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ASSESSMENT AND COMPARISON OF DISCRETE OPTIMAL MULTIRATE AND CONVENTIONAL ANALOGUE CONTROLLER DESIGN APPLIED TO A HYDROGENERATOR SYSTEM

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ABSTRACT

In the present work an algebraic and an optimal control method (in connection with linear continuous systems) as well as a new optimal multirate control method (in connection with linear discrete systems) are presented and applied, in order to design a suitable excitation controller (analogue and digital type respectively) and thus obtain a pertinent closed-loop system with enhanced dynamic stability characteristics. The obtained computer simulation results, based on a practical power system of the Greek national grid, show clearly the validity-suitability-effectiveness and implementability of the excitation controllers designed using these control procedures.

1. INTRODUCTION

It is a well known fact that the stability enhancement of an open-loop power system model linearized about the nominal operating point may be achieved by designing a suitable excitation controller, which results in the associated closed-loop system with pre-assigned dynamic stability characteristics. The design of such controllers is accomplished by using the various conventional control methods for linear continuous systems (e.g. optimal control methods [1-4], algebraic control methods [5-8]) and the new optimal multirate control method for linear discrete systems [9-11].

In the present work the successful design of the sought analogue and digital excitation controllers of the under study hydrogenerator system [12] (in order to give it enhanced dynamic stability characteristics over a wide range of operating conditions of the hydrogenerator unit) are attained, respectively, by: a) using the algebraic control method [8] and the optimal control method [3,4] and b) using the new optimal multirate control method [9-11].

The physical system (consisting of a 117 MVA hydrogenerator with single stage excitation system supplying power to an infinite grid via a step-up transformer and a double-circuit transmission line) along with its 6^{th} order, non-linear and linearealized model (in untransformed and transformed form for various operating points) are given in detail in [12].

2. OVERVIEW OF PERTINENT CONTROLLER DESIGN PROCEDURES

2.1 Conventional controller design procedure for linear continuous systems

The general description of a continuous-time linear time-invariant system model in state-space is

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \quad (1)$$

where $\mathbf{x}(t) \in \mathbf{R}^n$ is the state vector, $\mathbf{u}(t) \in \mathbf{R}^m$ is the control vector, $\mathbf{y}(t) \in \mathbf{R}^p$ is the output vector of the system and $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are constant matrices of appropriate dimensions.

Pole assignment procedure (Method 1) [8]

With this method one determines the (1xn) dimension constant value gain feedback vector, which leads to the following output feedback control law

$$\mathbf{u}(t) = -\mathbf{k}\mathbf{x}(t) + \mathbf{u}_0 \quad (2)$$

The combination of (1) and (2) gives the associated closed-loop system model

$$\begin{aligned} \dot{\mathbf{x}}(t) &= (\mathbf{A} - \mathbf{b}\mathbf{k}^T)\mathbf{x}(t) + \mathbf{u}_0 \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) \end{aligned} \quad (3)$$

with preassigned (desired) eigenvalues. The final form of $\mathbf{k}$ is given by $\mathbf{k} = \mathbf{T}^{-1}\eta$ (the exact derivation steps are shown in [8]).

Optimal control technique (Method 2) [1-4]

In the design of conventional optimal control systems, the control law is given by

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t) \quad (4)$$

where $\mathbf{K}$ is a (mxn) state feedback control matrix, designed to minimize the following quadratic performance index:

$$J = \frac{1}{2}\int_0^\infty (\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u})dt \quad (5)$$

In eq.(5) the weighting symmetric matrices are $\mathbf{Q} \geq 0$ and $\mathbf{R} > 0$. The $\mathbf{K}$ of eq.(4) is given by $\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}$, where $\mathbf{P}$ being a symmetric positive definite matrix which results from the solution of the Ricatti equation, i.e.

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = 0 \quad (6)$$

The eigenvalues of the so resulting closed-loop system may be positioned to desired locations on the open left halfplane of the complex s-plane.

2.2 New optimal multirate controller design procedure for linear discrete system (Method 3) [9-11]

The theory of this method is presented here in a very brief but concise form, whereas the pertinent details are found in [9-11].

With respect to system (1) are defined the $n_i, i = 1, 2, \ldots, p$ which comprise an observability index vector of the pair $(\mathbf{A},\mathbf{C})$, and $T_0 \in \mathbf{R}^+$ the sampling period.

Next, to system (1) is applied the Two Point Multirate Controller (TPMRC) strategy [9,11], where the inputs of the plant are constrained to the following piecewise constant control law

$$\begin{aligned} &\mathbf{u}(kT_0 + \mu T^* + \zeta) = T^{*-1}\Delta_\mu^T\mathbf{B}_N^\Gamma\hat{\mathbf{u}}(kT_0), \\ &\hat{\mathbf{u}}(kT_0) \in \mathbf{R}^{p_N} \end{aligned} \quad (7)$$

The ith plant output, $y_i(t)$, is detected at every $T_i = T_0 / M_i$, such that

$$\begin{aligned} &y_i(kT_0 + \rho T_i) = c_i^T x(kT_0 + \rho T_i), \\ &\rho = 0, 1, \ldots, M_i - 1 \end{aligned} \quad (8)$$

where, $M_i \in \mathbf{Z}^+, i = 1, 2, \ldots, p,$ are the output multiplicities of the sampling. In general $M_i \neq N$ (i.e. a multirate sampling of the plant inputs and output may be performed at a different rate) where N=input multiplicity of the sampling and $T^* = T_0 / N$.

The sampled values of the plant outputs obtained in the interval $[kT_0, (k+1)T_0)$, are stored in the M^*- dimensional column vector $\hat{\gamma}(kT_0)$ of the form

$$\hat{\gamma}(kT_0) = [y_1(kT_0)\ldots y_1(kT_0 + (M_1 - 1)T_1\ldots$$
$$\ldots y_p(kT_0)\ldots y_p(kT_0 + (M_p - 1)T_p)]^T$$

where $M^* = \sum_{i=1}^{p} M_i$. The vector $\hat{\gamma}(kT_0)$ is used in the control law of the form

$$\hat{\mathbf{u}}[(k+1)T_0] = \mathbf{L}_u\hat{\mathbf{u}}(kT_0) - \mathbf{K}\hat{\gamma}(kT_0) \quad (9)$$

where $\mathbf{L}_u \in \mathbf{R}^{p_N x p_N}$, $\mathbf{K} \in \mathbf{R}^{p_N x M^*}$.

The multirate optimal scheme is based on solving the continuous-time LQR problem, which is to find a controller of the form (7) and (9), when applied to system (1) with minimizing the following performance index

$$J = \frac{1}{2}\int_0^\infty [\mathbf{y}^T(t)\mathbf{Q}\mathbf{y}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)]dt \quad (10)$$

when $\mathbf{y}(0)$ and $\mathbf{Q}$ and $\mathbf{R}$ are symmetric matrices with $\mathbf{Q} \geq 0$, $\mathbf{R} > 0$.

From Lemma 1 of the [11] one has the multirate output sampling mechanism

$$\mathbf{H}\mathbf{x}[(k+1)T_o] = \hat{\gamma}(kT_o) - \mathbf{D}\hat{\mathbf{u}}(kT_o), \; k \geq 0 \qquad (11)$$

where $\mathbf{H} \in \mathbf{R}^{M^* \times n}$ and $\mathbf{D} \in \mathbf{R}^{M^* \times p_{N_o}}$.
From Theorem 1 of [11], for almost every T_o the control law (9) can be made equivalent to any static state feedback law of the form

$$\hat{\mathbf{u}}(kT_o) = -\mathbf{F}\mathbf{x}(kT_o), \quad \text{for } k \geq 1 \qquad (12)$$

by choosing the controller pair $(\mathbf{K}, \mathbf{L}_u)$ such that

$$\mathbf{KH} = \mathbf{F}, \quad \mathbf{L}_u = \mathbf{KD} \qquad (13)$$

The $\mathbf{K}$ and $\mathbf{L}_u$ gains are given by (14) and (15) respectively

$$\mathbf{K} = (\widetilde{\mathbf{R}}_N + \mathbf{B}_N^T\mathbf{P}\mathbf{B}_N)^{-1}(\widetilde{\mathbf{G}}_N + \mathbf{B}_N^T\mathbf{P}\Phi)\mathbf{H}^{\dagger} \qquad (14)$$

$$\mathbf{L}_u = (\widetilde{\mathbf{R}}_N + \mathbf{B}_N^T\mathbf{P}\mathbf{B}_N)^{-1}(\widetilde{\mathbf{G}}_N + \mathbf{B}_N^T\mathbf{P}\Phi)\mathbf{H}^{\dagger}\mathbf{D} \qquad (15)$$

where $\mathbf{H}^{\dagger}$ is the left pseudoinverse of matrix $\mathbf{H}$.

3. DESCRIPTION AND MODELLING OF LINEAR CONTINUOUS OPEN-AND CLOSED-LOOP HYDROGENERATOR SYSTEM

The system under study consists of a 117 MVA hydrogenerator connected to an infinite bus of the Greek national grid through a step-up transformer and a double-circuit transmission line (see Fig. 1).

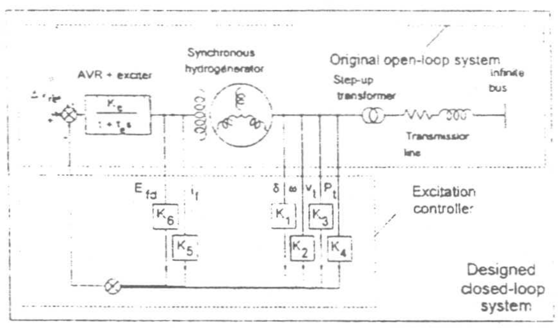


Fig.1. System representation with reference to Method 1 [13].

The numerical data of the system are given in [12,13], whereas the associated state space system modelling for the following operating points is presented in [13].

	v_t(p.u.)	P_t(p.u.)	Q_t(p.u.)
Op#1	1.0	0.9	0.436
Op#2	1.0	1.1	0.5
Op#3	1.0	0.5	0.58
Op#4	1.0	0.4	-0.68

The complete state and output vector of the transformed linear continuous open-loop system model (all states bein measurable quantities) of the system are given as

$$\mathbf{w}^T = \mathbf{y}^T = [\delta, \omega, v_t, P_t, i_f, E_{fd}]$$

The time responses of the system state (output) variables for the four operating points were obtained for the input step change $\Delta V_{ref} = 0.05$ p.u. and zero initial conditions. The time responses of δ and v_t corresponding to op#1 & op#2, and op#4 are shown in Figs. 2, and 3 respectively.
The application of the theory of Method 1 and Method 2 (of this paper) to the transformed linear continuous open-loop model of the hydrogenerator system yielded the associated closed-loop system models[13]. The computed closed loop systems (for the same input step change and initial conditions used in the time responses of the open-loop system models) are also shown in Fig. 2 and 3. From these figures it is clear that the designed closed-loop systems possess superior dynamic stability characteristics by comparison to the ones of the associated open-loop system model. The robustness of the controllers designed with respect to op#1 was tested by applying them to the other operating points (op#2, op#3 and op#4) and gave also significantly improved dynamic stability characteristics (e.g. see Figs 2 and 3).

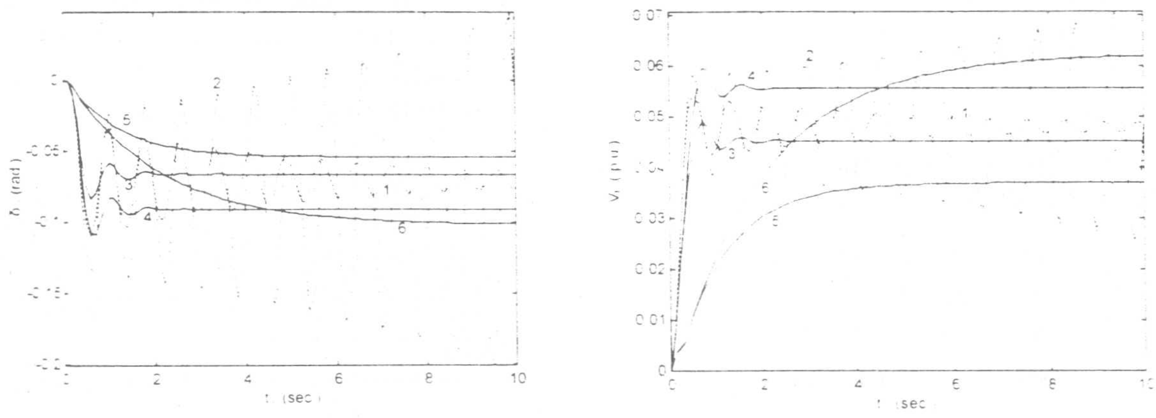


Fig. 2. δ and v_t time responses of linear continuous system model.

-------- open-loop system ——— closed-loop system

1: transformed of op#1

2: transformed of op#2

3: transformed of op#1 with Method 1

4: transformed of op#1 with Method 2

5: transformed of op#2 (with **k** of op#1) with Method 1

6: transformed of op#2 (with **k** of op#1) with Method 2

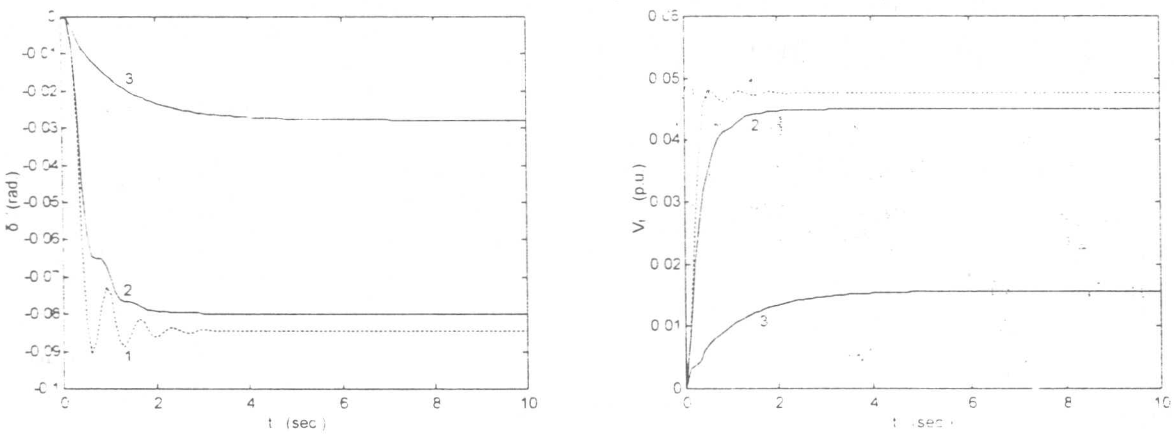


Fig. 3. δ and v_t time responses of linear continuous system model.

-------- open-loop system ——— closed-loop system

1: transformed of op#4

2: transformed of op#4 (with **k** of op#1) with Method 1

3: transformed of op#4 (with **k** of op#1) with Method 2

4. DESIGN AND SIMULATION OF OPEN-AND CLOSED-LOOP HYDROGENERATOR LINEAR DISCRETE SYSTEMS

Based on the 6^{th} -order transformed open-loop model of op#1 of the hydrogenerator system mentioned in § 3 and by using a special software program (when is based on the theory of § 2.2 [9-11] and runs in MATLAB program environment) the associated 6^{th}-order open- and closed-loop linear time-invariant discrete system models are determined.

The matrices $\mathbf{A}_d$. $\mathbf{b}_d$ and $\mathbf{C}_d$ of the obtained discrete open-loop system model with sampling period T_o=0.2 sec. are as follows

$$\mathbf{A}_d = \begin{bmatrix} -0.7823 & 0.1197 & -1.6435 & 0.3899 & -0.0970 & -0.0033 \\ -11.4315 & -0.0673 & -8.3943 & 2.0835 & -0.8719 & -0.0345 \\ 0.1933 & -0.0105 & 0.4518 & -0.0775 & 0.0756 & 0.0034 \\ -1.2529 & -0.2204 & -2.6607 & 0.8042 & -0.0396 & 0.0002 \\ 0.0961 & 0.0885 & -0.5009 & 0.0543 & 0.1091 & 0.0074 \\ -6.0999 & 0.5261 & -25.7354 & 3.0428 & -3.9612 & -0.1447 \end{bmatrix}$$

$$\mathbf{b}_d^T = [-0.1950 \quad -3.3053 \quad 0.5281 \quad 0.6223 \quad 2.5379 \quad 30.8293]$$

$$\mathbf{C}_d = \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

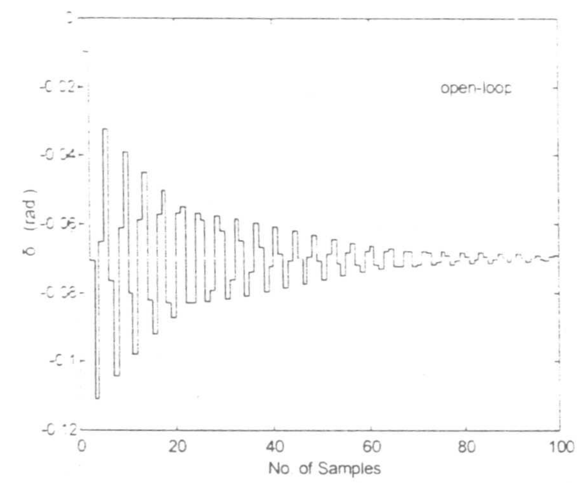
The numerical description of the resulting discrete closed-loop system model is not presented here, but it is mentioned that is based on the arrived weight matrices given below

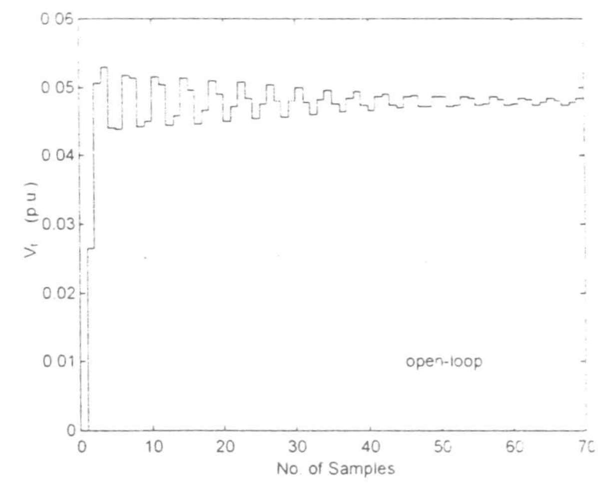
$$\mathbf{Q} = \mathrm{diag}\left[10^{-2} \quad 10^{-2} \quad 10^{-5} \quad 10^{-5} \quad 10^{-5} \quad 10^{-5}\right]$$

$$\mathbf{R} = 1$$

and the chosen output multiplicities of the sampling $M_i = [2, 3, 4, 6, 8, 12]$, while the input multiplicity of the sampling was taken as N=8.

The solution results of the discrete system models (i.e. eigenvalues, eigenvectors, time responses of system variables, etc.) were obtained by proper use of the MATLAB program. The time responses of the variables δ and v_t, corresponding to the linear discrete open-and closed-loop system models, are shown in Fig. 4 to 6. These figures show clearly that the application of the new optimal multirate control method lead to the design of a very efficient two-point multirate controller, i.e. to a discrete closed-loop system model with superior dynamic stability characteristics. The motivation for designing and using **two-point-multirate controllers** stems from the fact that they may be implemented directly using a digital computer, which makes them very useful in practical applications.





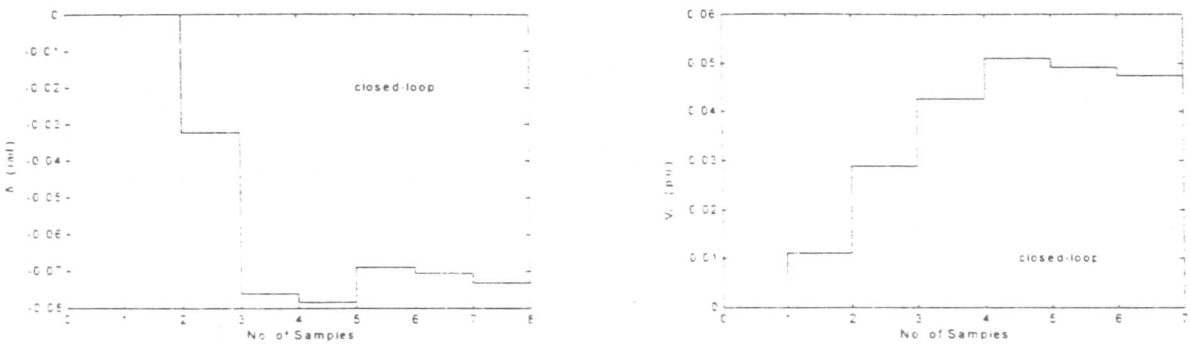


Fig. 4. δ and v_t time responses of discrete system model for op#1 (with its own gain matrix **K**)

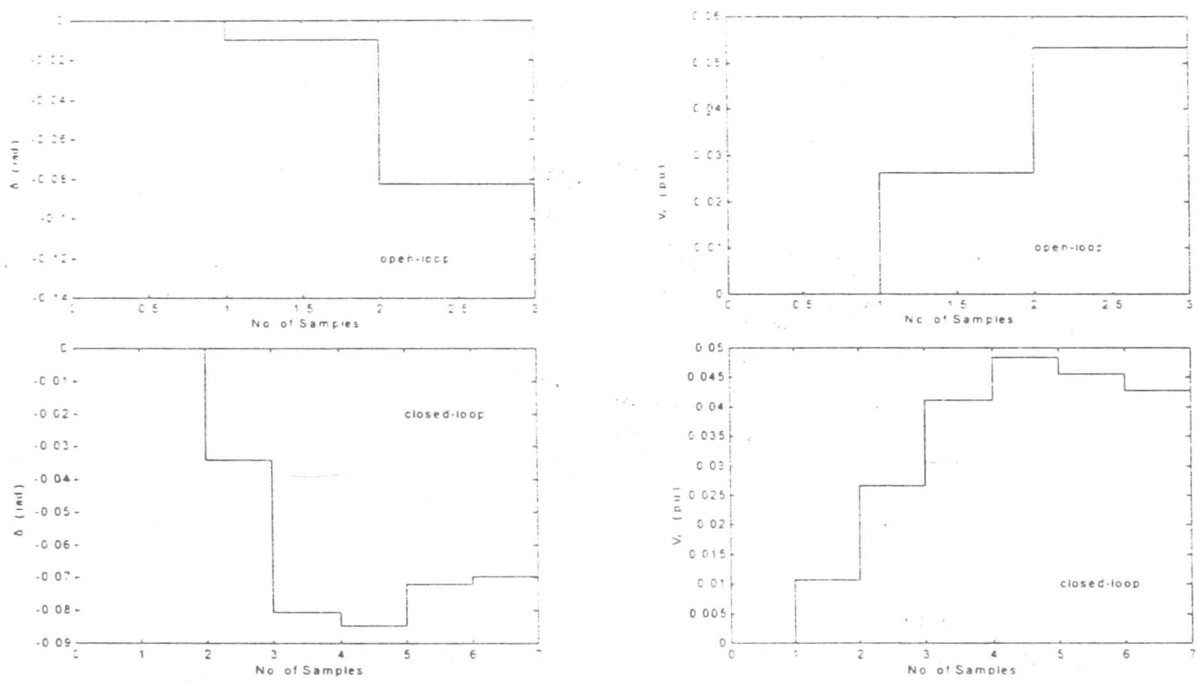


Fig. 5. δ and v_t time responses of discrete system model for op#2 (with gain matrix **K** of op#1).

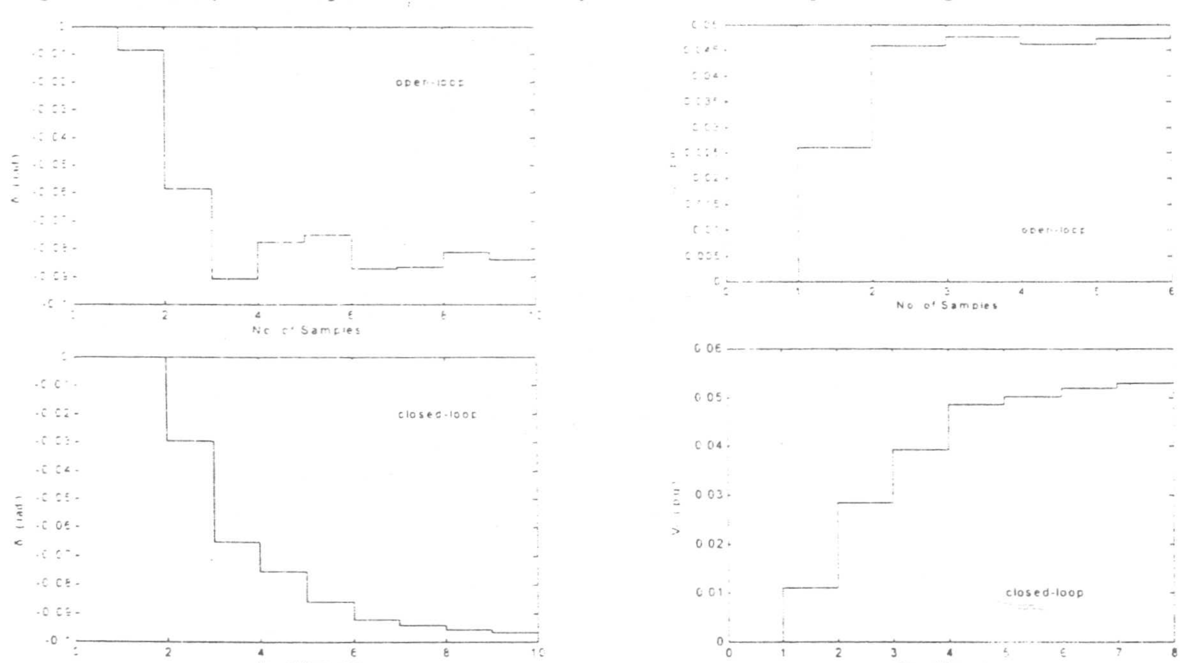


Fig. 6. δ and v_t time responses of discrete system model of op#4 (with gain matrix **K** of op#1).

5. CONCLUSIONS

Three efficient control methods have been presented in concise form and applied successfully in the design of robust excitation controllers (i.e. an algebraic and an optimal control method for linear continuous open-loop system models, and a new optimal multirate control method for linear discrete open-loop systems models) for a 6^{th}-order model of a hydrogenerator system. The designed linear continuous and discrete closed-loop system models (for a wide range of operating conditions) displayed superior dynamic stability characteristics by comparison to the associated ones of their open-loop systems. The demonstrated simplicity of the control methods used make them good and reliable tools for the design of suitable controllers.

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