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A DYNAMIC APPROACH FOR OPTIMIZATION OF OPEN PIT MINE INVESTMENTS WITH CAPITAL BUDGET

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Abstract- Mining investments require huge capitals. Since mining has an economical aspect, a reasonable profit is expected at the end of investment. In other words, a mining investment is reasonable only if it brings a desirable income. In fact, the objective of mine investment should be the optimum profit, which means the maximum profit that can be handled under certain technical conditions and constraints. In this study, dynamic programming procedures have been investigated whether they can be applied on mining investments having budget constraint. A general and brief review of dynamic programming technique and the theoretical base of capital budget problem have been presented. Two examples have been employed for the applications; first is selecting optimal mine investment alternative(s) through a group of choices and second is selection of optimum excavation-machine alternative(s) providing maximum excavation capacity. By this way, applicability of dynamic programming has been examined on both the whole system and a sub-system. The objective function and recursion functions have been defined and applied on numerical cases. The results have revealed the validity of the approach for optimization of mining investments with budget constraint.

1. INTRODUCTION

Mine investment is a complicated task including a number of parameters. A mine system is composed of many sub-systems like exploration, development, excavation, drilling and blasting, transportation, dumping, stockpiles, ore dressing, etc. Each sub-system may be divided into further sub-systems, too. Operations research techniques are applicable for the optimization of each sub-system and the whole system. The aim is to maximize profit, minimize costs and deflection from action plan. Optimization of mine system and its sub-systems can be performed by mathematical modeling and operations research applications.

Dynamic programming technique has a wide application field in the industry due to its stage by stage problem solving approach [1]. Since it is a mathematical technique, which does not have a standard formulation and rather, an approach having basic criteria, it can be applicable on much type of optimization problems regardless of study field [2]. Also, Dynamic programming technique is one of optimization methods having an application in mining, too. Most of optimization studies are on pit limits [3,4,5,6,7] or production planning [8,9,10,11,12,13]. However, dynamic programming technique can be used for many optimization items. A capital holder with a certain budget can make the optimal decision through a number of investment alternatives by dynamic approach. Such a decision is a global or whole-system decision. However, a decision about machine selection can be thought as a local or sub-system decision. In both cases, related recursion functions and mathematical model should be formed according to dynamic criteria.

In the general form of dynamic programming the problem is divided into **stages**, each of which is a step in sequencing [2]. Destination from one stage to another requires a **policy decision** that means there must be a criterion through the stages. At each stage there should exist some destination possibilities called as **states**. While moving forward through the stages, knowledge of the current stage conveys all the information about its previous behavior which provides a chain relation called Markovian property. There is every time a **recursive relationship** that identifies optimum policy for stage n and n+1. It is generally in the form of;

$$f_n^*(s_n) = \min_{x_n}\{f_n(s_n, x_n)\} \tag{1}$$

or

$$f_n^*(s_n) = \max_{x_n}\{f_n(s_n, x_n)\} \tag{2}$$

where,

s_n = current state for stage n

n = label for current stage *(n=1,2,...,N)*

N = number of stages

x_n = decision variable for stage n

x_n^* = optimum value of x_n

$f_n(s_n, x_n)$ = contribution of stages, n, $n+1$, ..., N

The optimum decision at stage s_n will be x_n^* which can be shown in function terms as;

$$f_n^*(s_n) = f_n(s_n, x_n^*) \tag{3}$$

Deterministic dynamic programming considers the state at the next stage is completely determined by the state and policy decision at the current stage (Fig. 1), while in probabilistic case the next state is defined using a probability distribution.

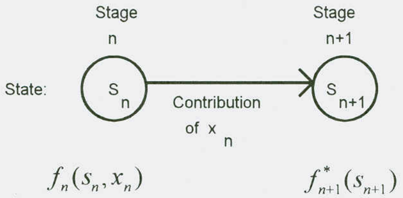

Figure 1. The basic structure of DP [2].

2. DYNAMIC PROGRAMMING WITH CAPITAL BUDGET

Decision making under limited budget is a capital budgeting type problem [14]. The aim is to reach at the best alternative through a set of choices while budget is a constraint [15]. Whitehouse and Wechsler [16] form the model such that, a_n is the present worth of project n and c_n is the investment required. There are N projects, $n=1,2,...,N$. Recursion equations are;

$$f_n(x)=f_{n-1}(x) \qquad x-c_n \quad 0 \tag{4}$$

$$f_n(x)=max[f_{n-1}(x),\ a_n+f_{n-1}(x-c_n)] \tag{5}$$

for $n=1,...,N$ and $0 \leq x \leq K$, where K is the total capital available and x is the budget available to allocate through projects 1 to n. a_n is the present worth of n'th project, which is a function of time (year) $i(i=1,...,I)$, annual net profit S_i and rate of return r:

$$a_n = \sum_{i=0}^{I} S_i / (1+r)^i \tag{6}$$

Recursion equations state that, for budget x, project n should be selected only if the present worth that it yields together with the present worth that can be provided by projects 1 to n-1 having budget x-c_n, is greater than the maximum present worth that can be obtained from projects 1 through n-1 with budget x [15]. Above model is suitable for mine investment problems, too. Selection problems favor the use of this approach. A capital holder can decide on which investment(s) are optimal for the capital in hand. Additionally, in subsystem perspective, investment problems requiring selection like equipment, process plant machinery, etc., capital budget approach is convenient.

3. OPTIMAL MINE INVESTMENT

A capital holder aims at the maximum profit by the capital in hand. This requires a crucial decision on the project(s) through choices. Table 1 presents investment alternatives, in the sorted form, with present worth that can be provided by the project and investment amounts in Turkish Lira (TL).

Table 1. Mine investment alternatives.

Project (n)	PW (10^9 TL)	Investment (10^9 TL)
1	750	600
2	700	550
3	500	400
4	350	250
5	150	150

Total capital, C, is 1.2×10^{12} TL. The objective is to maximize the profit a_n.

$$\max Z = \sum_{n=0}^{N} a_n$$

subjected to:

$\sum c_n \leq C$ and $n \geq 0$

Application of recursion functions (eqns. 4 to 6) is presented below;

total capital C=1,200x10^9 TL. Initiated $f(x)$ is zero, $f_o(x)=0$.

For n=1 (x10^9 TL),

$f_1(x) = max(0,750)$ $\quad$ $x-750 \geq 0$
$f_1(x) = 0$ $\quad$ $x-750 \quad 0$ than,

$f_1(x) = 750$ $\quad$ $750 \leq x \leq 1,200$ *(accept 1)*
$= 0$ $\quad$ $0 \leq x \leq 750$ *(reject 1)*

In Table 2 comparisons are presented.

Table 2. Comparisons for n=1.

(x10^9 TL)

$f_1(x)=max[f_0(x),\ 750+f_0(x-600)]$			
$a \leq x \leq b$		$f_1(x)$	Projects Accepted
a	B		
0	599	0	None
600	1,200	750	1

Functions and comparisons for n=2 to n=5 are calculated in Table 3-6 respectively.

Table 3. Comparisons for n=2.

(x10^9 TL)

$f_2(x)=max[f_1(x),\ 650+f_1(x-550)]$			
$a \leq x \leq b$		$f_2(x)$	Projects Accepted
a	b		
0	549	0	None
550	1,149	750	2
1,150	1,200	1,400	1, 2

Table 4. Comparisons for n=3.

(x10^9 TL)

$f_3(x)=max[f_2(x),\ 500+f_2(x-400)]$			
$a \leq x \leq b$		$f_3(x)$	Projects Accepted
a	B		
800	949	750	1
950	999	1,200	2,3
1,000	1,200	1,450	1,2

Note: x below 800x10^9 TL can not be taken into consideration because investment in projects 4 and 5 are 400x10^9 TL in total.

Table 5. Comparisons for n=4.

(x10^9 TL)

$f_4(x)=max[f_3(x),\ 350+f_3(x-250)]$			
$a \le x \le b$			*Projects*
a	*B*	$f_4(x)$	*Accepted*
1,050	1,200	1,550	2,3,4

Note: x below 1,050x10^9 TL can not be taken into consideration because investment in project 5 is 150x10^9 TL.

Table 6. Comparison for n=5.

(x10^9 TL)

$f_5(x)=max[f_4(x),\ 150+f_4(x-150)]$			
$a \le x \le b$			*Projects*
a	*B*	$f_5(x)$	*Accepted*
1,200	1,200	1,550	2,3,4

Note: x below 1,200x10^9 TL can not be taken into consideration because n=N.

As it is observed, the optimum investment combination is obtained for projects 2, 3 and 4 yielding 1,550x10^9 TL. In case of testing a large number of alternatives (N), this technique provides the optimum investment set quickly.

4. OPTIMAL DECISION ON EQUIPMENT INVESTMENT

Another optimization field of capital budget approach may be of purchasing problems. A capital holder having limited budget, wishes to purchase any machine park, with maximum facilities. As an example, an equipment selection problem could be given; an investment for excavators will be realized. The objective is to purchase the excavator(s) providing the maximum excavation capacity per hour. There is a limited budget separated for this purpose. In Table 7, numerical data is presented for alternatives.

Table 7. Excavator investment choices.

Machine (n)	Capacity (m^3/hr)	Investment (x10^9 TL)
1	1150	175
2	1055	165
3	1000	140
4	990	125
5	900	110
6	875	95

Total capital, K, is 500x10^9 TL. The objective is to maximize the excavation capacity in terms of m^3/hr, e_n.

$$\max Z = \sum_{n=0}^{N} e_n$$

subjected to:

$\sum c_n \leq K$

and

$n \geq 0$

Recursion functions (eqns. 4 to 6) are applied to the problem. Functions for each n have been presented in Table 8. Table 9 presents the current optimum solutions for $n=1,...,N$. Results obtained when n N are current bests. When $n=N$, dynamic process is ended and the overall optimum is obtained

Table 8. Functions for n=1 to n=N.

n	Functions
1	$f_1(x) = max\ [f_0(x),\ 1150 + f_0(x\text{-}175)]$
2	$f_2(x) = max\ [f_1(x),\ 1055 + f_1(x\text{-}165)]$
3	$f_3(x) = max\ [f_2(x),\ 1000 + f_2(x\text{-}140)]$
4	$f_4(x) = max\ [f_3(x),\ 990 + f_3(x\text{-}125)]$
5	$f_5(x) = max\ [f_4(x),\ 900 + f_4(x\text{-}110)]$
6	$f_6(x) = max\ [f_5(x),\ 875 + f_5(x\text{-}95)]$

Table 9. Comparison of machines.

	Optimum	
n	Machines	Exc. Capacity (m^3/hr)
1	1	1150
2	1,2	2205
3	1,2	2205
4	1,2,4	3195
5	1,2,4	3195
6	2,4,5,6	3820

In the example, optimum machine combination is machine 2, 4, 5 and 6 which providing 3820 m^3/hr of excavation capability. For huge numbers of N, fast and reliable solutions can be obtained. This approach can also be applied to minimization type problems with budget constraint.

5. CONCLUSION

Dynamic programming is an operations research technique, which has a wide application in mining, especially for the optimization of pit limits and production planning. However, the structure of the technique enables different applications, too. The approach used for capital budget type problems can be applied to mine investments. Any capital holder can make decision on which mine(s) or which machine(s) should be selected to obtain the optimum investment combinations. In this study, how limited capitals can be evaluated optimally has been researched. Two example studies have been performed for mine investment alternatives and machine selection choices. It is revealed that capital budget approach of dynamic programming is applicable and well suits to mining problems.

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