Mathematical and Computational Applications

MDPI

Article

New Exact Solutions of the System of Equations for the Ion Sound and Langmuir Waves by ETEM

Seyma Tuluce Demiray † and Hasan Bulut *,†

Department of Mathematics, Firat University, Elazig 23119, Turkey; seymatuluce@gmail.com
* Correspondence: hbulut@firat.edu.tr; Tel.: +90-424-237-0000
† These authors contributed equally to this work.

Academic Editor: Mehmet Pakdemirli
Received: 19 February 2016; Accepted: 29 March 2016; Published: 2 April 2016

Abstract: This manuscript focuses attention on new exact solutions of the system of equations for the ion sound wave under the action of the ponderomotive force due to high-frequency field and for the Langmuir wave. The extended trial equation method (ETEM), which is one of the analytical methods, has been handled for finding exact solutions of the system of equations for the ion sound wave and the Langmuir wave. By using this method, exact solutions including the rational function solution, traveling wave solution, soliton solution, Jacobi elliptic function solution, hyperbolic function solution and periodic wave solution of this system of equations have been obtained. In addition, by using Mathematica Release 9, some graphical simulations were done to see the behavior of these solutions.

Keywords: the system of equations for the ion sound wave and the Langmuir wave; extended trial equation method; exact solutions; Mathematica Release 9

1. Introduction

The survey of new exact solutions for the ion sound wave and the Langmuir wave has a highly important position among the scientists. Many authors have worked on the Langmuir solitons. Degtyarev *et al.* have presented some properties of Langmuir solitons [1]. Then, they have examined the Langmuir wave energy dissipation [2]. Some authors have obtained the numerical simulations of Langmuir collapse [3–6]. Benilov has demonstrated the stability of solitons by using the Zakharov equation which identifies the interaction between Langmuir and ion-sound waves [7]. Zakharov *et al.* have exhibited the modeling of Langmuir turbulence [8]. Dyachenko *et al.* have calculated computer simulations of Langmuir collapse [9]. Rubenchik *et al.* have studied strong Langmuir turbulence in laser plasma [10]. Musher *et al.* have submitted weak Langmuir turbulence [11]. In addition, some authors have focused on Langmuir waves [12–14]. Dodin *et al.* have tackled Langmuir wave evolution in nonstationary plasma [15]. Zavlavsky *et al.* have considered spatial localization of Langmuir waves [16]. In addition, Langmuir wave spectral energy densities have been derived from the electric field and compared to the weak turbulence results by Ratcliffe *et al.* [17].

We consider the system of equations for the ion sound wave under the action of the ponderomotive force due to high-frequency field and for the Langmuir wave [18]:

$$\begin{gathered} i\frac{\partial E}{\partial t}+\frac{1}{2}\frac{\partial^2 E}{\partial x^2}-nE=0, \\ \frac{\partial^2 n}{\partial t^2}-\frac{\partial^2 n}{\partial x^2}-2\frac{\partial^2 |E|^2}{\partial x^2}=0, \end{gathered} \tag{1}$$

where $Ee^{-iw_p t}$ is the normalized electric field of the Langmuir oscillation and n is the normalized density perturbation. The spatial variable x and the time variable t are also normalized appropriately [18]. The system of Equation (1) for the ion sound and Langmuir waves has been submitted by Zakharov [19]. Recently, this system has been investigated by some authors [20–26].

In this study, the basic interest is to constitute new exact solutions of the system of equations for the ion sound and Langmuir waves via extended trial equation method (ETEM). In Section 2, we mention basic facts of ETEM [27–32]. In Section 3, we get new exact solutions of the system of equations for the ion sound and Langmuir waves via ETEM.

2. Basic Facts of the ETEM

Step 1. For a common nonlinear partial differential equation (NLPDE),

$$P\left(u, u_t, u_x, u_{xx}, \ldots\right) = 0 \tag{2}$$

perform the wave transformation

$$u\left(x_1, x_2, \ldots, x_N, t\right) = u\left(\eta\right), \eta = \lambda\left(\sum_{j=1}^{N} x_j - ct\right), \tag{3}$$

where $\lambda \neq 0$ and $c \neq 0$. Replacing Equation (3) with Equation (2) reduces a nonlinear ordinary differential equation (NLODE),

$$N\left(u, u', u'', \ldots\right) = 0. \tag{4}$$

Step 2. Fulfill transformation and trial equation as the following:

$$u = \sum_{i=0}^{\delta} \tau_i \Gamma^i, \tag{5}$$

where

$$\left(\Gamma'\right)^2 = \Lambda\left(\Gamma\right) = \frac{\phi\left(\Gamma\right)}{\psi\left(\Gamma\right)} = \frac{\xi_\theta \Gamma^\theta + \ldots + \xi_1 \Gamma + \xi_0}{\zeta_\in \Gamma^\in + \ldots + \zeta_1 \Gamma + \zeta_0}. \tag{6}$$

Taking into consideration Equations (5) and (6), we can reach

$$\left(u'\right)^2 = \frac{\phi\left(\Gamma\right)}{\psi\left(\Gamma\right)}\left(\sum_{i=0}^{\delta} i\tau_i \Gamma^{i-1}\right)^2, \tag{7}$$

$$u'' = \frac{\phi'\left(\Gamma\right)\psi\left(\Gamma\right) - \phi\left(\Gamma\right)\psi'\left(\Gamma\right)}{2\psi^2\left(\Gamma\right)}\left(\sum_{i=0}^{\delta} i\tau_i \Gamma^{i-1}\right) + \frac{\phi\left(\Gamma\right)}{\psi\left(\Gamma\right)}\left(\sum_{i=0}^{\delta} i\left(i-1\right)\tau_i \Gamma^{i-2}\right), \tag{8}$$

where $\phi\left(\Gamma\right)$ and $\psi\left(\Gamma\right)$ are polynomials. Putting these terms into Equation (4) yields an equation of polynomial $\Omega\left(\Gamma\right)$ of Γ:

$$\Omega\left(\Gamma\right) = \sigma_s \Gamma^s + \ldots + \sigma_1 \Gamma + \sigma_0 = 0. \tag{9}$$

With regard to the balance principle, we can constitute a formula of $\theta, \in$ and δ. We can gain some values of $\theta, \in$ and δ.

Step 3. Letting the coefficients of $\Omega\left(\Gamma\right)$ all be zero will construct an algebraic equations system:

$$\sigma_i = 0, i = 0, \ldots s. \tag{10}$$

Solving this equation system (10), we will find the values of $\xi_0, \ldots, \xi_\theta; \zeta_0, \ldots, \zeta_\in$ and $\tau_0, \ldots, \tau_\delta$.

Step 4. Reduce Equation (6) to basic integral form,

$$\pm\left(\eta - \eta_0\right) = \int \frac{d\Gamma}{\sqrt{\Lambda\left(\Gamma\right)}} = \int \sqrt{\frac{\psi\left(\Gamma\right)}{\phi\left(\Gamma\right)}} d\Gamma. \tag{11}$$

Performing a complete discrimination system for polynomials to distinguish the roots of $\phi(\Gamma)$, we solve the infinite integral Equation (11) and classify the exact solutions of Equation (2) via Mathematica [33].

3. ETEM for the System of Equations for the Ion Sound and Langmuir Waves

In this section, we seek the exact solutions of the system of equations for the ion sound and Langmuir waves by using ETEM.

In an effort to find traveling wave solutions of the Equation (1), we get the transformation by use of the wave variables

$$E(x,t) = e^{i\theta} u(\xi),\ n(x,t) = v(\xi), \theta = kx + mt, \xi = px + rt, \tag{12}$$

where k, m, p and r are arbitrary constants.

Substituting Equations (13)–(15) into Equation (1),

$$iE_t = -me^{i\theta}u + ire^{i\theta}u', \tag{13}$$

$$E_{xx} = -k^2 e^{i\theta}u + 2ipke^{i\theta}u' + p^2 e^{i\theta}u'', \tag{14}$$

$$n_{tt} = r^2 v'',\ n_{xx} = p^2 v'',\ \left(|E|^2\right)_{xx} = p^2\left(u^2\right)''. \tag{15}$$

We obtain the following system:

$$i(r+pk)u'(\xi) = 0, \tag{16}$$

$$p^2 u'' - \left(2m + k^2\right)u - 2uv = 0, \tag{17}$$

$$\left(r^2 - p^2\right)v'' - 2p^2\left(u^2\right)'' = 0. \tag{18}$$

By setting the integration constant to zero, we integrate function v with respect to ξ, we find

$$v(\xi) = \frac{2p^2}{r^2 - p^2}u^2(\xi). \tag{19}$$

Putting Equation (19) into Equation (17) and by using Equation (16), we gain

$$p^2\left(k^2 - 1\right)u'' - \left(k^2 - 1\right)\left(2m + k^2\right)u - 4u^3 = 0, \tag{20}$$

where the prime remarks the derivative with respect to ξ.

Substituting Equations (5) and (8) into Equation (20) and using the balance principle, the formula is found as

$$\theta = 2\delta + \in + 2. \tag{21}$$

In order to gain exact solutions of Equation (1), if we take $\in = 0, \delta = 1$ and $\theta = 4$ in Equation (21), then

$$\left(v'\right)^2 = \frac{\tau_1^2\left(\zeta_0 + \Gamma\xi_1 + \Gamma^2\xi_2 + \Gamma^3\xi_3 + \Gamma^4\xi_4\right)}{\zeta_0}, v'' = \frac{\tau_1\left(\xi_1 + 2\Gamma\xi_2 + 3\Gamma^2\xi_3 + 4\Gamma^3\xi_4\right)}{2\zeta_0}, \tag{22}$$

where $\xi_4 \neq 0, \zeta_0 \neq 0$. Solving the algebraic equation system (10) provides

$$\begin{gathered}\xi_0 = \xi_0,\ \xi_2 = \xi_2,\ \xi_3 = \xi_3,\ \xi_4 = \xi_4, \xi_1 = -\frac{\xi_3^3 - 4\xi_2\xi_3\xi_4}{8\xi_4^2},\ \zeta_0 = \frac{p^2\left(k^2-1\right)\xi_3^2}{32\xi_4\tau_0^2},\\ \tau_0 = \tau_0, \tau_1 = \frac{4\xi_4\tau_0}{\xi_3},\ m = -\frac{k^2}{2} + \frac{2\left(-3\xi_3^2 + 8\xi_2\xi_4\right)\tau_0^2}{\left(k^2-1\right)\xi_3^2}.\end{gathered} \tag{23}$$

Setting these results into Equations (6) and (11), we have

$$\pm\left(\eta-\eta_0\right)=A\int\frac{d\Gamma}{\sqrt{\frac{\xi_0}{\xi_4}+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_3}{\xi_4}\Gamma^3+\Gamma^4}},\tag{24}$$

where $A=\sqrt{\frac{p^2\left(k^2-1\right)\xi_3^2}{32\xi_4\tau_0^2}}$.

Integrating Equation (24), we find the solutions of Equation (1) as the following:

$$\pm\left(\eta-\eta_0\right)=-\frac{A}{\Gamma-\alpha_1},\tag{25}$$

$$\pm\left(\eta-\eta_0\right)=\frac{2A}{\alpha_1-\alpha_2}\sqrt{\frac{\Gamma-\alpha_2}{\Gamma-\alpha_1}},\alpha_2>\alpha_1,\tag{26}$$

$$\pm\left(\eta-\eta_0\right)=\frac{A}{\alpha_1-\alpha_2}\ln\left|\frac{\Gamma-\alpha_1}{\Gamma-\alpha_2}\right|,\alpha_1>\alpha_2,\tag{27}$$

$$\pm\left(\eta-\eta_0\right)=\frac{2A}{\sqrt{\left(\alpha_1-\alpha_2\right)\left(\alpha_1-\alpha_3\right)}}\ln\left|\frac{\sqrt{\left(\Gamma-\alpha_2\right)\left(\alpha_1-\alpha_3\right)}-\sqrt{\left(\Gamma-\alpha_3\right)\left(\alpha_1-\alpha_2\right)}}{\sqrt{\left(\Gamma-\alpha_2\right)\left(\alpha_1-\alpha_3\right)}+\sqrt{\left(\Gamma-\alpha_3\right)\left(\alpha_1-\alpha_2\right)}}\right|,\alpha_1>\alpha_2>\alpha_3,\tag{28}$$

$$\pm\left(\eta-\eta_0\right)=\frac{2A}{\sqrt{\left(\alpha_1-\alpha_3\right)\left(\alpha_2-\alpha_4\right)}}F\left(\varphi,l\right),\alpha_1>\alpha_2>\alpha_3>\alpha_4,\tag{29}$$

where

$$F\left(\varphi,l\right)=\int_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}},\varphi=\arcsin\sqrt{\frac{\left(\Gamma-\alpha_1\right)\left(\alpha_2-\alpha_4\right)}{\left(\Gamma-\alpha_2\right)\left(\alpha_1-\alpha_4\right)}},l^2=\frac{\left(\alpha_2-\alpha_3\right)\left(\alpha_1-\alpha_4\right)}{\left(\alpha_1-\alpha_3\right)\left(\alpha_2-\alpha_4\right)}.\tag{30}$$

In addition, $\alpha_1,\alpha_2,\alpha_3$ and α_4 are the roots of the polynomial equation

$$\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4}=0.\tag{31}$$

Substituting the solutions Equations (25)–(29) into Equation (5) and by using Equation (12), the solutions of Equation (1) are obtained rational function solutions,

$$\begin{aligned}E_1\left(x,t\right)&=e^{i(kx+mt)}\left(\pm\frac{A_1}{px+rt}\right),\\ n_1\left(x,t\right)&=\left(\frac{2p^2}{r^2-p^2}\right)\left(\pm\frac{A_1}{px+rt}\right)^2,\end{aligned}\tag{32}$$

$$\begin{aligned}E_2\left(x,t\right)&=e^{i(kx+mt)}\left(\frac{4A^2\left(\alpha_2-\alpha_1\right)\tau_1}{4A^2-\left[\left(\alpha_1-\alpha_2\right)\left(px+rt\right)\right]^2}\right),\\ n_2\left(x,t\right)&=\left(\frac{2p^2}{r^2-p^2}\right)\left(\frac{4A^2\left(\alpha_2-\alpha_1\right)\tau_1}{4A^2-\left[\left(\alpha_1-\alpha_2\right)\left(px+rt\right)\right]^2}\right)^2,\end{aligned}\tag{33}$$

traveling wave solutions

$$\begin{aligned}E_3\left(x,t\right)&=e^{i(kx+mt)}\left(\frac{\left(\alpha_2-\alpha_1\right)\tau_1}{2}\left\{1\pm\coth\left[\frac{\left(\alpha_1-\alpha_2\right)}{A}\left(px+rt\right)\right]\right\}\right),\\ n_3\left(x,t\right)&=\left(\frac{2p^2}{r^2-p^2}\right)\left(\frac{\left(\alpha_2-\alpha_1\right)\tau_1}{2}\left\{1\pm\coth\left[\frac{\left(\alpha_1-\alpha_2\right)}{A}\left(px+rt\right)\right]\right\}\right)^2,\end{aligned}\tag{34}$$

soliton solutions

$$E_4(x,t) = e^{i(kx+mt)} \frac{A_2}{(D+\cosh[B(px+rt)])},$$
$$n_4(x,t) = \left(\frac{2p^2}{r^2-p^2}\right)\left(\frac{A_2}{D+\cosh[B(px+rt)]}\right)^2, \tag{35}$$

and Jacobi elliptic function solutions

$$E_5(x,t) = e^{i(kx+mt)} \frac{A_3}{(M+Nsn^2(\varphi,l))},$$
$$n_5(x,t) = \left(\frac{2p^2}{r^2-p^2}\right)\left(\frac{A_3}{M+Nsn^2(\varphi,l)}\right)^2, \tag{36}$$

where $A_1 = \tau_1 A, A_2 = \left(\frac{2\tau_1(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{\alpha_3-\alpha_2}\right), B = \frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A}, D = \frac{2\alpha_1-\alpha_2-\alpha_3}{\alpha_3-\alpha_2}$, $A_3 = (2\tau_1(\alpha_1-\alpha_3)(\alpha_4-\alpha_2))$, $M = \alpha_4-\alpha_2$, $N = \alpha_1-\alpha_4$, $l^2 = \frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}$, $\varphi = \frac{\pm\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(px+rt)$. Here, A_2 is the amplitude of the soliton, and B is the inverse width of the solitons. Thus, the solitons exist for $\tau_1 < 0$.

Remark 1. *When the modulus $l \to 1$, then by using Equation (12), the Solution (36) can be converted into the hyperbolic function solutions*

$$E_6(x,t) = e^{i(kx+mt)} \frac{A_3}{\left(M+N\tanh^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(px+rt)\right]\right)},$$
$$n_6(x,t) = \left(\frac{2p^2}{r^2-p^2}\right)\left(\frac{A_3}{M+N\tanh^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(px+rt)\right]}\right)^2, \tag{37}$$

where $\alpha_3 = \alpha_4$.

Remark 2. *When the modulus $l \to 0$, then by using Equation (12), the Solution (36) can be reduced to the periodic wave solutions*

$$E_7(x,t) = e^{i(kx+mt)} \frac{A_3}{\left(M+N\sin^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(px+rt)\right]\right)},$$
$$n_7(x,t) = \left(\frac{2p^2}{r^2-p^2}\right)\left(\frac{A_3}{M+N\sin^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}(px+rt)\right]}\right)^2, \tag{38}$$

where $\alpha_2 = \alpha_3$.

Remark 3. *The exact solutions of Equation (1) were found via ETEM, and have been calculated by using Mathematica 9. As far as we know, the solutions of Equation (1) obtained in this study are new and are not observable in former literature.*

4. Conclusions

In this paper, we obtain exact solutions of the system of equations for the ion sound and Langmuir waves by using ETEM. Then, for suitable parametric choices, we plot two and three dimensional graphics of some exact solutions of this system of equations by using Mathematica Release 9. This method supplies us to make complicated and tedious algebraic calculations. That is to say, the availability of computer programs such as Mathematica accelerates the tedious algebraic calculations.

The above results show that ETEM has been efficient for the analytical solutions of the system of equations for the ion sound and Langmuir waves. In addition, this method is a powerful mathematical tool in the way of finding new exact solutions. Thus, we can point out that ETEM has a key role in obtaining analytical solutions of NLPDEs. The graphical demonstrations such as Figures 1–6 clearly indicate effectiveness of the recommended method. We suggest that this method can also be applied to other NLPDEs.

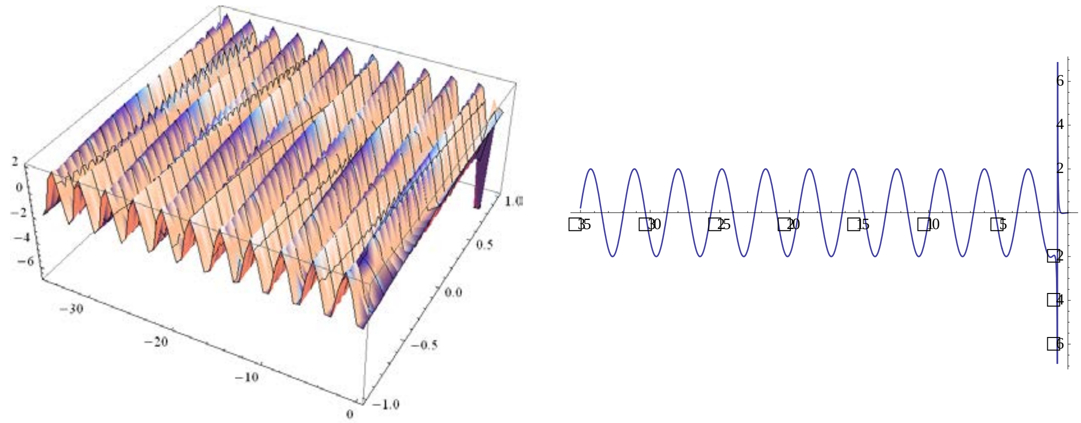

Figure 1. Graph of imaginary values of $E_3(x,t)$ in Equation (34) is indicated at $m = \tau_0 = 1$, $\tau_1 = k = \xi_3 = \xi_4 = \alpha_1 = 2, \alpha_2 = r = 3,\ p = 4, -35 < x < 0,\ -1 < t < 1$ and the second graph shows imaginary values of $E_3(x,t)$ in Equation (34) for $-35 < x < 0,\ t = 1$.

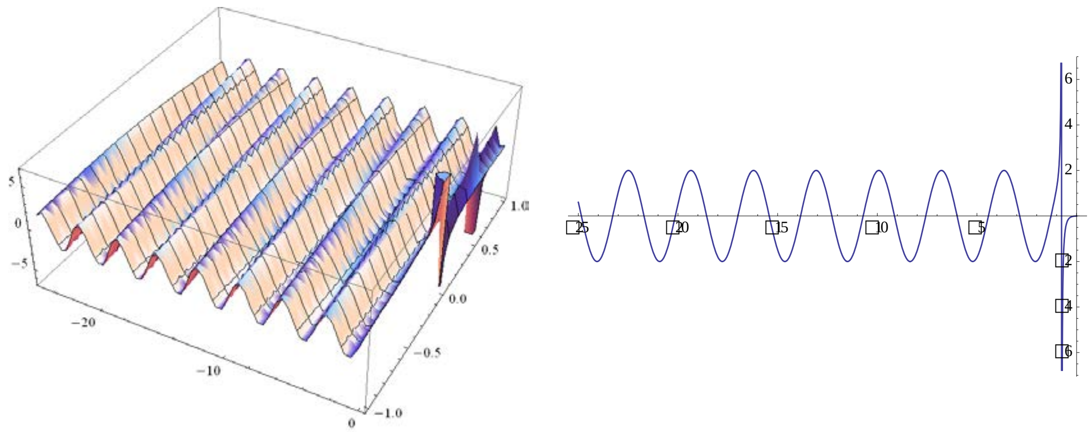

Figure 2. Graph of real values of $E_3(x,t)$ in Equation (34) is denoted at $m = \tau_0 = 1$, $\tau_1 = k = \xi_3 = \xi_4 = \alpha_1 = 2, \alpha_2 = r = 3,\ p = 4, -25 < x < 0,\ -1 < t < 1$ and the second graph remarks on real values of $E_3(x,t)$ in Equation (34) for $-25 < x < 0,\ t = 1$.

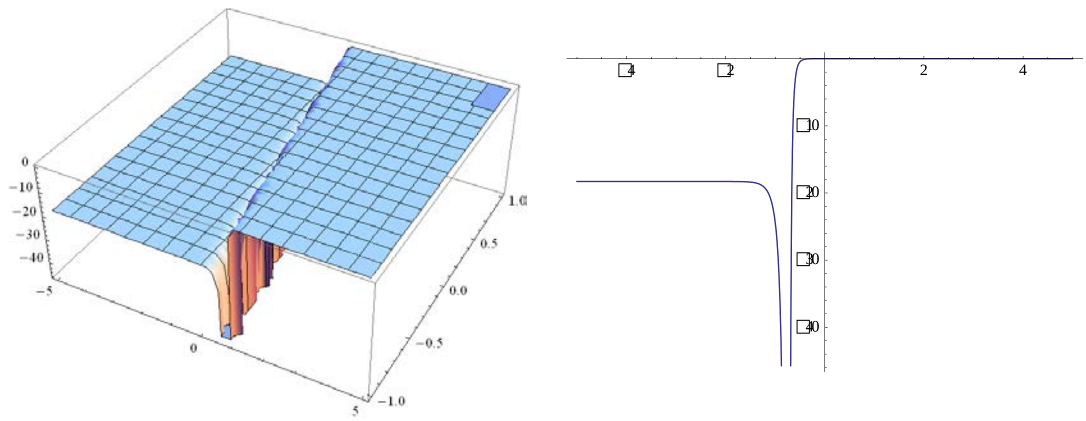

Figure 3. Graph of $n_3\,(x,t)$ in Equation (34) is drawn at $\tau_0 = 1,\ \tau_1 = k = \xi_3 = \xi_4 = \alpha_1 = 2,$ $\alpha_2 = r = 3,\ p = 4,$ $-5 < x < 5,\ -1 < t < 1$ and the second graph shows $n_3\,(x,t)$ in Equation (34) for $-5 < x < 5,\ t = 1$.

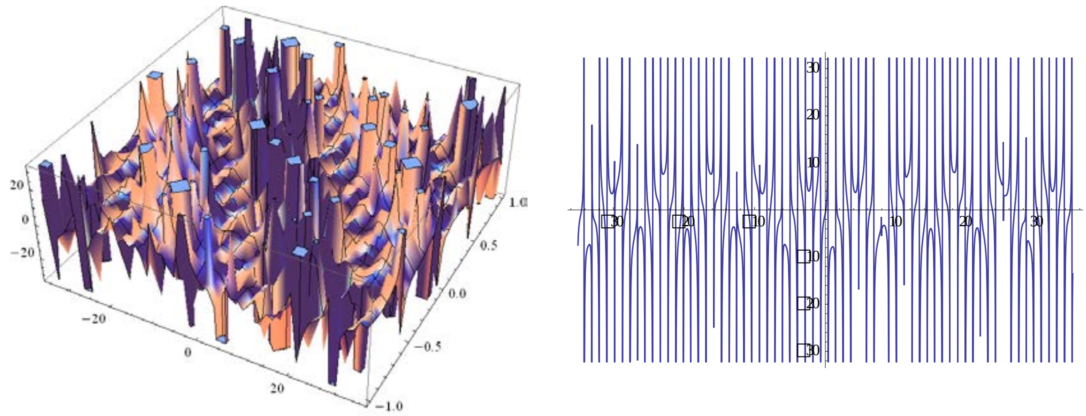

Figure 4. Graph of imaginary values of $E_5\,(x,t)$ in Equation (36) is indicated at $\tau_0 = \tau_1 = \alpha_1 = 1,$ $p = -1, k = r = \xi_4 = \alpha_2 = 2,\ m = \alpha_3 = 3,\ \xi_3 = \alpha_4 = 4,\ -35 < x < 35,\ -1 < t < 1$ and the second graph illustrates imaginary values of $E_5\,(x,t)$ in Equation (36) for $-35 < x < 35,\ t = 1$.

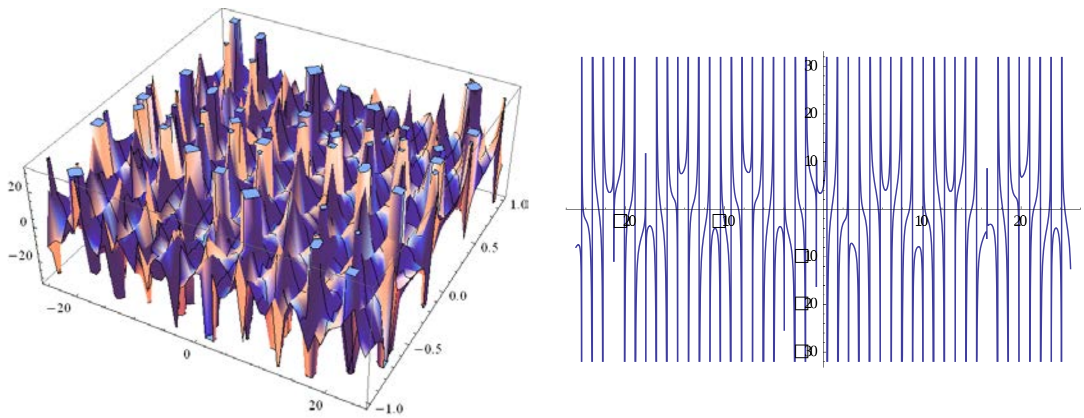

Figure 5. Graph of real values of $E_5\,(x,t)$ in Equation (36) is denoted at $\tau_0 = \tau_1 = \alpha_1 = 1,\ p = -1,$ $k = r = \xi_4 = \alpha_2 = 2,\ m = \alpha_3 = 3,\ \xi_3 = \alpha_4 = 4,\ -25 < x < 25,\ -1 < t < 1$, and the second graph remarks on real values of $E_5\,(x,t)$ in Equation (36) for $-25 < x < 25,\ t = 1$.

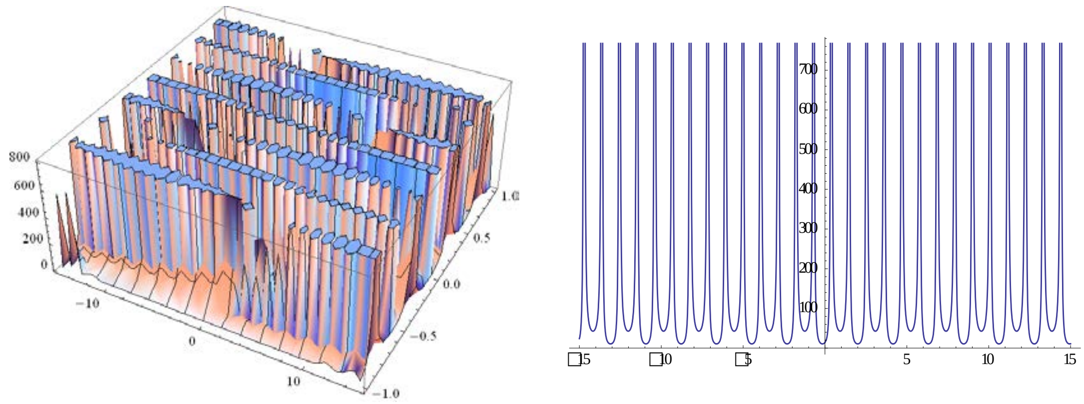

Figure 6. Graph of $n_5(x,t)$ in Equation (36) is drawn at $\tau_0 = \tau_1 = \alpha_1 = 1$, $p = -1$, $k = r = \xi_4 = \alpha_2 = 2$, $\alpha_3 = 3$, $\xi_3 = \alpha_4 = 4$, $-15 < x < 15$, $-1 < t < 1$, and the second graph illustrates $n_5(x,t)$ in Equation (36) for $-15 < x < 15$, $t = 1$.

Acknowledgments: The authors would like to thank the reviewers for their valuable comments and suggestions to improve the present work.

Author Contributions: All authors have equally contributed to this paper. They have read and approved the final version of the manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Degtyarev, L.M.; Nakhan'kov, V.G.; Rudakov, L.I. Dynamics of the formation and interaction of Langmuir solitons and strong turbulence. *Zhurnal Eksp. Teor. Fiz.* **1974**, *67*, 533–542.
2. Degtyarev, L.M.; Zakharov, V.E.; Sagdeev, R.Z.; Solov'ev, G.I.; Shapiro, V.D.; Shevchenko, I.V. Langmuir collapse under pumping and wave energy dissipation. *Zhurnal Eksp. Teor. Fiz.* **1983**, *85*, 1221–1231.
3. Anisimov, S.I.; Berezovskii, M.A.; Ivanov, M.F.; Petrov, I.V.; Rubenchick, A.M.; Zakharov, V.E. Computer simulation of the Langmuir collapse. *Phys. Lett. A* **1982**, *92*, 32–34. [CrossRef]
4. Anisimov, S.I.; Berezovskii, M.A.; Zakharov, V.E.; Petrov, I.V.; Rubenchik, A.M. Numerical simulation of a Langmuir collapse. *Zhurnal Eksp. Teor. Fiz.* **1983**, *84*, 2046–2054.
5. D'yachenko, A.I.; Zakharov, V.E.; Rubenchik, A.M.; Sagdeev, R.Z.; Shvets, V.F. Numerical simulation of two-dimensional Langmuir collapse. *Zhurnal Eksp. Teor. Fiz.* **1988**, *94*, 144–155.
6. Zakharov, V.E.; Pushkarev, A.N.; Rubenchik, A.M.; Sagdeev, R.Z.; Shvets, V.F. Numerical simulation of three-dimensional Langmuir collapse in plasma. *Zhurnal Eksp. Teor. Fiz.* **1988**, *47*, 287–290.
7. Benilov, E.S. Stability of plasma solitons. *Zhurnal Eksp. Theor. Fiz.* **1985**, *88*, 120–128.
8. Zakharov, V.E.; Pushkarev, A.N.; Sagdeev, R.Z.; Soloviev, S.I.; Shapiro, V.D.; Shvets, V.F.; Shevchenko, V.I. "Throughout" modelling of the one-dimensional Langmuir turbulence. *Sov. Phys. Dokl.* **1989**, *34*, 248–251.
9. Dyachenko, A.I.; Pushkarev, A.N.; Rubenchik, A.M.; Sagdeev, R.Z.; Shvets, V.F.; Zakharov, V.E. Computer simulation of Langmuir collapse. *Phys. D* **1991**, *52*, 78–102. [CrossRef]
10. Rubenchik, A.M.; Zakharov, V.E. *Strong Langmuir Turbulence in Laser Plasma*; Handbook of Plasma Physics; Elsevier Science Publishers: Amsterdam, The Netherlands, 1991; Volume 3, pp. 335–360.
11. Musher, S.L.; Rubenchik, A.M.; Zakharov, V.E. Weak Langmuir turbulence. *Phys. Rep.* **1995**, *252*, 178–274. [CrossRef]
12. Robinson, P.A.; Willes, A.J.; Cairns, I.H. Dynamics of Langmuir and ion-sound waves in type III solar radio sources. *Astrophys. J.* **1993**, *408*, 720–734. [CrossRef]
13. Chen, Y.-H.; Lu, W.; Wang, W.-H. The Nonlinear Langmuir Waves in a Multi-ion-Component Plasma. *Commun. Theor. Phys.* **2001**, *35*, 223–228.
14. Soucek, J.; Krasnoselskikh, V.; de Wit, T.D.; Pickett, J.; Kletzing, C. Nonlinear decay of foreshock Langmuir waves in the presence of plasma inhomogeneities: Theory and Cluster observations. *J. Geophys. Res.* **2005**, *110*, 1–10. [CrossRef]

15. Dodin, I.Y.; Geyko, V.I.; Fisch, N.J. Langmuir wave linear evolution in inhomogeneous nonstationary anisotropic plasma. *Phys. Plasmas* **2009**, *16*, 1–9. [CrossRef]
16. Zaslavsky, A.; Volokitin, A.S.; Krasnoselskikh, V.V.; Maksimovic, M.; Bale, S.D. Spatial localization of Langmuir waves generated from an electron beam propagating in an inhomogeneous plasma: Applications to the solar wind. *J. Geophys. Res.* **2010**, *115*, 1–11. [CrossRef]
17. Ratcliffe, H.; Brady, C.S.; Rozenan, M.B.C.; Nakariakov, V.M. A comparison of weak-turbulence and particle-in-cell simulations of weak electron-beam plasma interaction. *AIP Phys. Plasmas* **2014**, *21*, 1–9. [CrossRef]
18. Ajima, N.Y.; Wa, M.O. Formation and Interaction of Sonic-Langmuir Solitons. *Prog. Theor. Phys.* **1976**, *56*, 1719–1739. [CrossRef]
19. Zakharov, V.E. Collapse of Langmuir Waves. *Zhurnal Eksp. Teor. Fiz.* **1972**, *62*, 1745–1759.
20. Zhen, H.-L.; Tian, B.; Wang, Y.-F.; Liu, D.-Y. Soliton solutions and chaotic motions of the Zakharov equations for the Langmuir wave in the plasma. *AIP Phys. Plasmas* **2015**, *22*. [CrossRef]
21. Dubinov, A.E.; Kitayev, I.N. New solutions of the Zakharov's equation system for quantum plasmas in form of nonlinear bursts lattice. *AIP Phys. Plasmas* **2014**, *21*. [CrossRef]
22. Khan, Y.; Faraz, N.; Yildirim, A. New soliton solutions of the generalized Zakharov equations using He's variational approach. *Appl. Math. Lett.* **2011**, *24*, 965–968. [CrossRef]
23. Yang, X.-L.; Tang, J.-S. Explicit exact solutions for the generalized Zakharov equations with nonlinear terms of any order. *Comput. Math. Appl.* **2009**, *57*, 1622–1629.
24. Javidi, M.; Golbabai, A. Exact and numerical solitary wave solutions of generalized Zakharov equation by the variational iteration method. *Chaos Solitons Fractals* **2008**, *36*, 309–313. [CrossRef]
25. Wang, Y.-Y.; Dai, C.-Q.; Wu, L.; Zhang, J.-F. Exact and numerical solitary wave solutions of generalized Zakharov equation by the Adomian decomposition method. *Chaos Solitons Fractals* **2007**, *32*, 1208–1214. [CrossRef]
26. De Oliveira, G.I.; Rizzato, F.B. Scaling laws for breathing frequencies of solitary modes in the Zakharov equations. *Phys. Rev. E* **2001**, *65*. [CrossRef] [PubMed]
27. Bulut, H.; Pandir, Y.; Demiray, S.T. Exact Solution of Nonlinear Schrödinger's Equation with Dual Power-Law Nonlinearity by Extended Trial Equation Method. *Waves Random Complex Media* **2014**, *24*, 439–451. [CrossRef]
28. Demiray, S.T.; Pandir, Y.; Bulut, H. New Soliton Solutions for Sasa-Satsuma Equation. *Waves Random Complex Media* **2015**, *25*, 417–428. [CrossRef]
29. Demiray, S.T.; Pandir, Y.; Bulut, H. New Solitary Wave Solutions of Maccari System. *Ocean Eng.* **2015**, *103*, 153–159. [CrossRef]
30. Demiray, S.T.; Bulut, H. New Exact Solutions of the New Hamiltonian Amplitude Equation and Fokas Lenells Equation. *Entropy* **2015**, *17*, 6025–6043. [CrossRef]
31. Demiray, S.T.; Bulut, H. Some Exact Solutions of Generalized Zakharov System. *Waves Random Complex Media* **2015**, *25*, 75–90. [CrossRef]
32. Demiray, S.T.; Pandir, Y.; Bulut, H. All Exact Travelling Wave Solutions of Hirota Equation and Hirota-Maccari System. *Optik* **2016**, *127*, 1848–1859. [CrossRef]
33. Pandir, Y.; Gurefe, Y.; Kadak, U.; Misirli, E. Classification of exact solutions for some nonlinear partial differential equations with generalized evolution. *Abstr. Appl. Anal.* **2012**, *2012*. [CrossRef]