

PROPERTIES OF SYMMETRIC NUCLEAR MATTER WITH SKYRME INTERACTIONS

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Abstract-Symmetric nuclear matter properties such as binding energy, pressure, saturation density and incompressibility are investigated in the Skyrme Hartree-Fock model. A new set of Skyrme parameters for symmetric nuclear matter is obtained by the fitting of Variational Monte Carlo method results to density-dependent Skyrme type energy. The results obtained are in good agreement with those obtained with selected Skyrme parameter sets in the literature.

Key Words- Symmetric nuclear matter, Skyrme interaction, Variational Monte Carlo method.

1.INTRODUCTION

Nuclear matter is an idealized system of interacting protons and neutrons. It is not matter in a nucleus, but hypothetical system consisting of a huge number of protons and neutrons interacting by only nuclear force and no Coulomb force. Volume and particle number are infinite, but the ratio is finite. Infinite volume implies no surface effects and translational invariance (only differences in position matter, not absolute positions). A common idealization is symmetric nuclear matter, which consists of equal numbers of protons and neutrons. The investigation of ground state properties of nuclear matter is one of the fundamental subjects in nuclear physics. Many calculations have been performed using different methods to obtain properties of symmetric nuclear matter [1-7]. One of the most important approachs to nuclear matter calculations is the Skyrme-Hartree-Fock (SHF) method [8-21]. Because the Hartree Fock (HF) calculations with Skyrme's density-dependent effective nucleon-nucleon interaction are very useful and successful for describe the ground state properties of the symmetric nuclear matter and neutron matter. In order to determine the parameter sets of all Skyrme interactions, known experimental quantities such as binding energy and saturation density can be used. The pioneering work of Brink and Vautherin [8] of implementing the Skyrme-type effective force was carried out to reproduce the experimental data on the binding energy and charge rms radii. Many calculations on the new set of Skyrme parameters were performed by various authors [22-25]. The parameters of Skyrme interaction [26] are obtained in these self consistent HF calculations by fitting the experimental binding energies, charge radii and other singleparticle properties of the spherical nuclei [23]. Also, there is a lot of Skyrme interaction parameter set which were obtained by the fitting of the HF results to experimental data on bulk properties of a few stable closed-shell nuclei in the literature. Recently, studies involving the generalized Skyrme effective force (GSEF) have been revisited [27-30].

In our previous studies, we have obtained ground state properties of symmetric nuclear matter (SNM) [1] and some thermodynamics properties of asymmetric nuclear matter (ASNM) [31]. In this paper, we obtained the new Skyrme parameter set for

symmetric nuclear matter by fitting the obtained results from Variational Monte Carlo (VMC) calculations to the Skyrme energy density functional. The realistic Urbana two nucleon interaction potential of Lagaris and Pandharipande was used for VMC calculations of nuclear matter. However, we investigate the some properties of symmetric nuclear matter with the new Skyrme interaction parameters.

The rest of this paper is organized as follows. In Sec. 2, we describe the details of the calculations and present the new Skyrme parameter set. In Sec. 3, we give the results obtained with new Skyrme parameter set for symmetric nuclear matter. We conclude the paper in Sec. 4.

2. THE CALCULATION PROCEDURE

The calculations of this paper are twofold. Firstly, we calculate energy values as a function of density for symmetric nuclear matter by Variational Monte Carlo (VMC) method. These values will be used to obtain the new Skyrme parameter set. Secondly, we calculate symmetric nuclear matter properties such as binding energy, saturation density, pressure and incompressibility with the new Skyrme parameter set.

2.1. Nucleon-nucleon interaction Potential

The Hamiltonian operator for a system which consist of N nucleons interacting through a two body interaction potential V_{ii} is given by

$$H = -\frac{\hbar^2}{2m} \sum \nabla_i^2 + \sum_{i < j} V_{ij} \tag{1}$$

If this operator is known, one can determine the ground state properties of nuclear matter. However, an assumed realistic two-body potential V_{ij} should correctly reproduce the saturation point of nuclear matter. In order to obtain correct binding energy and saturation density of nuclear matter, two-body potential V_{ij} fitted to the phase shifts observed in scattering experiments. However, it is necessary various operator components, because the phase-shift data varies greatly from channel to channel requires. Lagaris and Pandharipande were proposed the Urbana V₁₄ potential which contains 14 operator components. These operator components were obtained by fitting the phase-shift data from low energy nucleon-nucleon scattering experiments and the properties of the deuteron [32,33] :

$$V_{ij} = V^{c} + V^{\sigma}(\sigma_{i}.\sigma_{j}) + V^{\tau}(\tau_{i}.\tau_{j}) + V^{\sigma\tau}(\sigma_{i}.\sigma_{j})(\tau_{i}.\tau_{j}) + V^{t}S_{ij} + V^{t\tau}S_{ij}(\tau_{i}.\tau_{j}) + V^{b}(\vec{L}.\vec{S})_{ij} + V^{b\tau}(\vec{L}.\vec{S})_{ij}(\tau_{i}.\tau_{j}) + V^{q}L^{2} + V^{q\sigma}L^{2}(\sigma_{i}.\sigma_{j}) + V^{q\tau}L^{2}(\tau_{i}.\tau_{j}) + V^{q\sigma\tau}L^{2}(\sigma_{i}.\sigma_{j})(\tau_{i}.\tau_{j}) + V^{bb}(\vec{L}.\vec{S})^{2} + V^{bb\tau}(\vec{L}.\vec{S})^{2}(\tau_{i}.\tau_{j}).$$

$$(2)$$

The terms depending on the relative angular momentum operator \overline{L} , do not considerably effect the binding energy due to the symmetrical nature of the nuclear matter. The contributions of the first four terms are much stronger than latter terms,

because the effect of latter terms is smaller than the statistical fluctuations inherent to the Monte Carlo technique so the inclusion of these terms was pointless. Therefore only the first four terms of Urbana potential were retained in the calculations of expectation value. Thus we have used the two body interaction

$$V_{ij} = V^{c} + V^{\sigma}(\sigma_{i}.\sigma_{j}) + V^{\tau}(\tau_{i}.\tau_{j}) + V^{\sigma\tau}(\sigma_{i}.\sigma_{j})(\tau_{i}.\tau_{j}), \qquad (3)$$

where $V^{c}, V^{\sigma}, V^{\tau}$, and $V^{\sigma\tau}$ depend only on the distance between the nucleons *i* and *j*. Each term in Eq. (3) has three parts

$$V^{i} = V_{\pi}^{i} + V_{I}^{i} + V_{S}^{i}$$
(4)

representing long-range (V_{π}^{i}) , intermediate-range (V_{I}^{i}) , and short-range (V_{s}^{i}) interactions. (V_{π}^{i}) is nonzero only for $i = \sigma \tau$ and is given by

$$V_{\pi}^{\sigma\tau} = 3.488 \frac{e^{-\mu r}}{\mu r} (1 - e^{-cr^2})$$
(5)

where $\mu = 0.7 \text{ fm}^{-1}$ is the inverse compton wavelength for pions. The intermediate and short range parts are

$$V_{I}^{i}(r) = I^{i} \left[\left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^{2}} \right) \frac{e^{-\mu r}}{\mu r} \left(1 - e^{-cr^{2}} \right) \right]^{2},$$

and

$$V_{S}^{i}(r) = \frac{S^{i}}{1 + e^{(r-R)/a}}$$
(6)

respectively. Values of the potential strengths I^1 and S^1 and the parameters c, R, a are given in Table 1.

In fact, all two nucleon interaction models estimate too large equilibrium densities for nuclear matter. However, there is not much known about the exact nature of the three nucleon interactions, and experimental data is not available. Therefore one has to consider three body interactions in a "somewhat" arbitrary way.

Significantly realistic two nucleon interactions seem to overbind nuclear matter at high densities ($\rho > 0.24 \text{ fm}^{-3}$) and slightly underbind at lower densities ($\rho < 0.15 \text{ fm}^{-3}$). Consideration of the three and more body effects should be able to correct this discrepancy.

	Table 1. I diameters of the Orbana	v 14 nucleon nucleon potential.	_
Ι	I^i	S^{i}	_
С	-5.7030	2575.3	_
σ	0.7628	-366.56	
τ	0.8892	-466.56	
$\sigma \tau$	-0.2790	402.81	
C	$x=0.2 \text{ fm}^{-2}$, $R=0.5 \text{ fm}$, $a=0.2 \text{ fm}$		

Table 1 Parameters of the Urbana V., nucleon-nucleon notential

The observations of overbinding at high densities suggests that the repulsive part of the two body interactions should be more effective at high densities. In order to have this effect we have used a potential including the three body interactions

$$v_{14} + TNI = v_{\pi} + v_{I} + v_{s} \left[1 + (\alpha \rho)^{\beta} \right].$$
⁽⁷⁾

 α and β in the above equation are free parameters and are adjusted so as to obtain the correct properties of the nuclear matter. The same parameters were also used for the calculations of the neutron matter.

2.2. The Variational Monte Carlo method

In Variational Monte Carlo method the ground state wave function $\Psi_0(\vec{R})$ is approximated by a variational wave function $\Psi_V(\vec{R})$ with many variational parameters, which are determined by minimizing $\langle H \rangle$. We will base our calculations on a Jastrow type wave function of the form

$$\Psi_j(\vec{R}) = \prod_{i < j} f_j(r_{ij}) \Phi , \qquad (8)$$

where Φ is the many particle wave function for the system of non-interacting particles and \vec{R} is a 3N dimensional vector representing the coordinates of particles, while f_j is the two particle correlation function. Jastrow suggests that this correlation function in general be an operator function [34]. However in most applications f_j is assumed to depend only on the interparticle distance, $r_{ij} = |\vec{r}_i - \vec{r}_j|$.

In order to simulate the nuclear matter, we will consider a system of N nucleons confined in a cube of side L with periodic boundary conditions. Therefore for the single particle wave functions we can use the plane waves

$$\phi(\vec{r}) = e^{i\vec{k}.\vec{r}} \tag{9}$$

where $\bar{k} = 2\pi \bar{n}/L$ and n is an integer vector. Because of the symmetry of the ground state we can use real plane waves

$$\phi(\vec{r}) = \begin{cases} \cos\left(\frac{2\pi}{L}\vec{n}.\vec{r}\right) \\ \sin\left(\frac{2\pi}{L}\vec{n}.\vec{r}\right) \end{cases}, \tag{10}$$

instead of complex plane waves in eq.(9). Under these conditions the many particle wave function in eq.(8) becomes

$$\Phi(R) = \prod_{s=1}^{g} D^{s}$$
(11)

where D^s is the slater determinant of single particle wave functions

$$d_{ij}^{s} = \phi_{j}((\vec{r},s)_{i}), \quad D^{s} = \det(d_{ij}^{s}).$$
 (12)

For the two particle correlation function f_j in eq.(8) we chose a function similar to the Woods-Saxon potential

$$f_{j}(r) = \left[\frac{1}{1 + e^{(r_{0} - r)/a}}\right]^{l}.$$
(13)

where t, r_0 and a are variational parameters. We define the pseudo potential u(r) for practical reasons as

$$f_{j}(r_{ij}) = \exp(-u(r_{ij})), \quad u(r_{ij}) = -\ln(f_{j}(r_{ij}))$$
(14)

then our variational wave function becomes

$$\Psi_j = \exp\left(-\sum_{i\langle j} u(r_{ij})\right) x \prod_{s=1}^g D^s .$$
(15)

The method of sampling the wave function is identical to that used for classical ensembles [35]. The only complication that arises is the evaluation of the determinant. Initial coordinates are chosen randomly for each particle from an initial Monte Carlo random walk. The particles are then moved one by one to new trial positions. Suppose particle 1 is being moved. Then its new trial position r_{new} is

$$\vec{r}_{new} = \vec{r}_1 + \vec{\xi} \tag{16}$$

where $\vec{\xi}$ is a random vector uniformly distributed in a cube of side Δ centered at the origin. The new position for particle 1 is accepted with a probability equal to

$$P = \min\left[1, \left|\Psi(\vec{r}_y) / \Psi(\vec{r}_1)\right|^2\right].$$
(17)

If the absolute value of the wave function at the new position is larger than that at the old position, the new coordinates are automatically accepted. This random walk is Markovian and by the usual argument [36] the set of coordinates generated by a sufficiently long calculation is an unbiased sample drawn from the probability distribution

$$\frac{\left|\Psi(\vec{R})\right|^2}{\int d\vec{R} \left|\Psi(\vec{R})\right|^2}.$$
(18)

The expectation value of any operator F is then simply the average value of the operator evaluated for the coordinates of the random walk with M moves

$$\langle \hat{F} \rangle = \frac{\int d\vec{r} \Psi^*(\vec{r}) F(\vec{r}) \Psi(\vec{r})}{\int d\vec{r} |\Psi(\vec{r})|^2} \cong \frac{1}{M} \sum_{i=1}^M F(\vec{r}_i) \,. \tag{19}$$

Total energy of the system is calculated using this approach. The contribution of the nucleon-nucleon interactions to total energy are calculated for interparticle separations up to a cut off distance of L/2. A reasonable approximation to include the contributions of the pairs farther apart is to assume that the density of particles is constant outside this interaction sphere.

For each density the total energy corresponding to the Hamiltonian of the system is calculated for various values of the parameters in the trial wave function. Then the variational parameters r_0 , a, and t are determined from these calculations so that the total energy is a minimum. Then a final Monte Carlo calculation of the system with the optimized parameter set is performed.

We consider a system of N nucleons confined in a cube of side L with periodic boundaries. For the symmetric nuclear matter there are four nucleons in each spatial state. In order for our wave function to represent all symmetries of the ground state the number of neutrons or protons must be chosen from the set (2, 14, 38, 54, 66, 114, ...). VMC method and calculations for nuclear matter are described in detail in ref. [1, 31]. **2.3. The Skyrme Hartree-Fock calculations**

The effective interaction proposed by Skyrme was desined for HF calculations of nuclei. It contains basically of a two-body part which is momentum dependent, and

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zero range three-body part. The role of the latter part is to simulate the effects of shortrange correlations since it is equivalent in HF calculations to a two-body force with a linear density dependence [8].

We use the Skyrme interaction as well as the corresponding HF equations which are described in detail in ref.[8]. For the Skyrme interaction the energy density $H(\bar{r})$ is given as follows [9]:

$$H(\vec{r}) = \frac{\hbar^2}{2m} \tau(\vec{r}) + \frac{1}{2} t_0 \left[\left(1 + \frac{1}{2} x_0 \right) \rho^2 - \left(x_0 + \frac{1}{2} \right) \left(\rho_n^2 + \rho_p^2 \right) \right] + \frac{1}{4} (t_1 + t_2) \rho^2 \tau + \frac{1}{8} \rho (t_2 - t_1) \left(\rho_n \tau_n - \rho_p \tau_p \right) \\ + \frac{1}{16} (t_2 - 3t_1) \rho \nabla^2 \rho + \frac{1}{32} (3t_1 + t_2) \left(\rho_n \nabla^2 \rho_n + \rho_p \nabla^2 \rho_p \right) + \frac{1}{16} (t_1 - t_2) \left(\vec{J}_n^2 + \vec{J}_p^2 \right) + \frac{1}{4} t_3 \rho_n \rho_p \rho$$

$$+ H_C(\vec{r}) - \frac{1}{2} W_0 \left(\rho \vec{\nabla} \cdot \vec{J} + \rho_n \vec{\nabla} \cdot \vec{J}_n + \rho_p \vec{\nabla} \cdot \vec{J}_p \right)$$
(20)

where $\rho_n(\rho_p)$ is the density of neutrons (protons) and $\rho_n + \rho_p = \rho$, while $\tau_n(\tau_p)$ and $\vec{J}_n(\vec{J}_p)$ are te kinetic energy and the spin-orbit densities of neutrons (protons), repectively. $\tau_n + \tau_p = \tau$, and $\vec{J} = \vec{J}_n + \vec{J}_p$. The direct part of the Coulomb interaction in $H_c(\vec{r})$ is $\frac{1}{2}V_c(\vec{r})\rho_p(\vec{r})$, where

$$V_{C}(\vec{r}) = \int \rho_{p}(\vec{r}) \frac{e^{2}}{\left|\vec{r} - \vec{r}\right|} d^{3}r'.$$
 (21)

 t_0 , t_1 , t_2 , t_3 , x_0 and W_0 in Eq.(20) are parameters of the Skyrme interactions. Table 2 shows selected Skyrme parameter sets [8].

FORCE	t_0 (MeV.fm ³)	t_1 (MeV.fm ⁵)	t_2 (MeV.fm ⁵)	t ₃ (MeV.fm ⁶)	X ₀	W_0
						(MeV.fm ⁵)
SI	-1057.3	235.9	-100	14463.5	0.56	120
SII	-1169.9	585.6	-27.1	9331.1	0.34	105
SIII	-1128.75	395.0	-95.0	14000.0	0.45	120
SIV	-1205.6	765.0	35.0	5000.0	0.05	150
SV	-1248.29	970.56	107.22	0.0	-0.17	150
SVI	-1101.81	271.67	- 138.33	17000.0	0.583	115

Table 2. Selected parameter sets of Skyrme interaction in the literature.

2.3.1. symmetric nuclear matter

As we have mentioned before, nuclear matter is a uniform hypothetical system with translational invariance and has a fixed ratio of neutrons and protons (ignoring the Coulomb forces). When the number of protons and neutrons are the same, the system is called symmetric nuclear matter :

$$\rho_n = \rho_p = \frac{1}{2}\rho, \qquad \tau_n = \tau_p = \frac{1}{2}\tau, \qquad \vec{J}_n = \vec{J}_p = 0, \quad (22)$$

and $\vec{\nabla}\rho = \vec{\nabla}.\vec{J} = 0$, $\rho = \left(\frac{2}{3\pi^2}\right)k_F^3$, $\tau = \frac{3}{5}k_F^2$. Thus, from Eq.(20) one can get the binding

energy per particle for SNM :

$$\frac{E}{A} = \frac{H}{\rho} = \frac{3}{5}T_F + \frac{3}{8}t_0\rho + \frac{1}{16}t_3\rho^2 + \frac{3}{80}(3t_1 + 5t_2)\rho k_F^2.$$
(23)

Where $T_F = \hbar^2 k_F^2 / 2m$ is the kinetic energy of a particle at the Fermi surface. From Eq.(23) one can write the expressions of pressure P and incompresibility K for symmetric nuclear matter:

$$P = \rho^2 \frac{\partial (E/A)}{\partial \rho} = \frac{2}{5} T_F \rho k_F^2 + \frac{3}{8} t_0 \rho^2 + \frac{1}{8} t_3 \rho^3 + \frac{1}{16} (3t_1 + 5t_2) \rho^2 k_F^2, \qquad (24)$$

$$K = k_F^2 \frac{\partial^2 (E/A)}{\partial k_F^2} = \frac{6}{5} T_F + \frac{9}{4} t_0 \rho + \frac{15}{8} t_3 \rho^2 + \frac{3}{4} (3t_1 + 5t_2) \rho k_F^2.$$
(25)

The resulting sets of Skyrme parameters are not six. Over seventy parametrizations of the Skyrme interaction have been published so far. The some of them are : SVII [13], SkM [12], SGI, SGII [37], Sk2 [8], Sk3 [9,16], Skb [10]. The aim of this study is to obtain the new Skyrme parameter set that can describe properties of nuclear matter. Firstly, we have calculated energy values as a function of density for SNM by VMC method. The realistic Urbana two nucleon interaction potential of Lagaris and Pandharipande was used for VMC calculations of SNM. Also many body interactions are included as a density dependent term in the potential. Then, the new parameter set of Skyrme interaction, which is t_0 =-982.424 MeV.fm³, t_3 =16938.92 MeV.fm⁶, and ($3t_1$ + $5t_2$)=-559.205 MeV.fm⁵, obtained by fitting the results obtained from VMC calculations to the Skyrme energy density functional. However, From Eq. (23), Eq. (24) and Eq. (25), we have obtained binding energy, saturation density, pressure and incompressibility for SNM with the new Skyrme parameter set. The given six parameter set of Skyrme interaction in table 2 are selected for the compare with the new Skyrme parameter set in this study.

3. RESULTS

In this section we present the new Skyrme parameter set and the results obtained by this parameter set. The obtained new Skyrme parameter set are given in Table 3 with selected Skyrme parameter set.

FORCE	t ₀ (MeV.fm ³)	$(3t_1+5t_2)$ (MeV.fm ⁵)	t_3 (MeV.fm ⁶)
SI	-1057.3	207.7	14463.5
SII	-1169.9	1621.3	9331.1
SIII	-1128.75	710	14000.0
SIV	-1205.6	2470	5000.0
SV	-1248.29	3447.78	0.0
SVI	-1101.81	123.36	17000.0
This study	-982.424	-559.205	16938.92

Table 3. New Skyrme parameter set with selected Skyrme parameter sets.

We have obtained the binding energy per nucleon at densities between $\rho = 0.01 \text{ fm}^{-3}$ and $\rho = 0.20 \text{ fm}^{-3}$ in 0.01 steps with the new Skyrme parameter set. The obtained values of the energy for SNM are given Table 4 From these data, we have obtained the saturation density, saturation energy, pressure and incompressibility of SNM.

parameter set.				
Density (fm ⁻³)	E/A (MeV)	Density (fm ⁻³)	E/A (MeV)	
0.01	-0.11310	0.11	-13.4769	
0.02	-1.53730	0.12	-14.1831	
0.03	-3.13570	0.13	-14.7346	
0.04	-4.75430	0.14	-15.1277	
0.05	-6.32760	0.15	-15.3593	
0.06	-7.82000	0.16	-15.4268	
0.07	-9.20885	0.17	-15.3279	
0.08	-10.4788	0.18	-15.0605	
0.09	-11.6188	0.19	-14.6229	
0.10	-12.6204	0.20	-14.0134	

Table 4. Values of energy for symmetric nuclear matter per nucleon obtained with new Skyrme

Table 5 shows binding energy, saturation density, Fermi momentum and incompressibility of SNM with selected Skyrme parameters in the literature. Figure 1 shows the energy results along with those obtained with selected Skyrme parameters in the literature.

Table 5. Binding energy, saturation density, Fermi momentum and incompressibility of symmetric nuclear matter with selected Skyrme parameter sets in the literature.

Skyrme Interactions	Binding Energy E/A (MeV)	Saturation density $ ho$ (fm ⁻³)	$\begin{array}{c} \text{Fermi momentum} \\ k_{\text{F}} (\text{fm}^{\text{-1}}) \end{array}$	Incompressibility K (MeV)
SI	-15.78	0.155	1.31	335
сп	-16.00 [9]	0.148 [9]	1.30 [9]	342 [9]
511	-15.811	0.149	1.30	352
SIII	-15.87 [9]	0.145 [9]	1.29 [9]	356 [9]
5111	-15.65	0.146	1.29	320
SIV	-15.98 [9]	0.152 [9]	1.31 [9]	325 [9]
51V	-15.790	0.151	1.30	319
SV	-16.06 [9]	0.155 [9]	1.32 [9]	306 [9]
51	-15.867	0.154	1.31	281
SVI	-15.77 [9]	0.145 [9]	1.29 [9]	364 [9]
311	-15.567	0.149	1.30	339
This study	-15.42	0.159	1.32	385



Figure 1. Binding energy per nucleon obtained with selected Skyrme parameter sets along with the results of this study (solid disks).

In Table 6, the pressures of the SNM at each density obtained from Eq. (5) are given. Our pressure results (solid disks) and the results obtained with selected Skyrme parameters in the literature are compared in Figure 2.

Density	Pressure	Density	Pressure
(fm ⁻³)	(MeV.fm ⁻³)	(fm^{-3})	(MeV.fm ⁻³)
0.01	-0.01805	0.11	-0.97461
0.02	-0.06920	0.12	-0.95635
0.03	-0.14861	0.13	-0.86691
0.04	-0.25085	0.14	-0.68871
0.05	-0.36979	0.15	-0.40233
0.06	-0.49851	0.16	0.013614
0.07	-0.62914	0.17	0.582597
0.08	-0.75277	0.18	1.330277
0.09	-0.85935	0.19	2.284626
0.10	-0.93754	0.20	3.476040

Table 6. Values of pressure of the symmetric nuclear matter obtained with new Skyrme parameter set.



Figure 2. Pressure of symmetric nuclear matter obtained with selected Skyrme parameter sets along with the results of this study (solid disks).

The obtained pressures for SNM are also compared with the pressures values obtained by Friedman and Pandharipande [38] in Figure 3.



Figure 3. Pressure of symmetric nuclear matter obtained by Fiedman and Pandharipande along with the results of this study (solid disks).

The incompressibility of the SNM at each density obtained from Eq. (6) are given in Table 7. A comparison similar to Fig.1 and Figure 2 is given Figure 4 for incompressibility of SNM. There is a good agreement between the seven curves.

The resulting empirical saturation point has an energy per particle between -15 and -17 MeV and Fermi momentum k_F in the range 1.29-1.44 fm⁻¹ [7]. In this study, the values of binding energy and Fermi momentum are found -15.42 MeV and 1.32 fm⁻¹ respectively. These results are in good agreement with empirical values.

It can be seen from Figure 1 and Figure 2 that values of binding energies and pressures obtained in this study are good agreement with those obtained with selected Skyrme parameter set (SI-SVI). Also, it is observed that both our results are in good agreement with those of Friedman and Pandharipande for the pressures of SNM.

Table 7. Values of incompressibility of the symmetric nuclear matter obtained with new Skyrme

Density (fm ⁻³)	Incompressibility (MeV)	Density (fm ⁻³)	Incompressibility (MeV)
0.02	-24.041	0.12	155.261
0.03	-30.389	0.13	204.054
0.04	-31.665	0.14	258.594
0.05	-27.662	0.15	318.898
0.06	-18.256	0.16	384.983
0.07	-3.360	0.17	456.864
0.08	17.087	0.18	534.555
0.09	43.137	0.19	618.067
0.10	74.829	0.20	707.412



Figure 4. Incompressibility of symmetric nuclear matter obtained with selected Skyrme parameter sets along with the results of this study (solid disks).

To sum up, in this paper we have the new Skyrme parameter set for SNM calculations. As an application of the new Skyrme parameter set we have calculated the binding energy, saturation density, pressure and incompressibility of SNM.

4. CONCLUSION

The empirically known bulk properties of nuclear matter, such as the binding energy, saturation density, incompressibility, etc., which starting from the underlying two-body interactions, are one of the fundamentals of nuclear matter theory [39]. Calculations of binding energy as a function of density of symmetric nuclear matter have been made for many nucleon-nucleon potentials [32, 33, 40]. However, the Skyrme potential and other phenomenological potential models are very convenient and useful in the calculations of the bulk properties of nuclear matter, however before using such potentials the reliability of the potential model should be established. Monte Carlo simulation results obtained in this study from a nucleon-nucleon potential may serve as a means to evaluate relative merits of various phenomenological models. Also the obtained data used to optimize the parameter set of Skyrme interaction potential.

There are many known the Skyrme nucleon-nucleon interaction of it which reproduce experimental data for the ground state of finite nuclei and for the observables of infinite nuclear matter at the saturation density, giving more and less comparable agreement with experimental or expected empirical data. Here, we have presented a new Skyrme parameter set that can describe properties of symmetric nuclear matter as described by realistic approach of nuclear matter. However, the ground state properties of symmetric nuclear matter are investigated with the new Skyrme parameter set. If our results compared with the obtained results by other selected Skyrme parameter sets, it is observed that no any serious differences between our results and selected results.

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