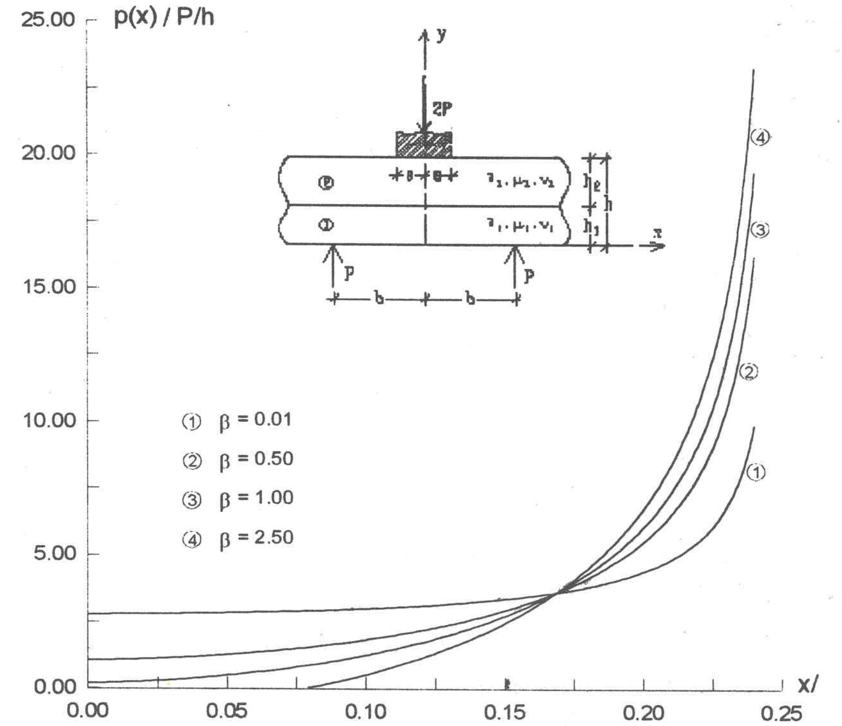

Figure 7. Contact pressure distribution for a flat-ended rigid stamp

($a / h = 0.25$, $h_1 / h = 0.50$ and $b / h = 1.0$)

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