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THE TANGENT CONOIDS FAMILY WHICH DEPENDS ON THE RULED SURFACE

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Abstract - In this study, a new congruence $[A_{**}]$ has been defined by putting a tangent right conoid on each line of a ruled surface $(A_1(s))$ of a line congruence $[A]$. Then, by considering special case of the congruence $[A_{**}]$ which has been defined in the previous part, the concepts of tangent congruence, drall and the relation among Blaschke vectors of Blaschke trihedrons, having common line A_o, has been examined for this special case. At the end of this study, the concept of tangent congruence for some special congruences has been examined.

I. BASIC CONCEPTS

ID- Module theory is very important for the space kinematics. Especially, the Euclidian motions in IR^3 are represented in ID^3 by orthogonal 3x3 matrices.

Let ID be a commutative ring with unit element. $(ID^3,+)$ is a module on the dual number ring. We call it ID-Module. This modul's elements are dual vectors. We denote dual unit vector $\mathbf{A}$ as

$$\mathbf{A} = (\mathbf{a},\mathbf{a}_o) = \mathbf{a} + \varepsilon\, \mathbf{a}_o \quad ; \quad \mathbf{a}.\mathbf{a} = 1 \quad , \quad \mathbf{a}.\mathbf{a}_o = 0 \quad , \quad \mathbf{a},\mathbf{a}_o \in IR^3. \tag{1}$$

A ruled surface $(A(s))$ is given by a unit dual vector depending on one real parameter as

$$\mathbf{A}(s) = \mathbf{a}(s) + \varepsilon\, \mathbf{a}_o(s) \quad , \quad \mathbf{A}^2(s) = 1. \tag{2}$$

Let $(\mathbf{A}_1,\mathbf{A}_2,\mathbf{A}_3)$ be Blaschke trihedron at the striction point of the line $\mathbf{A}_1$ on the ruled surface $(\mathbf{A}_1(s))$. We give Blaschke vector (dual instantaneous rotation vector) of this trihedron as

$$\mathbf{B} = Q\, \mathbf{A}_1 + P\, \mathbf{A}_3 \tag{3}$$

,[1].

If we take a ruled surface $(\mathbf{A}_1(s))$ of the congruence

$$\mathbf{A}(u,v) = \mathbf{a}(u,v) + \varepsilon\, \mathbf{a}_o(u,v) \tag{4}$$

we may write Blaschke trihedrons of parameter ruled surfaces $(\mathbf{A}_{11}(s))$ and $(\mathbf{A}_{21}(s))$ at the common line $\mathbf{A}_o$ as

$$(\mathbf{A}_o=\mathbf{A}_{11},\mathbf{A}_{12},\mathbf{A}_{13}) \quad , \quad (\mathbf{A}_o=\mathbf{A}_{21},\mathbf{A}_{22},\mathbf{A}_{23}) \tag{5}$$

Blaschke vectors of these trihedrons are given by

$$\mathbf{B}_1 = Q_1\, \mathbf{A}_{11} + P\, \mathbf{A}_{13} \quad , \quad \mathbf{B}_2 = Q_2\, \mathbf{A}_{21} + P\, \mathbf{A}_{23} \tag{6}$$

, respectively.

If parameter ruled surface are taken as principal ruled surface, we have

$$\mathbf{A}_{12}.\mathbf{A}_{22} = 0 \tag{7}$$

Theorem 1 : The edges of Blaschke trihedrons of the parameter ruled surfaces coincide with each other, under the condition that their directions and orders are not the same, [2].

Definition 1 : Let $d\Phi = d\varphi + \varepsilon\, d\varphi^*$ be dual angle between the lines $\mathbf{A}(t)$ and $\mathbf{A}(t+dt)$. The relation

$$\frac{1}{d} = \frac{d\mathbf{a}.d\mathbf{a}^*}{d\mathbf{a}^2} = \frac{d\varphi}{d\varphi^*} \tag{8}$$

is called drall along the line of the ruled surface.

On the other hand, the dual angle of pitch is another invariant of the ruled surface $(\mathbf{A}_1(s))$ and can be given by the relation:

$$\Lambda_{a_1} = \oint \frac{\left(\mathbf{A}_1, \mathbf{A}_1', \mathbf{A}_1''\right)}{(\mathbf{A}_1')^2} \tag{9}$$

Theorem 2: The dual angle of pitch of the closed ruled surface $(\mathbf{A}_1(s))$, corresponds to the dual spherical surface area described by the dual spherical image of the closed ruled surface $(\mathbf{A}_1(s))$, [3].

$$\Lambda_{a_1} = 2\pi - A_{a_1} \tag{10}$$

The right conoids are the ruled surfaces whose all lines intersect with the constant line perpendicularly.

Another invariant of ruled surface is

$$\Sigma = \frac{Q}{P} \tag{11}$$

which is known as dual spherical curvature of the ruled surface. One of the characterization of the right conoids is

$$\Sigma = 0 \tag{12}$$

, [4].

The right conoids at the line $\mathbf{A}_1(s_o)$ of the ruled surface $(\mathbf{A}_1(s))$ is given by the relation:

$$\mathbf{Z}\,(\mathrm{s}) = \frac{\mathbf{A}_1(\mathrm{s}_o) + \mathrm{s}\,\mathbf{A}_1'(\mathrm{s}_o)}{\sqrt{1+P_o^2 s^2}} \tag{13}$$

The ruled surface ($\mathbf{A}_1$(s)) and the right conoid ($\mathbf{Z}$(s)) have the first degree coupling at the line $\mathbf{A}_1(\mathrm{s}_o)$,[5].

2. COUPLING OF TWO LINE CONGRUENCE

Let

$$[\mathrm{A}]:\ \mathbf{A}=\mathbf{A}(\mathrm{u},\mathrm{v}) \qquad [\mathrm{Y}]:\ \mathbf{Y}=\mathbf{Y}(\mathrm{u},\mathrm{v}) \tag{14}$$

be two line congruences having a common regular line $\mathbf{Ao}$:$\mathbf{A}(\mathrm{u}_o,\mathrm{v}_o)=\mathbf{Y}(\mathrm{u}_o,\mathrm{v}_o)=\mathbf{A}_o$. If the congruences have

$$\left(\frac{\partial^{\,i+j}\mathbf{A}}{\partial u^i v^j}\right)_o = \left(\frac{\partial^{\,i+j}\mathbf{Y}}{\partial u^i v^j}\right)_o \qquad 1 \le i+j \le n \tag{15}$$

we say that two congruences have n th degree coupling at least, at the line $\mathbf{A}_o$,[6].

Let us take a congruence [A]. If we use the Maclaurin series of (14) at the line

$$\mathbf{A}_{10}\mathbf{:}\ \mathbf{A}_{10} = \mathbf{A}\ (\,\mathrm{u}_o = 0\ ,\ \mathrm{v}_o = 0\,) \tag{16}$$

and take the norm of the first three terms of it, we can define the line congruence as

$$[Y]:\ \ Y = \frac{A_{10} + u\,A_u + v\,A_v}{\sqrt{B}} \tag{17}$$

$$\mathrm{B} = 1 + \mathrm{E}\,\mathrm{u}^2 + 2\,\mathrm{F}\,\mathrm{u}\,\mathrm{v} + \mathrm{G}\,\mathrm{v}^2$$

Definition 2: The congruence [Y] is called " The Tangent Congruence " of the congruence [A], [6].

3. A NEW TRIHEDRON AND CONGRUENCE

Let a line congruence [A] be given by (4) on two real parameters. The line congruence [A*] which contains a ruled surface ($\mathbf{A}_1$(s)) on the line congruence [A] can be defined as follows:

$$\left[\mathbf{A}_*\right] = \mathbf{A}_*(\lambda,\mathrm{s}) = \frac{\mathbf{A}_1(\mathrm{s}) + \lambda\,\mathrm{P}\,(\mathbf{A}_2(\mathrm{s})\sin\varphi - \mathbf{A}_3(\mathrm{s})\cos\varphi)}{\sqrt{1+\lambda^2\mathrm{P}^2}} \ . \tag{18}$$

where, $\{\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3\}$ is Blaschke trihedron at the striction point of the ruled surface $(\mathbf{A}_1(\mathrm{s}))$, P is dual curvature and $\varphi(s)$ is the real angle between the line

$$\mathbf{N}=\mathbf{A}_2\cos\varphi+\mathbf{A}_3\sin\varphi \tag{19}$$

and second axis $\mathbf{A}_2$ of the Blaschke trihedron.

Let the first axis be $\mathbf{A}_1(s_o)=\mathbf{A}_o$ at each line $\mathbf{A}_1(s_o)$ of the ruled surface $(\mathbf{A}_1(s))$ on the line congruence $[A]$, the third axis be N, and the second axis be

$$\mathbf{G}=\mathbf{N}\mathbf{x}\mathbf{A}_1=\mathbf{A}_2\sin\varphi-\mathbf{A}_3\cos\varphi \quad . \tag{20}$$

where $\mathbf{G}$ is accomplished a positive trihedron with the directed lines $\mathbf{A}_1$ and $\mathbf{N}$. So, we established a new dual trihedron $\{\mathbf{A}_1,\mathbf{G},\mathbf{N}\}$ at the striction point of the ruled surface $(\mathbf{A}_1(s))$ called as **dual Darboux trihedron.**

The geometric interpretation of dual vector $\mathbf{N}$ is the axis of the right conoid given by the following relation:

$$\mathbf{Z}_*(\lambda)=\frac{\mathbf{A}_1(s_o)+\lambda P_o(\mathbf{A}_2(s_o)\sin\varphi_o-\mathbf{A}_3(s_o)\cos\varphi_o)}{\sqrt{1+\lambda^2P_o^2}} \quad . \tag{21}$$

4. THE RIGHT CONOIDS FAMILY (R.C.F)

Let us take $\sin\varphi=1$ and $\cos\varphi=0$ in (18). Thus, we have the R.C.F. which is connecting the ruled surface $(\mathbf{A}_1(s))$ as follows:

$$\mathbf{A}_{**}(s,\lambda)=\frac{\mathbf{A}_1(s)+\lambda\mathbf{A}_1'(s)}{\sqrt{1+\lambda^2P^2}} \quad , \quad \mathbf{A}_{**}^2=1 \tag{22}$$

Definition 3 : R.C.F. is called right conoid family which is obtained by putting a right conoid to each line of the ruled surface $(\mathbf{A}_1(s))$.

If we take the partial derivative from (22) with respect to s and λ

$$\left.\begin{aligned}
\mathbf{A}_{**s}&=\frac{\mathbf{A}_1(-\lambda^2PP'-P^2(\lambda^3P^2+\lambda))+\mathbf{A}_2(P+P^3\lambda^2+P'\lambda)+\mathbf{A}_3(\lambda^3P^3Q+\lambda PQ)}{(1+\lambda^2P^2)^{3/2}}\\
\mathbf{A}_{**\lambda}&=\frac{-\lambda P^2\mathbf{A}_1+P\mathbf{A}_2}{(1+\lambda^2P^2)^{3/2}}
\end{aligned}\right\} \tag{23}$$

can be written, P and Q are dual curvature and dual torsion which belongs to the ruled surface

$(\mathbf{A_1}(s))$. By taking

$$C=(1+\lambda^2 P^2)^{3/2} \tag{24}$$

$$\Lambda_1=\frac{-\lambda^2 P P'-P^2(\lambda^3 P^2+\lambda)}{C^{3/2}}, \quad \Lambda_2=\frac{P(1+\lambda^2 P^2)+\lambda P'}{C^{3/2}}, \quad \Lambda_3=\frac{\lambda PQ}{C^{1/2}} \tag{25}$$
$$\Psi_1=\frac{-\lambda P^2}{C^{3/2}}, \quad \Psi_2=\frac{P}{C^{3/2}},$$

, we may write equations (23) as

$$\begin{aligned} \mathbf{A}_{**s} &= \Lambda_1 \mathbf{A_1}+\Lambda_2 \mathbf{A_2}+\Lambda_3 \mathbf{A_3} \\ \mathbf{A}_{**\lambda} &= \Psi_1 \mathbf{A_1}+\Psi_2 \mathbf{A_2} \end{aligned} \tag{26}$$

If we open Taylor series of (22) at the line $\mathbf{A}_{10}$ with the relation $s-s_o=\alpha$, $\lambda-\lambda_o=\beta$, we have tangent congruence as:

$$Y_{**}[\alpha,\beta]=\frac{\mathbf{A_{10}}\left[\alpha\Lambda_{10}+\beta\Psi_{10}+C_o^{-1/2}\right]+\mathbf{A_{20}}\left[\alpha\Lambda_{20}+\beta\Psi_{20}+\lambda_o P_o C_o^{-1/2}\right]+\mathbf{A_{30}}(\alpha\Lambda_{30})}{\sqrt{1+\alpha^2(\Lambda_{10}^2+\Lambda_{20}^2+\Lambda_{30}^2)+2\alpha\beta(\Lambda_{10}\Psi_1+\Lambda_{20}\Psi_{20})+\beta^2(\Psi_{10}^2+\Psi_{20}^2)}} \tag{27}$$

This tangent congruence is geometric definition and depends on the line $\mathbf{A_1}(s_o)$, but does not depend on s and λ. Thus, we have the following theorem :

Theorem 3: The congruences $[A_{**}]$ and $[Y_{**}]$ have the first coupling.

Proof: If we use Taylor series of (22) and take $s-s_o=\alpha$, $\lambda-\lambda_o=\beta$, we can write the following relation:

$$Y_{**}[\alpha,\beta]=\frac{A_{**}(s_o,\lambda_o)+\alpha\, A_{**s_o}+\beta\, A_{**\lambda_o}}{\left\|A_{**}(s_o,\lambda_o)+\alpha\, A_{**s_o}+\beta\, A_{**\lambda_o}\right\|} \tag{28}$$

By using transformation $s-s_o=\alpha$, $\lambda-\lambda_o=\beta$ in (28), we easily show that $[Y_{**}]$ and $[A_{**}]$ have common line at the parameter (s_o,λ_o) as follows:

$$\mathbf{Y}_{**}\left[s_o,\lambda_o\right]=\mathbf{A}_{**}\left[s_o,\lambda_o\right]=\frac{\mathbf{A_1}(s_o)+\lambda_o\mathbf{A_1}'(s_o)}{\sqrt{1+\lambda_o^2 P_o^2}} \tag{29}$$

Then, if we put transformation $s-s_o=\alpha$, $\lambda-\lambda_o=\beta$ in (28) and take derivative according to s and λ at (s_o,λ_o), we can easily see second condition of coupling

$$(\mathbf{Y}_{**_s})_{\substack{s=s_o\\ \lambda=\lambda_o}} = (\mathbf{A}_{**_s})_{\substack{s=s_o\\ \lambda=\lambda_o}}$$

$$(\mathbf{Y}_{**_\lambda})_{\substack{s=s_o\\ \lambda=\lambda_o}} = (\mathbf{A}_{**_\lambda})_{\substack{s=s_o\\ \lambda=\lambda_o}} \tag{30}$$

Theorem 4: Let $\{\mathbf{A_1, A_2, A_3}\}$, $\{\mathbf{A_{11}, A_{12}, A_{13}}\}$ and $\{\mathbf{A_{21}, A_{22}, A_{23}}\}$ be Blaschke trihedrons of any ruled surface $(A(t))$ and parameter ruled surfaces (A_{11}) and (A_{21}) of the congruence $(A_{**}(s,\lambda))$. There exist following relations among the dual curvatures P_1, P_{11} and P_{21} of these ruled surface:

$$P_1^2 = \left(\frac{d\mathbf{A}_{**}}{dt}\right)^2 = P_{11}^2\, s'^2 + P_{21}^2\, \lambda'^2 + 2(\Lambda_1\Psi_1 + \Lambda_2\Psi_2)\, s'\lambda' \tag{31}$$

Proof: If we consider definition of dual curvature and take partial derivative according to s and λ from the congruence $[\mathbf{A}_{**}]$, we may easily see (31).

Theorem 5: Let $(A(t))$, (A_{11}) and (A_{21}) be a ruled surface and parameter ruled surfaces of the congruence $[A_{**}]$. These ruled surface have common line as $\mathbf{A_1}$. There is following relation among the Blaschke vectors $\mathbf{B}$, $\mathbf{B_1}$ and $\mathbf{B_2}$ at the common line $\mathbf{A_1}$ of these ruled surface

$$\mathbf{B} = P\left[\cos\Phi\frac{\mathbf{B_1}}{P_1} + \sin\Phi\left(\frac{\mathbf{B_2} - Q_2\mathbf{A_{21}}}{P_2}\right)\right] + Q\mathbf{A_1} \tag{32}$$

Proof: According to (7) and theorem 1, we can write ,[3].

$$\mathbf{A_{12}}.\mathbf{A_{22}} = 0 \tag{33}$$

$$\mathbf{A_3} = \cos\Phi\,\mathbf{A_{13}} + \sin\Phi\,\mathbf{A_{23}} \tag{34}$$

On the other hand, we may write Blaschke vectors of the ruled surfaces $(A(t))$, (A_{11}) and (A_{21}) as follows

$$\mathbf{B} = Q\,\mathbf{A_1} + P\,\mathbf{A_3}\ , \qquad \mathbf{B_1} = P_1\,\mathbf{A_{13}}\ , \qquad \mathbf{B_2} = Q_2\,\mathbf{A_{21}} + P_2\,\mathbf{A_{23}} \tag{35}$$

where, Q_1 is equal to zero, because, the ruled surface (A_{11}) is a right conoid. Then, if we take $\mathbf{A_3}$, $\mathbf{A_{13}}$ and $\mathbf{A_{23}}$ from (35) and insert into (34), we have (32).

Result 1 : There exists following relation among the dralls of the ruled surfaces $(A(t))$, (A_{11}) and (A_{21}) of the congruence $[A_{**}]$.

$$\frac{1}{d}=\frac{p_{11}^{2}}{p_{1}^{2}}s'\frac{1}{d_{1}}+\frac{p_{21}^{2}}{p_{1}^{2}}\lambda'\frac{1}{d_{2}}+\frac{2(\lambda_{1}\overset{*}{\varphi}_{1}+\overset{*}{\lambda}_{1}\varphi_{1}+\lambda_{2}\overset{*}{\varphi}_{2}+\overset{*}{\lambda}_{2}\varphi_{2})s'\lambda'}{p_{1}^{2}} \tag{36}$$

Proof: Seperating (31) into the real and dual parts, we may write

$$p_1^2=p_{11}^2 s'^2+p_{21}^2\lambda''^2+2(\lambda_1\varphi_1+\lambda_2\varphi_2)s'\lambda'' \tag{37}$$

$$p_1 p_o=p_{10}p_{11}s'+p_{20}p_{21}\lambda'+2(\lambda_1\overset{*}{\varphi}_1+\overset{*}{\lambda}_1\varphi_1+\lambda_2\overset{*}{\varphi}_2+\overset{*}{\lambda}_2\varphi_2)s'\lambda'. \tag{38}$$

Then, dividing both side of the equality (38) by $p_1^2\, p_{11}^2\, p_{21}^2$, (36) is obtained.

5. THE TANGENT CONGRUENCE CONCEPT ON THE SPECIAL CONGRUENCE

Definition 4 : If all the lines of a line congruence orthogonally intersect a constant line then the congruence is called a recticongruence.

Let **A** and **B** be unit dual vectors. The equation and the axes of recticongruence, which is passing through constant two lines **A** and **B**, can be given as the following, respectively.

$$\mathbf{K}=\frac{\mathbf{A}\sin(\Theta-\Phi)+\mathbf{B}\sin\Phi}{\sin\Theta}\quad,\quad \mathbf{A}\,\mathbf{B}=\cos\Theta\ ,\quad \mathbf{K}\,\mathbf{A}=\cos\Phi \tag{39}$$

$$\mathbf{U}=\frac{\mathbf{A}\times\mathbf{B}}{\|\mathbf{A}\times\mathbf{B}\|} \tag{40}$$

where Θ is a constant angle between **A** and **B**, [7].

If we take derivative according to φ and $\overline{\varphi}$ at the line $\mathbf{K}_o$ of the congruence [K],

$$\mathbf{K}_{\varphi}=\frac{-\mathbf{A}\cos(\Theta-\Phi)+\mathbf{B}\cos\Phi}{\sin\Theta}\qquad \text{ve}\quad \mathbf{K}_{\overline{\varphi}}=\varepsilon\,\mathbf{K}_{\varphi} \tag{41}$$

is obtained.

Thus, we may write tangent congruence of the recticongruence as

$$\mathbf{Y}[\lambda,\mu]=\frac{\mathbf{A}[\sin(\Theta-\Phi)-(\lambda+\varepsilon\mu)\cos(\Theta-\Phi)]+\mathbf{B}[\sin\Phi+(\lambda+\varepsilon\mu)\cos\Phi]}{\|\mathbf{A}[\sin(\Theta-\Phi)-(\lambda+\varepsilon\mu)\cos(\Theta-\Phi)]+\mathbf{B}[\sin\Phi+(\lambda+\varepsilon\mu)\cos\Phi]\|} \tag{42}$$

Theorem 6: The recticongruence which is passing through constant unit dual vector

A and **B,** and its tangent congruence has the same axis.

Proof: It is clear to see that the dual unit vector **U** is ortogonal to each lines of tangent congruence [Y]. We know that from (39), U is axes of the congruence [K]. These show us that [Y] and [K] has the same axis.

Result 2: The dual angle of pitch is zero for any ruled surface (A(t)) of the recticongruence and its tangent congruence.

Proof: The dual angle of pitch is given by [3] as

$$\Lambda_{a_1} = \oint \frac{(\mathbf{A_1}, \mathbf{A_1}', \mathbf{A_1}'')}{(\mathbf{A_1}')^2} \tag{43}$$

Any ruled surface of recticongruence and its tangent congruence are right conoids. We know that, the dual torsion

$$Q = \frac{(\mathbf{A_1}, \mathbf{A_1}', \mathbf{A_1}'')}{(\mathbf{A_1}')^2} \tag{44}$$

is zero for right conoids. Thus, (43) is equal to zero.

Result 3: The dual spherical area of the spherical indicatrix of right conoids is

$$A = 2\pi \tag{45}$$

Proof: Using equation (11),(12) and result 2, we just get (45).

Theorem 7: The tangent congruence of isotrop congruence is also isotrop.

Proof: Let (A(u,v)) be a congruence. The dual arc element of this congruence is

$$dS^2 = E\, du^2 + 2\, F\, du\, dv + G\, dv^2 \tag{46}$$

where,

$$E = A_u^2 = e + \varepsilon\, e_o \ , \quad F = A_u A_v = f + \varepsilon\, f_o \ , \quad G = A_v^2 = g + \varepsilon\, g_o \tag{47}$$

From definition of isotrop congruence, if (A(u,v)) is isotrop, we may write , [6].

$$\frac{e_o}{e} = \frac{f_o}{f} = \frac{g_o}{g} \quad , \tag{48}$$

On the other hand, we know that relation (48) is also isotrop condition for tangent congruence, [6]. This completed the proof.

Theorem 8: Let (A(u,v)) be a isotrop congruence and [Y] be a tangent congruence of it. The ruled surface which is satisfies the relation $\mu = c\,\lambda + c_1$ of [Y] are developable isotrop tangent right conoids.

Proof: We know that, all ruled surfaces of isotrop congruence are developable and the relation $\mu = c\,\lambda + c_1$ determines tangent right conoid of [Y] , [6]. Moreover, we proved that [Y] is isotrop by theorem 7. Thus, the proof is completed.

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