Mathematical & Computational Applications, Vol. 2, No. 1, pp. 1-11, 1997.

SUM RULE APPROACH TO NUCLEAR COLLECTIVE VIBRATION

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Abstract - The NEWSR and the EWSR are studied microscopically within TDA and RPA methods. By exploiting the analytic properties of the electromagnetic and allowed β- transitions matrix elements the theory of residue and contour integrals is used to show that exact calculation of the NEWSR and EWSR is possible . In TDA (non - degenerate limit) we deduce formula for the EWSR of $E\lambda$ transition strength. We have given different way of the proof of Thouless theorem about exactly exhausting the EWSR of $E\lambda$ transitions matrix elements in RPA. The unlike particle-hole formalism is developed for allowed β transitions. We find that TDA and RPA conserve the NEWSR and EWSR for Fermi and G-T β-decays.

1.INTRODUCTION

Microscopic nuclear models are successfully used to investigate the properties of nuclear collective excitations [1]. This model involves the concept of effective nuclear interaction. For each class of nuclear effects, one finds the corresponding most important component of nuclear forces , only this component is then used in the calculation. The nuclear many-body problem is thus reduced to a problem with a limited nuclear degrees of freedom .The equations for the problem are defined by means of the Green functions method [2] , the finite Fermi system theories [3] the Tamm-Dancoff approximation (TDA) and the random-phase approximation (RPA)[4]. All of them contain description of collective motion in many body system. However exactly solvable models are very important in nuclear physics for the prediction of the reduced probability of the electric and magnetic multipole radiation, α and β- transition probabilities .

In quantum mechanics, the transition probabilities of the system from one state to the other one are restricted by certain relations which are valid for the matrix elements and these relations are called the sum rules The sum rules are often used in the atomic, nuclear, and particle physics.

The sum rule approach is not exclusive to nuclear physics ,this is quite general and often used in other theoretical physics applications . The sum rule description was first used by H.Bethe (1930) to get the general formula for the effective retardation of fast electrons by hydrogen atoms . The sum rule approach has been widely used in dispersion theory of resonance nuclear reactions, which is well known for reduced width of nuclear levels. According to the rule, the sum of reduced width of nuclear levels for given reaction products has a certain value. The sum rule method has been extensively employed in microscopic nuclear theory in order to investigate the properties of nuclear collective excitations[5-8] .For example, the total cross-section for Eλ gamma absorption can be deduced from the energy-weighted sum rule (EWSR) for the electric 2^{λ}-pole transition matrix elements. The sum rules allow us to obtain the approximate oscillator estimate of the giant dipole and quadrupole resonance energy in the case of an arbitrary potential in nuclei [9,10].To check the accuracy of the TDA and RPA solutions, we can use the sum rule approach for nuclear transition matrix elements .

Numerical calculations of the sum rules within the framework of modern microscopic models of nucleus are simple with a small number of the phonon states. However, in many real cases the spectrum of such states is characterized by high density. This gives rise to considerable difficulties in exact calculations of all the eigenvalues ω_n and correct evaluation of transition matrix elements .In view of this ,it is useful to consider the approach in which the evaluation of sum rules can be made analytically .Alternative and more direct method of calculation of the sum of beta transition matrix elements was proposed in article[11].We note that similar difficulties can be avoided in the case of the beta transition properties of highly excited states in double beta decay by using the analytic properties of nuclear matrix elements [12].

In this paper, using the analytic properties of the electromagnetic and beta transition nuclear matrix elements and the theory of residues and contour integrals, we have derived formulas for the energy- weighted and none energy-weighted sum rules (NEWSR) for matrix elements of the electromagnetic and allowed beta transition operators in TDA and RPA approximations .

2.SUM RULES

In quantum mechanics, the sum rules for the transition matrix elements from one state to the other one are obtained by using the commutation relations of the transition operators and their hermitic conjugates with each other, and with the system Hamiltonian by making explicit use of the closure relation of exact eigenstates of the system. Sum rules have two types: EWSR and NEWSR. First, we will show the use of sum rules, and then the calculation of these sum rules in TDA and RPA method.

None energy-weighted sum rule ,is obtained from the law of matrix multiplication

$$(fg)_{mn} = \sum_{k} f_{mk} g_{kn} \tag{1}$$

For any operator f , the transition probability from the ground state to the excited states of the system is given by the sum rules

$$\sum_{n} \left|< n \left|f\right| 0 >\right|^{2} = \sum_{n} < 0 \left| f^{+} \right| n > < n \left|f\right| 0 > = < 0 \left|f^{+} f\right| 0 > \tag{2}$$

Here $|0>$ and $|n>$ are the wave-functions of ground and excited states of many-body particle system, respectively.

Sum-rule (2) is widely used in the processes in which the electric charge is conserved. As it is seen from the formula, transition probability from ground state to all excited states is equal to the expectation value of square module of the transition operator in the ground state.

We apply (2) to the commutator of operators

$$F=[f,f^{+}] \tag{3}$$

A direct calculation gives

$$\sum_{n}\left(\left|<n\left|f\right|0>\right|^{2}-\left|<n\left|f^{+}\right|0>\right|^{2}\right)=<0\left|F\right|0> \tag{4}$$

Since beta-transition operators are not hermitic, this sum rule is widely used in the allowed beta transitions.

Using the relation between the matrix elements of some transition operators and its derivative with respect to time, the general formula for the EWSR can be calculated by means of summation theorem [13] which is

$$\sum_{n}(E_{n}-E_{0})\left|<n\left|f\right|0>\right|^{2}=\frac{1}{2}<0\left|\left[f^{+}\left[H,f\right]\right]\right|0> \tag{5}$$

Here E_0 and E_n are energies of the ground state and the excited state , respectively.

The matrix elements of the quantity f and of its commutation with the Hamiltonian H are related

$$<n\left|f\right|0>=\frac{<n\left|\left[H,f\right]\right|0>}{E_{n}-E_{0}} \tag{6}$$

Direct calculation of the left hand side of (5) using (6),we have the required theorem (5).

The right-hand sides of the sum rules in (2),(4) and (5) for transition operators are independent from the properties of the exciting energy levels considered and their calculation methods, and are calculated with the help of the ground state wave-function. On the other hand, since the left-hand side of the sum rules contains the excited states wave functions their values depend on model and the accuracy of the methods used. Thus, sum rules simplify the results obtained without calculating the matrix elements numerically, and to check the accuracy degree of the methods used.

Another importance of sum rules is that they are independent from the model for the particular transitions. For example, the EWSR (5) for electric-dipole and electric-quadrupole transitions are of definite value, because they can be compared with model independent estimate, obtained by ignoring the effects of exchange and velocity-dependent interactions [7]. This also provides to understand whether the models applied to many-body system are useful or not.

3.BASIC EQUATIONS

The key problem in the program for investigating accuracy of TDA and RPA approaches is the calculation of the EWSR and NEWSR. A detailed description of the use of the RPA and TDA in microscopic nuclear model was given in many articles [1,2] The equations of the TDA may be obtained from the more general equations of RPA. In this section, we shall discuss the nuclear processes that give an information about the integral characteristics of the nuclear vibrations. Here we are mainly interested in E1,E2,M1 and allowed Fermi and Gamov-Teller (GT) β-transitions. We shall use the model with pairing and multipole-multipole interaction for electric dipole and quadrupole vibrations and spin -isospin interaction for magnetic-dipole and G-T excitation. The dipole-dipole interaction describes relative motion of the protons against the neutrons. Such a vibration determines the properties of the giant dipole resonance with $I^{\pi} = 1^-$ [7,9]. The quadrupole-quadrupole interactions generate the vibration with the quantum number $I^{\pi} = 0^+, 2^+$ (K=0,2) in deformed nuclei [1,14].

In the RPA, the Hamiltonian of system is reduced to the form[1]:

$$H = \sum_n \omega_n Q_n^+ Q_n$$

Where the phonon operators Q_n^+ describe the intrinsic I^{π} excitations generating by multipole-multipole interactions and eigenenergies $\omega_n > 0$ are solutions to the equation

$$D(\omega_n) = 1 + \chi\left[F_n(\omega_n) + F_p(\omega_n)\right] = 0 \tag{7}$$

where

$$F_{\tau}(\omega_n) = \sum_{ss} \frac{2\varepsilon_{ss'} f_{ss'}^{(\lambda)2} U_{ss'}^2}{\varepsilon_{ss}^2 - \omega_n^2}, \tag{8}$$

Here $U_{ss'} = u_s v_{s'} - u_{s'} v_s$ and τ denotes the summation over the states of identical particles (neutrons or protons). $\varepsilon_{ss'}$ and $f_{ss'}^{(\lambda)} \equiv <s'\left|r^{\lambda} Y_{\lambda\mu}\right|s>$ are single quasiparticle energies and reduced matrix elements of multipole transition operators $f_{\lambda\mu} = r^{\lambda} Y_{\lambda\mu}$, respectively. u_s and v_s are quasiparticle occupation parameters.

4.ELECTRIC MULTIPOLE TRANSITIONS

The explicit form of the matrix element of electric 2^{λ}-pole transition from the ground state to one phonon state with spin and parity I^{π} is [1,14]

$$M = <n\left|\sum_{i=1}^{z}(r^{\lambda} Y_{\lambda\mu})_i\right| 0> = \frac{F_p(\omega_n)}{\sqrt{Z(\omega_n)}} \tag{9}$$

where

$$Z(\omega_n) \equiv 4\omega_n\left(\sum_{prot} \frac{\varepsilon_{ss'} f_{ss'}^{(\lambda)2} U_{ss'}^2}{(\varepsilon_{ss'}^2 - \omega_n^2)^2} + \sum_{neut} \frac{\varepsilon_{ss'} f_{ss'}^{(\lambda)2} U_{ss'}^2}{(\varepsilon_{ss'}^2 - \omega_n^2)^2}\right) \tag{10}$$

Following the usual procedure for generalised nuclear model calculations, we find the reduced $E\lambda$ transitions probability

$$B(E\lambda, 0^+ \rightarrow \lambda^{\pi}) = |M|^2 = \frac{F_p^2(\omega_n)}{Z(\omega_n)} \tag{11}$$

Now using Eqs. (11), we can write the EWSR (5) and NEWSR (2) in classical form

$$S_{E\lambda} = \sum_n \omega_n B(E\lambda, 0^+ \rightarrow \lambda_n^{\pi}) = \frac{1}{2} <0\left|\left[f^+,[H,f]\right]\right|0> \tag{12}$$

$$S_{NE\lambda} = \sum_n \left|<n|f|0>\right|^2 = <0\left|f^+ f\right|0> \tag{13}$$

To emphasize the physical importance of the EWSR we should mention that it defines the total integrated cross sections of electric 2^{λ} pole photon absorption [9]

$$\sigma_{E\lambda} = \int \sigma(E) E dE = \frac{32\pi^4}{9hc} S_{E\lambda}$$

According to [7] we can write the model-independent estimate of $S_{E\lambda}$,obtained by ignoring the effects of exchange and velocity dependent interactions considering all $I^{\pi} =\lambda^{\pi}$ excitations of the nucleus up to the threshold for meson formation

$$S_{E\lambda} = \frac{\lambda(2\lambda+1)^2}{32\pi^3}\frac{e^2h^2}{m} < r^{2\lambda-2} > Z \tag{14}$$

where Z is the number of protons in nuclei, m is the mass of proton . $<r^{2\lambda-2}>$is the mean value of $r^{2\lambda-2}$ at the Hartree-Fock (HF) ground state, an estimate of it is obtained by approximating the nuclear density to be constant throughout and of radius R_0 .This gives

$$< r^{2\lambda-2} >= \frac{3}{2\lambda+1}R_0^{2\lambda-2}$$

The sum rule (14) includes both $\Delta T=0$ and $\Delta T=1$ excitations. If isobaric spin is conserved,it splits between to

$$S_{E\lambda,T=0} = \frac{\lambda(2\lambda+1)^2}{8\pi}\frac{e^2h^2}{m}\frac{Z^2}{A} < r^{2\lambda-2} >$$

and

$$S_{E\lambda,T=1} = \frac{\lambda(2\lambda+1)^2}{8\pi}\frac{e^2h^2}{m}\frac{NZ}{A} < r^{2\lambda-2} >$$

Let us calculate right-hand side of (12) and (13) in the quasiparticle representation
(averaging with respect to the Hartree-Fock-Bogolyubov (HFB) ground state wave function):

$$< 0|[f^+,[H,f]]|0 >_{HFB} = 2\sum_{ss'} \varepsilon_{ss'} f_{ss'}^{(\lambda)2} U_{ss'}^2 \tag{15}$$

$$< 0|f^+f|0 >_{HFB} = \sum_{ss'} f_{ss'}^{(\lambda)2} U_{ss'}^2 \tag{16}$$

Thouless[15] showed that left hand side of (12) calculated with RPA is equal to the right hand side of (12) calculated using the HF(HFB) ground state wave function (Eq. (14)).Without numerical calculations it is not easy to see which approximations TDA or RPA is more reliable. However using the analytic properties of transition matrix elements we give another proof of Thouless theorem for the case $E\lambda$ transition operators. We can show that the following useful relation is valid

$$Z_n = \frac{1}{\chi}\frac{dD(\omega_n)}{d\omega} = \frac{D'(\omega_n)}{\chi} \tag{17}$$

If we use (11) and (17), we find that the general expression for $S_{E\lambda}$ given by (12) assumes the form

$$S_{E\lambda} = \chi \sum_{n=1,2,..} \frac{\omega_n F(\omega_n)^2}{D'(\omega_n)} = \frac{1}{2}\chi \sum_{n=\pm1,\pm2,..} \frac{\omega_n F(\omega_n)^2}{D'(\omega_n)} \tag{18}$$

This sum is very laborious for evaluation. Since ω_n are the zeros of the function $D(\omega_n)$ the basic theorem of the theory of residues [16] now allows us to write the expression for $S_{E\lambda}$ in the form of the contour integral

$$S_{E1} = \frac{\chi}{4\pi i}\oint_{L_n} \frac{zF(z)^2}{D(z)}dz \tag{19}$$

The contour L_n contains first-order singularities of the integrands at $z=\pm\omega_n$ which are the zeros of the corresponding function D(z). Analysis show that, outside L_n, the integrands in (19) have singularities at $z=\pm\varepsilon_\mu$ and the corresponding residues can be evaluated relatively simply. Using the main theorem of the residue theory (the sum of all residues of the analytic function f(z) is equal to zero) we have

$$\sum_{n=\pm1,\pm2,..} \frac{\omega_n F^2(\omega_n)}{D'(\omega_n)} + \sum_\mu \mathrm{Res}\,\varphi(+\varepsilon_\mu) + \sum_\mu \mathrm{Res}\,\varphi\,(-\varepsilon_\mu) = 0 \qquad (20)$$

where

$$\varphi(z) = \frac{z\,F^2(z)}{D(z)}\,,$$

$$\mathrm{Res}\,\varphi(-\varepsilon_\mu) = \mathrm{Res}\,\varphi(\varepsilon_\mu) = -\sum_\mu \varepsilon_\mu\, f_\mu^2\, u_\mu^2 \qquad (21)$$

Using (20) and (21) we obtain the following expression for left-hand side of the EWSR(12) in the RPA

$$S_{E\lambda} = \sum_{n>0} \omega_n\, B(E\lambda, 0^+ \to \lambda_n^\pi) = \sum_{ss'} \varepsilon_{ss'}\, f_{ss'}^{(\lambda)2}\, U_{ss'}^2 \qquad (22)$$

It is the proof of Thoules theorem [15], Eq.(22) exactly exhausts the EWSR calculated using the HFB ground state wave function (Eq.(15)).

If we use the TDA method in similar calculations, the EWSR becomes model dependent

$$S_{E\lambda}^{TDA} = \sum_n \omega_n\, B(E\lambda, 0^+ \to \lambda_n^\pi) = \sum_{ss'} f_{ss'}^{(\lambda)2}\, U_{ss'}^2\, \varepsilon_{ss'} + \chi \left|\sum_{ss'} f_{ss'}^{(\lambda)2}\, U_{ss'}^2\right|^2 \qquad (23)$$

and violates the EWSR. This expression for the EWSR is first obtained in this paper and in the degenerate limit when all the quasi-particle energies are equal to common i.e. $\varepsilon_{ss'}=\bar{\varepsilon}$ gives the well known result of [7].

In the degenerate limit Eq. (23) predicts a pure collective single state

$$\omega_{coll} = \bar{\varepsilon} + \chi \sum_{ss'} f_{ss'}^{(\lambda)2} U_{ss'}^2$$

which exactly exhausts the non- energy weighted sum rule (13):

$$B(E\lambda, 0^+ \to \lambda_{coll}^\pi) = \sum_{ss'} f_{ss'}^{(\lambda)2} U_{ss'}^2$$

Thus the TDA method in general(see Eq.(23)) gives small energy weighted sum rule for attractive interactions ($\chi<0$) and to a large value for the repulsive interactions ($\chi>0$). The TDA satisfies the NEWSR but violates the EWSR, whereas exactly the opposite was true for the RPA .The energy weighted sum rule in TDA method is not so much physical value as the one for RPA method ,since its magnitude depends on the constant χ of the effective interactions. For this reason RPA method is usually preferred, since EWSR in this method is almost model-independent and therefore more reliable .

5.MAGNETIC DIPOLE TRANSITION

Since the deformed nucleus is assumed to be rotating about the x axis, we must separate the collective magnetic moment connected with this motion from the x component of the magnetic-moment operator[16]

$$\vec{\mu}=\frac{1}{2}\sum_{i,\tau}\left[(g_s^{\tau}-g_l^{\tau})\vec{\sigma}_i+g_l^{\tau}\vec{j}_i\right] \tag{24}$$

where σ are the Pauli matrices, g_s^{τ} and g_e^{τ} are the free nucleon spin and orbital gyro-magnetic ratios, and the sum rule over all nucleons .The key problem for investigating 1^+ states is the isolation of the rotational branch from the internal excitation spectrum. A detailed description of the use RPA for 1^+ excitation was given in[16].Here we are mainly interested in the consequences for internal I^{π} $K=1^+1$ excitations and its M1 transition probability when allowance is made for the conservation of angular momentum .

A more important characteristic of the one phonon 1+ states is the reduced M1 transition probability for the ground state ,which in the RPA has the form[16]

$$B(M1,0^+\to 1_n^+)=\frac{3}{4\pi}\frac{\omega_n}{4Z(\omega_n)}\left[J_p(\omega_n)+\sum_{\tau}(g_s^{\tau}-g_l^{\tau})X^{\tau}(\omega_n)\right]\mu_N^2 \tag{25}$$

where for one kind of nucleon

$$J_{\tau}(\omega_n)=\sum_{ss'}\frac{2E_{ss'}j_{ss'}^2L_{ss'}^2}{E_{ss'}^2-\omega^2}$$

$$Z(\omega_n)=\frac{1}{4}\omega_n\frac{dJ(\omega_n)}{d\omega_n}$$

Here μ_N is the Bohr magneton, $j_{ss'}\equiv\langle s'|j_x|s\rangle$ are the single-particle matrix elements of the angular momentum operator and in the usual notation $L_{ss'}=u_sv_{s'}-u_{s'}v_s$.The excitation energies ω_n - are solutions of the dispersion equation

$$\omega_n^2\left[J_n(\omega_n)+J_p'(\omega_n)\right]=\omega_n^2J(\omega_n)=0 \tag{26}$$

In HFB approximation direct calculation of the commutator in the right-hand side of the EWSR (12) gives[16]

$$\left[\mu^+,[H,\mu]\right]_{HFB}=\frac{3}{4\pi}\left[\gamma_p+\sum_{\tau}(g_s^{\tau}-g_l^{\tau})\gamma_{\sigma}^{\tau}-\frac{\gamma_p^2}{\gamma_p+\gamma_n}\right]\mu_N^2 \tag{27}$$

where

$$\gamma=4\sum_{ss'}E_{ss'}j_{ss'}^2L_{ss'}^2,\quad \delta_{\sigma}=\sum_{ss'}E_{ss'}\sigma_{ss'}^2L_{ss'}^2$$

In RPA approach using the analytical properties of the B(M1) and dispersion Eq.(26), the following formulas for EWSR are obtained using the residue theory and contour integrals:

$$S_{M1}^{RPA} = \frac{3}{8\pi}\left[\ \gamma_p + \sum_{\tau}(g_s^{\tau} - g_l^{\tau})\gamma_{\sigma}^{\tau} - \frac{\gamma_p^2}{\gamma_p + \gamma_n}\ \right]\mu_N^2 \qquad (28)$$

and exactly exhausts the EWSR calculated using the HFB ground state wave function. Thus in the RPA for the magnetic dipole transitions we get

$$\sum_{n=0,1,..}\omega_n B(M1, 0^+ \to 1_n^+) = \frac{1}{2} < 0\left|\left[M^+, [H, M]\right]\right|0 >_{HFB} \qquad (29)$$

The last term in the square brackets of (28) represents the contribution related to ground state correlations caused by the non-spherical nature of the deformed nucleus and the effective interactions restoring the rotational invariance of the Hamiltonian. It can be shown by a similar calculation that TDA method do not conserve EWSR (29) and gives the uncorrect result for M1-transition.

6.GAMOV-TELLER AND FERMI β-DECAY MATRIX ELEMENTS

In this section, we shall apply the method developed before for the calculation of sum rule for allowed GT and Fermi β decay matrix elements. It is interesting to study in the microscopic approach (unlike particle-hole RPA)the properties of 0^+ and 1^+ states generated charge-exchange interactions of the form $\chi_F \bar{\tau}\tau + \chi_{GT}\bar{\sigma}'\bar{\sigma}\tau\tau$ in odd-odd nucleus and to estimate their beta transitions sum rules (σ and τ are the spin and isospin matrices). The excitation energies ω_n are obtained from [11]

$$D(\omega_n) = \left(\frac{1}{2\chi_\beta} + f(\omega)\right)\left(\frac{1}{2\chi_\beta} + \bar{f}(\omega)\right) - t^2(\omega) = 0 \qquad (30)$$

$$f = \sum_v\left(\frac{b_v^2}{\varepsilon_v - \omega_n} + \frac{\bar{b}_v^2}{\varepsilon_v + \omega_n}\right),\ \bar{f} = \sum_v\left(\frac{\bar{b}_v^2}{\varepsilon_v - \omega_n} + \frac{b_v^2}{\varepsilon_v + \omega_n}\right)$$

$$t = \sum_v b_\mu \bar{b}_v\left(\frac{1}{\varepsilon_v - \omega_n} + \frac{1}{\varepsilon_v + \omega_n}\right)$$

where $b_\mu \equiv u_p v_n \langle p\|\bar{\sigma}\|n\rangle, \bar{b}_\mu \equiv u_n v_p \langle p\|\bar{\sigma}\|n\rangle$; and $\varepsilon_\mu = \varepsilon_p + \varepsilon_n$ is the two quasiparticle energy of a neutron -proton pair. The RPA treatment of the collective Fermi 0^+ states is identical with the treatment given by GT oscillation if $\langle n\|\ \bar{\sigma}\ \|p\rangle$ of the latter is simply replaced by the overlap $\langle n|p\rangle$. If we discard factors containing $\frac{1}{\varepsilon_v + \omega_n}$ in (30) we obtain the formulas corresponding to the TDA approximation. The matrix elements of the β- transitions from the ground state of $|0\rangle$ of an even -even nucleus to different one phonon states $|n\rangle$ of an odd-odd nucleus

$$M_{\beta^-}^n = < n|\beta_-|0 > = \frac{L_n}{2\chi_\beta\sqrt{Y_n}} \qquad (31)$$

$$M_{\beta^+}^n = < n|\beta_+|0 > = \frac{1}{2\chi_\beta}\frac{1}{\sqrt{Y_n}}$$

$$Y(\omega_n) = \frac{D'(\omega_n)}{1/2\chi + f} \quad (32)$$

$$L = -\frac{t}{1/2\chi_\beta + f} = -\frac{1/2\chi_\beta + \bar{f}}{t} \quad (33)$$

Here, in the allowed beta transition, operator $\beta^{\pm}$ may be written

$$\beta_{\pm} = \begin{cases} \sum_i t_{\pm}^i & \text{for Fermi decay} \\ \sum_i \vec{\sigma}_i t_{\pm}^i & \text{for GT decay} \end{cases} \quad (34)$$

where $t_{\pm}$ changes a proton (neutron) into a neutron (proton). According to (3) and (4) the matrix elements (31) and (32) satisfy the following sum rule

$$\sum_{n>0} \left(\left| M_{\beta^-}^n \right|^2 - \left| M_{\beta^+}^n \right|^2 \right) = < 0 \left| \left[\beta_+, \beta_- \right] \right| 0 > \quad (35)$$

Let us calculate right-hand side of the (35) in the quasiparticle representation (averaging with respect HFB ground)

$$\left[\beta_+, \beta_- \right]_{qp} = < 0 \left| \left[\beta_+, \beta_- \right] \right| 0 >_{HFB} = \sum_{\mu} (b_\mu^2 - \bar{b}_\mu^2) \quad (36)$$

The sum rule (35) is physically valuable because it can be compared with the following model independent estimate containing the neutron and proton numbers of nuclei :

$$< 0 \left[\beta^{(+)}, \beta^{(-)} \right] 0 > = \begin{cases} N - Z & \text{for Fermi transition} \\ 3(N - Z) & \text{for GT transition} \end{cases} \quad (37)$$

The calculation of the left-hand side of the double commutator (35) using (31)-(33) we get

$$S_\beta = \sum_{n>0} \left(\left| M_{\beta^-}^n \right|^2 - \left| M_{\beta^+}^n \right|^2 \right) = \frac{1}{4\chi_\beta^2} \sum_{n=1,2} \frac{f(\omega_n) - \bar{f}(\omega_n)}{D'(\omega_n)} \quad (38)$$

The basic theorem of the theory of residues now allows us to write the expression S_β in the form of the contour integrals ;

$$S_\beta = \frac{1}{2\pi i} \frac{1'}{8\chi_\beta^2} \oint_{L_n} \frac{\bar{f}(z) - f(z)}{D(z)} dz \quad (39)$$

The integral S_β are very laborious for evaluation. Analysis show that ,outside L_n the integrand have singularities at $z = \infty$ and the corresponding residues can be evaluated relatively simple .We note that $z = \pm\varepsilon_\mu$ are removable singularities of the integrands in (39). Since $z = \infty$ is an isolated singularity (fig.1.) of the integrand in (39) and using the basic theorem of the theory of residues[16], we find

$$S_\beta^{RPA} = S_\beta^{TDA} = \sum_{\mu} (b_\mu^2 - \bar{b}_\mu^2) \quad (40)$$

We see that unlike particle-hole RPA and TDA exactly exhausts the NEWSR (36).

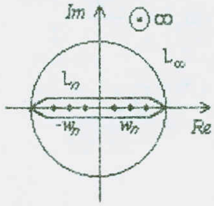

Fig 1 Contours of integration in the complex plane for (39)

In order to establish the degree of collectivization of the phonon states under investigation and the position of the GT and Fermi resonances we shall use the following EWSR :

$$\sum_{n} \omega_n (|M^{n}_{\beta^-}|^2 + |M^{n}_{\beta^+}|^2) = < 0|[\beta_+,[H,\beta_-]]|0 > \qquad (41)$$

where H is the model Hamiltonian for the problem. To verify the validity of the solutions obtained by the RPA (TDA) we can evaluate the right- hand side of (41) in the quasiparticle representation (averaging over the quasiparticle vacuum)

$$< 0|[\beta_+,[H,\beta_-]]|0 >_{HFB} = \sum_{\mu}\left[E_\mu (b^2_\mu + \bar{b}^2_\mu) + 2\chi_\beta (b^2_\mu - \bar{b}^2_\mu)\right] \qquad (42)$$

The last term in (42) represents the contribution of the effective interaction and violates the EWSR.

Thus the EWSR for β transitions is not of so much physical value as the one for E1 and E2 transitions,sinse it is magnitude depends on the model and form of the effective interactions. However ,it can be used effectively to estimate the position of Fermi and Gammov-Teller resonances, whose properties are basically determined by isovector spin forces.

7.SUMMARY

The method of calculating of the quantum mechanical sum of nuclear matrix elements with the use of theory of residues and contour integrals is used for evaluating the sum rules . The method was demonstrated for the case of the electro-magnetic and allowed beta transitions. We have given different way of the proof of Thouless theorem. Our calculations in accordance with this theorem exactly support the conclusion of Thouless about exactly exhausting the energy weighting sum rule for electromagnetic transitions in RPA. In TDA we deduce formula for the EWSR of Eλ transitions matrix elements and showed that unlike the RPA, the TDA do not predict full exhausting of the whole energy weighted sum rule and therefore is unreliable for exact calculations.

The usual like particle RPA has been extended to the calculation of sum rules for matrix elements of beta transition operator. We find that the unlike particle-hole RPA and TDA conserves the NEWSR, and showed that NEWSR (35) for allowed beta transition matrix elements is satisfied if the left-hand side is evaluated with RPA or TDA and the right-hand side is calculated with HFB ground state wave function.

In the unlike particle-hole case we deduced formula for the EWSR and showed that the sum rule is not of so much physical value as one for like particle RPA method, since its magnitude depends on the constant of effective interaction. However, it can be used effectively to estimate the position of magnetic dipole, Fermi and Gamov-Teller resonances .

Thus the RPA and TDA methods are used with the theory of residues and contour integrals to show that the exact analytical calculations of physical quantities such as the NEWSR and the EWSR of the nuclear matrix elements for the electromagnetic transitions and Fermi and Gamov-Teller β-decays are possible. We have concluded whether the methods used conserve or not sum rules without the numerical calculations, and we can see the beneficence of the methods used.

Acknowledgement-one of the auhtors (A.K.) thanks the Scientific and Technical Research Council of Turkey (TUBITAK) for their support and Sakarya University for the hospitality.

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