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JOINT DETERMINATION OF THE PRODUCTION LOT SIZE AND NUMBER OF SHIPMENTS FOR EPQ MODEL WITH REWORK

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Abstract– This paper is concerned with joint determination of the optimal lot size and optimal number of shipments for an economic production quantity (EPQ) model with the reworking of random defective items produced. The classic EPQ model assumes a continuous issuing policy for satisfying product demand and perfect quality production for all items produced. However, in a real life vendor-buyer integrated-production-inventory system, a multi-delivery policy is used practically in lieu of the continuous issuing policy, and it is inevitable to generate defective items during a production run. In this study, all nonconforming items produced are considered to be repairable and are reworked in each cycle after the end of a production run. The fixed-quantity multiple installments of the finished batch can only be delivered to customers if the whole lot is quality assured at the end of the rework. Mathematical modeling is used and the long-run average integrated cost per unit time is derived. Convexity of the cost function is proved by the use of the Hessian matrix equations. A closed-form optimal production-shipment policy to the problem is obtained. A special case to the model is discussed. Finally, a numerical example is provided to demonstrate the model's practical usage.

Key Words–Multiple shipments; EPQ model; Rework; Lot sizing; Random defective rate; Manufacturing

1. INTRODUCTION

In the production and inventory management field, "when should a replenishment lot be initiated?" and "how many to be produced in a lot?" are two fundamental questions that must be answered by production-inventory practitioners for the products they routinely manufacture [1-2] in order to minimize the long-run average costs per unit time. The economic production quantity (EPQ) model [1-3] is often utilized to assist production and inventory managers in addressing the aforementioned questions on "when to start a production run" and "how many items to be produced each time".

The classic EPQ model assumes a continuous inventory issuing policy for satisfying product demand. However, in a real life vendor-buyer integrated production-inventory system, multiple or periodic deliveries of finished products are commonly adopted at customer's request. Goyal [4] first considered integrated inventory model for a single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. Example was given to illustrate the method proposed. Studies have since been carried out to address various aspects of supply chain optimization [5-13]. Banerjee [5] studied a joint economic-lot-size model for purchaser and vendor. Sarker and Parija [6] considered a manufacturing system which procures raw materials from suppliers and processes them to convert to finished products. They proposed a model that was used to determine an optimal ordering policy for procurement of raw materials, and the manufacturing batch size to minimize the total cost for meeting equal shipments of the finished products, at fixed intervals, to the buyers. Hill [8] examined a model in which a manufacturing company purchases a raw material, manufactures a product (at a finite rate) and ships a fixed quantity of the product to a single customer at fixed and regular intervals of time, as specified by the customers. The objective of his model is to determine a purchasing and production schedule which minimizes the total cost of purchasing, manufacturing and stockholding. Viswanathan [9] reexamined an integrated vendor -buyer inventory models with two different strategies that had been proposed in the literature for the problem: one where each replenishing quantity delivered to the buyer is identical and the other strategy where at each delivery all the inventory available with the vendor is supplied to the buyer. He showed that there is no one strategy that obtains the best solution for all possible problem parameters. His study presented the results of a detailed numerical investigation that analyzed the relative performance of the two strategies for various problem parameters. Sarker and Khan [10] considered a manufacturing system that procures raw materials from suppliers in a lot and processes them into finished products which are then delivered to outside buyers at fixed points in time. A general cost model was formulated considering both raw materials and finished products. Using this model, a simple procedure was developed to determine optimal ordering policy for procurement of raw materials as well as manufacturing batch size, to minimize the total cost of meeting customer demands in time. Khouja [12] formulated and solved the two-stage supply chain inventory models in which the proportion of defective products increases with increased production lot sizes. He showed that the quality considerations can lead to significant reduction in production lot sizes. In addition, his models showed that most benefits to the supply chain are attained from the suppliers producing on a just-in-time basis rather than delivering to their customers just-in-time. The closed-form expressions for the optimal lot sizes for a two-stage supply chain under deterministic and stochastic demand were derived, respectively.

Another unrealistic assumption of the classic EPQ model is that “all items produced are of perfect quality”. In a real-life production system, due to process deterioration and/ or other factors, generation of imperfect quality items is inevitable. Studies have been carried out to enhance the classic EPQ model by addressing the issue of defective items produced [14-20]. The nonconforming items sometimes can be reworked and repaired hence overall production costs can be significantly reduced [21-29]. For instance, manufacturing processes in printed circuit board assembly, or in plastic injection molding, etc., sometimes employs rework as an acceptable process in terms of level of quality. Hayek and Salameh [22] assumed that all of the defective items produced are repairable and derived an optimal operating policy for EPQ model under the effect of rework of all defective items. Jamal et al. [24] studied the optimal manufacturing batch size with rework process at a single-stage production system. Cases of rework being completed within the same production cycle as well as rework being done after N cycles are examined. They developed mathematical models for each case, and derived total system costs and the optimal batch sizes accordingly. For the reason that little attention was paid to investigate the joint effect of a multi-shipment policy and a rework process on the optimal replenishment lot size and optimal number of shipments of EPQ model, this paper intends to bridge the gap.

2. MATHEMATICAL MODELLING

The aim of this study is to jointly determine the optimal lot size and optimal number of shipments for an EPQ model with rework. Consider that during regular production time, there is an x portion of defective items produced randomly at a production rate d. All defective items are assumed to be repairable and are reworked at a rate P_1 in each cycle after the end of a production run. In order to prevent shortages from occurring, the constant production rate P has to be larger than the sum of demand rate λ and production rate of defective items d. That is: $(P-d-\lambda)>0$ or $(1-x-\lambda/P)>0$; where $d=Px$. Unlike the classic EPQ model assuming a continuous issuing policy for satisfying demand λ, this paper considers a multi-delivery policy. It is assumed that the finished items can only be delivered to customers if the whole lot is quality assured at the end of rework. The fixed-quantity n installments of finished batch are delivered to customers, at a fixed interval of time during the production downtime t_3 (see Figure 1).

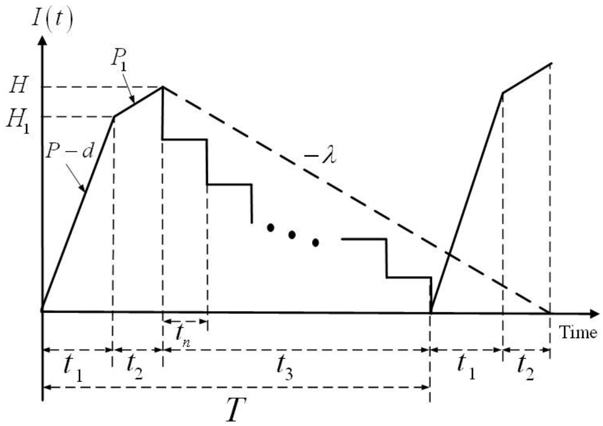

Figure 1.On-hand inventory of perfect quality items in an EPQ model with a multi-shipment policy and rework

Cost parameters considered in the proposed model include unit production cost C, unit holding cost h, setup cost K, unit rework cost C_R, holding cost h_1 for each reworked item, fixed delivery cost K_1 per shipment, and delivery cost C_T per item shipped to customers. Additional notation used is listed as follows.

Q = a decision variable, stands for the production batch size for each run,

n = a decision variable, denotes the number of fixed-quantity installments of the finished batch to be delivered to the customers,

H_1 = maximum level of on-hand inventory in units when production process ends,

H = maximum level of on-hand inventory in units when rework process finishes,

t_1 = the production uptime for the proposed EPQ model,

t_2 = time required for reworking of defective items,

t_3 = time required for delivering all quality assured finished products,

t_n = a fixed interval of time between each installment of finished products delivered during production downtime t_3,

T = cycle length,

$I(t)$ = on-hand inventory of perfect quality items at time t,

$I_d(t)$ = on-hand inventory of defective items at time t,

$TC(Q,n)$ = total production-inventory-delivery costs per cycle for the proposed model,

$TC_1(Q,n)$ = total production-inventory-delivery per cycle when no defective items produced (i.e. the special case, it is the same as the classic EPQ model with a multi-delivery policy),

$E[TCU(Q,n)]$ = the long-run average costs per unit time for the proposed model,

$E[TCU_1(Q,n)]$ = the long-run average costs per unit time for model in the special case.

From Figure 1, the following parameters can be obtained directly [22-23]:

$$t_1 = \frac{Q}{P} = \frac{H_1}{P-d} \tag{1}$$

$$t_2 = \frac{xQ}{P_1} \tag{2}$$

$$t_3 = nt_n = T - (t_1 + t_2) = Q\left(\frac{1}{\lambda} - \frac{1}{P} - \frac{x}{P_1}\right) \tag{3}$$

$$T = t_1 + t_2 + t_3 \tag{4}$$

$$H_1 = (P-d)t_1 = (P-d)\frac{Q}{P} = (1-x)Q \tag{5}$$

$$H = H_1 + P_1 t_2 = Q \tag{6}$$

The on-hand inventory of defective items during the production uptime t_1 and the reworking time t_2 is illustrated in Figure 2. It may be noted that maximum level of on-hand defective items is dt_1, and

$$dt_1 = Pxt_1 = xQ. \tag{7}$$

Cost for each delivery and total delivery costs for n shipments in a cycle are:

$$K_1 + C_T\left(\frac{H}{n}\right) \tag{8}$$

$$n\left[K_1 + C_T\left(\frac{H}{n}\right)\right] = nK_1 + C_T H = nK_1 + C_T Q \tag{9}$$

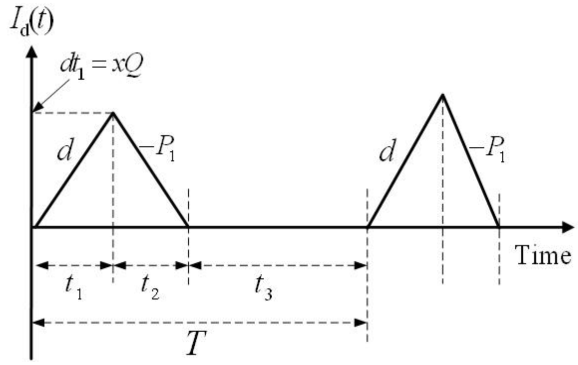

Figure 2.On-hand inventory of defective items in an EPQ model with multi-shipment policy and rework

The variable holding costs for finished products kept by the manufacturer, during the delivery time t_3 where n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time, are as follows (see Appendix A for derivations).

$$h\left(\frac{n-1}{2n}\right)Ht_3 \tag{10}$$

The variable holding costs for finished products kept by the customer during the delivery time t_3, are as follows (see Appendix B for the detailed computations).

$$\frac{h_2}{2}\left[\frac{H}{n}t_3 + T\left(H-\lambda t_3\right)\right] \tag{11}$$

Total costs per cycle $TC(Q,n)$ consists of setup cost, variable production cost, variable rework cost, fixed and variable delivery cost, holding cost during production uptime t_1 and reworking time t_2, variable holding cost for items reworked, and holding cost for finished goods kept by both the manufacturer and the customer during the delivery time t_3 where n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time. Therefore, $TC(Q,n)$ is

$$\begin{aligned} TC(Q,n) &= CQ + K + C_R[xQ] + C_T[Q] + nK_1 + h_1\cdot\frac{P_1\cdot t_2}{2}\cdot(t_2) \\ &+ h\left[\frac{H_1+dt_1}{2}(t_1) + \frac{H_1+H}{2}(t_2) + \left(\frac{n-1}{2n}\right)Ht_3\right] + \frac{h_2}{2}\left[\frac{H}{n}t_3 + T\left(H-\lambda t_3\right)\right] \end{aligned} \tag{12}$$

In order to take the randomness of defective rate into account, the expected values of x can be used in the inventory cost analysis. Substituting equations (1)-(11) in $TC(Q,n)$, E[$TCU(Q,n)$] can be obtained as follows:

$$\begin{aligned} E\left[TCU(Q,n)\right] &= \frac{E\left[TC(Q,n)\right]}{E[T]} = C\lambda + \frac{(K+nK_1)\lambda}{Q} + C_R E[x]\lambda + C_T\lambda + \frac{hQ\lambda}{2P} \\ &+ \frac{hQ\lambda}{2P_1}\left[2E[x] - \left(E[x]\right)^2\right] + \left(\frac{n-1}{n}\right)\left[\frac{hQ}{2} - \frac{hQ\lambda}{2P} - \frac{hQE[x]\lambda}{2P_1}\right] \\ &+ \frac{h_1\left(E[x]\right)^2 Q\lambda}{2P_1} + \left(\frac{1}{n}\right)\frac{h_2Q}{2} + \left(1-\frac{1}{n}\right)\frac{h_2Q\lambda}{2P} + \frac{h_2Q}{2}\left[\left(1-\frac{1}{n}\right)\frac{E[x]\lambda}{P_1}\right] \end{aligned} \tag{13}$$

3. PROOF OF CONVEXITY AND DERIVING OPTIMAL SOLUTIONS

For the proof of convexity of E[$TCU(Q,n)$], one can use Hessian matrix equation [30] and obtain the following:

$$[Q \quad n]\cdot\begin{pmatrix}\frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q^2} & \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n}\\ \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial Q\partial n} & \frac{\partial^2 E\left[TCU(Q,n)\right]}{\partial n^2}\end{pmatrix}\cdot\begin{bmatrix}Q\\ n\end{bmatrix}=\frac{2K\lambda}{Q}>0 \tag{14}$$

Equation (14) is resulting positive, because K, λ, and Q are all positive. Hence, it follows that the expected integrated costs $E[TCU(Q,n)]$ is a strictly convex function for all Q and n different from zero. Then for deriving the optimal production lot size Q^* and optimal number of shipments n^*, one can differentiate $E[TCU(Q,n)]$ with respect to Q and with respect to n, and solve the linear system of equations (15)-(16) by setting these partial derivatives equal to zero.

$$\begin{aligned}\frac{\partial E\left[TCU(Q,n)\right]}{\partial Q}=&-\frac{(K+nK_1)\lambda}{Q^2}+\frac{h\lambda}{2P}+\frac{h\lambda}{2P_1}\left[2E[x]-(E[x])^2\right]+\left(1-\frac{1}{n}\right)\left[\frac{h}{2}-\frac{h\lambda}{2P}-\frac{hE[x]\lambda}{2P_1}\right]\\&+\frac{h_1(E[x])^2\lambda}{2P_1}+\left(\frac{1}{n}\right)\frac{h_2}{2}+\left(1-\frac{1}{n}\right)\frac{h_2\lambda}{2P}+\left(1-\frac{1}{n}\right)\frac{h_2E[x]\lambda}{2P_1}\end{aligned} \tag{15}$$

$$\frac{\partial E\left[TCU(Q,n)\right]}{\partial n}=\frac{K_1\lambda}{Q}-\frac{1}{n^2}(h_2-h)\left[\frac{Q}{2}-\frac{Q\lambda}{2P}-\frac{QE[x]\lambda}{2P_1}\right] \tag{16}$$

By setting equation (15) equal to zero, one has:

$$\begin{aligned}\frac{(K+nK_1)\lambda}{Q^2}=&\frac{h\lambda}{2P}+\frac{h\lambda}{2P_1}\left[2E[x]-(E[x])^2\right]+\left(1-\frac{1}{n}\right)\left[\frac{h}{2}-\frac{h\lambda}{2P}-\frac{hE[x]\lambda}{2P_1}\right]\\&+\frac{h_1(E[x])^2\lambda}{2P_1}+\left(\frac{1}{n}\right)\frac{h_2}{2}+\left(1-\frac{1}{n}\right)\frac{h_2\lambda}{2P}+\left(1-\frac{1}{n}\right)\frac{h_2E[x]\lambda}{2P_1}\end{aligned} \tag{17}$$

With further rearrangements, one obtains the optimal production lot size Q^*.

$$Q^*=\sqrt{\frac{2(K+nK_1)\lambda}{\left\{\frac{h\lambda}{P}+\frac{h\lambda}{P_1}\left[2E[x]-(E[x])^2\right]+\left(\frac{n-1}{n}\right)\left[h+(h_2-h)\left(\frac{\lambda}{P}+\frac{E[x]\lambda}{P_1}\right)\right]+\left(\frac{1}{n}\right)h_2+\frac{h_1(E[x])^2\lambda}{P_1}\right\}}} \tag{18}$$

By setting equation (16) equal to zero, one has:

$$n^2=Q^2(h_2-h)\left[1-\left(\frac{\lambda}{P}+\frac{E[x]\lambda}{P_1}\right)\right]/2K_1\lambda \tag{19}$$

Substituting equation (18) in equation (19), the following can be obtained:

$$n^2=\frac{(K+nK_1)(h_2-h)\left[1-\left(\frac{\lambda}{P}+\frac{E[x]\lambda}{P_1}\right)\right]}{K_1\left\{\frac{h\lambda}{P}+\frac{h\lambda}{P_1}\left[2E[x]-(E[x])^2\right]+\frac{h_1(E[x])^2\lambda}{P_1}+\frac{h_2}{n}+\left(\frac{n-1}{n}\right)\left[h+(h_2-h)\left(\frac{\lambda}{P}+\frac{E[x]\lambda}{P_1}\right)\right]\right\}} \tag{20}$$

With further arrangements, one obtains the following optimal number of delivery n^*:

$$n^*=\sqrt{\frac{K(h_2-h)\left[1-(\lambda/P+E[x]\lambda/P_1)\right]}{K_1\left\{\frac{h\lambda E[x]}{P_1}(1-E[x])+h+\frac{h_1(E[x])^2\lambda}{P_1}+h_2\left(\frac{\lambda}{P}+\frac{E[x]\lambda}{P_1}\right)\right\}}} \tag{21}$$

4. SPECIAL CASE

Suppose that all items produced are of perfect quality (i.e. x=0), the proposed model becomes the same as the classic EPQ model with a multi-shipment policy. Total cost per cycle is:

$$TC_1(Q,n)=CQ+K+C_TQ+nK_1+h\frac{H}{2}(t_1)+h\left(\frac{n-1}{2n}\right)Ht_2+\frac{h_2}{2}\left[\frac{H}{n}t_2+T(H-\lambda t_2)\right] \tag{22}$$

The expected production-inventory-delivery cost per unit time for this special model can be derived as follows:

$$E[TCU_1(Q,n)]=C\lambda+\frac{(K+nK_1)\lambda}{Q}+C_T\lambda+\frac{hQ\lambda}{2P}+\left(\frac{n-1}{n}\right)\left(\frac{hQ}{2}-\frac{hQ\lambda}{2P}+\frac{h_2Q\lambda}{2P}\right)+\left(\frac{1}{n}\right)\frac{h_2Q}{2} \tag{23}$$

The convexity of E[$TCU_1(Q,n)$] can also be proved and the optimal solutions to this special model can be obtained as follows:

$$Q^*=\sqrt{\frac{2(K+nK_1)\lambda}{\left\{h+\left(\frac{1}{n}\right)\left(1-\frac{\lambda}{P}\right)(h_2-h)+(h_2)\left(\frac{\lambda}{P}\right)\right\}}} \tag{24}$$

$$n^*=\sqrt{\frac{K(h_2-h)\left[1-(\lambda/P)\right]}{K_1\left[h+h_2(\lambda/P)\right]}} \tag{25}$$

5. NUMERICAL EXAMPLE

Consider the demand rate of a manufactured item is 3,400 units per year. It can be produced at a rate of 60,000 units per year. During the production uptime, a random defective rate x is assumed and which follows a uniform distribution over the interval [0, 0.3]. All defective items produced are considered to be repairable and a rate of rework P_1=2,200 units per year is assumed. Other parameters used in this example are:

C_R = \$60, repaired cost for each item reworked,
C = \$100 per item,
h = \$20 per item per year,
h_1 = \$40 per item reworked per unit time (year),
h_2 = \$80 per item kept at the customer's end per unit time,

K = \$20,000 per production run,
C_T = \$0.1 per item delivered,
K_1 = \$4,350 per shipment, a fixed cost,

From equations (21), (18), and (13), the optimal number of delivery $n^*=2$, the optimal production lot size $Q^*=1{,}673$, and the long-run average cost $E[TCU(Q^*,n^*)]=\$487{,}617$ can be obtained. It may be noted that n^* should practically be an integer number, so in this example $n^*=2$ is rounded off from its original value 2.0136 computed by equation (21). The effect of variation of the number of shipments n on the long-run cost function $E[TCU(Q,n)]$ is depicted in Figure 3. It is noted that in this numerical example, the optimal integer number of shipments $n^*=2$.

Also, since $E[TCU(Q^*,n^*)]$ is not necessarily symmetrical on both sides of n^*, in the case of n^* falling closer to the midpoint of two integers, we suggest that both integer numbers should be plugged into equation (13), and select whichever integer gives the minimum cost as n^*. Variation of the random defective rate x effects on the cost function $E[TCU(Q^*,n^*)]$ is displayed in Figure 4. It is noted that as the random defective rate x increases, the value of the long-run cost function $E[TCU(Q^*,n^*)]$ increases significantly.

The optimal solutions for the special case (i.e. situation when all items produced are of perfect quality) can be obtained are by using equations (25), (24) and (23) as follows: the optimal number of delivery $n^*=3$ (is rounded off from 3.257), the optimal lot size $Q^*=2{,}276$, and the long-run average cost $E[TCU_1(Q^*,n^*)]=\$439{,}101$.

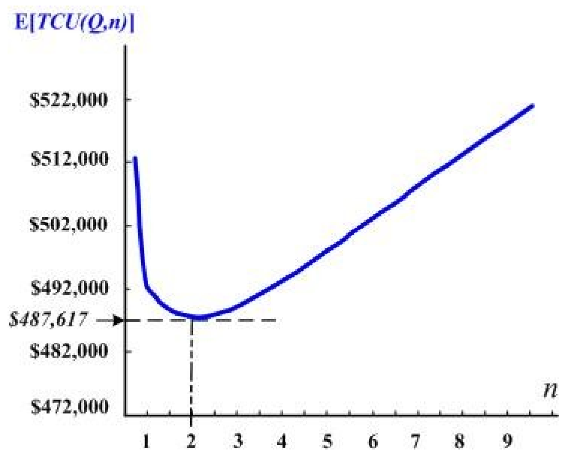

Figure 3. Variation of the number of shipments effects on the long-run cost function E[*TCU(Q,n)*]

6. CONCLUSION

This paper is concerned with jointly determining the optimal production lot size and optimal number of shipments for an EPQ model with the rework of random defective items produced. It is intended to address the unrealistic assumptions of the classic EPQ model in regards to a continuous issuing policy and the perfect production.

The mathematical modeling is used, and an integrated production-inventory-delivery cost function is derived. Hessian matrix equations are employed to prove the convexity of this cost function, and the closed-form solutions to the problem in terms of the optimal lot-size and optimal number of shipments are obtained. It may be noted that without an in-depth investigation and robust analysis of such a realistic EPQ model, the optimal production- shipment policy cannot be obtained. Neither can the insight regarding the effects of system parameters be gained (refer to Figures 3 and 4).

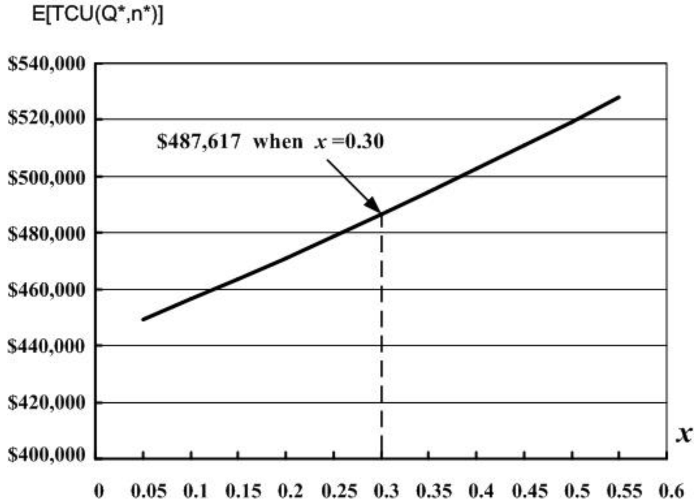

Figure 4. Variation of the random defective rate effects on the long-run cost function E[TCU(Q^*,n^*)]

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APPENDIX – A

Computation of the holding cost of finished products kept by the manufacturer during delivery time t_3 (i.e. equation (10)).

(1) When n=1, total holding cost in delivery time t_3 =0.

(2) When n=2, total holding costs in delivery time t_3 are (see Figure 5):

$$h\left(\frac{H}{2}\times\frac{t_3}{2}\right)=h\left(\frac{1}{2^2}\right)Ht_3 \tag{A-1}$$

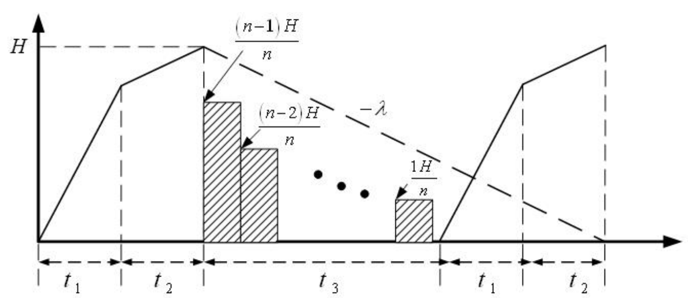

Figure 5: On-hand inventory of the finished items kept by manufacturer during t_3 in EPQ model with a multi-shipment policy and rework

(3) When n=3, total holding costs in delivery time t_3 become:

$$h\left(\frac{2H}{3}\times\frac{t_3}{3}+\frac{1H}{3}\times\frac{t_3}{3}\right)=h\left(\frac{2+1}{3^2}\right)Ht_3 \tag{A-2}$$

(4) When n=4, total holding costs in delivery time t_3 are:

$$h\left(\frac{3H}{4}\times\frac{t_3}{4}+\frac{2H}{4}\times\frac{t_3}{4}+\frac{1H}{4}\times\frac{t_3}{4}\right)=h\left(\frac{3+2+1}{4^2}\right)Ht_3 \tag{A-3}$$

Therefore, the following general term (the same as given in equation (10)) for total holding costs during delivery time t_3 can be obtained:

$$h\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)Ht_3=h\left(\frac{1}{n^2}\right)\left[\frac{n(n-1)}{2}\right]Ht_3=h\left(\frac{n-1}{2n}\right)Ht_3 \tag{A-4}$$

APPENDIX – B

Computations of the customer's holding cost during t_3 are as follows.

Because n installments (fixed quantity D) of the finished lot are delivered to customer at a fixed interval of time t_n, one has the following:

$$D=\frac{H}{n} \tag{B-1}$$

$$t_n=\frac{t_3}{n} \tag{B-2}$$

At the customer's end, the demand between shipments is (λt_n), if we let I denote number of items that will be left over after satisfying the demand during each fixed interval of time t_n (refer to Figure 6), then:

$$I=D-\lambda t_n \tag{B-3}$$

From Figure 6, one can calculate the average inventory as follows:

$$\begin{aligned}\text{Average inventory } &=\left[\left(\frac{D+I}{2}\right)t_n\right]+\left[\frac{(D+I)+\left[(D+I)-\lambda t_n\right]}{2}t_n\right]+\left[\frac{(D+2I)+\left[(D+2I)-\lambda t_n\right]}{2}t_n\right]\\ &+\ \dots\ +\left[\frac{\left[D+(n-1)I\right]+\left[\left[D+(n-1)I\right]-\lambda t_n\right]}{2}t_n\right]+\left(\frac{nI}{2}\right)(t_1+t_2)\end{aligned} \tag{B-4}$$

Substituting equation (B-3) in equation (B-4), the average inventory becomes:

$$\begin{aligned}\text{Average inventory } &=\left(D-\frac{\lambda}{2}t_n\right)t_n+\left(D+I-\frac{\lambda}{2}t_n\right)t_n+\left(D+2I-\frac{\lambda}{2}t_n\right)t_n+\dots\\ &+\left(D+(n-1)I-\frac{\lambda}{2}t_n\right)t_n+\left(\frac{nI}{2}\right)(t_1+t_2)\\ &=n\left(D-\frac{\lambda}{2}t_n\right)t_n+\frac{n(n-1)}{2}It_n+\frac{nI}{2}(t_1+t_2)\end{aligned} \tag{B-5}$$

Substituting equations (B-1) through (B-3) in equation (B-5), the following general term for average inventory at the customer's end can be obtained:

$$\begin{aligned}\text{Average inventory } &=n\left(\frac{H}{n}-\frac{\lambda}{2}t_n\right)t_n+\frac{n(n-1)}{2}\left(\frac{H}{n}-\lambda t_n\right)t_n+\frac{n}{2}\left(\frac{H}{n}-\lambda t_n\right)(t_1+t_2)\\ &=Ht_n-\frac{n\lambda}{2}t_n^{\,2}+Ht_n\frac{(n-1)}{2}-\frac{n(n-1)}{2}\lambda t_n^{\,2}+\frac{H}{2}(t_1+t_2)-\frac{n}{2}(\lambda t_n)(t_1+t_2)\\ &=1/2\left[(Ht_3/n)+T(H-\lambda t_3)\right]\end{aligned} \tag{B-6}$$

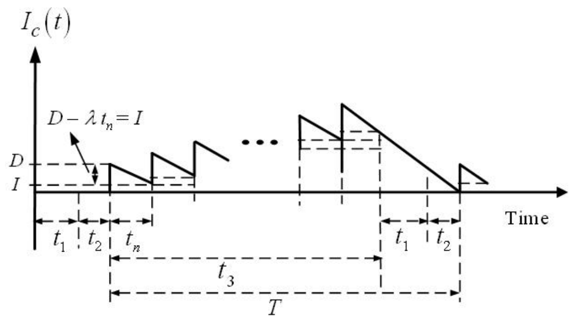

Figure 6. On-hand inventory of the finished items kept by customer during t_3 in EPQ model with a multi-shipment policy and rework

REFERENCES

1. E. A. Silver, D. F. Pyke, and R. Peterson, *Inventory Management and Production Planning and Scheduling*, John Wiley & Sons, Inc., New York, 151-172, 1998.
2. S. Nahmias, *Production & Operations Analysis*, McGraw-Hill Inc., New York, 195-223, 2005.
3. P. H. Zipkin, *Foundations of Inventory Management*, McGraw-Hill Co. Inc., New York, 30-60, 2000.
4. S. K. Goyal, Integrated Inventory Model for a Single Supplier - Single Customer Problem, *International Journal of Production Research* **15**, 107-111, 1977.
5. A. Banerjee, A joint economic-lot-size model for purchaser and vendor, *Decision Sciences* **17**, 3, 292-311, 1986.
6. B. R. Sarker and G. R. Parija, An Optimal Batch Size for a Production System Operating Under a Fixed-Quantity, Periodic Delivery Policy, *Journal of the Operational Research Society* **45**, 891–900, 1994.
7. L. Lu, A one-vendor multi-buyer integrated inventory model, *European Journal of Operational Research* **81**, 312-323, 1995.
8. R. M. Hill, Optimizing a production system with a fixed delivery schedule”, *Journal of the Operational Research Society* **47**, 954-960, 1996.
9. S. Viswanathan, Optimal strategy for the integrated vendor-buyer inventory model, *European Journal of Operational Research* **105**, 38-42, 1998.
10. R. A. Sarker and L. R. Khan, Optimal batch size for a production system operating under periodic delivery policy, *Computers and Industrial Engineering* **37**, 711-730, 1999.
11. P. C. Yang and H. M. Wee, A single-vendor and multiple-buyers production-inventory policy for a deteriorating item, *European Journal of Operational Research* **143**, 3, 570-581, 2002.
12. M. Khouja, Optimizing inventory decisions in a multi-stage multi-customer supply chain, *Transportation Research Part E: Logistics and Transportation Review* **39**, 3, 193-208, 2003.
13. L-Y. Ouyang, K-S. Wu, and C-H. Ho, Integrated vendor-buyer cooperative models with stochastic demand in controllable lead time, *International Journal of Production Economics* **92**, 3, 255-266, 2004.
14. M. J. Rosenblatt and H. L. Lee, Economic production cycles with imperfect production processes, *IIE Transactions* **18**, 48-55, 1986.
15. T. C. E. Cheng, An economic order quantity model with demand-dependent unit

production cost and imperfect production processes, *IIE Transactions,* **23**, 1991, 23-28.

16. H. Groenevelt, L. Pintelon, and A. Seidmann, Production lot sizing with machine breakdowns, *Management Sciences* **38**, 104-123, 1992.
17. K. Moinzadeh and P. Aggarwal, Analysis of a production/inventory system subject to random disruptions, *Management Science* **43**, 1577-1588, 1997.
18. H. Kuhn, A dynamic lot sizing model with exponential machine breakdowns, *European Journal of Operational Research* **100**, 514-536, 1997.
19. M. A. Rahim and M. Ben-Daya, Joint determination of production quantity, inspection schedule and quality control for imperfect process with deteriorating products, *Journal of the Operational Research Society* **52**, 1370-1378, 2001.
20. Y-S. P. Chiu, The effect of service level constraint on EPQ model with random defective rate, *Mathematical Problems in Engineering*, **2006**, Article ID 98502, 13 pages, 2006.
21. T. Dohi, N. Kaio, and S. Osaki, Minimal repair policies for an economic manufacturing process, *Journal of Quality in Maintenance Engineering* 4, 248-262, 1998.
22. P. A. Hayek and M. K. Salameh, Production lot sizing with the reworking of imperfect quality items produced, *Production Planning and Control* **12**, 584-590, 2001.
23. Y. P. Chiu, Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging, *Engineering Optimization* **35**, 427-437, 2003.
24. A. M. M. Jamal, B. R. Sarker, and S. Mondal, Optimal manufacturing batch size with rework process at a single-stage production system, *Computers & Industrial Engineering* **47**, 77-89, 2004.
25. S. W. Chiu and Y-S. P. Chiu, Mathematical modeling for production system with backlogging and failure in repair, *Journal of Scientific & Industrial Research,* **65**, 499-506, 2006.
26. Y-S. P. Chiu, H-D. Lin, and F-T. Cheng, Optimal production lot sizing with backlogging, random defective rate, and rework derived without derivatives, *P I Mech E Part B: Journal of Engineering Manufacture* **220**, 9, 1559-1563, 2006.
27. Y-S. P. Chiu, C-Y. Tseng, W-C. Liu, and C-K. Ting, Economic manufacturing quantity model with imperfect rework and random breakdown under abort/resume policy, *P I Mech E Part B: Journal of Engineering Manufacture* **223**, 2, 183-194, 2009.
28. G. C. Lin and D. E. Kroll, Economic lot sizing for an imperfect production system subject to random breakdowns, *Engineering Optimization* **38**, 73-92, 2006.
29. Y-S. P. Chiu, S.W. Chiu, and H-D. Lin, Solving an EPQ model with rework and service level constraint, *Mathematical & Computational Applications* **11**, 1, 75-84, 2006.
30. R. L. Rardin, *Optimization in Operations Research*, Prentice-Hall, New Jersey, 739-741, 1998.