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Finite-Time Static Output-Feedback H_∞ Control for Discrete-Time Singular Markov Jump Systems Based on Event-Triggered Scheme

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Abstract: The problem of finite-time static output feedback H_∞ control for a class of discrete-time singular Markov jump systems is studied in this paper. With the consideration of network transmission delay and event-triggered schemes, a closed-loop model of a discrete-time singular Markov jump system is established under the static output feedback control law, and the corresponding sufficient condition is given to guarantee this system will be regular, causal, finite-time bounded and satisfy the given H_∞ performance. Based on the matrix decomposition algorithm, the output feedback controller can be reduced to a feasible solution of a set of strict matrix inequalities. A numerical example is presented to show the effectiveness of the presented method.

Keywords: discrete-time Markov jump system; event-triggered scheme; finite-time stability; static output feedback

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1. Introduction

Over the past few years, a lot of attention has been attracted to the research of Markov jump systems due to the fact that those systems can effectively describe the stochastic dynamics of physical systems suffering from some structural and parametric changes, random abrupt variations and unexpected environment disturbances. The Markov jump systems can better model a class of engineering systems, such as mechanical systems, biological systems, economic systems, electric systems and networked systems. Since it was presented in 1961, some fundamental concepts and theories, including stability analysis, controller synthesis and observer design, have been studied. By analyzing the relation among transition rates, the problem of stability and stabilization of Markov jump systems with generally uncertain transition rates was studied in [1], where a sufficient stability criterion was given. The concept of mean-square stability was also introduced in [2], where an adaptive control algorithm was presented to study the problem of a class of Markov jump systems with Lévy noise. This concept was also applied to study the stabilization problem of positive Markov jump systems in both continuous-time and discrete-time contexts [3]. When the system state was unavailable, the observer-based asynchronous control problem was studied for Markov jump systems with external disturbance and time delay [4]. By using the average dwell time method, the state observer was also designed for a class of homogeneous Markov jump systems with random communication delays, stochastic nonlinearity, and piecewise-constant transition probabilities in [5], where a passive control algorithm was given.

Singular systems, also called generalized state-space systems, algebraic-differential systems, implicit systems and semi-state space systems, are complex systems described as both dynamic systems and algebraic systems [6–8]. It is well known that the regularity and absence of impulse problems should be considered simultaneously when the stability problem of singular systems is studied, which are not required to be considered in state-space systems. Via an observer-based technique, the sliding mode control problem was investigated for discrete-time Takagi–Sugeno fuzzy networked singularly perturbed systems [9]. By using the slow-state feedback control method, the discrete-time singularly perturbed switched systems with persistent dwell-time switching law were studied in [10]. With the development of singular systems, the singular Markov systems have been put forward in the past few years. The bounded real lemma was presented for discrete-time Markov jump linear singular systems in [11], and the robust fault tolerant control problem was studied when the mixed time delays appeared in the system state. When the system input also involves time delay, the H_∞ control problem was considered for singular Markov systems with both input delay and state delay [12]. When saturation nonlinearity was contained in the system input, the robust stochastic stability problem was considered for a class of uncertain singular Markov jump systems with actuator saturation nonlinearity [13]. The discontinuity problem introduced by the mode-dependent singular matrix and Markov jump was studied in [14], where a new Lyapunov functional was designed to present the switching characteristics, and the corresponding stability criterion was given. By using dynamics decomposition and Weierstrass decomposition, respectively, the stochastic stability problem was also studied for discrete-time Markov jump linear singular systems with partially known transition probabilities [15]. The H_∞ control problem was also considered for singular Markov jump neural networks with mode-dependent time-varying delays by using singular value decomposition and hidden Markov model [16].

On the other research area, the networked control systems had been widely used in intelligent transportation systems, mobile robots and other fields since this class of systems has the characteristics of flexible structure, low cost, convenient expansion and maintenance [17,18]. However, in practical applications, the factors such as network delays, packet loss and packet disorders might result in declined system performance or system instability. For this case, the event-triggered strategy has been proposed in recent years and has received much attention by scholars. Compared with the traditional time-triggered strategy, the sampled data are not released to the network until the event-triggered condition is violated. In recent years, much research has been done based on event-triggered schemes. For example, Wang et al. [19] studied reliable controller design for singular Markov jump systems with partly known transition probabilities based on event-triggered schemes, where two event-triggered schemes were introduced. The event-triggered H_∞ controller problem of networked control systems with network channel delay was also studied where the network channel delay is modeled as a distributed delay with a probability density function [20]. By applying the distributed delay system method, the weighted memory-event-triggered H_∞ static output control problem was investigated for Takagi–Sugeno fuzzy wind turbine systems with uncertainty [21]. When the system information was transmitted through networks, the event-triggered asynchronous H_∞ filtering for singular Markov jump systems with redundant channels [22]. The event-triggered dissipative filtering problem of discrete singular neural networks with time-varying time delay and Markov jump parameters was also investigated in [23]. By using the delay partitioning technology, the H_∞ filtering problem was studied for a discrete-time singular Markov jump system with an event-triggered scheme [24].

All the above contributions considered the Lyapunov stability problem, while the dynamic performance may be preferred in some special application cases. In those cases, the attention is focused on whether the system trajectory exceeds a certain value in a finite time interval, and thus the Lyapunov functional method, where the focus is placed on the system performance in infinite-time interval, is not applicable. To resolve this problem, the finite-time stabilization problem was studied for discrete-time singular Markov jump systems using event-triggered and quantification mechanisms [25]. Via an optimization algorithm, the finite-time H_∞ control problem was also considered for stochastic singular systems with partly known transition rates [26]. The robust finite-time H_∞ control problem was also considered for discrete-time singular Markovian jump systems with time-varying delay and actuator saturation [27]. When the transition was time-varying, the finite-time stabilization problem was considered for nonlinear discrete-time singular Markov jump systems subject to average dwell time [28]. The finite-time event-triggered stabilization was introduced, where a mode-dependent event-triggered scheme was constructed to study the problem for discrete-time nonlinear Markov jump singularly perturbed models with partially unknown transition probabilities [29].

This paper considers the finite-time static output-feedback H_∞ control problem for discrete-time singular Markov jump systems based on an event-triggered scheme, where the transition probabilities are exactly known while the external disturbance is amplitude-bounded. Considering the merits of network transmission such as low cost, high flexibility, and the merit of event-trigger schemes such as saving network resources, we consider both the network transmission and event-trigger scheme, build the closed-loop discrete-time singular Markov system model, give a sufficient criterion guaranteeing the closed-loop system to be regular, causal, bounded and satisfy H_∞ performance in a finite-time interval based on a new Lyapunov–Krasovskii functional. Based on matrix decomposition, the corresponding static output feedback controller synthesis algorithm is given explicitly. A DC-motor is used finally to demonstrate the effectiveness of the presented method.

The main contributions can be organized as

1. According to the event-triggered strategy, a sufficient criterion is deduced such that the closed-loop discrete-time singular Markov systems are finite-time bounded and satisfy H_∞ performance based on a new Lyapunov–Krasovskii functional. By utilizing the augmented matrix methods, the system is also verified to be regular and causal.
2. Based on martrix decomposition techniques, the corresponding event-triggered static output feedback controller synthesis algorithm is given explicitly in terms of a group of feasible linear matrix inequalities.
3. The presented approach is capable to the event-triggered finite-time controller and filter analysis and design for discrete-time singular systems. Finally, an application to DC-motor is used to demonstrate the effectiveness of the presented method.

Notation: Throughout this paper, $\mathbb{R}^n$ represents the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the sets of all $n \times m$ real matrices. X^T denotes the transpose of X; $X > 0$ means X is real symmetric and positive definite. $\mathrm{He}[X]$ means that $X + X^T$; $\lambda_{\min}(X)$ represents the minimum eigenvalue of X; $\Pr\{x\}$ represents the occurrence probability of x, and $\mathcal{E}\{\cdot\}$ represents mathematical expectation operator. $l_2[0,\infty)$ refers to the space of square-integrable vector functions over $[0,\infty)$; $\|x\|$ is the standard l_2 norm of x; $(*)$ in LMIs represents the symmetric term of the matrix; and $\mathrm{diag}\{\cdots\}$ denotes a block diagonal matrix.

2. Problem Formulation

Consider a class of discrete-time singular Markov jump system described by

$$\begin{aligned} Ex(k+1) &= A(r_k)x(k) + B(r_k)u(k) + D(r_k)\omega(k), \\ y(k) &= C(r_k)x(k), \\ z(k) &= F(r_k)x(k) + G(r_k)\omega(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state vector, $u(k) \in \mathbb{R}^m$ is the control input vector, $y(k) \in \mathbb{R}^{q_1}$ is the system output vector, $z(k) \in^{q_2}$ is the measured output vector, $\omega(k) \in \mathbb{R}^{q_3}$ is the disturbance input vector. $r(k)$ is a Markov chain taking values in the finite space $\mathcal{I} = \{1,2,3,\ldots,M\}$, which is governed by the following probability transitions:

$$\Pr\{r_{k+1} = j | r_k = i\} = \pi_{ij}, \tag{2}$$

where $0 \le \pi_{ij} \le 1$, $\sum_{j=1}^{M} \pi_{ij} = 1$, and $\forall i, j \in \mathcal{I}$. The matrix E is singular and satisfies $\mathrm{rank}(E) = r \le n$. $A(r_k)$, $B(r_k)$, $C(r_k)$, $D(r_k)$, $F(r_k)$, and $G(r_k)$ are known matrices with compatible dimensions, which depend on $r(k)$. For arbitrary given $r(k) = i$, $A(r_k)$, $B(r_k)$, $C(r_k)$, $D(r_k)$, $F(r_k)$, and $G(r_k)$ can be simplified as A_i, B_i, C_i, D_i, F_i, and G_i.

The event-triggered scheme adopted in this paper can be described by

$$[y(k) - y(k_l)]^T \Omega_i [y(k) - y(k_l)] > \delta_i y^T(k) \Omega_i y(k), \tag{3}$$

where $\Omega_i > 0$ and δ_i are weighting matrix and threshold constant, respectively, $y(k)$ is the current sampled signal and k_l describes the release time constant, and $k_{l+1} - k_l$ is the transmission interval of the event trigger. For the existence of delay introduced by signal transmission, those signals will arrive at the controller at time $k_0 + d_{k_0}, k_1 + d_{k_1}, \ldots$. When $k_{l+1} + d_{k_{l+1}} \leq k_l + \tilde{d} + 1$, it can be shown that $d(k) = k - d_{k_l}$ satisfies $d_{k_l} \leq d(k) \leq (d_{k+1} - d_k) + d_{k_{l+1}} - 1$ for $k \in [k_l + d_{k_l}, k_{l+1} + d_{k_{l+1}} - 1]$. When $k_{l+1} + d_{k_{l+1}} < k_l + \tilde{d} + 1$, we split this interval into two subintervals, that is, $[k_l + d_{k_l}, k_{l+1} + \tilde{d}]$ and $[k_l + \tilde{d} + q, k_{l+1} + \tilde{d} + q + 1]$, where $q \in Z_+$. It can be easily verified that there exists a scalar $p > 0$ such that $k_l + \tilde{d} + p < k_{l+1} + d_{k_{l+1}} \leq k_l + \tilde{d} + p + 1$, and

$$[y(k_l + q) - y(k_l)]^T \Omega_i [y(k_l + q) - y(k_l)] \leq \delta_i y^T(k_l + q) \Omega_i y(k_l + q), \tag{4}$$

where $q = 1, 2, \ldots, p$.

Then, we can divide the interval $[k_l + d_{k_l}, k_{l+1} + d_{k_{l+1}} - 1]$ into some subintervals:

$$[k_l + d_{k_l}, k_{l+1} + d_{k_{l+1}} - 1] = \Xi_0 \cup \left\{ \bigcup_{q=1}^{p-1} \Xi_q \right\} \cup \Xi_p, \tag{5}$$

where $\Xi_0 = [k_l + d_k, k_1 + \tilde{d} + 1)$, $\Xi_q = [k_l + \tilde{d} + q, k_l + \tilde{d} + q + 1)$, $\Xi_p = [k_l + \tilde{d} + p, k_{l+1} + d_{k_{l+1}} - 1)$. Now, we can define $d(k)$ as

$$d(k) = \begin{cases} k - k_l, & k \in \Xi_0, \\ k - k_l - q, & k \in \Xi_q, \\ k - k_l - p, & k \in \Xi_p, \end{cases} \tag{6}$$

and $e(k)$ as

$$e(k) = \begin{cases} 0, & k \in \Xi_0, \\ y(k_l) - y(k_l + q), & k \in \Xi_q, \\ y(k_l) - y(k_l + p), & k \in \Xi_p, \end{cases} \tag{7}$$

and then, it follows $d_1 \leq d(k) \leq d_2$ for any $k \in [k_l + d_{k_l}, k_{l+1} + d_{k_{l+1}} - 1]$, where $d_1 = \max\{d_{k_l}\}$, $d_2 = \tilde{d} + 1$, $\tilde{d} = \max\{d_{k_l}\}$. With this observation, it can be obtained that

$$e^T(k) \Omega_i e(k) \leq \delta_i y^T(k - d(k)) \Omega_i y(k - d(k)), \tag{8}$$

for any $k \in [k_l + d_{k_l}, k_{l+1} + d_{k_{l+1}} - 1]$, and the following static output feedback control law can be designed:

$$u(k) = K_i y(k_l) = K_i C_i (x(k - d(k)) + e(k)), \tag{9}$$

and then we obtain the following closed-loop system:

$$\begin{aligned} Ex(k+1) &= A_i x(k) + A_{di} x(k - d(k)) + A_{yi} e(k) + D_i \omega(k), \\ z(k) &= F_i x(k) + G_i \omega(k), \\ x(\theta) &= \varphi(\theta), \theta \in [-d_2, 0], \end{aligned} \tag{10}$$

where $A_{di} = B_i K_i C_i$, $A_{yi} = B_i K_i$, the time delay $d(k)$ satisfies $d_1 \leq d(k) \leq d_2$ with $d_{12} = d_2 - d_1$. $\varphi(\theta)$ is the initial state condition of $x(k)$ specified in the interval $[-d_2, 0]$. Denote $\bar{y}(\theta) = x(\theta + 1) - x(\theta)$ and $\bar{y}^T(\theta) \bar{y}(\theta) \leq \sigma$ with $\sigma > 0$.

It is also assumed that the disturbance input $\omega(k)$ satisfies the following amplitude-bounded condition

$$\omega^T(i)\omega(i) \le \rho, \tag{11}$$

for any $i \in \mathcal{I}$ or $i \in [-d_2, 0]$ and $\rho > 0$.

To facilitate the following discussion, some definitions and lemmas are required.

Definition 1 ([30]).

(1) The matrix pair (E, A) is said to be regular, if $\det(zE - A) \not\equiv 0$ for any $i \in \mathcal{I}$.

(2) The matrix pair (E, A) is said to be causal, if $deg(\det(zE - A)) = rank(E)$ for any $i \in \mathcal{I}$.

(3) The system (10) is said to be regular and causal when $\omega(k) = 0$, if the matrix pair (E, A) is regular and causal for any $i \in \mathcal{I}$.

Definition 2 ([31]). *The discrete-time singular Markov system (10) is said to be finite-time bounded with respect to (c_1, c_2, N, R, ρ), if for any given scalars $c_2 > c_1 > 0$, and symmetric positive-definite matrix R, this system is regular, causal and*

$$\mathcal{E}\{x^T(k_1)E^T Rx(k_1), \|E\|^2 x^T(k_1)Rx(k_1)\} \le c_1^2 \Rightarrow \mathcal{E}\{x^T(k_2)E^T REx(k_2)\} \le c_2^2. \tag{12}$$

where $k_1 \in \{-d_2, -d_2 + 1, \ldots, 0\}$, $k_2 \in \{1, 2, \ldots, N\}$.

Remark 1. *When $\omega(k) = 0$, the definition of finite-time bounded can be reduced to finite-time stability with respect to (c_1, c_2, N, R, ρ).*

Definition 3. *The discrete-time singular Markov system (10) is said to be H_∞ finite-time bounded with respect to (c_1, c_2, N, R, ρ), if for any given scalars $c_2 > c_1 > 0$ and symmetric positive-definite matrix R, this system is finite-time bounded respect to (c_1, c_2, N, R, ρ), and the condition*

$$\mathcal{E}\left\{\sum_{k=0}^{N} z^T(k)z(k)\right\} < \gamma^2 \mathcal{E}\left\{\sum_{k=0}^{N} \omega^T(k)\omega(k)\right\} \tag{13}$$

under zero initial condition of $x(k)$.

Lemma 1 ([32]). *For given integers $a_2 > a_1 > 0$ and symmetric positive-definite matrix W, the following inequality holds:*

$$\left(\sum_{i=a_1}^{a_2} x(i)\right)^T W \left(\sum_{i=a_1}^{a_2} x(i)\right) \le (a_2 - a_1 + 1) \sum_{i=a_1}^{a_2} x^T(i)Wx(i). \tag{14}$$

Lemma 2 ([33]). *Let $B \in \mathbb{R}^{n\times m}$ to be a full column rank matrix with singular value decomposition $B = \mathcal{U}\mathcal{B}\mathcal{V}^T$, where $\mathcal{U}$ and $\mathcal{V}$ are orthogonal matrices and $\mathcal{B} \in \mathbb{R}^{n\times m}$ is an rectangular diagonal matrix with positive real numbers on the diagonal. For matrix $Z \in \mathbb{R}^{n\times n}$, there exists a matrix $L \in \mathbb{R}^{m\times m}$ such that $ZB = BL$, if and only if the matrix Z is of the form of*

$$Z = \mathcal{U}\begin{bmatrix} Z_1 & Z_3 \\ 0 & Z_2 \end{bmatrix}\mathcal{U}^T$$

with $Z_1 \in \mathbb{R}^{m\times m}$, $Z_2 \in \mathbb{R}^{(n-m)\times(n-m)}$ and $Z_3 \in \mathbb{R}^{n\times(n-m)}$.

3. Results

In this section, the stochastic finite-time boundedness analysis is given for the discrete-time singular Markov system (10) under the event-triggerd scheme, where sufficient criteria are first established.

3.1. Stochastic Finite-Time Boundedness

Theorem 1. *For given scalars $c_2 > c_1 > 0$, $N > 0$, $\sigma > 0$, $\delta_i \in [0,1)$, and symmetric positive-definite matrix R, the closed-loop discrete-time singular Markov system (10) is finite-time bounded with respect to (c_1, c_2, N, R, ρ), if there exist scalars τ_{ki}, $(k = 1,2)$, $\mu > 1$, $\varepsilon_j > 0$, $(j = 1,2)$, symmetric positive-definite matrices P_i, Q_1, Q_2, Q_3, M_1, M_2, Ω_i, and matrices H_i, V_i and K_i such that the following matrix inequalities hold:*

$$\Phi = \begin{bmatrix} \Phi_1 & \Phi_2 & \Phi_3 \\ * & \Phi_4 & \Phi_5 \\ * & * & \Phi_6 \end{bmatrix} < 0, \tag{15a}$$

$$\varepsilon_1 R < P_i < \varepsilon_2 R, \tag{15b}$$

$$0 < Q_j < \theta_j R (j = 1,2,3), \tag{15c}$$

$$0 < M_1 < \theta_4 R, \tag{15d}$$

$$0 < M_2 < \theta_5 R, \tag{15e}$$

$$\mu^N((\alpha_1 + \varepsilon_2)c_1^2 + \alpha_2\sigma) + \lambda_{\max}(W_i)\bar{\rho} < \varepsilon_1 c_2^2, \tag{15f}$$

where $S \in \mathbb{R}^{n\times n}$ fulfills $E^T S = 0$ with rank$(S) = n - r$, and

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & 0 & \tau_{1i}H_iB_iK_iC_i \\ * & \Phi_{12} & \mu^{d_1+1}E^TM_2E \\ * & * & \Phi_{13} \end{bmatrix},$$
$$\Phi_{11} = E^T(\bar{P}_i - \mu P_i - \mu M_1)E + Q_1 + Q_2 + (d_{12}+1)Q_3 + He\{\tau_{1i}H_i(A_i - E)\},$$
$$\Phi_{12} = -\mu^{d_1}Q_1 - \mu^{d_1+1}E^TM_2E,$$
$$\Phi_{13} = -\mu^{d_2}Q_3 - 2\mu^{d_1+1}E^TM_2E + \delta_iC_i^T\Omega_iC,$$
$$\Phi_2 = \begin{bmatrix} 0 & \tau_{1i}H_iB_iK_i \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \Phi_3 = \begin{bmatrix} \tau_{1i}H_iD_i & \Phi_{32} \\ 0 & 0 \\ 0 & \tau_{2i}C_i^TK_i^TB_i^TH_i^T \end{bmatrix},$$
$$\Phi_{32} = -\tau_{1i}Z_i + \tau_{2i}(A_i - E)^TH_i^T + E^T\bar{P}_i + V_i^TS^T,$$
$$\Phi_4 = \begin{bmatrix} -\mu^{d_2}Q_2 - \mu E^TM_1E - \mu^{d_1+1}E^TM_2E & 0 \\ 0 & -\Omega_i \end{bmatrix},$$
$$\Phi_5 = \begin{bmatrix} 0 & 0 \\ 0 & \tau_{2i}K_i^TB_i^TH_i^T \end{bmatrix}, \Phi_6 = \begin{bmatrix} -W_i & \tau_{2i}D_i^TH_i^T \\ 0 & \bar{P}_i + d_2^2M_1 + d_{12}^2M_2 - He\{\tau_{2i}H_i\} \end{bmatrix},$$
$$\mathcal{T}_1 = \begin{bmatrix} \tau_{1i}I_n & 0 & 0 & 0 & 0 & 0 & \tau_{2i}I_n \end{bmatrix},$$
$$\mathcal{T}_2 = \begin{bmatrix} A_i - E & 0 & B_iK_iC_i & 0 & B_iK_i & D_i & -I_n \end{bmatrix},$$
$$\bar{P}_i = \textstyle\sum_{j=1}^M \pi_{ij}P_j, \bar{\rho} = \frac{\mu^N - 1}{\mu - 1}\rho, \alpha_1 = \frac{\theta_1\eta_1 + \theta_2\eta_2 + \theta_3\eta_2 + \theta_3\eta_3}{\|E\|^2}, \alpha_2 = \theta_4\eta_4 + \theta_5\eta_5.$$
$$\tilde{P}_i = R^{-\frac{1}{2}}P_iR^{-\frac{1}{2}}, \theta_j = \lambda_{\max}\{R^{-\frac{1}{2}}Q_jR^{-\frac{1}{2}}\}, j = 1,2,3,4,5,$$
$$\eta_1 = \frac{\mu^{d_1} - 1}{\mu - 1}, \eta_2 = \frac{\mu^{d_2} - 1}{\mu - 1}, \eta_3 = \frac{\mu^{d_2} - \mu^{d_1} - d_{12}\mu + d_{12}}{(\mu - 1)^2},$$
$$\eta_4 = \frac{\mu^{d_2+1} - (d_2+1)\mu + d_2}{(\mu - 1)^2}, \eta_5 = \frac{\mu^{d_2+1} - \mu^{d_1+1} - d_{12}\mu + d_{12}}{(\mu - 1)^2}$$

Proof of Theorem 1. We first prove the regularity and casuality of this system. From (15) and the Schur complement lemma, it gives

$$\Pi_i = \begin{bmatrix} \Pi_{i11} & \Pi_{i12} \\ * & \Pi_{i22} \end{bmatrix} < 0, \tag{16}$$

where

$$\Pi_{i11} = E^T(\bar{P}_i - \mu P_i - \mu M_1)E + Q_1 + Q_2 + (d_{12}+1)Q_3 + \mathrm{He}\{\tau_{1i}H_i(A_i - E)\},$$
$$\Pi_{i12} = E^T\bar{P}_i^T + V_iS^T + \tau_{2i}(A_i - E)^TH_i - \tau_{1i}H_i,$$
$$\Pi_{i22} = \bar{P}_i - \mathrm{He}\{\tau_{2i}H_i\} + d_2^2M_1 + d_{12}^2M_2.$$

Noting $Q_j > 0$, $(j = 1,2,3)$, $M_1 > 0$ and $M_2 > 0$, we can obtain

$$\bar{\Pi}_i = \begin{bmatrix} \bar{\Pi}_{i11} & \bar{\Pi}_{i12} \\ * & \bar{\Pi}_{i22} \end{bmatrix} < 0, \tag{17}$$

where
$\bar{\Pi}_{i11} = E^T(\bar{P}_i - \mu P_i - \mu M_1)E + \mathrm{He}\{\tau_{1i} H_i(A_i - E)\}$,
$\bar{\Pi}_{i22} = \bar{P}_i - \mathrm{He}\{\tau_{2i} H_i\}$.
Let

$$\tilde{A}_i = \begin{bmatrix} E & I_n \\ A_i - E & I_n \end{bmatrix}, \tilde{V}_i = \begin{bmatrix} V_i & \tau_{1i} H_i \\ 0_{n\times n} & \tau_{2i} H_i \end{bmatrix},$$
$$\tilde{E} = \mathrm{diag}\{E, 0_{n\times n}\}, \tilde{S} = \mathrm{diag}\{S, I_n\}, \tilde{P}_0 = \mathrm{diag}\{\bar{P}_i, 0_{n\times n}\}, \tilde{P}_i = \mathrm{diag}\{P_i + \mu M_1, 0_{n\times n}\},$$

and $\bar{\Pi}_i < 0$ can be decomposed as

$$\mathrm{He}\{\tilde{A}_i^T \tilde{S} \tilde{V}_i^T\} - \tilde{E}^T \tilde{P}_i \tilde{E} + \tilde{A}_i^T \tilde{P}_0 \tilde{A}_i < 0. \tag{18}$$

From $P_i \geq 0$, it follows $\tilde{A}_i^T \tilde{P}_0 \tilde{A}_i \geq 0$, and then

$$\mathrm{He}\{\tilde{A}_i^T \tilde{S} \tilde{V}_i^T\} - \tilde{E}^T \tilde{P}_i \tilde{E} < 0. \tag{19}$$

Since $\mathrm{rank}(E) = r \leq n$, there must exist two nonsingular matrices $\bar{M}$, $\bar{N}$ such that

$$\bar{M}\tilde{E}\bar{N} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \bar{M}\tilde{A}_i\bar{N} = \begin{bmatrix} \bar{A}_{i11} & \bar{A}_{i12} \\ \bar{A}_{i21} & \bar{A}_{i22} \end{bmatrix},$$

and accordingly

$$\bar{M}^{-T}\tilde{P}_i\bar{M}^{-1} = \begin{bmatrix} \bar{P}_{i11} & \bar{P}_{i12} \\ \bar{P}_{i21} & \bar{P}_{i22} \end{bmatrix}, \bar{N}^T\tilde{V}_i = \begin{bmatrix} \bar{V}_{i1} \\ \bar{V}_{i2} \end{bmatrix}, \bar{M}^{-T}\tilde{S} = \begin{bmatrix} 0 \\ \bar{S}_{21} \end{bmatrix}.$$

Multiplying (19) by $\bar{N}^T$ and $\bar{N}$, on the left and on the right, respectively, gives

$$\begin{bmatrix} \diamond & \diamond \\ \diamond & \bar{A}_{i22}^T \bar{S}_{21} \bar{V}_{i2}^T + \bar{V}_{i2} \bar{S}_{21}^T \bar{A}_{i22} \end{bmatrix} < 0, \tag{20}$$

where $\diamond$ represents the matrix element irrelevant to the following and thus neglected here. It follows from (19) that $\bar{A}_{i22}$ is nonsingular and thus

$$\det(zE - A) = \det(z\tilde{E} - \tilde{A}_i) = \det(\bar{M}^{-1}(z\bar{E} - \bar{A}_i)\bar{N}^{-1}), \tag{21}$$

and thus the system (10) is regular and causal.

We are now in the position to prove the finite-time boundedness of this system. We construct the following functional:

$$V(k, x(k), r_k) = \sum_{i=1}^{4} V_i(k, x(k), r_k), \tag{22}$$

where
$V_1(k, x(k), r_k) = x^T(k)E^T P_i E x(k)$,
$V_2(k, x(k), r_k) = \sum_{i=k-d_1}^{k-1} \mu^{k-i-1} x^T(i) Q_1 x(i) + \sum_{i=k-d_2}^{k-1} \mu^{k-i-1} x^T(i) Q_2 x(i)$,
$V_3(k, x(k), r_k) = \sum_{m=-d_2+2}^{-d_1+1} \sum_{i=k+m-1}^{k-1} \mu^{k-i-1} x^T(i) Q_3 x(i) + \sum_{i=k-d(k)}^{k-1} \mu^{k-i-1} x^T(i) Q_3 x(i)$,

$$V_4(k,x(k),r_k) = d_2 \sum_{m=-d_2+1}^{0} \sum_{i=k+m-1}^{k-1} \mu^{k-i-1} \bar{y}^T(i) E^T M_1 E \bar{y}(i) + d_{12} \sum_{m=-d_2+1}^{-d_1} \sum_{i=k+m-1}^{k-1} \mu^{k-i-1} \bar{y}^T(i) E^T M_2 E \bar{y}(i).$$

Let

$$\mathcal{E}\{\Delta V(k)\} = \mathcal{E}\{V(x(k+1), r_{k+1} = j, | r_k = i)\} - V\{x(k), r_k = i\}, \tag{23}$$

and obtain

$$\begin{aligned}
\mathcal{E}\{\Delta V_1(k)\} - (\mu-1)V_1(k) &= x^T(k+1)E^T \tilde{P}_i E x(k+1) - \mu x^T(k) E^T P_i E x(k) \\
&= (E\bar{y}(k) + Ex(k))^T \tilde{P}_i (E\bar{y}(k) + Ex(k)) - \mu x^T(k) E^T P_i E x(k) \\
&= x^T(k)(E^T \tilde{P}_i E - \mu E^T P_i E) x(k) + 2\bar{y}^T(k) E^T \tilde{P}_i E^T x(k) \\
&- \mu x^T(k) E^T P_i E x(k)
\end{aligned} \tag{24}$$

$$\begin{aligned}
\mathcal{E}\{\Delta V_2(k)\} - (\mu-1)V_2(k) \le\ & x^T(k)(Q_1+Q_2)x(k) \\
&- \mu^{d_1} x^T(k-d_1) Q_1 x(k-d_1) \\
&- \mu^{d_2} x^T(k-d_2) Q_2 x(k-d_2),
\end{aligned} \tag{25}$$

$$\begin{aligned}
\mathcal{E}\{\Delta V_3(k)\} - (\mu-1)V_3(k) \le\ & (d_{12}+1) x^T(k) Q_3 x(k) \\
&- \mu^{d_1} x^T(k-d(k)) Q_3 x(k-d(k)),
\end{aligned} \tag{26}$$

$$\begin{aligned}
\mathcal{E}\{\Delta V_4 k)\} - (\mu-1)V_4(k) =\ & \bar{y}^T(k) E^T (d_2^2 M_1 + d_{12}^2 M_2) E \bar{y}(k) \\
&- d_2 \sum_{i=k-d_2}^{k-1} \mu^{k-i} \bar{y}^T(k) E^T M_1 E \bar{y}(k) \\
&- d_{12} \sum_{i=k-d_2}^{k-d_1-1} \mu^{k-i} \bar{y}^T(k) E^T M_2 E \bar{y}(k),
\end{aligned} \tag{27}$$

and

$$-d_2 \sum_{i=k-d_2}^{k-1} \mu^{k-i} \bar{y}^T(k) E^T M_1 E \bar{y}(k) \le -\mu d_2 \sum_{i=k-d_2}^{k-1} \bar{y}^T(k) E^T M_1 E \bar{y}(k), \tag{28}$$

$$\begin{aligned}
-d_{12} \sum_{i=k-d_2}^{k-d_1-1} \mu^{k-i} \bar{y}^T(k) E^T M_2 E \bar{y}(k) &\le -\mu^{d_1+1} d_{12} \sum_{i=k-d_2}^{k-d_1-1} \bar{y}^T(k) E^T M_2 E \bar{y}(k) \\
&= -\mu^{d_1+1} d_{12} \sum_{i=k-d(k)}^{k-d_1-1} \bar{y}^T(k) E^T M_2 E \bar{y}(k) \\
&- \mu^{d_1+1} d_{12} \sum_{i=k-d_2}^{k-d(k)-1} \bar{y}^T(k) E^T M_2 E \bar{y}(k).
\end{aligned} \tag{29}$$

From Lemma 1, it also follows

$$
\begin{aligned}
-d_{12}\sum_{i=k-d_2}^{k-d_1-1}\bar{y}^T(k)E^TM_1\bar{y}(k) &\leq -\left(\sum_{i=k-d_2}^{k-1}E\bar{y}(k)\right)^T M_1\left(\sum_{i=k-d_2}^{k-1}E\bar{y}(k)\right) \\
&= \zeta_1^T(k)\begin{bmatrix}-E^TM_1E & E^TM_1E \\ E^TM_1E & -E^TM_1E\end{bmatrix}\zeta_1(k),
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
-d_{12}\sum_{i=k-d(k)}^{k-d_1-1}\bar{y}^T(k)E^TM_2E\bar{y}(k) &\leq -\frac{d_{12}}{d(k)-d_1}\left(\sum_{i=k-d(k)}^{k-d_1-1}E\bar{y}(k)\right)^T M_2\left(\sum_{i=k-d(k)}^{k-d_1-1}E\bar{y}(k)\right) \\
&\leq \zeta_2^T(k)\begin{bmatrix}-E^TM_2E & E^TM_2E \\ E^TM_2E & -E^TM_2E\end{bmatrix}\zeta_2(k),
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
-d_{12}\sum_{i=k-d_2}^{k-d(k)-1}\bar{y}^T(k)E^TM_2E\bar{y}(k) &\leq -\frac{d_{12}}{d_2-d(k)}\left(\sum_{i=k-d_2}^{k-d(k)-1}E\bar{y}(k)\right)^T M_2\left(\sum_{i=k-d_2}^{k-d(k)-1}E\bar{y}(k)\right) \\
&\leq -\zeta_3^T(k)\begin{bmatrix}-E^TM_2E & E^TM_2E \\ E^TM_2E & -E^TM_2E\end{bmatrix}\zeta_3(k),
\end{aligned}
\tag{32}
$$

where
$\zeta_1 = [x^T(k) \quad x^T(k-d_2)]^T$,
$\zeta_2 = [x^T(k-d_1) \quad x^T(k-d(k))]^T$,
$\zeta_3 = [x^T(k-d(k)) \quad x^T(k-d_2)]^T$,
From $E^TS = 0$ and $\bar{y}(k) = x(k+1) - x(k)$, it follows

$$
\begin{aligned}
2x^T(k+1)E^TSV_ix(k) &\equiv 0, \\
2\zeta^T(k)\mathcal{T}_1^TH_i\mathcal{T}_2\zeta(k) &\equiv 0,
\end{aligned}
\tag{33}
$$

where $\zeta = [x^T(k) \quad x^T(k-d_1) \quad x^T(k-d(k)) \quad x^T(k-d_2) \quad e^T(k) \quad \omega^T(k) \quad \bar{y}^T(k)E^T]^T$.
From event trigger scheme (3), it also follows

$$
\begin{aligned}
&\mathcal{E}\{\Delta V(k) - (\mu-1)V(k) - \omega^T(k)W_i\omega(k)\} \\
&\leq \mathcal{E}\{\Delta V(k) - (\mu-1)V(k) - \omega^T(k)W_i\omega(k) - e^T(k)\Omega_ie(k) \\
&+ \delta_iy^T(k-d(k))\Omega_iy(k-d(k)) + 2x^T(k+1)E^TSV_ix(k) + 2\zeta^T(k)\mathcal{T}_1^TH_i\mathcal{T}_2\zeta(k)\} \\
&\leq \zeta^T(k)\Phi\zeta(k),
\end{aligned}
\tag{34}
$$

and then

$$
\mathcal{E}\{\Delta V(k) - (\mu-1)V(k) - \omega^T(k)W_i\omega(k)\} < 0. \tag{35}
$$

From this, it further follows

$$
\begin{aligned}
&\mathcal{E}\{V(k)\} \\
&\quad <\mu\mathcal{E}\{V(k-1)\} + \lambda_{\max}(W_i)\mathcal{E}\{\omega^T(k-1)\omega(k-1)\} \\
&\quad <\mu^2\mathcal{E}\{V(k-2)\} + \mu\lambda_{\max}(W_i)\mathcal{E}\{\omega^T(k-2)\omega(k-2)\} \\
&\quad\quad + \lambda_{\max}(W_i)\mathcal{E}\{\omega^T(k-1)\omega(k-1)\} \\
&\quad < \cdots \\
&\quad <\mu^k\mathcal{E}\{V(0)\} + \lambda_{\max}(W_i)\sum_{j=0}^{k-1}\mu^{k-j-1}\omega^T(j)\omega(j),
\end{aligned}
\tag{36}
$$

and then $\mathcal{E}\{V(k)\} < \mu^k\mathcal{E}\{V(0)\} + \lambda_{\max}(W_i)\bar{\rho}$.

From

$$\mathcal{E}\{V_1(0)\} = \mathcal{E}\{x^T(0)E^T P_i E x(0)\} \leq \lambda_{\max}(\tilde{P})c_1, \tag{37}$$

$$\begin{aligned}\mathcal{E}\{V_2(0)\} &= \sum_{i=-d_1}^{-1} \mu^{-i-1} x^T(i) Q_1 x(i) + \sum_{i=-d_2}^{-1} \mu^{-i-1} x^T(i) Q_2 x(i) \\ &\leq (\theta_1\eta_1 + \theta_2\eta_2)c_1,\end{aligned} \tag{38}$$

$$\begin{aligned}\mathcal{E}\{V_3(0)\} &= \sum_{m=-d_2+2}^{-d_1+1} \sum_{i=m-1}^{-1} \mu^{-i-1} x^T(i) Q_3 x(i) + \sum_{i=-d(0)}^{-1} \mu^{-i-1} x^T(i) Q_3 x(i) \\ &\leq (\theta_3\eta_2 + \theta_3\eta_3)c_1,\end{aligned} \tag{39}$$

$$\begin{aligned}\mathcal{E}\{V_4(0)\} &= d_2 \sum_{m=-d_2+1}^{0} \sum_{i=m-1}^{-1} \mu^{-i-1} \bar{y}^T(i) E^T M_1 E \bar{y}(i) \\ &\quad + d_{12} \sum_{m=-d_2+1}^{-d_1} \sum_{i=m-1}^{-1} \mu^{-i-1} \bar{y}^T(i) E^T M_2 E \bar{y}(i) \\ &\leq (\theta_4\eta_4 + \theta_5\eta_5)\sigma,\end{aligned} \tag{40}$$

we can obtain

$$\mathcal{E}\{V(0)\} \leq \alpha_1 + \lambda_{\max}(\tilde{P}_i)c_1^2 + \alpha_2\sigma, \tag{41}$$

$$\mathcal{E}\{V(k)\} \leq \mu^N((\alpha_1 + \lambda_{\max}(\tilde{P}_i))c_1^2 + \alpha_2\sigma) + \lambda_{\max}(W_i)\bar{\rho}, \tag{42}$$

Then, we can obviously obtain

$$\mathcal{E}\{V(k)\} \geq \mathcal{E}\{x^T(k)E^T P_i E x(k)\} \geq \lambda_{\min}(\tilde{P}_i)\mathcal{E}\{x^T(k)E^T R E x(k)\}, \tag{43}$$

from (15f), we can obtain $\mathcal{E}\{x^T(k)E^T R E x(k)\} \leq c_2^2$. This shows that this system is finite-time bounded with respect to (c_1, c_2, N, R, ρ), which concludes the proof. □

Remark 2. *A power function μ^{k-i} has been taken into consideration in the Lyapunov function. With the advantage of the design, we can obtain $V(k) < \mu V(k-1)$, then $V(k) < \mu^N V(0)$. However, the power function is ignored in many works, such as [26], and replaced by $\Delta V(k) < (\mu - 1)V_1(k) < (\mu - 1)V(k)$, which results in more conservatism. In Section 4, the less conservatism of the result can be confirmed by a numerical example.*

3.2. Stochastic Finite-Time Boundedness with H_∞ Performance

According to Theorem 1, the stochastic finite-time boundedness with H_∞ performance analysis of the discrete-time singular Markov system (10) is easily derived.

Theorem 2. *For given scalars $c_2 > c_1 > 0$, $N > 0$, $\sigma > 0$, $\delta_i \in [0, 1)$, $\gamma > 0$, and symmetric positive-definite matrix R, the closed-loop discrete-time singular Markov system (10) is finite-time bounded with respect to (c_1, c_2, N, R, ρ) with H_∞ performance γ, if there exist scalars τ_{ki}, $(k = 1, 2)$, $\mu > 1$, $\varepsilon_j > 0$, $(j = 1, 2)$, symmetric positive-definite matrices P_i, Q_1, Q_2, Q_3, M_1, M_2, Ω_i, and matrices H_i, V_i and K_i such that the matrix inequalities (15b)–(15e) and the following inequality hold:*

$$\bar{\Phi} = \begin{bmatrix} \bar{\Phi}_1 & \Phi_2 & \bar{\Phi}_3 \\ * & \Phi_4 & \Phi_5 \\ * & * & \bar{\Phi}_6 \end{bmatrix} < 0, \tag{44a}$$

$$\mu^N((\alpha_1 + \varepsilon_2)c_1^2 + \alpha_2\sigma) + \gamma^2\rho < \varepsilon_1 c_2^2, \tag{44b}$$

where $S \in \mathbb{R}^{n \times n}$ fulfills $E^T S = 0$ with rank$(S) = n - r$, and

$\bar{\Phi}_{11} = E^T(\bar{P}_i - \mu P_i - \mu M_1)E + Q_1 + Q_2 + (d_{12} + 1)Q_3 + He\{\tau_{1i} H_i (A_i - E)\} + F_i^T F_i,$

$$\bar{\Phi}_3 = \begin{bmatrix} \tau_{1i}H_iD_i + F_i^T G_i & \Phi_{32} \\ 0 & 0 \\ 0 & \tau_{2i}C_i^T K_i^T B_i^T H_i^T \end{bmatrix},$$
$$\Phi_{32} = -\tau_{1i}Z_i + \tau_{2i}(A_i - E)^T H_i^T + E^T \bar{P}_i + V_i^T S^T,$$
$$\bar{\Phi}_6 = \begin{bmatrix} -\bar{\gamma}^2 I + G_i^T G_i & \tau_{2i}D_i^T H_i^T \\ 0 & \bar{P}_i + d_2^2 M_1 + d_{12}^2 M_2 - He\{\tau_{2i}H_i\} \end{bmatrix},$$
$$W_i = \bar{\gamma}^2 I,\ \gamma = \sqrt{\frac{\mu^N - 1}{\mu - 1}\bar{\gamma}^2}$$

and other variables follow the same definition as those in Theorem 1.

Proof of Theorem 2. From inequality (44), it follows

$$\mathcal{E}\{\Delta V(k) - (\mu - 1)V(k) + z^T(k)z(k) - \bar{\gamma}^2\omega^T(k)\omega(k)\} < 0, \tag{45}$$

and

$$\mathcal{E}\{V(k)\} < \mu^N \mathcal{E}\{V(k)\} - \mathcal{E}\left\{\sum_{j=0}^{k-1} \mu^{k-j-1} z^T(j)z(j)\right\} + \bar{\gamma}^2 \mathcal{E}\left\{\sum_{j=0}^{k-1} \mu^{k-j-1}\omega^T(j)\omega(j)\right\}. \tag{46}$$

Under zero initial condition, it follows from (46) that

$$\mathcal{E}\left\{\sum_{j=0}^{k-1} \mu^{k-j-1} z^T(j)z(j)\right\} < \bar{\gamma}^2 \mathcal{E}\left\{\sum_{j=0}^{k-1} \mu^{k-j-1}\omega^T(j)\omega(j)\right\}.$$

Since $\mu > 1$, the following inequalities holds,

$$\mathcal{E}\left\{\sum_{k=0}^{N} z^T(k)z(k)\right\} < \mathcal{E}\left\{\sum_{j=0}^{k-1} \mu^{k-j-1} z^T(j)z(j)\right\},$$
$$\bar{\gamma}^2 \mathcal{E}\left\{\sum_{j=0}^{k-1} \mu^{k-j-1}\omega^T(j)\omega(j)\right\} < \gamma^2 \mathcal{E}\left\{\sum_{k=0}^{N} \omega^T(k)\omega(k)\right\},$$

and then

$$\mathcal{E}\left\{\sum_{k=0}^{N} z^T(k)z(k)\right\} < \gamma^2 \mathcal{E}\left\{\sum_{k=0}^{N} \omega^T(k)\omega(k)\right\}. \tag{47}$$

This completes the proof. □

3.3. Controller Synthesis

We are now in the position to give a output feedback gain design algorithm for the discrete-time singular Markov system (10), where a variable separation approach is adopted.

Theorem 3. *For given scalars $c_2 > c_1 > 0$, $N > 0$, $\sigma > 0$, $\delta_i \in [0,1)$, $\gamma > 0$, and symmetric positive-definite matrix R, the closed-loop discrete-time singular Markov system (10) is finite-time bounded with respect to (c_1, c_2, N, R, ρ) with H_∞ performance γ, if there exist scalars τ_{ki}, $(k = 1,2)$, $\mu > 1$, $\varepsilon_j > 0$, $(j = 1,2)$, symmetric positive-definite matrices P_i, Q_1, Q_2, Q_3, M_1, M_2, Ω_i, and matrices $H_i = \mathcal{M}_i \begin{bmatrix} H_{i11} & H_{i12} \\ 0 & H_{i22} \end{bmatrix} \mathcal{M}_i^T$, V_i and K_i such that the matrix inequalities (15b)–(15e) and the following inequality hold:*

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Phi}_1 & \tilde{\Phi}_2 & \tilde{\Phi}_3 \\ * & \Phi_4 & \tilde{\Phi}_5 \\ * & * & \bar{\Phi}_6 \end{bmatrix} < 0, \tag{48a}$$

$$\mu^N((\alpha_1 + \varepsilon_2)c_1^2 + \alpha_2\sigma) + \gamma^2\rho < \varepsilon_1 c_2^2, \tag{48b}$$

where $S \in \mathbb{R}^{n\times n}$ *fulfills* $E^T S = 0$ *with* $rank(S) = n - r$, *and*

$$\tilde{\Phi}_1 = \begin{bmatrix} \tilde{\Phi}_{11} & 0 & \tau_{1i}B_iY_iC_i \\ * & \tilde{\Phi}_{12} & \mu^{d_1+1}E^T M_2 E \\ * & * & \tilde{\Phi}_{13} \end{bmatrix},$$

$$\tilde{\Phi}_{12} = -\mu^{d_1}Q_1 - \mu^{d_1+1}E^T M_2 E,$$

$$\tilde{\Phi}_{13} = -\mu^{d_2}Q_3 - 2\mu^{d_1+1}E^T M_2 E + \delta C_i^T \Omega_i C_i,$$

$$\tilde{\Phi}_2 = \begin{bmatrix} 0 & \tau_{1i}B_iY_i \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tilde{\Phi}_3 = \begin{bmatrix} \tau_{1i}H_iD_i & \tilde{\Phi}_{31} \\ 0 & 0 \\ 0 & \tau_{2i}C_i^T Y_i^T B_i^T \end{bmatrix},$$

$$\tilde{\Phi}_{31} = -\tau_{1i}H_i + \tau_{2i}(A_i - E)^T H_i^T + E^T\bar{P}_i + V_i^T S^T,$$

$$\tilde{\Phi}_5 = \begin{bmatrix} 0 & 0 \\ 0 & \tau_{2i}Y_i^T B_i^T \end{bmatrix},$$

and other variables follow the same definition as those in Theorem 1. Furthermore, a suitable static output feedback controller gain matrix is given by

$$K_i = (B_i^T H_i B_i)^{-1} B_i^T B_i Y_i. \tag{49}$$

Proof of Theorem 3. Since B_i is with full column rank, it follows from lemma 2 that there must exist two Orthogonal matrices $\mathcal{M}_i$ and $\mathcal{N}_i$ such that

$$B_i = \mathcal{M}_i \begin{bmatrix} B_{i1} \\ 0 \end{bmatrix} \mathcal{N}_i^T, H_i B_i = B_i L_i,$$

and then

$$B_i^T B_i = \mathcal{N}_i B_{i1}^2 \mathcal{N}_i^T, \tag{50}$$

$$B_i^T H_i B_i = \mathcal{N}_i B_{i1} H_{i11} B_{i1} \mathcal{N}_i^T, \tag{51}$$

$$(B_i^T B_i)^{-1} B_i^T H_i B_i = \mathcal{N}_i B_{i1}^{-1} H_i B_{i1} \mathcal{N}_i^T, \tag{52}$$

and further gives

$$Y_i = (B_i^T B_i)^{-1} B_i^T H_i B_i K_i, \tag{53}$$

and

$$\begin{aligned} B_i L_i K_i &= B_i Y_i \\ &= B_i (B_i^T B)^{-1} B_i^T H_i B_i K_i \\ &= \mathcal{M}_i \begin{bmatrix} H_{i11} B_{i1} \\ 0 \end{bmatrix} \mathcal{N}_i^T K_i \\ &= H_i B_i K_i. \end{aligned} \tag{54}$$

This completes the proof. □

Remark 3. *In this paper, (48a), (15b) and (15f) in Theorem 3 are not strictly linear matrix inequalities, they can be converted to linear ones by given scalars* τ_{1i} *and* τ_{2i}*. Therefore, the finite-time event-triggered output feedback gain design algorithm for the discrete-time singular Markov* *system can be translated into the following optimization problem based on linear matrix inequalities.*

$$\begin{aligned}&\min_{\tau_{1i},\tau_{2i}} \quad \gamma^2\\ &P_i, H_i, K_i, V_i, \delta_i, \varepsilon_j, i \in \mathcal{I}\\ &s.t. \quad (48a), (15b)\text{–}(15f)\end{aligned}$$

4. Numerical Examples

In this section, we use a DC motor model to verify the correctness and effectiveness of the proposed method. The typical DC motor model can be described as a discrete-time singular Markov jump system with two jump modes,

$$\begin{aligned} Ex(k) &= A_i x(k) + B_i u(k) + D_i \omega(k) \\ y(k) &= C_i x(k) \\ z(k) &= F_i x(k) + G_i x(k) \end{aligned} \tag{55}$$

where

$$A_i = \begin{bmatrix} R_s & K_l \\ \frac{K_s T^*}{J_i} & 1 - \frac{b}{J_i} \end{bmatrix}, B_i = D_i = \begin{bmatrix} 0 \\ \frac{T^*}{J_i} \end{bmatrix}, \pi_{ij} = \begin{bmatrix} 0.4 & 0.6 \\ 0.7 & 0.3 \end{bmatrix}$$

and $E = \mathrm{diag}\{0, 1\}$, $C_i = [0.5 \quad 1]^T$, $F_i = [1 \quad 1]^T$, $G_i = 0.2$, $J_1 = 6.1$, $J_2 = 6.2$, $K_l = 2.4$, $K_s = 1$, $b = 1$, $T^* = 0.1$, $R_s = 3$.

According to E, we can choose $S = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$. The other parameters can be chosen as $d_1 = 1$, $d_2 = 3$, $\delta_1 = 0.08$, $\delta_2 = 0.12$, $c_1 = 1$, $c_2 = \sqrt{5}$, $N = 8$, $\rho = 0.56$, $\sigma = 0.1$, $R = \mathrm{diag}\{1, 1\}$, $\mu = 1.11$, $\gamma = 0.7235$. Based on those parameters and using Theorem 3, we can obtain the pair of static output feedback controller gains as

$$K_1 = -3.7008, K_2 = -3.5634$$

The initial state is set to be $x(k_0) = [-0.6 \quad 0.75]^T$, and the disturbance is assumed to be $\omega(k) = e^{-0.2k} \sin 0.5k$. The Markov jump modes and event trigger release instants and intervals are shown in Figures 1 and 2, which show that only 17 times are triggered and transmitted. Compared with other works, such as [33], the network transmission frequency is greatly decreased.

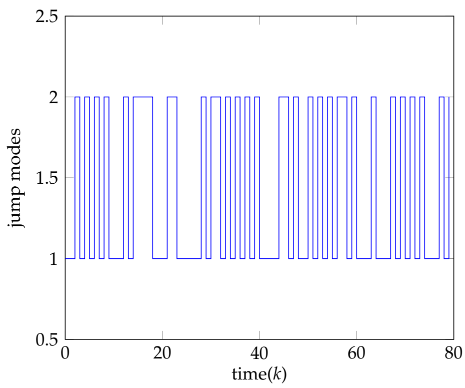

Figure 1. The Markov jump modes.

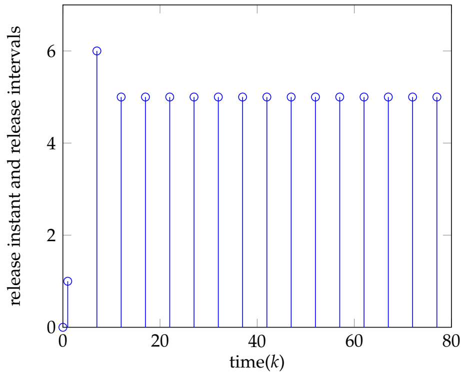

Figure 2. The event trigger release instants and intervals.

The system trajectory is shown in Figure 3 and the trajectory of $x^T(k)E^TREx(k)$ is shown in Figure 4, which indicate that the DC motor model is finite-time bounded and also satisfies H_∞ performance.

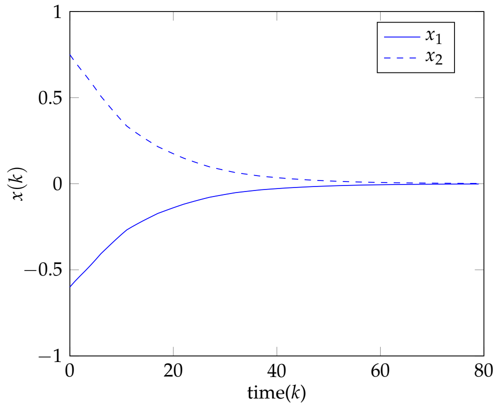

Figure 3. The state trajectory.

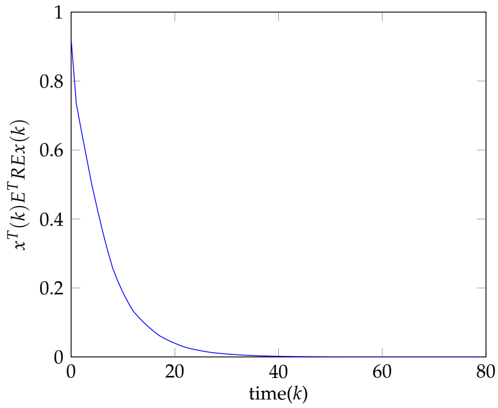

Figure 4. The trajectory of $x^T(k)E^TREx(k)$.

Remark 4. *If the minimum value of γ^2 is chosen as the optimization objective, then the optimal values γ can change with the different values μ, which are presented in Table 1.*

Table 1. Comparison of γ_{min} with different values μ.

μ	**1.05**	**1.15**	**1.20**
γ_{min}	0.8313	0.6625	0.6251

5. Conclusions

This paper studied the finite-time static output feedback H_∞ control for discrete-time singular Markov jump systems based on an event-triggered scheme. We considered both the network transmission and event-trigger scheme, built the closed-loop discrete-time singular Markov system model, and gave a static output feedback H_∞ controller synthesis method guaranteeing the closed-loop to be regular, causal, finite-time bounded and satisfy a given H_∞ performance in a finite time interval. A DC-motor was used finally to demonstrate the effectiveness of the presented method. In the future, based on the finite-time event-triggered scheme, the issues of H_∞ control or filtering for neutral Markov jump systems or neutral singular Markov jump systems with random delays and cyber-attacks will be discussed.

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