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RETURNS TO SCALE AND SCALE ELASTICITY IN TWO-STAGE DEA

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Abstract- Data Envelopment Analysis (DEA) provides a method to evaluate the relative efficiency of peer Decision Making Units (DMUs) that have multiple inputs and outputs. Production process in two-stage DEA is performed in the two consecutive phases and DMUs have intermediate measures, in addition to their inputs and outputs. A unique feature of the intermediate measures is that the outputs in the first stage are being treated as inputs in the second stage. The aim of this paper is to determine the returns to scale (RTS) classification and scale elasticity (SE) in two-stage DEA. Therefore an approach is introduced for estimating the RTS situation of DMUs with two-stage structure based on the consideration of SE quantity in each of the individual stages. The utilization of the proposed approach is demonstrated with a real data set.

Key Words- Returns to scale, Scale elasticity, Two-stage data envelopment analysis

1. INTRODUCTION

Data envelopment analysis is a scientific method for the performance analysis of peer decision making units, in the presence of multiple inputs and outputs. In the recent years, a number of DEA studies have focused on measuring the relative efficiency of DMUs with a two-stage structure (e.g. see [1-6]). In the two-stage DEA, DMUs have a two-stage structure and intermediate measures exist between two consecutive stages. Namely, the first stage uses the inputs to generate intermediate measures and later on the second stage uses them to produce outputs. Consequently, the intermediate measures which were determined by the first stage are all of the second stage inputs.

Meanwhile returns to scale and scale elasticity are two important topics in the production theory and since the beginning of DEA research, RTS has been widely discussed as an important economic implication of DEA efficiencies. These two concepts can determine the optimal size of efficient DMUs under variable returns to scale technology. Most of the previous attempts to deal with the two-stage DEA have only addressed measuring the performance of such two-stage processes. Therefore, this research attempts to measure RTS in the analytical framework of two-stage DEA. To achieve this goal, the production space in two-stage DEA under variable returns to scale (VRS) technology is investigated and a method for measuring RTS quality in this field, regarding the SE quantity and RTS classification in each of the individual stages, is proposed.

The structure of this research unfolds as follows: In section 2, the two-stage DEA under variable returns to scale (VRS) technology is introduced and also the cost minimization model is applied for evaluating each DMU with two-stage structure. The proposed method for estimating RTS and SE in two-stage DEA is discussed in section 3. Section 4 includes an application of the proposed method to 26 branches of an Iranian commercial bank. Finally, the concluding remarks are provided in section 5.

2. TWO-STAGE DEA UNDER VARIABLE RETURNS TO SCALE

It is assumed that there are n DMUs which their production activities are performed in two phases. In the first stage, each DMU_j $(j=1,...,n)$ uses m inputs, x_{ij} $(i=1,...,m)$, in order to produce D outputs, z_{dj} $(d=1,...,D)$.These D outputs are used as inputs to the second phase and are called intermediate measures. These intermediate measures produce s outputs, y_{rj} $(r=1,...,s)$, in the second stage. Therefore, DMU_j is characterized by the two consecutive production stages, the first stage from $\mathbf{x}\in R^m$ to $\mathbf{z}\in R^D$ and the second one from $\mathbf{z}\in R^D$ to $\mathbf{y}\in R^s$.Figure 1 visually describes this production process for DMU_j. The Production Possibility Set (PPS) under variable returns to scale (VRS) technology for each stage can be defined by the following DEA formulations:

$$PPS\,1=\left\{(\mathbf{x},\mathbf{z})\in R^m\times R^D \,\middle|\, X\lambda^1\le\mathbf{x}, Z\lambda^1\ge\mathbf{z}, \mathbf{e}\lambda^1=1, \mathbf{x},\mathbf{z}\ge\mathbf{0}, \lambda^1\ge\mathbf{0}\right\}$$
$$PPS\,2=\left\{(\mathbf{z},\mathbf{y})\in R^D\times R^s \,\middle|\, Z\lambda^2\le\mathbf{z}, Y\lambda^2\ge\mathbf{y}, \mathbf{e}\lambda^2=1, \mathbf{z},\mathbf{y}\ge\mathbf{0}, \lambda^2\ge\mathbf{0}\right\}$$

Where $X\in R^{m\times n}$, $Z\in R^{D\times n}$ and $Y\in R^{s\times n}$ are the given data set related to inputs, intermediate measures and outputs, respectively. Semi-positive vectors of weights, $\lambda^1=(\lambda_1^1,\lambda_2^1,...,\lambda_n^1)^t$ and $\lambda^2=(\lambda_1^2,\lambda_2^2,...,\lambda_n^2)^t$ are used to connect inputs and intermediate measures of n DMUs in stage 1 and intermediate measures and outputs of n DMUs in stage 2, respectively. $\mathbf{e}$ is a row vector with all elements equal to 1. $\mathbf{0}$ is a zero vector, whose dimension depends upon its corresponding vector comparison.

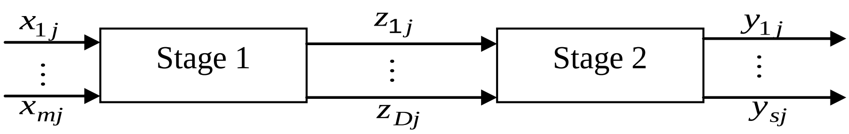

Figure 1- The production process of DMU_j in two-stage DEA.

Now the cost minimization model [7, 8] is suggested for minimizing a total production cost of DMU_o over two stages. DMU_o $(o\in\{1,...,n\})$ is the DMU under assessment.

$$
\begin{array}{lll}
Min & \mathbf{w}^t\mathbf{x}+\mathbf{v}^t\mathbf{z} & \\
s.t. & X\lambda^1 \le \mathbf{x}, & \\
 & Z\lambda^1 \ge \mathbf{z}, & \\
 & Z\lambda^2 \le \mathbf{z}, & (1) \\
 & Y\lambda^2 \ge \mathbf{y}_o, & \\
 & \mathbf{e}\lambda^1 = 1, & \\
 & \mathbf{e}\lambda^2 = 1, & \\
 & \mathbf{x},\mathbf{z} \ge \mathbf{0}, \qquad \lambda^1,\lambda^2 \ge \mathbf{0}. &
\end{array}
$$

Where $\mathbf{w} \in R^m$ and $\mathbf{v} \in R^D$ are two column vectors of inputs cost in stage 1 and intermediate measures cost in stage 2, respectively. Based on an optimal solution $\left(\mathbf{x}^*,\mathbf{z}^*,\lambda^{1^*},\lambda^{2^*}\right)$ of this model, the cost efficiency can be defined by

$$E_C = \frac{\mathbf{w}^t\mathbf{x}^* + \mathbf{v}^t\mathbf{z}^*}{\mathbf{w}^t\mathbf{x}_o + \mathbf{v}^t\mathbf{z}_o}$$

It is trivial that $0 < \mathbf{w}^t\mathbf{x}^* + \mathbf{v}^t\mathbf{z}^* \le \mathbf{w}^t\mathbf{x}_o + \mathbf{v}^t\mathbf{z}_o$ therefore $0 < E_C \le 1$ and DMU_o will be cost efficient if $E_c = 1$. If $E_c < 1$, then $\left(\mathbf{x}^*,\mathbf{z}^*,\mathbf{y}_o\right)$ will be considered as the cost efficient projection of DMU_o. For introducing a common $\mathbf{z}^*$ between stage 1 and stage 2, the values of slack variables have been ignored in the cost efficient projection. The dual model of (1) is expressed by

$$
\begin{array}{lll}
Max & \mu\mathbf{y}_o + \sigma_1 + \sigma_2 & \\
s.t. & \alpha \le \mathbf{w}, & \\
 & \beta - \theta \le \mathbf{v}, & (2) \\
 & -\alpha X + \theta Z + e\sigma_1 \le 0, & \\
 & -\beta Z + \mu Y + e\sigma_2 \le 0, & \\
 & \alpha,\beta,\theta,\mu \ge \mathbf{0}, \qquad \sigma_1,\sigma_2 \ \ are \ \ free. &
\end{array}
$$

Here $\alpha = \left(\alpha_1,\ldots,\alpha_m\right)$ and $\theta = \left(\theta_1,\ldots,\theta_D\right)$ are two row vectors of dual variables related to the first and second sets of constraints in model (1). Similarly, $\beta = \left(\beta_1,\ldots,\beta_D\right)$ and $\mu = \left(\mu_1,\ldots,\mu_s\right)$ are those related to the third and fourth sets of constraints in model (1). Finally, the dual variables, σ_1 and σ_2, are due to the fifth and sixth constraints of model (1), respectively. If $\left(\mathbf{x}^*,\mathbf{z}^*,\lambda^{1^*},\lambda^{2^*}\right)$ is an optimal solution of model (1) and $\left(\alpha^*,\theta^*,\beta^*,\mu^*,\sigma_1^*,\sigma_2^*\right)$ is an optimal solution of model (2), then the following lemmas hold.

Lemma1. For each feasible solution $\left(\bar{\mathbf{x}},\bar{\mathbf{z}},\overline{\lambda^1},\overline{\lambda^2}\right)$ of (1), we have:

$$(i) -\alpha^*\bar{\mathbf{x}}+\theta^*\bar{\mathbf{z}}+\sigma_1^* \le 0 \qquad (ii) -\beta^*\bar{\mathbf{z}}+\mu^*\mathbf{y}_o+\sigma_2^* \le 0$$

Furthermore, $-\alpha^*\mathbf{x}+\theta^*\mathbf{z}+\sigma_1^*=0$ is the supporting hyper-plane on PPS1 at point $\left(\mathbf{x}^*,\mathbf{z}^*\right)$ and $-\beta^*\mathbf{z}+\mu^*\mathbf{y}+\sigma_2^*=0$ is the supporting hyper-plane on PPS2 at point $\left(\mathbf{z}^*,\mathbf{y}_o\right)$.

Proof. Suppose $\left(\bar{\mathbf{x}},\bar{\mathbf{z}},\overline{\lambda^1},\overline{\lambda^2}\right)$ is a feasible solution of model (1) therefore the following inequalities are conducted:

$$-X\overline{\lambda^1} \ge -\bar{\mathbf{x}}\ , \quad Z\overline{\lambda^1} \ge \bar{\mathbf{z}}, \tag{3}$$

$$-Z\overline{\lambda^2} \ge -\bar{\mathbf{z}}\ , \quad Y\overline{\lambda^2} \ge \mathbf{y}_o \tag{4}$$

Considering the third sets of constraints in (2) and $\overline{\lambda^1} \ge 0$, we have

$$-\alpha^* X\overline{\lambda^1}+\theta^* Z\overline{\lambda^1}+e\overline{\lambda^1}\sigma_1^* = \left(-\alpha^* X+\theta^* Z+e\sigma_1^*\right)\overline{\lambda^1} \le 0 \tag{5}$$

It follows from (3) and (5) that $-\alpha^*\bar{\mathbf{x}}+\theta^*\bar{\mathbf{z}}+\sigma_1^* \le 0$.
Similarly, it is obtained from the fourth sets of constraints in (2), $\overline{\lambda^2} \ge 0$ and the relation (4) that $-\beta^*\bar{\mathbf{z}}+\mu^*\mathbf{y}_o+\sigma_2^* \le 0$.

Mathematically, the complementary slackness conditions can be specified as follows:

$$\alpha^*\left(\mathbf{x}^* - X\lambda^{1^*}\right)=0 \tag{6}$$

$$\theta^*\left(Z\lambda^{1^*} - \mathbf{z}^*\right)=0 \tag{7}$$

$$\beta^*\left(\mathbf{z}^* - Z\lambda^{2^*}\right)=0 \tag{8}$$

$$\mu^*\left(Y\lambda^{2^*} - \mathbf{y}_o\right)=0 \tag{9}$$

$$\left(-\alpha^* X+\theta^* Z+e\sigma_1^*\right)\lambda^{1^*}=0 \tag{10}$$

$$\left(-\beta^* Z+\mu^* Y+e\sigma_2^*\right)\lambda^{2^*}=0 \tag{11}$$

Now, it follows from (6), (7), (10) that

$$-\alpha^*\mathrm{X}\lambda^{1^*}+\theta^*\mathrm{Z}\lambda^{1^*}+e\lambda^{1^*}\sigma_1^* = -\alpha^*\mathbf{x}^*+\theta^*\mathbf{z}^*+\sigma_1^*=0$$

Also, from (8), (9) and (11) follows that

$$-\beta^*\mathrm{Z}\lambda^{2^*}+\mu^*\mathrm{Y}\lambda^{2^*}+e\lambda^{2^*}\sigma_2^* = -\beta^*\mathbf{z}^*+\mu^*\mathbf{y}_o+\sigma_2^*=0.$$

Lemma2. If $X > \mathbf{0}$ and $Z > \mathbf{0}$ then the following items are valid:

(i) $\mathbf{x}^* > \mathbf{0}$ and $\mathbf{z}^* > \mathbf{0}$ $\qquad$ (ii) $\alpha^* = \mathbf{w}$ and $\beta^* - \theta^* = \mathbf{v}$

Proof. The proof is not difficult regarding the constraints of model (1) and the complementary slackness conditions.

Remark1. Note that $(\mathbf{x}^*, \mathbf{z}^*)$ and $(\mathbf{z}^*, \mathbf{y}_o)$ are the coordinates of the points on the efficiency frontier of stages 1 and 2, respectively, and we want to measure the RTS at $(\mathbf{x}^*, \mathbf{z}^*, \mathbf{y}_o)$ in the two-stage DEA.

Remark2. The cost minimization model is a non-radial DEA model. Therefore a problem of multiple projections can be found and this issue is an effective factor on RTS measurement. Sueyoshi et al. [9] have investigated how to solve this difficulty. In this study, it is assumed that the projection is unique.

Remark3. Scale elasticity (SE) is an important topic in performance analysis related to (RTS). In fact, it represents the quantitative part of (RTS) which is the proportional change in outputs resulting from the equi-proportionate change in inputs. Considering the optimal solution of model (2), the scale elasticity of each stage is simply determined as follows [9, 10]:

$$SE_1 = \frac{\alpha^* \mathbf{x}^*}{\theta^* \mathbf{z}^*} \qquad\qquad SE_2 = \frac{\beta^* \mathbf{z}^*}{\mu^* \mathbf{y}_o} \qquad\qquad (12)$$

Therefore for each real number γ the following equations hold:

$$-\alpha^*(\mathbf{x}^* + \gamma\, \mathbf{x}^*) + \theta^*(\mathbf{z}^* + \gamma SE_1\, \mathbf{z}^*) + \sigma_1^* = 0 \qquad (13)$$

$$-\beta^*(\mathbf{z}^* + \gamma SE_1\, \mathbf{z}^*) + \mu^*(\mathbf{y}_o + \gamma SE_1 SE_2\, \mathbf{y}_o) + \sigma_2^* = 0 \qquad (14)$$

A problem associated with the RTS measurement is that sometimes a supporting hyper-plane of each stage could not be uniquely determined. In other words, it is necessary to consider an occurrence of multiple optimal solutions on σ_1^* and σ_2^* , in model (2). For dealing with this problem see [9]. In this paper it is assumed that model (2) has unique optimal solution.

3. MESUREMENT OF RTS AND SE IN TWO-STAGE DEA

To introduce a new approach for RTS measurement in two-stage DEA, consider the following brief description on the relation among a supporting hyper-plane, RTS and SE in DEA [8, 9, 11].

The relation among SE_1, σ_1^* and the type of RTS in stage 1 at $(\mathbf{x}^*, \mathbf{z}^*)$ is as follows:

(i) Increasing RTS (IRS) $\leftrightarrow \sigma_1^* > 0 \leftrightarrow SE_1 > 1$
(ii) Decreasing RTS (DRS) $\leftrightarrow \sigma_1^* < 0 \leftrightarrow SE_1 < 1$
(iii) Constant RTS (CRS) $\leftrightarrow \sigma_1^* = 0 \leftrightarrow SE_1 = 1$

The similar relation exists among SE_2, σ_2^* and the type of RTS in stage 2 at $(\mathbf{z}^*, \mathbf{y}_o)$. Now for determining the two-stage overall RTS at $(\mathbf{x}^*, \mathbf{z}^*, \mathbf{y}_o)$ based on the consideration of SE quantity in stages 1 and 2, 5 different cases must be perused.

Case 1) If $SE_1 = 1$ and $SE_2 = 1$, then the two- stage overall RTS will be considered as CRS and $SE_O = 1$; (SE_O denotes the overall SE of two–stage).

Case 2) If $SE_1 = 1$ and $SE_2 < 1$, then the overall RTS will depend on the existence or non-existence of contraction possibility in stage 1. Consequently, we have proposed the following model to investigate this possibility.

$$
\begin{array}{lll}
Max & \gamma & \\
s.t. & X\lambda^1 \le \mathbf{x}^* - \gamma\,\mathbf{x}^*, & \\
 & Z\lambda^1 \ge \mathbf{z}^* - \gamma SE_1\,\mathbf{z}^*, & \\
 & Z\lambda^2 \le \mathbf{z}^* - \gamma SE_1\,\mathbf{z}^*, & (15) \\
 & Y\lambda^2 \ge \mathbf{y}_o - \gamma SE_1 SE_2\,\mathbf{y}_o, & \\
 & \mathbf{e}\lambda^1 = 1, & \\
 & \mathbf{e}\lambda^2 = 1, & \\
 & \lambda^1 \ge 0, \quad \lambda^2 \ge 0, \quad \gamma \ge 0. &
\end{array}
$$

One of the following conditions is held at optimality:

(i) If $\gamma^* = 0$, then there will be no contraction possibility in stage 1 and CRS with $SE_O = 1$ will be considered as the two- stage overall RTS.

(ii) If $\gamma^* > 0$ then the two- stage overall RTS will be DRS with $SE_O = SE_1 \times SE_2$ at $(\mathbf{x}^*, \mathbf{z}^*, \mathbf{y}_o)$.

Also regarding to γ^*, we can resize the scale of $(\mathbf{x}^*, \mathbf{z}^*, \mathbf{y}_o)$ for improving its productivity as follows:

$$\left(\left(1-\gamma^*\right)\mathbf{x}^*, \left(1-\gamma^* SE_1\right)\mathbf{z}^*, \left(1-\gamma^* SE_1\, SE_2\right)\mathbf{y}_o\right) \qquad (16)$$

The discussion like above is held when $SE_1 < 1$ and $SE_2 = 1$.

Case 3) If $SE_1 = 1$ and $SE_2 > 1$, then the existence or non-existence of expansion possibility in stage 1 is the recognition criterion of RTS measurement. In this case, model (17) examines this possibility.

$$
\begin{aligned}
Max \quad & \gamma \\
s.t. \quad & X\lambda^1 \le \mathbf{x}^* + \gamma\, \mathbf{x}^*, \\
& Z\lambda^1 \ge \mathbf{z}^* + \gamma SE_1\, \mathbf{z}^*, \\
& Z\lambda^2 \le \mathbf{z}^* + \gamma SE_1\, \mathbf{z}^*, \qquad (17) \\
& Y\lambda^2 \ge \mathbf{y}_o + \gamma SE_1 SE_2 \mathbf{y}_o, \\
& \mathbf{e}\lambda^1 = 1, \\
& \mathbf{e}\lambda^2 = 1, \\
& \lambda^1 \ge 0,\ \lambda^2 \ge 0,\ \gamma \ge 0.
\end{aligned}
$$

Considering the value of optimal objective function:

(i) If $\gamma^* = 0$, then there will not be any expansion possibility in stage 1, so CRS with SE_O =1 will be considered as the two- stage overall RTS.

(ii) If $\gamma^* > 0$ then the two- stage overall RTS will be IRS with SE_O = $SE_1 \times SE_2$ at $(\mathbf{x}^*, \mathbf{z}^*, \mathbf{y}_o)$.

By resizing the scale of $(\mathbf{x}^*, \mathbf{z}^*, \mathbf{y}_o)$, we have:

$$\left(\left(1+\gamma^*\right)\mathbf{x}^*, \left(1+\gamma^* SE_1\right)\mathbf{z}^*, \left(1+\gamma^* SE_1\, SE_2\right)\mathbf{y}_o\right) \qquad (18)$$

Similarly, the type of RTS at $(\mathbf{x}^*, \mathbf{z}^*, \mathbf{y}_o)$ and its corresponding SE_O are identified when SE_1 >1 and SE_2 =1.

Case 4) If SE_1 <1 and SE_2<1 (SE_1 >1 and SE_2>1) then DRS (IRS) with SE_O =$SE_1 \times SE_2$ will be considered as the two-stage overall RTS and for resizing the $(\mathbf{x}^*, \mathbf{z}^*, \mathbf{y}_o)$ model(15) (model(17)) must be solved.

Case 5) If SE_1 >1 and SE_2<1, three possibility for overall RTS can occur. In this case, first the value of k is calculated according to the following equation:

$$k = SE_1 \times SE_2 \qquad (19)$$

Therefore, three options may happen:

(a) If k=1 then CRS with SE_O =1 will be considered as an overall RTS.

(b) If k<1 then the type of RTS will be determined by solving model (15), like case2.

(c) If k>1 then the type of RTS will be determined by solving model (17), similar case3.

The discussion similar above is held when SE_1 <1 and SE_2>1.

4. APPLICATION

In this section the proposed method is applied to 26 branches of an Iranian commercial bank that each branch has a two-stage structure. The two inputs to the first stage are personnel privilege and interest on deposit. The two intermediate measures (or the outputs from the first stage) are the total sum of four deposits and bank commission. The three outputs from the second stage are facilities, bank interest and other resources. Table 1 reports the data set and the last row of it indicates the unit costs associated with the inputs and intermediate measures.

Note that the first input i.e. the personnel privilege is composed of some effective factors on quality of the personnel including the record, university degree, educational major, skills, salary, and so on. This input is obtained after normalizing these factors and therefore it is a non-dimensional quantity. Moreover, dollar has been considered as the unit of the other inputs and outputs. The interest on deposit is an amount that each branch pays to the clients for long-term and short-term deposit accounts. The first intermediate measure contains the sum of four deposits which are opened by the clients in each branch. These deposits are long-term and short-term deposit accounts and current and savings accounts. Since each branch allocates 20 percent of each deposit as

Table 1. Data Set

Branch	*Personnel Privilege*	*Interest on deposit*	*The total sum of four deposits*	*Bank commission*	*Facilities*	*Bank interest*	*Other resources*
1	16.07	99272.12	4143731.78	12867.52	5860446.51	36842.48	269955.30
2	4.51	22352.29	1297892.47	3453.76	2580211.64	6252.27	19003.02
3	2.70	26618.04	1575703.62	6119.34	5059604.70	4381.66	179007.30
4	9.39	119015.92	5168841.25	5736.58	11952750.00	135671.57	2461.72
5	4.88	26845.16	1059214.24	1353.09	1457197.88	16292.55	127688.08
6	3.46	17949.54	1268440.57	1347.21	1155743.90	6229.55	17244.36
7	32.73	227103.63	9237666.67	20134.05	13465583.30	199134.83	189855.85
8	16.19	148001.68	5445175.15	6169.16	6634759.41	154826.53	48816.70
9	2.93	59020.12	1972679.62	3697.98	1761978.91	10117.50	28404.79
10	16.59	203862.72	7345164.97	15879.25	10089416.60	141897.84	96653.20
11	3.59	31241.74	1410375.56	3524.54	2614268.06	18070.47	41221.33
12	4.53	50118.70	2416600.94	3651.77	2310034.80	29392.00	21100.06
13	6.61	69974.13	3798898.97	5815.61	4297721.84	75786.22	34569.69
14	7.86	69715.60	2788445.39	10232.90	3110072.00	20333.53	43421.08
15	13.04	101411.85	4888892.41	16179.22	5247780.49	46160.11	559297.40
16	3.94	27250.03	1405237.22	2101.81	1504436.96	16665.64	9076.39
17	3.41	29349.74	1397802.73	4695.45	842394.62	9263.58	15249.70
18	23.18	427888.97	12776333.33	25000.39	26562416.60	298220.31	1306725.69
19	13.61	92975.89	3285315.38	4526.60	3312143.54	16195.50	36950.77
20	14.51	126342.59	3847575.67	14015.86	3831995.13	25919.06	282562.11
21	9.17	60606.61	2194132.21	7145.59	3393991.92	12540.40	510783.58
22	9.97	131315.43	4456843.56	12261.77	2251233.49	11812.49	150713.77
23	6.09	61520.92	1951258.99	5201.19	1551428.19	9801.87	4509.37
24	16.60	190186.41	5985318.54	16007.76	5788983.09	8715.10	793441.23
25	8.59	96531.47	4207147.44	4625.65	5761584.53	33098.30	61601.55
26	10.58	110215.82	3662413.36	5889.99	4672044.83	15736.23	255814.65
Unit cost	$833.4	$1	$0.2	$1	_	_	_

Table 2. RTS Measurement

Branch	x_1^*	x_2^*	z_1^*	z_2^*	E_c	$SE1$	$SE2$	γ^*	RTS
1	4.39	44368.46	2472671.00	6263.22	0.57	0.79	1.10	0	CRS
2	3.22	20659.21	1220225.00	2838.92	0.93	4.88	3.03	–[1]	IRS
3	2.70	26618.04	1575703.62	6119.34	1.00	1.00	1.10	0	CRS
4	13.19	109552.60	5168841.00	5736.58	0.99	0.71	1.00	0	CRS
5	3.46	17960.22	1059214.00	1353.09	0.96	9.19	1.28	–[1]	IRS
6	3.46	17960.22	1059214.00	1353.09	0.85	9.19	1.28	–[1]	IRS
7	25.68	184695.80	7769791.00	12163.69	0.83	1.00	1.00	–[1]	CRS
8	14.52	117536.10	5445175.00	6169.16	0.97	0.72	1.00	0	CRS
9	3.39	18692.71	1102912.00	1756.34	0.53	7.28	3.97	–[1]	IRS
10	14.09	114969.80	5356346.00	6571.67	0.71	0.72	1.06	68×10^{-7}	DRS
11	3.27	20113.26	1319844.00	2538.36	0.90	5.34	1.59	–[1]	IRS
12	3.72	22626.22	1493821.00	1822.54	0.60	0.80	1.50	0	CRS
13	5.59	53066.83	2976534.00	3445.22	0.78	0.90	1.20	79×10^{-7}	IRS
14	3.35	22532.24	1456901.00	2953.67	0.50	0.80	1.51	86×10^{-7}	IRS
15	6.87	62700.73	3293314.00	8555.27	0.66	0.70	1.06	0	CRS
16	3.45	18014.43	1074939.00	1382.93	0.76	8.97	1.85	–[1]	IRS
17	3.46	17960.22	1059214.00	1353.09	0.74	9.14	1.28	–[1]	IRS
18	23.18	427889.00	1277633.33	25000.39	1.00	1.00	1.00	–[1]	CRS
19	3.17	22218.59	1427211.00	3435.04	0.41	0.79	1.52	0	CRS
20	3.47	33707.95	1975082.00	4808.07	0.47	0.85	1.14	0	CRS
21	4.26	39754.16	2194132.00	7145.59	0.95	0.76	1.00	16×10^{-7}	DRS
22	3.27	20061.89	1201304.00	2510.09	0.25	5.39	1.86	–[1]	IRS
23	3.44	18186.69	1072724.00	1477.76	0.51	8.46	4.37	–[1]	IRS
24	17.27	132167.90	5952119.00	13486.25	0.95	0.75	0.56	–[1]	DRS
25	4.04	38313.22	2201062.00	5133.87	0.51	0.86	1.29	17×10^{-7}	IRS
26	3.28	32465.36	1880447.00	5905.58	0.47	0.84	1.14	85×10^{-7}	DRS

1. It is not necessary to compute γ^* for determining the RTS classification of the units which they have the same RTS situation in their two stages.

the interest to the depositor, therefore $0.2 is considered as the unit cost associated with the first intermediate measure. The bank commission is an amount that each branch receives from the clients for providing different services. The facilities are the loans and other credit facilities that each branch pays to the clients. The second output i.e., bank interest is the amount of interest that each branch receives from the customers for providing facilities. The last output i.e., other resources are the revenues that each branch makes by investing on different projects.

Table 2 reports the results from models (1), (2), (15) and (17). The optimal inputs and intermediate measures from model (1) for each branch are reported under columns 2, 3, 4 and 5. The cost efficiency corresponding to each branch appears in 6th column. Using model (2), the columns 7 and 8 of table 2 represent the SE quantity for stages 1 and 2, respectively. The 8th column reports the value of γ^* for RTS measurement of some branches based upon models (15) or (17). Note that each branch needs the value of γ^* for improving their size according to relations (16) and (18). The last column of table 2 reports the state of overall RTS for cost efficient projection of each branch.

5. CONCLUDING REMARKS

The current paper discusses the problems of SE and RTS measurement in two-stage DEA. In a two-stage process, the first stage outputs which are the intermediate measures, will serve as the inputs of the second stage. In fact, this research has extended the RTS concept from classical DEA to the two-stage DEA. The proposed method determines the SE quantity and the type of overall RTS for the two-stage process considering the SE quantity in each stage.

Congestion indicates an economic state where inputs are overly invested. In other words, congestion is identified when an increase in one or some inputs causes the worsening of one or more outputs. From an economic theory, the issues of RTS and congestion are closely interrelated. Therefore investigating the congestion concept in two-stage DEA can be the future research issue.

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