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A COMPLEMENTARY ON PROPOSING A NEW MODEL ON DATA ENVELOPMENT ANALYSIS BY CONSIDERING NON DISCRETIONARY FACTORS AND A REVIEW ON PREVIOUS MODELS

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Abstract- As the title suggests, this paper constitutes a modification and improvement of the paper by Gholam Abri et al, "PROPOSING A NEW MODEL ON DATA ENVELOPMENT ANALYSIS BY CONSIDERING NON DISCRETIONARY FACTORS AND A REVIEW ON PREVIOUS MODELS".

Key Words- Data Envelopment Analysis, Non-Discretionary, Efficiency.

1. INTRODUCTION

Standard DEA assumes that the assessed units (DMUs) are homogeneous, i.e. they perform the same tasks with similar objectives, consume similar inputs and produce similar outputs, and operate in similar operational environments. Often, the assumption of homogeneous environments is violated and the factors describing the differences in the environments need to be included in the analysis. These factors and others outside the control of the DMUs are frequently called non-discretionary factors.There are some approaches which seems to be more general.These approaches are developed to controlling the nondiscretionary inputs. Gholam Abri et al (2010) addressed five existing models, their strengths and weaknesses. Moreover, they introduced a new DEA model that overcomes the problems identified. But, their proposed method have some difficulties, too. In this paper, we constitute a modification and improvement.

2. MODIFICATION AND IMPROVEMENT

As obviously seen in the paper [1], model "9"is non-linear as follows because of two reasons.

$$Min \quad \Gamma = \frac{\frac{1}{t}\sum_{i=1}^{t}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\phi_r}$$

$$\begin{aligned}
s.t. \ & \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_i x_{io} && ,i=1,\dots,t \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \ge \phi_r y_{ro} && ,r=1,\dots,s \\
& \lambda_j = 0 \ \ if \ \ \sum_{i=1}^{k}\delta_i^* z_{io} < \sum_{i=1}^{k}\delta_i^* z_{ij} && , j=1,\dots,n \\
& \lambda_j \ge 0 && , j=1,\dots,n \\
& \theta_i \le 1 && ,i=1,\dots,t \\
& \phi_r \ge 1 && ,r=1,\dots,s
\end{aligned} \qquad (9)$$

At first, the objective function is fractional and it can be easily transformed to the linear one (see [1]) and then because of existing constraints of the third group.
It means that following group of constraints makes model "9" a non-linear one.

$$\lambda_j = 0 \qquad if \quad \sum_{i=1}^{k}\delta_i^* z_{io} < \sum_{i=1}^{k}\delta_i^* z_{ij}, \quad j=1,...,n \qquad (*)$$

In above constraints, because of concluding δ_i^* $(i=1,...,k)$ from model (8) and $z_{ij}(i=1,\dots,k,\ j=1,...,n)$ are scalar values, so calculating and comparing the above inner products can easily be done before solving model "9".
So, the following method can be introduced. After solving model " 8 " as follows:

$$Min \quad \Gamma = \frac{\frac{1}{m}(\sum_{i=1}^{t}\theta_i + \sum_{i=1}^{k}\delta_i)}{\frac{1}{s}\sum_{r=1}^{s}\phi_r}$$

$$\begin{aligned}
s.t. \ & \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_i x_{io} && ,i=1,\dots,t \\
& \sum_{j=1}^{n}\lambda_j z_{ij} \le \delta_i z_{io} && ,i=1,\dots,k \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \ge \phi_r y_{ro} && ,r=1,\dots,s \\
& \lambda_j \ge 0 && , j=1,\dots,n \\
& \theta_i \le 1 && ,i=1,\dots,t \\
& \delta_i \le 1 && ,i=1,\dots,k \\
& \phi_r \ge 1 && ,r=1,\dots,s
\end{aligned} \qquad (8)$$

First, $\sum_{i=1}^{k} \delta_i^* z_{io}$ is calculated for each oand we consider the (*) constraints.

In continue, we assume DMU_o a unit under evaluation. In order to obtain the real efficiency of DMU_o in the presence of non-discretionary factors, we will define the set (L) as following:

L=

But, on the other hand we know that:

$$\begin{cases} \sum_{j=1}^{n} \lambda_j x_{ij} = \sum_{j \in L} \lambda_j x_{ij} + \sum_{j \notin L} \lambda_j x_{ij} \\ \sum_{j=1}^{n} \lambda_j y_{rj} = \sum_{j \in L} \lambda_j y_{rj} + \sum_{j \notin L} \lambda_j y_{rj} \end{cases}$$

So, according to the definition of the set (L) we place:

$\sum_{j \notin L} \lambda_j = 0.$

Then, we apply the following modified model:

$$\begin{aligned} Min \quad & \Gamma = \frac{\frac{1}{t}\sum_{i=1}^{t}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\phi_r} \\ s.t. \; & \sum_{j \in L} \lambda_j x_{ij} \le \theta_i x_{io} && , i = 1, \ldots, t \\ & \sum_{j \in L} \lambda_j y_{rj} \ge \phi_r y_{ro} && , r = 1, \ldots, s \\ & \lambda_j \ge 0 && , j = 1, \ldots, n \\ & \theta_i \le 1 && , i = 1, \ldots, t \\ & \phi_r \ge 1 && , r = 1, \ldots, s \end{aligned} \qquad (9')$$

This modification has to very important values:

First, It causes the above model to change linear.

Second, It leads to reduce n constraint of the problem constraints that is vital in large size and practical problems.

3.REFERENCES

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