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DIFFRACTION OF WAVES BY A WEDGE WITH PHASE CONJUGATE FACES

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Abstract- The scattering process of waves by a wedge with phase conjugate faces is investigated with the aid of the modified diffraction theory of Kirchhoff. The scattering integral is evaluated asymptotically. The uniform diffracted waves and geometrical optics fields are plotted and examined numerically.
Key Words- Diffraction theory, Phase conjugation

1.INTRODUCTION

A phase conjugate surface reflects the incoming field in the direction of incidence with the same angle of the incident wave [1]. Such a property is important in the distortion correction of waves, being distorted by weak scatterers [2, 3]. Friberg introduced an integral equation method for the interaction of homogeneous waves with the phase conjugate mirrors [4]. Friberg and Drummond studied the reflection of the linearly polarized waves by a phase conjugate mirror, placed behind a stratified dielectric plane mirror [5]. The quantum optical theory of the phase conjugate surfaces is outlined by some authors [6, 7]. Zheng *et al* proposed an electromagnetic model for the scattering of waves by a phase conjugate mirror [8].

The phenomenon of wedge diffraction is a canonical problem that is used for the investigation of the wave scattering by more complex geometries. For example, a general problem in optics and electromagnetics is the transmission of waves by an aperture in a screen. Since the edge of the aperture has a nonzero width, the usage of the edge diffraction model is more realistic. For this reason, the scattering problem of waves by a wedge with phase conjugate faces provides an in depth vision for the diffraction process by phase conjugate edges. Furthermore, the effect of the un-illuminated face to the total scattered face can be investigated by the canonical problem of wedge diffraction.

The aim of this paper is to investigate the scattering characteristics of the plane waves by a phase conjugate wedge. To our knowledge, this problem has not yet been studied. We will formulate the scattering process of waves with the aid of the improved diffraction theory of Kirchhoff [9-11]. The scattering integral will be evaluated by using the uniform methods, introduced by us [12]. The evaluated geometrical optics (GO) and wedge diffracted waves will be examined numerically.

A time factor of $\exp(j\omega t)$ is suppressed throughout the paper. *w* is the angular frequency.

2.THEORY

The geometry, in Fig. 1, is considered. A plane wave of $u_0 \exp[jk(x\cos\alpha + y\sin\alpha)]$ is illuminating the wedge. u_0 is the complex amplitude. α

is the angle of incidence. P is the observation point. The outer angle of wedge is ψ. Since the incident wave reflects from a phase conjugate surface backwards with the angle of incidence, we will model the problem by defining a phase conjugate wave by the expression of $u_{PC}\big|_S = u_i^*\big|_S$. Here S denotes the surface of the wedge.

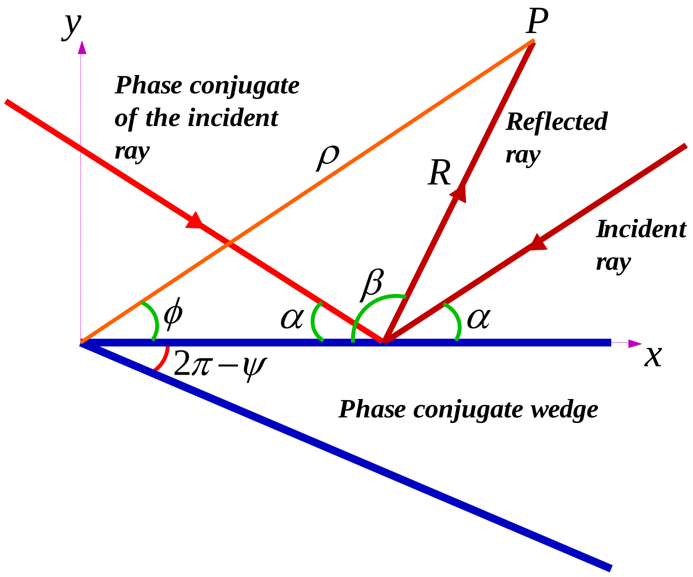

Fig. 1. Geometry of the phase conjugate wedge

The angle of incidence of the phase conjugate wave is $\pi-\alpha$. Thus the surface current, which causes the radiation of the scattered waves, can be written as

$$J_s = -\frac{\sin(\pi/n)}{n}\frac{\cos\alpha+\cos\beta}{\cos(\pi/n)-\cos[(\beta+\alpha)/n]}u_{PC}\big|_S \tag{1}$$

for n is ψ/π [13, 14]. Thus the scattering integral, by the phase conjugate wedge, can be constructed as

$$u_{rs}(P) = f(n)\int_0^\infty u_{PC}(Q)\frac{\cos\alpha+\cos\beta}{\cos(\pi/n)-\cos[(\beta+\alpha)/n]}\frac{\exp(-jkR)}{\sqrt{kR}}dx' \tag{2}$$

where Q is the integration point. $f(n)$ has the expression of

$$f(n) = -\frac{\exp(j\pi/4)k\sin(\pi/n)}{n\sqrt{2\pi}}. \tag{3}$$

The incident scattered wave can be directly written as

$$u_{is}(P) = u_i(P) - f(n)\int_0^\infty u_i(Q)\frac{\cos\alpha-\cos\beta}{\cos(\pi/n)-\cos[(\pi-\beta-\alpha)/n]}\frac{\exp(-jkR)}{\sqrt{kR}}dx' \tag{4}$$

according to Refs. [13] and [14]. R is equal to $\sqrt{(x-x')^2+y^2}$. The incident scattered wave transmits through the semi-infinite aperture at $x\in(-\infty,0)$ and only the edge diffracted waves occur at $x=0$. For this reason, we have used the actual scattering integral, introduced in Refs. [10] and [14] for the incident waves. The stationary phase point of Eq. (2) is located at $\beta_s = \pi - \alpha$. β is equal to $\pi - \phi$ at the edge point. Thus the reflected scattered waves by the phase conjugate wedge are found to be

$$u_{rs}(P) = u_0 \exp[-jk\rho\cos(\phi-\alpha)][I_{rGO} - I_{rd}] \tag{5}$$

where I_{rGO} and I_{rd} can be defined as

$$I_{rGO} = U(-\xi_r) \tag{6}$$

and

$$I_{rd} = \frac{2\sin(\pi/n)}{n} \frac{\sin\frac{\phi-\alpha}{2}}{\cos(\pi/n) - \cos[(\pi-\phi+\alpha)/n]} sign(\xi_r) F[|\xi_r|] \tag{7}$$

respectively [12, 14]. ξ_r is the detour parameter and has the expression of

$$\xi_r = \sqrt{2k\rho}\sin\frac{\phi-\alpha}{2}. \tag{8}$$

$U(x)$ is the unit step function, which is equal to one for $x > 0$ and zero otherwise. $sign(x)$ is the signum function that is equal to 1 for $x > 0$ and -1 for $x < 0$. $F[x]$ is the Fresnel function and can be defined by the integral of

$$F[x] = \frac{\exp(j\pi/4)}{\sqrt{\pi}} \int_x^\infty \exp(-jt^2)dt. \tag{9}$$

The uniform evaluation of the incident scattered field was performed in Ref. [14], and can be introduced as

$$u_{is}(P) = u_0 \exp[jk\rho\cos(\phi-\alpha)][I_{iGO} - I_{id}] \tag{10}$$

where I_{iGO} and I_{id} can be defined as

$$I_{iGO} = U(-\xi_i) \tag{11}$$

and

$$I_{id} = \frac{2\sin(\pi/n)}{n} \frac{\cos\frac{\phi-\alpha}{2}}{\cos(\pi/n) - \cos[(\phi-\alpha)/n]} sign(\xi_i) F[|\xi_i|] \tag{12}$$

for ξ_i is $-\sqrt{2k\rho}\cos[(\phi-\alpha)/2]$.

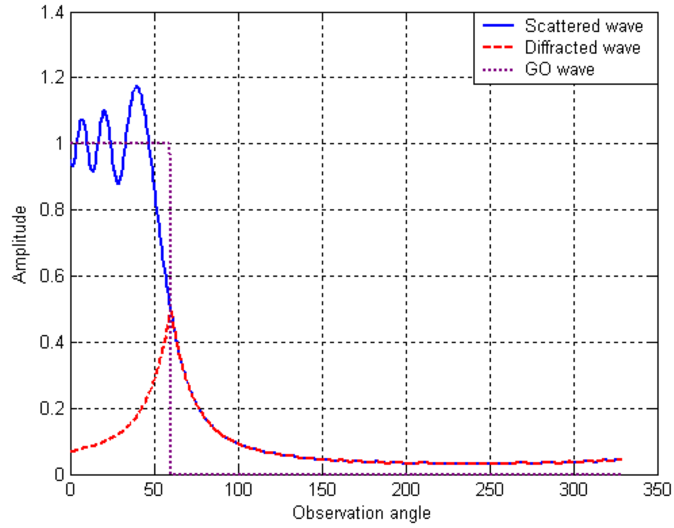

Fig. 2. Reflected scattered waves

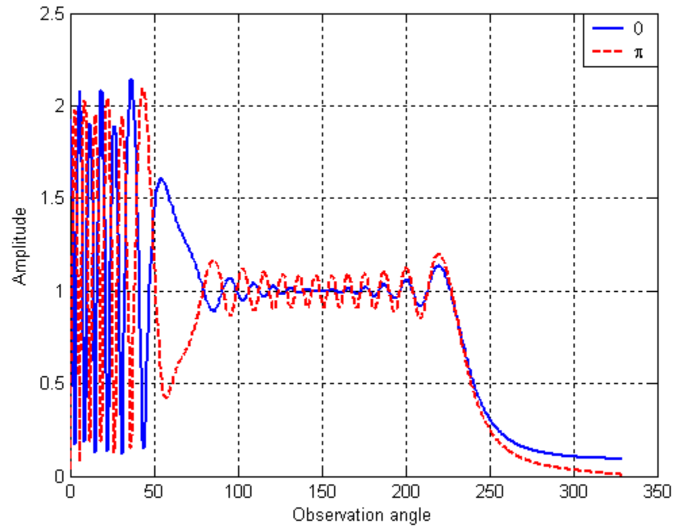

Fig. 3. Total scattered field according to phase shift

Figure 2 shows the variation of the reflected scattered, diffracted and GO fields with respect to the observation angle. The distance of observation is 6λ for λ is the wavelength. ψ is 330^0. The angle of incidence is equal to 60^0. Figure 2 gives important insights about the behavior of the phase conjugate surfaces. The GO field is discontinuous at $\phi = 60^0$. The discontinuity of the GO field is compensated by the diffracted wave. The total scattered field, which is the sum of the diffracted and GO waves, is continuous everywhere. It is important to note that the reflection angle of the field is equal to 60^0, which is also the angle of incidence. Thus it can be seen from Fig. 2 that the incident waves are reflected backwards with the angle of incidence on a phase conjugate surface. This result shows the validity of our mathematical model.

Figure 3 depicts the variation of the total scattered fields that are the sum of the incident and reflected scattered waves versus the observation angle. The values of the arguments are the same with the ones, in Fig. 2. We have taken into account two cases. The blue plot shows the total scattered wave when there is not any phase shift of the waves, reflected from the phase conjugate surface. The red graphic represents the case when the reflected waves have a phase shift of 180^0. It can be seen from the figure that the maxima and minima of the scattered waves coincide for $\phi < 150^0$. The maximum of a wave is located at the same point with the minimum of the second wave. The value of the total scattered field for the phase shift of π is equal to zero at the surfaces of the wedge.

3.CONCLUSION

In this paper, we studied the scattering process of plane waves by a wedge with phase conjugate surfaces. We used the modified diffraction theory in order to construct the scattering integrals. We have given also the field expressions that were obtained by the uniform evaluation of the scattering integrals. The scattered fields are plotted numerically. The results are in harmony with the theoretical expectations.

4. REFERENCES

1. A. Brignon and J. P. Huignard, *Phase Conjugate Laser Optics*, Wiley-Interscience, New Jersey, 2004.
2. G. S. Agarwal and E. Wolf, Theory of phase conjugation with weak scatterers, *Journal of the Optical Society of America* **72**, 312-326, 1982.
3. G. S. Agarwal, A. T. Friberg and E. Wolf, Effect of backscattering in phase conjugation with weak scatterers, *Journal of the Optical Society of America* **72**, 861-863, 1982.
4. A. T. Friberg, Integral equation for the scattered field in the presence of a phase-conjugate mirror, *Journal of the Optical Society of America* **73**, 405-407, 1983.
5. A. T. Friberg and P. D. Drummond, Reflection of a linearly polarized plane wave from a lossless stratified mirror in the presence of a phase-conjugate mirror, *Journal of the Optical Society of America* **73**, 1216-1219, 1983.
6. E. J. Bochove, Quantum theory of phase-conjugate mirrors, *Journal of the Optical Society of America B* **9**, 266-280, 1992.
7. M. Y. Lanzerotti and A. L. Gaeta, Theory of quantum-optical measurements with a phase-conjugate mirror, *Physical Review A* **51**, 4057-4061, 1995.
8. G. Zheng, L. Ran and C. Yang, Electromagnetic equivalent model for phase conjugate mirror based on the utilization of left-handed material, *Optics Express* **15**, 13877-13885, 2007.
9. Y. Z. Umul, Modified theory of physical optics, *Optics Express* **12**, 4959-4972, 2004.
10. Y. Z. Umul, Modified theory of physical optics approach to wedge diffraction problems, *Optics Express* **13**, 216-224, 2005.
11. Y. Z. Umul, Modified diffraction theory of Kirchhoff, *Journal of the Optical Society of America A* **25**, 1850-1860, 2008.
12. Y. Z. Umul, Diffraction of evanescent plane waves by a resistive half plane, *Journal of the Optical Society of America A* **24**, 3226-3232, 2007.
13. Y. Z. Umul, The theory of the boundary diffraction wave for wedge diffraction, *Journal of Modern Optics* **55**, 1417-1426, 2008.
14. Y. Z. Umul, Diffraction of plane waves by a black wedge, *Optics & Laser Technology* **42**, 32-36, 2010.